MATH 375 Numerical Linear Algebra

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1 Background

This course is the second linear algebra class students take while enrolled in the AMCS program. It is designed for the Scientific Computing concentration but would be of value to many of the other concentrations as well. The pedagogical approach recommended for this course is to provide students with the theoretical background for algorithms encountered in numerical linear algebra (NLA). The course would focus on mathematical proof, error analysis, and stability analysis. Some programming or use of NLA packages would be encouraged to enhance the theory but the focus would be on mathematical foundations versus best programming practices. The motivation for this course-philosophy is that students would go on to take MSCS 446 and 447 where they would learn to put scientific computing into practice.

2 Recommended Texts

- Trefethen and Bau *Numerical Linear Algebra* (TB)
- Demmel *Applied Numerical Linear Algebra* (JD)
- Supplementary: Golub and Van Loan *Matrix Computations* (GV)

3 Topics

Topics covered in a conventional numerical linear algebra course are listed below. These topics are chosen from TB, JD, and GV.

1. Vectors, Matrices, and Matrix, Matrix-vector Operations
2. Eigenvalues and Singular Values
3. Floating Point Arithmetic
4. Norms and Measuring Errors

5. The Singular Value Decomposition
6. Gaussian Elimination
7. Pivoting
8. Conditioning and Condition Numbers
9. Error, Backward Error, and Residual
10. Solvability and Numerically Singular
11. Stability of Gaussian Elimination
12. Stability of Back Substitution
13. Tridiagonal and Banded Matrices
14. The LU Decomposition
15. Matrix Factorizations that Solve the Linear Least Squares Problem
16. Normal Equations
17. QR Decomposition
18. Orthogonal Matrices
19. Householder Transformations
20. Givens Rotations (omit?)
21. Round-off Error Analysis for Orthongal Matrices
22. Rank-Deficient Least Squares Problems
23. Eigenvalue Problems
24. The Gershgorin Theorem and Gershgorin Circles (omit?)
25. The Power Method
26. Rayleigh Quotient Inverse Iteration
27. QR Iteration (without shifts)
28. Reduction to Hessenberg or Tridiagonal Form
29. QR Algorithm with Shifts
30. Algorithms for Symmetric Eigenvalue Problems (omit?)

4 Other Resources

- Sage
- Jupyter Notebooks
- MATLAB (Octave)