Article

Uncertainty in Pricing and Risk Measurement of Survivor Contracts

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Abstract: As life expectancy increases, pension plans face growing longevity risk. Standardized longevity-linked securities such as survivor contracts allow pension plans to transfer this risk to capital markets. However, more consensus is needed on the appropriate mortality model and premium principle to price these contracts. This paper investigates the impact of the mortality model and premium principle choice on the pricing, risk measurement, and modeling of survivor contracts. We present a framework for evaluating risk measures associated with survivor contracts, specifically survivor forwards (S-forward) and survivor swaps (S-swaps). We analyze how the mortality model and premium principle assumptions affect pricing and risk measures (value-at-risk and expected shortfall). Four mortality models (Lee-Carter, Renshaw-Haberman, Cairns-Blake-Dowd, and M6) and eight premium principles (Wang, proportional, dual, Gini, exponential, standard deviation, variance, median absolute deviation) are considered. We found that risk-neutral premium principles generally have smaller risk measure values than real-world premium principles. Furthermore, the RH model produced the smallest spread between values for both risk measures among various premium principles. Lastly, we found that insurers may want to consider the ES in the risk measurement of survivor contracts over the VaR when using risk-neutral premium principles.

Keywords: longevity risk management; longevity risk measure; value at risk; expected shortfall; survivor contracts

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1. Introduction

Longevity risk refers to the risk that people live longer relative to expectation or the lifespan assumed in the specification and valuation of insurance policies. Longevity risk poses a significant financial risk to pension and life annuity providers as they are at risk of paying out pensions and annuities for longer than anticipated. In 2013, the estimated potential size of the global longevity risk market for pension liabilities is around USD 60 trillion to USD 80 trillion (Blake et al. 2019). These calculations were based on the accumulated assets of private pension systems, the US social security system, the aggregate liability of the US State retirement system, and the public social security systems in 170 countries.

This longevity risk exposure can strain the finances of pension providers. Providers seek risk transfer solutions to mitigate this risk, which involve transferring some of the longevity risk to a third party in exchange for a premium payment. Longevity reinsurance is the most common type of longevity risk transfer. However, there is increasing interest in transferring longevity risk to capital markets as reinsurers become concentrated and need some place to lay off their longevity risk exposure. Securitization and longevity risk transfer to the financial market from pension funds to the financial market are becoming increasingly attractive solutions. Pension funds can offload some of their longevity risk exposure by transferring longevity risk through longevity-linked securities to capital markets. Transferring longevity risk to capital markets allows for greater diversification of longevity risk across a broader range of investors rather than concentrating it within a few reinsurers. Securitization also improves liquidity by enabling providers to hedge their exposure through trading standardized longevity-linked securities. An active market for longevity-linked securities increases price transparency and reduces counterparty credit risk compared to reinsurance contracts. Compared to a customized longevity risk transfer, a standardized longevity-linked security is more desirable due to cheaper costs and liquidity potential (Coughlan et al. 2011; Lin and Cox 2005). These benefits have led to an emerging market for standardized longevity-linked securities. However, the market for longevity-linked securities is smaller than typical financial markets, resulting in a slow development for standardized longevity-linked securities.

Two primary longevity-linked securities investigated in the literature are the survivor forward and survivor swap (Dowd et al. 2006). A survivor forward (S-forward) is a contract between two parties to exchange an amount proportional to the realized survival rate of a given population for an amount proportional to the fixed survival rate agreed upon by both parties at inception to be payable at a future date, which is the maturity. On the other hand, a survivor swap (S-swap) involves a buyer of the swap paying a pre-arranged fixed level of cash flows to the swap provider in exchange for multiple cash flows linked to the realized mortality experience. A pension of life insurance fund will purchase an S-swap to exchange multiple cash flows linked to a floating survival rate for a fixed cash flow payment to hedge longevity risk. Survivor contracts can remove longevity risk without either party needing an upfront payment, allowing pension plans to retain control of the asset allocation (Blake et al. 2019; Zeddouk and Devolder 2019). In essence, while a S-forward is a single survival rate exchange at one future time, a S-swap can be described as a portfolio or series of many S-forwards covering all the relevant payment periods.

