CIVE60005: Fluid Mechanics III

Coursework:
Controlling outdoor and indoor spread of COVID-19

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1.0 Introduction

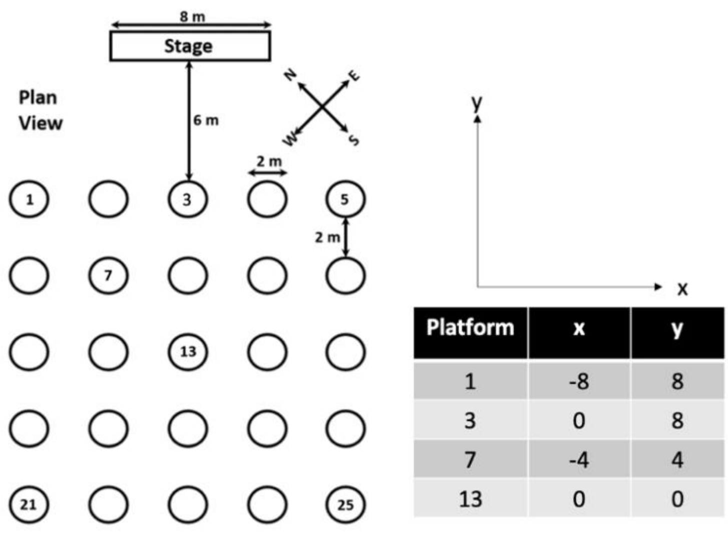
This report is in preparation for the government's Scientific Advisory Group for Emergencies (SAGE). It aims to refine government guidelines to reduce the spread of COVID-19 by airborne (aerosol) transmission of the SARS-CoV-2 virus in outdoor and indoor spaces. Transport of SARS-CoV-2 (the virus responsible for COVID-19) is measured in terms of quanta. One quantum roughly corresponds to the amount of virus required for infection. Quanta are dimensionless numbers; quanta emission rates G therefore have dimensions T^{-1}, where T is a representation of time, and quanta concentration C per unit volume has dimensions L^{-3}, where L is a representation of length. Quanta emission rates are estimates, depending on action and level of activity (see Table 1.1). The coordinates and dimensions of the platform are shown in Figure 1.1.

Activity	Quanta emission rate (approximate)
Breathing at rest	$1\,\text{hr}^{-1}$
Breathing during heavy activity	$8\,\text{hr}^{-1}$
Speaking during light activity	$15\,\text{hr}^{-1}$
Speaking loudly/singing	$98\,\text{hr}^{-1}$

Table 1.1: The quanta emission rate of different activities

The report is broken down into two sections: (i) outdoor transmission of COVID-19 and (ii) indoor transmission of COVID-19.

2.0 Question 1: outdoor transmission



Platform	x	y
1	-8	8
3	0	8
7	-4	4
13	0	0

Figure 2.1 Coordinate system and dimensions of platforms

Figure 2.1 shows the layout of the outdoor concert arena, which consists of the public area with 25 individual viewing platforms, and the stage. Within the public area, each viewing platform can hold up to 4 people and has a diameter of 2 m. The viewing platforms are at ground level and arranged in a regular grid, with a separation distance of 4 m (centre to centre) between adjacent platforms. The width of the stage is 8 m and is at a height of 3 m above the ground. The concert duration is 3 hours. In this report, assume that all 4 people on the central viewing platform (platform 13) are infected with SARS-CoV-2 and are contagious. All the calculations shall assume that the centre position of each platform is a good representation of the platform exposure.

2.1 Part a: temporal evolution

This section shall evaluate the temporal evolution of the virus concentration at head height at platforms 1, 3, 7 and 13 for conditions of no wind. The assumptions for this exercise are:

- Mass continuity and conservation holds.

- The average human height, *h*, is 1.68m (BBC News, 2010) and the variance in height is negligible.

- No wind is present. Therefore, the virus spreads in 3D diffusion, with no advection. A continuous release is modelled to mirror the continuous spreading of the virus by an individual. The governing equation is given as:

$$C(x, y, z, t) = \int_0^t \frac{\dot{M}}{(2\pi)^{3/2}\sigma^3} e^{-r^2/2\sigma^2} dt'$$

- For continuous release problems, we usually define a pollutant release rate M, which in 3-D, has dimensions [M T^{-1}]. This problem deals with quanta of the virus, and quanta concentrations can be interpreted as number concentrations, with dimensions [L^{-3}]. Furthermore, this report considers the quanta emission rate, G, to be equivalent to M in 3-D, noting the appropriate change in dimensions (i.e., T^{-1}).

