Pricing Longevity Linked Instruments

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```
library(demography)
library(StMoMo)
library(lifecontingencies)

source("MortalityFunctions.R")
source("Credentials.R")
```

Preparation

```
# Converting cenrtal exposure to initial exposure for logit models
EWMaleIniData = central2initial(EWMaleData)

# Restrict the Data
ages.fit = 60:89
years.fit = EWMaleIniData$years

# Matrix of age weights. See StMoMo vignette page 16 (https://cran.r-project.org/web/packages/StMoMo/vi
wxt = genWeightMat(ages = ages.fit, years = EWMaleIniData$years)

# InitialiZe mortality models
LC = lc(link = "log")
RH = rh(link = "log",
        cohortAgeFun = "1") # cohortAgeFun = "NP" sets the coefficient of the cohort term to be a varia
CBD = cbd(link="logit") # Logit is the q_{x,t}/{1 - q_{x,t}}
M6 = m6(link="logit")

# Fit Mortality Models
LCfit = fit(LC, data=EWMaleData, ages.fit=ages.fit, years.fit=years.fit)
RHfit = fit(RH, data=EWMaleData, ages.fit=ages.fit, years.fit=years.fit)
CBDfit = fit(CBD, data = EWMaleIniData, ages.fit = ages.fit, years.fit=years.fit, wxt = wxt)
M6fit = fit(M6, data = EWMaleIniData, ages.fit = ages.fit, years.fit=years.fit, wxt = wxt)
```

Goodness of Fit

```
### Goodness of Fit
table = data.frame(matrix(nrow = 4, ncol =2, c(AIC(LCfit),
```

```
      AIC(RHfit),
      BIC(LCfit),
      BIC(RHfit),
      AIC(CBDfit),
      AIC(M6fit),
      BIC(CBDfit),
      BIC(M6fit))
  )
)
colnames(table) = c("AIC","BIC")
rownames(table) = c("LC","RH","CBD","M6")
table
```

Market Price of Risk

```
interest_rate = 0.045
discount_factor = 1/(1+interest_rate)
annuitants = 100

LC_qxt = LCfit$Dxt / LCfit$Ext
LC_pxt = 1 - LC_qxt

# https://www.sharingpensions.co.uk/
payment = 6845
total = 100000

#Define K as the total number of years
K = 89-60

# Minimize SSE to fit lambda parameter
LC_wang_sse = function(lambda) {
  sum( payment * sum( discount_factor^(0:K-1) * pnorm(qnorm(LC_pxt[,1] ) - lambda))  - total )^2
}

LC_lambda_wang = nlm(LC_wang_sse, 0.5)$estimate
LC_lambda_wang
```

```
## [1] 0.4987625
```

Pricing

```
# Average of 30 year change in force of mortality
LC_mxt = LCfit$Dxt/LCfit$Ext

years_for = 20
sixty_to_seventy_mortality_improvement = mean(LC_mxt[1:10,years_for] - LC_mxt[1:10,1])
seventy_to_eighty_mortality_improvement = mean(LC_mxt[10:20,years_for] - LC_mxt[10:20,1])
eighty_to_ninety_mortality_improvement = mean(LC_mxt[20:30,years_for] - LC_mxt[20:30,1])
```

```r
LC_getPrice = function(k, years_for){
  LCfor = forecast(LCfit, h=years_for)

  # First year
  if (years_for == 1){
    # Set as global variables
    forecasted_qxt <<- LCfor$rates[k]
    forecasted_pxt <<- 1 - LCfor$rates[k]
  }
  # Other years
  else{
    forecasted_qxt <<- LCfor$rates[k,]
    forecasted_pxt <<- 1 - LCfor$rates[k,]
  }

  LC_wang_risk_adjusted_pxt = pnorm(qnorm(forecasted_pxt) - LC_lambda_wang)

  # Floating Leg
  S_t = sum( annuitants * LC_wang_risk_adjusted_pxt * discount_factor^(1:years_for) )

  # Fixed-Leg
  K_t = sum( X_k(k, years_for) * discount_factor^(1:years_for) ) # X_k(k, t)

  price = S_t - K_t
  riskprem = (S_t/K_t) - 1

  return(price)
}

# get the prices for the first "years_for" years
LC_prices = as.numeric(lapply(1:years_for, function(years_for) LC_getPrice(5, years_for)))
LC_prices
```

```
##  [1]   -2.949954   -6.969206  -12.000442  -17.989365  -24.884549  -32.637308
##  [7]  -41.201558  -50.533693  -60.592466  -71.338875  -82.736052  -94.749157
## [13] -107.345283 -120.493357 -134.164054 -148.329705 -162.964217 -178.042996
## [19] -193.542872 -209.442023
```