1.1. Mortality Model and Premium Principle Uncertainty

The primary problem researchers and practitioners face in standardizing longevity-linked securities is determining appropriate prices (Bauer et al. 2010; Denuit et al. 2007; Lin and Cox 2005; Tang and Li 2021; Zeddouk and Devolder 2019). Pricing longevity-linked securities involves two key components: projecting future mortality rates and selecting an appropriate valuation principle. Projecting future mortality rates for the reference population cohort requires fitting historical mortality data and trends to model and forecast future mortality trends. The projected mortality rates are then used to calculate expected future survival rates that form the basis for valuing the security's cash flows. The second key component in pricing longevity-linked securities is selecting an appropriate premium principle to calculate risk-adjusted premiums. The choice of premium principle, whether risk-neutral or pricing by market expectations (real-world), impacts the valuation. The premium principle dictates the framework for transforming the projected future survival rates and cash flows to arrive at a valuation for longevity-linked securities.

The lack of a universally accepted mortality model and premium principle contributes to the difficulty in pricing longevity-linked securities (Bauer et al. 2010; Tang and Li 2021; Wang et al. 2019). There is currently no single prevailing approach that is universally accepted by market participants for mortality models or premium principles. On the mortality projection front, various models incorporate multiple risk factors ranging from discrete to continuous stochastic models. Furthermore, there is no standard, consistent premium principle that has been adopted across the industry and agreed upon as the methodology to value longevity-linked cash flows. This absence of standardized mortality models and premium principles result in a lack of transparency and makes the valuation processes opaque.

1.2. Pricing Survivor Contracts

The primary work in model uncertainty for survivor contracts by Tang and Li (2021) investigated the impact of different mortality models and premium principles on the pricing of S-forwards and S-swaps using UK mortality data. Tang and Li (2021) examined how different mortality models and premium principles affect the pricing of S-forwards and S-swaps using UK mortality data. They found that mortality model choice has a greater impact on risk premiums than premium principles. There exists other literature that investigate mortality model or premium principles in longevity risk management, but most have focused solely on mortality model uncertainty (Atance et al. 2020; Cairns et al. 2008; Dowd et al. 2010; Haberman and Renshaw 2011; Leung et al. 2018; Li et al. 2020; Pelsser 2008; Yang et al. 2015). There is scant literature on how mortality model and premium principle uncertainty affect the valuation of survivor contracts.

This paper examines four mortality models: the Lee-Carter (LC) model that incorporates historical age-specific mortality rates to forecast future rates. The Renshaw-Haberman (RH) model extends the LC model by adding a cohort effect variable. The Cairns, Blake, Dowd (CBD) model that assumes age, period, and cohort effects are different and randomness exists between years. Finally, the M6 model that extends the CBD model by incorporating cohort effects.

Premium principles can be broadly categorized into two categories: risk-neutral and real-world. Under the risk-neutral measure, the price of a contract is equal to the expected present value of the cash flows under a distorted version of the real-world probability measure. Meanwhile, real-world valuation principles use the historical probability measure and assume that mortality rates and prices will repeat historical trends. This paper examines five risk-neutral premium principles: the Wang transform, proportional hazard transform, dual power transform, Gini transform, and exponential transform. The risk-neutral premium principles apply a distortion function to the cumulative distribution function of the risk to produce a risk-adjusted cumulative distribution function. Risk-neutral pricing relies on risk replication, which is only possible for highly liquid and deeply traded assets. Since the longevity market is immature, there is a lack of liquidity, and risk-neutral pricing methods cannot be used carelessly (Barrieu et al. 2012). Hence, under the real-world measure, the price of a security is determined using real-world probabilities derived from historical data. This paper examines three real-world premium principles: the standard deviation, the variance principle, and the median absolute deviation principle.