- The temperature remains constant and the turbulent diffusion coefficient, D_T, remains constant at 1 m^2s^{-1}.

- The four concertgoers on Platform 13 are assumed to be speaking loudly/singing to model the worst-case scenario. Therefore, the quanta emission rate is assumed to be:

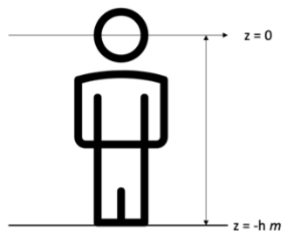
$$\dot{G} = \frac{4 \times G_{speak\ loud}}{60 \times 60} = 0.1089\ s^{-1}$$

- To reflect on how the floor is a solid boundary and no flow is expected to flow in and out of it, this report assumes the floor is a perfect reflector. Therefore, Neumann Boundary condition must be applied at z = - h, with z = 0 at head height. This report assumes that the distance between the point of release (mouth and nose) and head height is negligible. To satisfy the boundary condition, we must define an image source at z = -2h, and sum the contributions from both real source, r_r, and image source, r_i. This report also assumes that the top boundary requires zero net flux and both sources diffuse at the same rate. Therefore, the equation governing the concentration is:

$$C(x, y, z, t) = \frac{\dot{G}}{4\pi D_T r_r} erfc\left(\frac{r_r}{\sqrt{4Dt}}\right) + \frac{\dot{G}}{4\pi D_T r_i} erfc\left(\frac{r_i}{\sqrt{4Dt}}\right)$$

$$where\ r_r = \sqrt{x^2 + y^2 + z^2}, \qquad r_i = \sqrt{x^2 + y^2 + (z + 2h)^2},$$

$$given\ \left.\frac{dC}{dz}\right|_{z=-h} = 0$$



After substituting the centreline x and y coordinates of each platform and z at head height (z = 0), the temporal evolution of the virus concentration is plotted. As the source is described using the Dirac delta function, the concentration at platform 13's centreline is infinitely large. Therefore, to perform a meaningful comparison, the concentration at the edge of the platform is calculated instead (x=0, y=1). This is illustrated in Figure 2.2.

Based on the graph, it can be concluded that, platform 13 has the largest concentration (0.011 quanta per m^3). The concentration decreases radially as more distance is placed between platform 13 and the selected platform. Within the first 30 minutes of the concert, the concentration plateaus and reaches maximum concentration. This is consistent with expectations that the concentration reaches a constant value as time approaches infinity and diffusion becomes negligible.

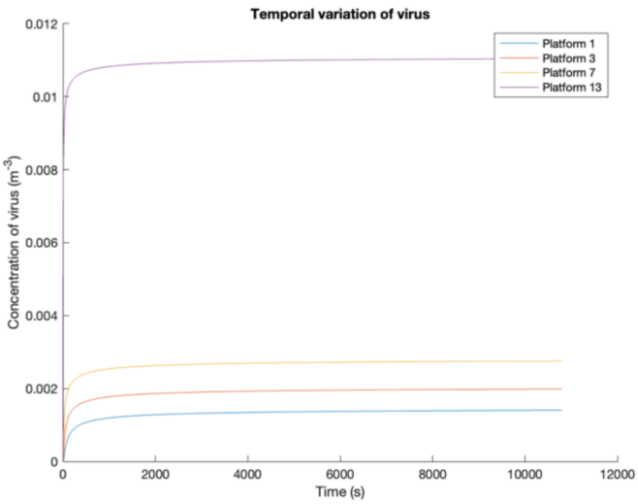


Figure 2.2: The temporal variation of virus at head height

2.2 Part b: effect of wind

In this section, the spatial distribution of the virus concentration within the public area at head height is evaluated. At the end of the concert, two conditions are modelled: (i) no wind and (ii) a spatially uniform wind with a speed of 3 ms^{-1} blowing from the south-west. The near-field equations governing the situations are:

$$C_{no\,wind,reflect}(x,y,z) = \frac{\dot{G}}{4\pi D_T r_r} erfc\left(\frac{r_r}{\sqrt{4Dt_{end}}}\right) + \frac{\dot{G}}{4\pi D_T r_i} erfc\left(\frac{r_i}{\sqrt{4Dt_{end}}}\right)$$

$$C_{U_y=3\,ms^{-1},reflect}\,(x,y,z) = \frac{\dot{G}}{4\pi D_T r_r} e^{\frac{-U_y}{2D}(r_r-y)} + \frac{\dot{G}}{4\pi D_T r_i} e^{\frac{-U_y}{2D}(r_i-y)}$$

The key assumptions for this exercise are:

- The streamwise diffusion term is not neglected.
- The diffusion of the virus is steady, and concentration is independent of time.
- The wind component, U_y, does not have an effect in the x and z direction. Its direction is also not affected by the density of people and remains uniform throughout the area.

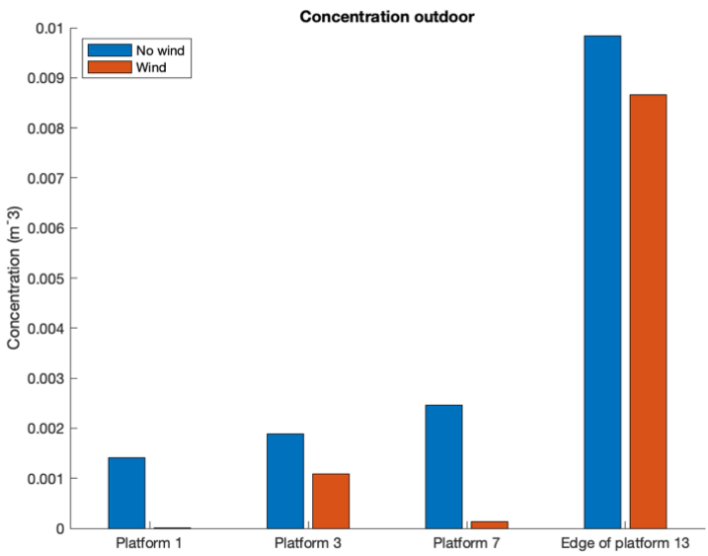


Figure 2.3: Comparison of maximum concentration at different platforms

The effect of wind on the concentration distribution is shown in Figure 2.3. When wind is present, the region northeast to platform 13 experiences a lower decrease in concentration, whereas the region southwest to the platform will experience a larger drop in concentration. This is aligned with the direction of U_y and demonstrates that the virus advects in the direction of the wind. The performers at head height are exposed to a higher concentration of the virus when wind is present and should be accounted for (Figure 2.4) and will be elaborated in section 2.4. However, the platforms near platform 13 generally experience a drop in concentration as the virus is spread across a larger area (Figure 2.4).

2.3 Part c: how effective are disinfectants?

If sufficient disinfectant can be applied to all ground surfaces within the public area prior to the concert, it will kill the SARSCoV-2 virus upon contact for the duration of the concert. Therefore, the floor should be modelled as a perfect absorber. To model the Dirichlet boundary condition, an image source of the opposite strength (i.e., a sink) is modelled instead. Key assumptions for the Neumann boundary condition are also applied here, except that mass is not conserved. Two conditions are modelled: (i) no wind and (ii) a spatially uniform wind with a speed of 3 ms^{-1} blowing from the south-west. The Dirichlet boundary condition and governing equations are:

$$C_{no\,wind,absorb}(x,y,z) = \frac{\dot{G}}{4\pi D_T r_r} erfc\left(\frac{r_r}{\sqrt{4Dt_{end}}}\right) - \frac{\dot{G}}{4\pi D_T r_i} rfc\left(\frac{r_i}{\sqrt{4Dt_{end}}}\right)$$

$$C_{U_y=3ms^{-1},absorb}(x,y,z) = \frac{\dot{G}}{4\pi D_T r_r} e^{\frac{-U_y}{2D}(r_r-y)} - \frac{\dot{G}}{4\pi D_T r_i} e^{\frac{-U_y}{2D}(r_i-y)}, \; where\; C_{z=-h} = 0$$

A comparison of concentrations, before and after disinfectants are applied, shows that disinfectants generally reduce the spatial distribution of the virus. This reduces the exposure of the virus to the northeast of platform 13 and the performers. The floor as a perfect absorber has a larger effect on the concentration when wind is not present. The reduction is explained by the perfectly absorbing floor that does not allow for mass conservation. Besides that, the perfectly absorbing properties also reflects on an earlier spatial decrease in concentration northeast to platform 13, decreasing the advective effects of wind.

