Investigation of Mortality Model and Premium Principle Uncertainty in Longevity-Linked Securities

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Introduction

Longevity risk is the risk that people live longer than expected, which poses a significant financial risk for pension plan and life annuity providers to pay more pensions and benefits than they expected. By transferring this risk using securities in capital markets, pension providers can diversify this risk across a broad range of investors.

A survivor forward is an agreement between two parties to exchange a cash flow based on how many people from a specific group are alive at a later date compared to a predetermined survival rate they agreed on initially. A survivor swap is similar to an S-forward but involves swapping cash flows every year instead of a single exchange at maturity.

The primary problem researchers and practitioners face in standardizing longevity-linked securities is determining appropriate prices which involves two key components: projecting future mortality rates and selecting an appropriate valuation principle. However, there is currently no single prevailing approach that is universally accepted by market participants for mortality models or premium principles, and this results in a lack of transparency and makes pricing processes opaque.

Statement of the Problem

Given the lack of literature systematically analyzing how mortality model and premium principle selection affect these securities, our objectives are as follows: To investigate the impact of using different mortality models and of using different premium principles on the valuation of survivor-linked contracts and; To quantify the relative impact of mortality model selection versus premium principle selection.

Conclusions

Here are some **key takeaways**:

There is a clear and predictable pattern from the results of the various premium principles. The magnitude of these differences across the mortality models was significantly larger than the differences observed when varying the premium principle (risk-neutral vs. real-world) within a fixed mortality model. Hence we conclude that the choice of mortality model matters more than the choice of premium principle.

References

[1] Ronald D. Lee and Lawrence R. Carter. Modeling and forecasting u. s. mortality. *Journal of the American Statistical Association*, 87(419):659–671, 1992. `http://www.jstor.org/stable/2290201`.

[2] A.E. Renshaw and S. Haberman. A cohort-based extension to the leecarter model for mortality reduction factors. *Insurance: Mathematics and Economics*, 38(3):556–570, 2006. `doi.org/10.1016/j.insmatheco.2005.12.001`.

[3] Andrew J. G. Cairns, David Blake, and Kevin Dowd. A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *Journal of Risk and Insurance*, 73(4):687–718, 2006. `doi.org/10.1111/j.1539-6975.2006.00195.x`.

Methodology

A mortality model is a statistical model used to fit and predict the probability of death for individuals based on various demographic and actuarial factors, such as age, and time. The Lee-Carter model [1] given by:

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t,$$

where $m_{x,t}$ is the central mortality rate at age x and period t, α_x is the average level of mortality at age x, κ_t is the time-index of mortality, and β_x represents the age sensitivity of mortality to changes in κ_t. We model the mortality index κ_t as a random walk with drift.
The Renshaw-Haberman model [2] is given by,

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_{t-x}$$

where γ_{t-x} is the cohort parameter. We model the cohort parameter γ_{t-x} as an AR(1) process
The CBD model [3] expresses the logit transform of one-year initial mortality rates $q_{x,t}$ of a life aged x in year t is given by,

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x})$$

where $\kappa_t^{(1)}$ represent the level of the mortality curve in year t , $\kappa_t^{(2)}$ represents the gradient of the mortality curve in year t , and $\bar{x}$ is the mean across sample age range.
The M6 model incorporates a cohort effect parameter into to CBD model

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_{t-x}.$$

Define $V_0[X]$ as the valuation at time 0 of a future liability or cash flow given by the random variable X under a specific premium principle. Premium principles involve distorting survival rates to reflect various market conditions. We consider eight premium principles, the first five are risk-neutral and the last three are real-world premium principles.

Premium Principle	**Distorted Form**
Wang	$F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$
Proportional Hazard	$F^*(x) = 1 - [1 - F(x)]^{1/\lambda}$
Dual Power	$S^*(x) = 1 - [1 - S(x)]^{\lambda}$
Gini	$S^*(x) = (1+\lambda)S(x) - \lambda[S(x)]^2$
Exponential	$S^*(x) = \frac{1-e^{-\lambda S(x)}}{1-e^{-\lambda}}$
Standard Deviation	$V_0[X] = \mathbb{E}[X] + [\lambda SD[X]]$
Variance	$V_0[X] = \mathbb{E}[X] + [\lambda VAR[X]]$
Median Absolute Deviation (MAD)	$V_0[X] = S^{-1}(0.5) + [\lambda MAD[X]]$

Here, $F(x)$ is the liability cdf, $F^*(x)$ is the risk-neutral liability cdf, $S(x) = 1 - F(x)$, $\Phi(\cdot)$ refers to the standard Gaussian cdf, $S^{-1}(0.5)$ refers to the median value of the survivor cdf, and MAD[X] = median($|X - S^{-1}(0.5)|$). The valuation equation of an S-forward and S-swap, respectively, is given by:

$$V_0[\underbrace{s_{x,T}^{realized}}_{\text{floating leg}} - \underbrace{(1+\pi)s_{x,T}^{anticipated}}_{\text{fixed leg}}] = 0. \text{ and } V_0\left[\underbrace{\sum_{t=1}^{T} s_{x,t}^{realized}}_{\text{floating leg}} - \underbrace{(1+\pi)\sum_{t=1}^{T} s_{x,t}^{anticipated}}_{\text{fixed leg}}\right] = 0.$$

Where $s_{x,t}^{\text{realized}}$ is the average of simulated one-year survival probability that an individual aged x survives from time $t-1$ to time t under the premium principle considered, and $s_{x,t}^{\text{anticipated}}$ is obtained by setting the pricing parameter λ equal to zero.

Results and Discussion

S-forward contracts generally have a higher risk-adjustment term compared to S-swap contracts of the same maturity due to the structure of the latter, which involves an annual exchange of cash flows as opposed to a single exchange at maturity. The values of the risk-adjustment term π increase as the contract term length increases, as there is greater uncertainty in longevity levels when comparing longer time periods versus shorter periods.

The risk-adjustment term π values generated under the real-world premium principles are generally lower than those produced using the risk-neutral premium principles. A clear pattern or ranking emerges in the relative magnitudes of the risk-adjustment terms across the different valuation principles. The dual hazard transform produced the highest risk-adjustment terms, followed by (in descending order): the Wang transform, the Gini transform, the proportional hazard transform, the standard deviation principle, and finally, the variance principle generated the lowest risk-adjustment terms.