1.3. Risk Measurement of Survivor Contracts

Risk measures allow institutions to understand the market risk exposure from their portfolios and determine appropriate capital to buffer against potential losses. Risk measures such as value at risk (VaR) and expected shortfall (ES) have been used in financial markets to estimate how much an investment might lose with a given probability under normal market conditions under a set time period. Solvency II is the supervisory framework for insurers and reinsurers in Europe since 2016. A capital requirement called the solvency capital requirement (SCR) aims to reduce the risk of an insurer being unable to meet claims. The European Insurance and Occupational Pensions Authority (EIOPA) defines the SCR as the 99.5% VaR of the basic own funds over a 1-year period, meaning that enough capital is available to cover the market-consistent losses that may occur over the next year with 99.5% confidence. While Solvency II defined the SCR as the 99.5% VaR, some authors propose that ES is a more appropriate risk measure to determine capital allocation (Boonen 2017; Frey and McNeil 2002; Sandström 2007; Wagner 2014; Yamai and Yoshiba 2005).

There have been some efforts to create a framework for VaR to estimate longevity risk, but most have focused on analyzing the longevity risk associated with mortality projections or annuity products rather than the uncertainty associated with survivor contracts (Börger 2010; Christiansen and Niemeyer 2014; Devolder and Lebègue 2017; Diffouo et al. 2020; Gylys and Šiaulys 2019; Pfeifer and Strassburger 2008; Plat 2011; Richards 2021; Richards et al. 2014). With the increased interest in utilizing survivor contracts through securitization as a means of transferring longevity risk, there arises a need to establish methods for calculating the SCR of survivor contracts, particularly in the framework of Solvency II. To effectively gauge the potential risks associated with these contracts, the calculation of risk measures such as VaR and ES is important. These metrics not only provide insights into the potential downside risks for both issuers and investors, but also play a role in regulatory compliance efforts. Insurers can ensure they maintain sufficient capital reserves to meet the requirements set forth by Solvency II by accurately quantifying the risk exposures of survivor contracts.

This paper presents a framework for evaluating the VaR and ES associated with survivor contracts. We discuss the uncertainty associated with the choice or mortality model and premium principle on the VaR and ES of survivor contracts and compare the risk measure values obtain under 1- and 2-year horizons. From an asset-liability management perspective, pension and life annuity funds hold portfolios of survivor contracts to hedge longevity risk; each contracted at a fair value at inception. As mortality experience evolves, the contracts may become unfavorable, exposing the hedger to potential future losses. By analyzing risk measures like VaR and ES estimated from the distribution of possible future contract values, the annuity fund can quantify potential downside losses on its survivor contract portfolio at a given confidence level. This informs capital allocation and reserving decisions to withstand adverse longevity experiences in line with regulatory limits.

1.4. Framework Overview

This paper builds upon the literature as follows: first, we investigate the impact of the choice of four mortality models on the valuation of S-forward and S-swaps. While the paper by Tang and Li (2021) investigated the LC model with cohort effect and CBD model with quadratic terms and cohort effect, we analyzed the LC and CBD model with and without the cohort effect term. Through our analysis of the results from the valuation process, we provide an alternative lens and a point of comparison to the results obtained by Tang and Li (2021). Additionally, we discuss a procedure for calibrating the pricing parameter λ necessary for each premium principle using publicly available data. Second, we adopt a simulation-based approach to pricing and risk measure calculation. We value the contracts using the simulation-based procedure in Boyer and Stentoft (2013). The simulation-based approach is more flexible and allows the pricing of a wide range of longevity-linked securities. Extending the framework used in this paper to price other longevity-linked contracts is possible because of the simulation-based approach. This simulation-based framework is adopted from the finance literature in pricing derivatives. Though it may be possible to derive closed-form solutions for the price of derivatives, closed-form solutions may require restrictive assumptions about the dynamics of the underlying factors. Simulation-based frameworks for pricing are particularly appealing in financial markets because they can be extended to incorporate complex contract features. Furthermore, simulation-based methods are suited for pricing derivatives with multiple risk factors because the computational complexity grows linearly with the number of risk factors.