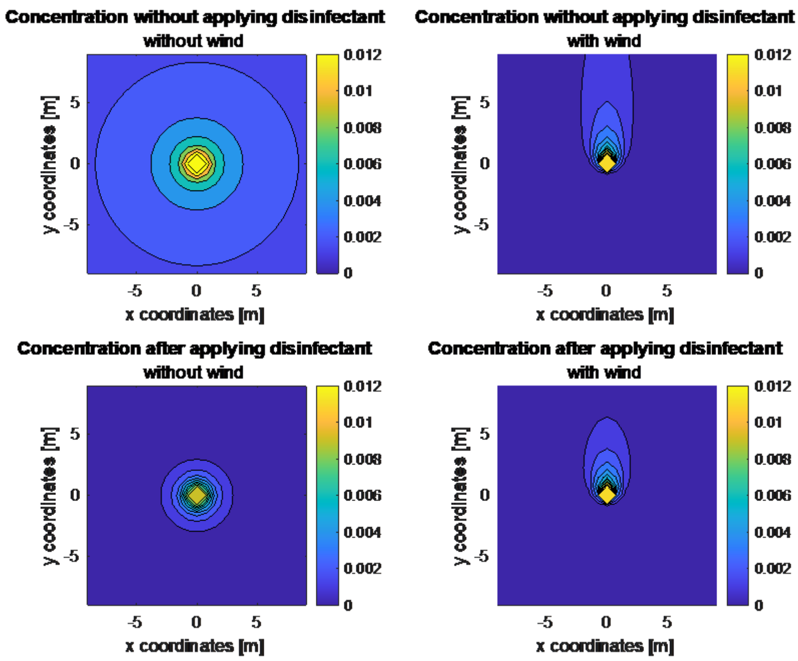


Figure 2.4: Concentration distribution at ground level

2.4 Part d: variation of concentration at stage

In this section, the vertical distribution of the virus concentration from ground level to the head height of a performer is evaluated (h = 4.68*m*). Based on the results from sections 2.3 and 2.4, the centre of the stage is most prone to exposure to the virus. Therefore, the position evaluated is directly in front of the centre of the stage, with coordinates x = 0 and y = 15 *m*.

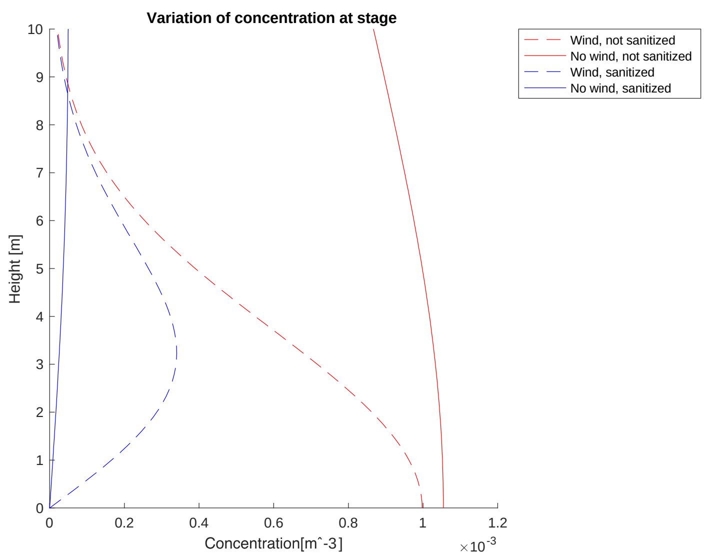


Figure 2.5: Variation of concentration with height at stage

The results for all four scenarios are shown in Figure 2.5. When disinfectants are applied, the concentration at floor-level is zero and this is aligned with the perfectly absorbing properties of the boundary condition. However, the concentration increases with height when wind travels towards the platform, and begins to decrease after 3 *m*. When the disinfectants are not applied, mass is conserved. This will make the concentration decrease when wind is applied as the advective effects are applied onto constant mass. The wind will spread the mass across a larger area, making the concentration decrease. The highest concentration in all four scenarios will be when both wind and disinfectants are not present (Figure 2.5). Another significant observation is that the concentrations with and without disinfectant converges as height increases. This suggests that the perfect absorber will eventually become negligible at greater heights when advection is present.