Finally, we extend the work of Tang and Li (2021) by examining not only the pricing of survivor contracts but also by investigating the risk measures and modeling of survivor contracts. Moreover, this paper makes a novel contribution by developing a framework for assessing VaR and ES for survivor contracts like S-forwards and S-swaps. Through Monte Carlo simulation across four mortality models and eight premium principles, our analysis provides insight into how modeling assumptions affect risk measures for survivor contracts. We investigate which models and principles lead to more conservative estimates of potential losses over different horizons. This highlights the implications of modeling choices for insurers using survivor contracts to hedge longevity exposure and transfer risk. Furthermore, our paper examines the sensitivity of the risk measures for different horizons. Specifically, we analyzed the dispersion of the risk measure values obtained for a fixed mortality model and premium principle under different maturities and determine the sensitivity of the risk measure values obtained. We found that risk-neutral premium principles generally yielded lower risk measure values compared to real-world premium principles for both contracts. Risk measure values are larger for S-swaps compared to S-forwards. Lastly, the difference between VaR and ES is generally larger for risk-neutral premium principles under the S-swap compared to the S-forward. We found that using the ES over the VaR may be beneficial to insurers for the risk-measurement of survivor contracts.

This paper is organized as follows: Section 2 presents the mortality models and the premium principles, while Section 3 discusses the survivor contracts. Section 4 analyzes the results obtained from valuation. Section 5 sets forth the risk measure framework and examines the results obtained from the risk measures. Section 6 concludes the paper.

2. Mortality Models and Premium Principles

Mortality models are statistical models that describe how mortality rates and life expectancy change over time in a population. This paper considers four mortality models: LC, RH, CBD, and M6. Premium principles refer to the pricing formulas used to determine the fair price charged for survivor contracts. This paper considers eight premium principles: Wang, proportional, dual, Gini, exponential, standard deviation, variance, and median absolute deviation principle.

2.1. Mortality Models

The LC model (Lee and Carter 1992) expresses the natural logarithm of the central death rate $m_{x,t}$ as

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t, \tag{1}$$

where α_x is the average level of mortality at age x, κ_t is the time-index of mortality, and β_x represents the age sensitivity of mortality to changes in κ_t. We model the mortality index κ_t as a random walk with drift to forecast future mortality values. That is,

$$\kappa_t = \kappa_{t-1} + \theta + u_t, \tag{2}$$

where θ is an estimated drift term, and u_t is a sequence of independent and identically distributed random variables following the standard Gaussian distribution.

Renshaw and Haberman (2006) extend the Lee-Carter model to include the cohort effect. The cohort effect term, denoted by γ_{t-x} captures the long-term impact of events on people born in different periods and does not change with one's age. The natural logarithm of the central death rate $m_{x,t}$ is given by

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_{t-x}. \tag{3}$$

To forecast future mortality rates, we model the cohort effect parameter γ_{t-x} as an AR(1) process,

$$\gamma_{t-x} = a_0 + a_1 \gamma_{t-x-1} + e_t, \tag{4}$$

where a_0 is an estimated drift term, a_1 is the estimated sensitivity of the previous cohort step, and the standard Gaussian error term e_t is assumed to be independent of u_t.

The CBD model (Cairns et al. 2006) is a two-factor parametric mortality model. In contrast to the non-parametric age structure in the LC and RH model, the CBD treats age as a continuous variable that varies linearly with the logit of the force of mortality. The CBD model has two latent factors, $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ that allow for more flexibility in capturing the dynamics of mortality changes. Furthermore, the CBD model has the advantage of modeling mortality at higher ages. The CBD model expresses the logit transform of one-year mortality rates $q_{x,t}$ of a life aged x in year t as

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}). \tag{5}$$

Here, $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ represent the estimated level and gradient, respectively, of the mortality curve in year t, and $\bar{x}$ is the mean across the sample age range. The two indices $\kappa_t^{(1)}, \kappa_t^{(2)}$ are modelled by a multivariate random walk with drift,

$$\boldsymbol{K_t} = \boldsymbol{K_{t-1}} + \boldsymbol{\Theta} + \boldsymbol{\epsilon_t}, \tag{6}$$

where $\boldsymbol{K_t} = (\kappa_t^{(1)}, \kappa_t^{(2)})'$, and $\boldsymbol{\Theta}$ is a 2×1 vector which contains two estimated drift coefficients, and the 2×1 error vector $\boldsymbol{\epsilon_t}$ is assumed to follow the standard multivariate Gaussian distribution.

Finally, the M6 model incorporates the cohort effect parameter into the CBD model. The logit transform of one-year mortality rates $q_{x,t}$ of a life aged x in year t is given by

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_{t-x}. \tag{7}$$

Like the Renshaw-Haberman model, the cohort effect parameter is modeled as an AR(1) process.

2.2. Premium Principles

Define $V_0[X]$ as the valuation at time 0 of a future liability or cash flow given by the random variable X. Assuming that the loss random variable X is non negative in insurance contexts is usually appropriate. The choice of $V_0[\cdot]$ is equivalent to choosing a valuation principle. Define the probability density function (pdf) as $f(x)$ and the cumulative distribution function (cdf) as $F(x)$. Define the de-cumulative function $S(x) = 1 - F(x)$. The risk premium is the expectation of the loss random variable X given by,

$$\mathbb{E}[X] = \int_0^\infty x f(x)dx = \int_0^\infty [1-F(x)]dx = \int_0^\infty S(x)dx. \tag{8}$$

Define $f^*(x), F^*(x), S^*(x), \mathbb{E}^*(x)$ as the risk-neutral pdf, cdf, decumulative function, and expectation of the risk respectively. This paper considers eight premium principles; the first five are risk-neutral, and the last three are real-world premium principles.

2.2.1. Risk-Neutral Probability Measures

Risk-neutral valuation principles take the expected present value of the cash flows under a distorted version of the real-world prices. The distortion of the real-world valuation into the risk-neutral valuation is parameterized by the pricing parameter λ. The Wang transform embeds a Gaussian distortion function that returns a distorted cdf (Wang 2002),

$$F^*(x) = \Phi\Big\{\Phi^{-1}[F(x)] - \lambda\Big\}, \quad \lambda \geq 0. \tag{9}$$

Here, $\Phi(\cdot)$ represents the cdf of a standard Gaussian distribution, and $\Phi^{-1}(\cdot)$ is the inverse standard Gaussian cdf. For a given risk X with cdf $F(X)$, the Wang transform produced a risk-adjusted cdf $F^*(X)$. The mean value under $F^*(X)$, denoted as $\mathbb{E}^*[X]$, is the risk-adjusted fair value of X at time T, which will be further discounted to time zero using the risk-free interest rate. One advantage of the Wang transform is that it is reasonably quick to evaluate numerically.

The proportional hazard transform has the advantage of having a simple distortion function of the following form (Wang 1995):

$$F^*(x) = 1 - [1-F(x)]^{1/\lambda}, \quad \lambda \geq 1. \tag{10}$$

The proportional hazard transform is quite sensitive to the choice of λ. Wang (1995) used the proportional hazard transform to price insurance risk.

The dual-power transform (Wang 1996) is given by

$$F^*(x) = F(x)^\lambda, \quad \lambda \geq 1. \tag{11}$$

The Gini principle (Denneberg 1990) has a risk-adjusted cdf given by,

$$F^*(x) = 1 - \left\{[1+\lambda][1-F(x)] - \lambda[1-F(x)]^2\right\}, \quad 0 \leq \lambda \leq 1. \tag{12}$$

Lastly, the exponential transform uses weighted probabilities to map the liability denoted by the random variable X from $[0,1]$ onto $[0,1]$. The risk-adjusted cdf is given by

$$F^*(x) = 1 - \frac{1-e^{-\lambda[1-F(x)]}}{1-e^{-\lambda}}, \quad \lambda > 0. \tag{13}$$

2.2.2. Real-World Probability Measures

The immaturity of the longevity risk transfer market results in low liquidity for survivor contracts, making it difficult to apply risk-neutral premium principles. Real world premium principles that use historical mortality rates offer an alternative methodology to price survivor contracts. The price under the standard deviation principle is given by

$$V_0[X] = \mathbb{E}[X] + [\lambda \mathrm{SD}(X)], \quad \lambda > 0. \tag{14}$$

A pure premium is defined as $V_0[X] = \mathbb{E}[X]$. Hence, the standard deviation principle is equal to the pure premium plus a risk-loading term proportional to the standard deviation of the liability.