2.5 Part e: would raising the platform help?

Based on Chris Whitty's request, the viewing platforms with a height of 2*m* from the ground in the public area are modelled. It is to assess the possibility of lowering the exposure of concert goers to the SARS-CoV-2 virus. The assumptions in this task are that the wind is uniform across the height and the elevated platforms do not disrupt wind flow. The results are shown in Figure 2.6.

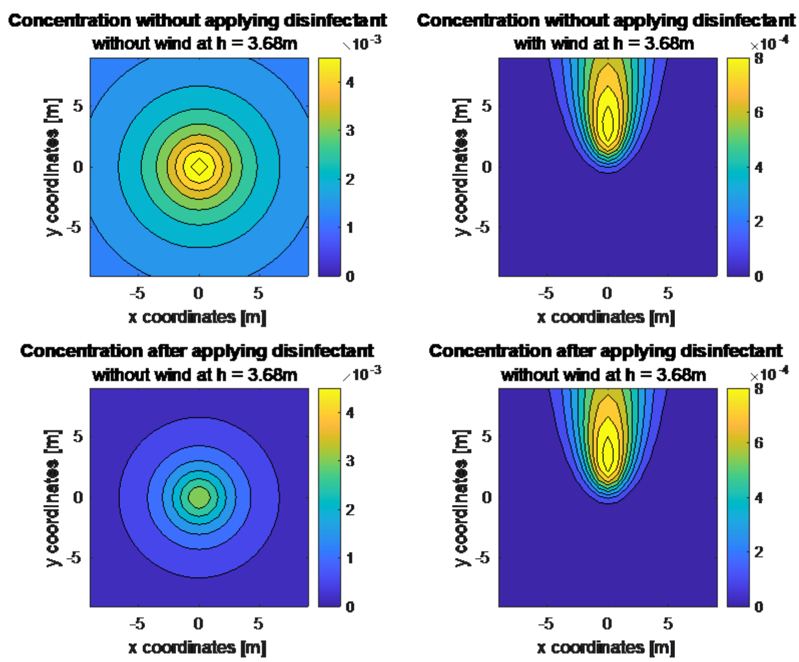


Figure 2.6 Variation of concentration with elevated platforms

In all scenarios, the concentration magnitude decreases in comparison to unelevated platforms. However, the virus spread is larger, affecting more platforms with or without wind. This is due to a larger initial height added by the elevated platform. When wind is not present, the application of disinfectants was effective in limiting the spread. On the other hand, the disinfectants have a minimal effect on concentration distribution when wind is present. This finding is aligned with section 2.4, where the Dirichlet boundary condition becomes negligible at greater heights when advection is present. This report recommends that the platform should not be raised as it increases the number of platforms at risk of exposure to the virus.

3.0 Question 2: indoor transmission

In a classroom with well-mixed air, the quanta per unit volume (concentration, *c*) of SARS-CoV-2 due to emission from one infected person satisfies the equation:

$$\frac{dc}{dt} = \frac{G}{V} - \frac{Qc}{V} \qquad [1]$$

where *V* is the volume of the classroom, *G* is the quanta emission rate per infected person and *Q* is the volume flux of outdoor air entering the classroom.

Here, we assume that $G = 1\ hr^{-1}$ based on breathing without speaking (Table 1.1). This assumption is not ideal as students are prone to ask questions during class. Nonetheless, the author will comply to the brief set by the client. The concentration, *c*, determines the probability, *p(t)*, of someone in the classroom becoming infected, which, summing over $N - 1$ uninfected occupants of a space, satisfies

$$\frac{dp}{dt} = r(N-1)(1-p)c \qquad [2]$$

given that $N \geq 1$ and r = 0.0002 $m^3s^{-1,}$ where r is the volume flux of respiration per person. Further assumptions to the report are that:

- $\frac{dp}{dt} = 0$ for N < 1
- The outdoor temperature is $T_0 = 7^{\circ}C$.
- The door remains closed throughout the lesson.
- N occupants of the space contribute a heat load of 100 W, in addition to a constant 1000 W associated with central heating. The heat is a source of buoyancy, *F*, to replenish the buoyancy that is removed from the space due to ventilation.