The price under the variance principle is given by

$$V_0[X] = \mathbb{E}[X] + [\lambda \mathrm{VAR}(X)], \quad \lambda > 0. \tag{15}$$

Like the standard deviation principle, the variance principle is a pure premium plus a risk-loading term proportional to the liability variance.

Finally, the price under the median absolute deviation principle is given by

$$V_0[X] = S^{-1}(0.5) + [\lambda \mathrm{MAD(X)}], \quad \lambda > 0. \tag{16}$$

Here, $\mathrm{MAD}[X] = \mathrm{median}(|X - S^{-1}(0.5)|)$. Since mean-variance statistics tend to be sensitive to outliers, we include a premium principle that uses the median. The MAD principle is better suited to datasets with small sample sizes and potential outliers.

3. Survivor Contracts

We assume a pension/annuity fund enters a long position in a survivor contract to hedge longevity risk. In a long position, the fund pays a fixed amount K for floating cash flows linked to a future survival rate S. If survivors exceed expectations, the contract payouts hedge the fund's larger liabilities. If fewer survivors occur, the negative payouts are offset by reduced liabilities. As previously mentioned, this paper considers two survivor contracts, the S-forward and S-swap.

3.1. Survivor Forward

A survivor forward (S-forward) is an agreement between two counterparties to exchange a payment linked to the number of survivors in a reference population at a predetermined future date T. The buyer of an S-forward pays a fixed forward rate K to the seller and receives a floating rate $S(T)$. The forward rate is specified at the contract's start and reflects the expected future longevity level. Assume a notional amount equal to one. The fixed leg K must be determined such that the fair value of the survivor forward at $t = 0$ is zero. Mathematically, this is given by

$$V_0[S(T) - K] = 0. \tag{17}$$

Here, the $V_0[\cdot]$ is a value function, which refers to a valuation principle, and the fixed forward rate is determined such that the S-forward has zero value at the start of the contract.

We follow Boyer and Stentoft (2013) and assume that the fixed leg is the known anticipated one-year survival probability scaled to some unknown constant linear risk-adjustment term π. The concrete formulation for the survival forward is given by

$$V_0[s_{x,T}^{\text{realized}} - (1+\pi)s_{x,T}^{\text{anticipated}}] = 0. \tag{18}$$

Here, $s_{x,T}^{\text{realized}}$ is the average of the distorted simulated one-year survival probabilities that an individual aged $x + T$ is alive at time T under the premium principle considered with calibrated pricing parameter λ. On the other hand, $s_{x,T}^{\text{anticipated}}$ is average of the distorted survival probabilities under a specific premium principle obtained by setting the pricing parameter λ equal to either zero or one. Based on the above formulation, the risk-adjustment term is given by

$$\pi = \frac{s_{x,T}^{\text{realized}}}{s_{x,T}^{\text{anticipated}}} - 1. \tag{19}$$

3.2. Survivor Swap

A survivor swap (S-swap) involves two counterparties exchanging a stream of future cash flow linked to the difference between the floating and fixed rates periodically (i.e., for every $t = 1, 2, \ldots, T$). We assume that the forward rate K is constant for all periods. A S-swap consists of a series of S-forwards with different maturities and can be interpreted as a portfolio of S-forwards. Assume a notional principal is equal to one. Analogous to the S-forward, the fixed leg of a S-swap K is determined such that the S-swap has zero value at the onset of the contract,

$$V_0\left[\sum_{t=1}^{T} S(t) - K\right] = 0. \tag{20}$$

Similar to the S-forward, we assume that the fixed leg for a S-swap is the sum of known anticipated one-year survival probability scaled to some unknown constant linear risk-adjustment term π. The concrete formulation for the survival forward is given by

$$V_0\left[\sum_{t=1}^{T} s_{x,t}^{\text{realized}} - (1+\pi)\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}\right] = 0. \tag{21}$$