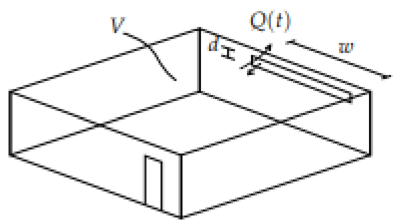


Figure 3.1: Dimensions of the classroom

- The classroom has a volume V = 200 m^3 and is naturally ventilated via a single high-level window of width w = 6 m and adjustable depth d, as shown in Figure 2.1.

This report aims to develop guidelines for the operation of the classroom. For each 1-hour class, the probability p of infection does not exceed 0.01 and the classroom should not fall below 17°C for comfort.

3.1 Part a: first class of the day

Based on Figure 2.1, this report shall model it as a box with a single opening. The formation of a warm upper layer is not possible because cool air enters at high level and would mix (and hence dilute) the upper layer; hence we assume that the air inside the box is 'well mixed' and of uniform temperature/buoyancy. Therefore, the ventilation is represented with the formula:

$$Q = \frac{k}{3} w\sqrt{bd^3} \qquad [3]$$

where k ≈ 0.5 for an opening in a vertical plane, *b* is the buoyancy of the air and *d* denotes the height of the opening, which has area A. Buoyancy, *b,* is represented by:

$$\frac{db}{dt} = \frac{F}{V} - \frac{Qb}{V} \qquad [4]$$

Where F is the buoyancy flux, represented by:

$$F = \frac{Wg}{c_p \rho_{air} T_0}$$

Where g = 9.81 ms^{-2}, ρ_{air} = 1225 gm^{-3}, c_p = 1.012 $Jg^{-1}K^{-1}$and *W* is represented by:

$$W = 1000 + 100N$$

Equations [1], [2], [3] and [4] form a system of ordinary differential equations. Therefore, the *ode45* function in MATLAB is used to obtain the buoyancy, *b*, probability, *p*, and concentration, *c*. With the buoyancy, *b*, the temperature, *T*, is back solved with the equation:

$$T = \frac{bT_0}{g} + T_0 \qquad [5]$$

The initial probability and concentration of the room is set to be zero to reflect on the empty occupancy. The initial temperature, however, is set to be 17°C, assuming it is preheated. The corresponding buoyancy, b_0, is used as the initial buoyancy where T = 17°C is substituted into [5].

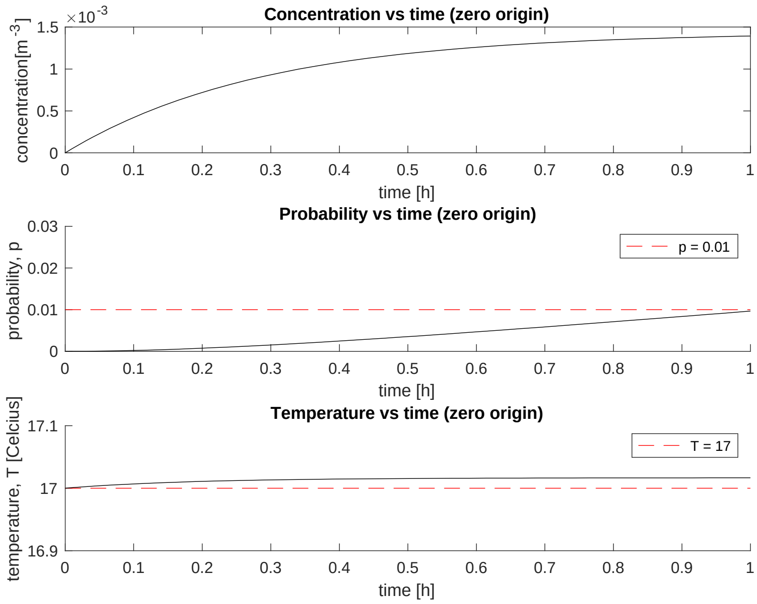


Figure 3.2: Variation of temperature, probability, and concentration against time (first class of the day)

Through trial and error, the optimal depth of the window for the first class of the day is 0.474 m and the classroom can hold 14 students at maximum capacity. The temporal variation of temperature, probability and concentration is shown in Figure 3.2. The results shown is validated by ensuring the temperature does not go below 17°C and the probability does not go above 0.01. Besides that, the temperature and concentration eventually reach a steady state, which is aligned with the behaviour of a well-mixed air.