Based on the above formulation, the risk-adjustment term is given by

$$\pi = \frac{\sum_{t=1}^{T} s_{x,t}^{\text{realized}}}{\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}} - 1. \tag{22}$$

4. Valuation of Survivor Contracts

As of the first quarter of 2011, the annuity rate is level payments of £6,000 per £100,000 funds for a single life aged 66 with level payments. The risk-free rate is assumed to be the 15-year Gilt rate quoted at 2.04% for the first quarter of 2011.

4.1. Model Calibration

Since survivor contracts prices are not publicly traded, there is scant information regarding transaction details. This paper overcomes the model calibration problem by linking the S-forwards and S-swaps to annuity rates. There are no closed-form solutions to the pricing parameter λ for risk-neutral premium measures. Hence, we resort to numerical root-finding algorithms. We apply a Newton-Raphson type of algorithm to obtain the values for the pricing parameter λ. We refer the interested reader to Appendix A for details regarding evaluating the pricing parameter λ

The results obtained for the pricing parameter λ are given in Table 1. The pricing parameters for the LC and RH models are larger than the the CBD and M6 models. Based on the table, it is implied that the survival rates implied by the premium principles from the LC and RH models do not capture an accurate view of the market view of longevity risk compared to the CBD and M6 models.

Table 1. Values obtained for the pricing parameter λ

Premium	LC	RH	CBD	M6
Wang	4.373e-01	4.346e-01	3.993e-01	3.906e-01
Proportional	2.300e+00	2.290e+00	2.155e+00	2.125e+00
Dual	1.386e+00	1.383e+00	1.344e+00	1.334e+00
Gini	6.344e-01	6.317e-01	5.951e-01	5.858e-01
Exponential	1.602e+00	1.593e+00	1.479e+00	1.451e+00
Std. Dev.	9.804e-01	9.746e-01	8.971e-01	8.786e-01
Variance	1.586e-04	1.579e-04	1.487e-04	1.460e-04
MAD	7.516e-01	7.491e-01	7.418e-01	7.964e-01

Table 2. Expressions for $s^{\text{realized}}_{x,T}$ and $s^{\text{anticipated}}_{x,T}$ used in pricing S-forwards. Here, $\text{DF}_T := e^{-rT}$ refers to the discount factor at time T under the risk-free rate r.

Premium Principle	$s^{\text{realized}}_{x,T}$	$s^{\text{anticipated}}_{x,T}$
Wang	$\text{DF}_T\left\{1-\Phi\left[\Phi^{-1}(1-\bar{p}_{x,T})-\lambda\right]\right\}$	$\text{DF}_T\left\{1-\Phi\left[\Phi^{-1}(1-\bar{p}_{x,T})\right]\right\}$
Proportional	$\text{DF}_T(\bar{p}_{x,T})^{\frac{1}{\lambda}}$	$\text{DF}_T(\bar{p}_{x,T})$
Dual	$\text{DF}_T\left[1-(1-\bar{p}_{x,T})^{\lambda}\right]$	$\text{DF}_T[1-(1-\bar{p}_{x,T})]$
Gini	$\text{DF}_T\left[(1+\lambda)\bar{p}_{x,T}-\lambda(\bar{p}_{x,T})^2\right]$	$\text{DF}_T[(1+\lambda)\bar{p}_{x,T}]$
Exponential	$\text{DF}_T\left[\frac{1-e^{-\lambda\bar{p}_{x,t}}}{1-e^{-\lambda}}\right]$	$\text{DF}_T\left[\frac{1-e^{-\bar{p}_{x,t}}}{1-e^{-1}}\right]$
Std. Dev.	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})+\lambda\text{SD}(\bar{p}_{x,T})]$	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})]$
Variance	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})+\lambda\text{VAR}(\bar{p}_{x,T})]$	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})]$
MAD	$\text{DF}_T\left[\text{S}^{-1}(0.5)+\lambda\text{MAD}(\bar{p}_{x,T})\right]$	$\text{DF}_T\left[\text{S}^{-1}(0.5)\right]$