3.2 Part b: every subsequent class

For every subsequent class from the first, the room is simulated with a periodic concentration and temperature after purging the space for 0.5 hour. During the purging periods, the following changes are made to the quanta emission, G, and the number of students, N:

$$G\begin{cases} 1\,hr^{-1} & if\ t\ \in [0,1] \\ 0 & if\ t\ \in (1,1.5] \end{cases} \quad \text{and} \quad N\begin{cases} \geq 1 & if\ t\ \in [0,1] \\ 0 & if\ t\ \in (1,1.5) \end{cases}$$

To achieve a periodic concentration and temperature, an iterative approach is taken to obtain the appropriate initial conditions. An initial condition is valid only if the initial and final temperature and concentration are less than 10^{-12} in difference. Based on the analysis, it can be observed that temperature and concentration decrease to its initial conditions when the classroom is emptied. However, the probability of contracting the virus does not decrease and remains constant. Therefore, more measures need to be taken on top of vacating the room. Through trial and error, every subsequent class after the first would achieve maximum occupancy when the window depth is 0.298 m and can only hold 8 students.

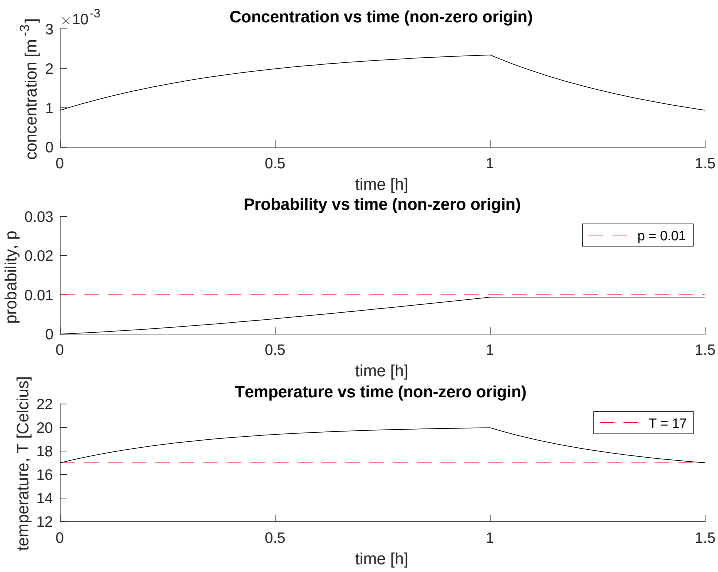


Figure 3.3: Variation of temperature, probability, and concentration against time (every subsequent class of the day)

3.3 Part c: would having a second opening help?

If a second opening is added to the room, like opening the door, the air flow in the room will transition from a mixing ventilation to a displacement ventilation. This will change the ventilation rate of the room and an illustration of this is tabulated in Table 3.1.

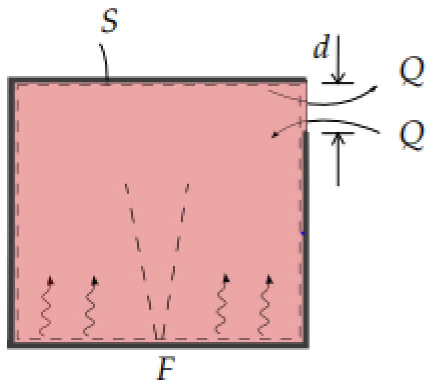
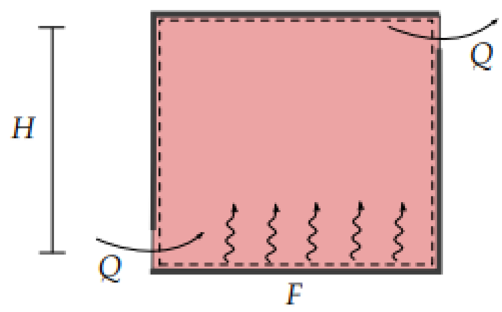
Type of ventilation	Mixing ventilation	Displacement ventilation
Ventilation rate, Q	$Q = \frac{k}{3} w \sqrt{bd^3}$	$Q = A^* \sqrt{2b(H-h)}$
2D representation		

Table 3.1: Comparison between mixing and displacement ventilation

Based on the equations governing the ventilation rate, displacement ventilation provides a higher rate. This is due to the relatively higher value of distance between openings, H, against window depth, d, and the larger effective area of the room, A^*, against coefficient k ≈ 0.5. With a higher ventilation rate, Q, the concentration, and temperature will be lowered at a quicker rate based on equations [1] and [4]. However, the increased rate of temperature decrease is not favourable because the room needs to be maintained at 17°C. This is supported by the conservation of buoyancy flux:

$$\underbrace{F}_{\text{buoyancy flux in}} = \underbrace{Qb.}_{\text{buoyancy flux out}}$$

Therefore, more power is required to maintain buoyancy, maintaining the room at comfort temperature. This demonstrates that mixing ventilation is the best way forward. However, this report assumes that the room is a perfect insulator – which is never the case for the built environment. Hence, more monitoring is needed to study the room and the above-mentioned ventilation rate is an underestimate.

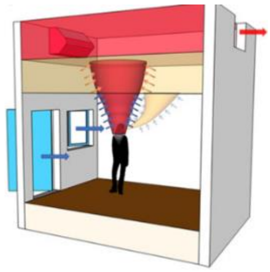


Figure 3.4: Spread of the virus when an inlet is installed, and no face mask worn (Bhagat et al., 2020)

Besides that, when an inlet is present, the spread of SARS-CoV-2 from an infected person is dependent on the location of inlet and outlet. The virus will advect in the direction of the flow and it is essential to minimize the spread by ensuring the flow advect directly upwards. Displacement ventilation is more ideal in comparison to mixing ventilation as the direction of spread can be controlled. Nonetheless, the challenge of heating the room is still present. An alternative, lower-cost method is to wear a face mask. A comparison is shown in Figure 3.4 and 3.5. By wearing a face mask, the spread is closer to the origin and will add value to both mixing and displacement ventilation (Bhagat et al., 2020).

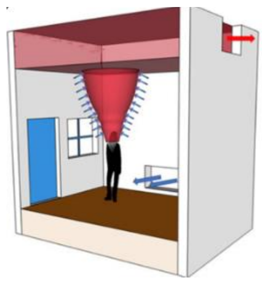


Figure 3.5: Spread of the virus when an inlet is installed, and face mask worn (Bhagat et al., 2020)

4.0 Conclusion

The report has simulated both indoor and outdoor spread of the SARS-CoV-2 virus to draw rational plans to curb the pandemic. For the outdoor concert, platforms at ground-level with disinfected surfaces are the ideal scenario for minimal spread. For the indoor classroom, a window of depth 0.298 m is necessary for maximum occupancy of 8 students. If the classroom needs to accommodate a larger class, the opening needs to be increased 0.474 m and it must be the first class of the day. The classroom can only hold 14 students at maximum capacity. Nonetheless, more measures need to be taken to eliminate the probability of getting infected. The class needs to be cautious of time as well because the probability of infection will go above 0.01 as soon as the hour is exceeded. Besides that, this report has ignored symptoms such as coughing and sneezing. They can be modelled as turbulent plumes and would change the results obtained above. More research is required to assess the assumptions made. In all scenarios, a face mask is recommended to limit the spread.

5.0 References

BBC News (2010) *Statistics reveal Britain's "Mr and Mrs Average."* 13 October 2010. https://www.bbc.co.uk/news/uk-11534042.

Bhagat, R.K., Davies Wykes, M.S., Dalziel, S.B. & Linden, P.F. (2020) Effects of ventilation on the indoor spread of COVID-19. *Journal of Fluid Mechanics*. 903. doi:10.1017/jfm.2020.720.