Table 3. Expressions for $\sum_{t=1}^{T} s_{x,t}^{\text{realized}}$ and $\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}$ used in pricing S-swaps. here, $\text{DF}_t := e^{-rt}$ refers to the discount factor at time t under the risk-free rate r.

Premium Principle	$\sum_{t=1}^{T} s_{x,t}^{\text{realized}}$	$\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}$
Wang	$\sum_{t=1}^{T} \text{DF}_t \{1 - \Phi[\Phi^{-1}(1-\bar{p}_{x,t}) - \lambda]\}$	$\sum_{t=1}^{T} \text{DF}_t \{1 - \Phi[\Phi^{-1}(1-\bar{p}_{x,t})]\}$
Proportional	$\sum_{t=1}^{T} \text{DF}_t (\bar{p}_{x,t})^{\frac{1}{\lambda}}$	$\sum_{t=1}^{T} \text{DF}_t (\bar{p}_{x,t})$
Dual	$\sum_{t=1}^{T} \text{DF}_t \left[1 - (1-\bar{p}_{x,t})^{\lambda}\right]$	$\sum_{t=1}^{T} \text{DF}_t [1 - (1-\bar{p}_{x,t})]$
Gini	$\sum_{t=1}^{T} \text{DF}_t \left[(1+\lambda)\bar{p}_{x,t} - \lambda(\bar{p}_{x,t})^2\right]$	$\sum_{t=1}^{T} \text{DF}_t [(1+\lambda)\bar{p}_{x,t}]$
Exponential	$\sum_{t=1}^{T} \text{DF}_t \left[\frac{1-e^{-\lambda \bar{p}_{x,t}}}{1-e^{-\lambda}}\right]$	$\sum_{t=1}^{T} \text{DF}_t \left[\frac{1-e^{-\bar{p}_{x,t}}}{1-e^{-1}}\right]$
Std. Dev.	$\sum_{t=1}^{T} \text{DF}_t [\mathbb{E}(\bar{p}_{x,t}) + \lambda \text{SD}(\bar{p}_{x,t})]$	$\sum_{t=1}^{T} \text{DF}_t [\mathbb{E}(\bar{p}_{x,t})]$
Variance	$\sum_{t=1}^{T} \text{DF}_t [\mathbb{E}(\bar{p}_{x,t}) + \lambda \text{VAR}(\bar{p}_{x,t})]$	$\sum_{t=1}^{T} \text{DF}_t [\mathbb{E}(\bar{p}_{x,t})]$
MAD	$\sum_{t=1}^{T} \text{DF}_t \left[\text{S}^{-1}(0.5) + \lambda \text{MAD}(\bar{p}_{x,t})\right]$	$\sum_{t=1}^{T} \text{DF}_t \left[\text{S}^{-1}(0.5)\right]$

4.2. Pricing Survivor Contracts

Consider a S-forward contract with maturity T. We begin by simulating future mortality scenarios using the fitted mortality model. Define the average simulated value at time t for an individual aged x as $\bar{p}_{x,t}$. We solve for the risk-adjustment term π such that the S-forward has zero value at inception. Mathematically, this is given by Equation 18. The variable of interest is π, which can be evaluated using Equation 19. Expressions for $s_{x,T}^{\text{realized}}$ and $s_{x,T}^{\text{anticipated}}$ can be found in Table 2.

On the other hand, consider a S-swap with maturity T that exchanges cash flows annually. Similar to an S-forward, the risk-adjustment term π of the S-swap is determined such that the contract is fair at inception. Mathematically, this is given by Equation 21. The variable of interest is π, which can be evaluated using Equation 22. Expressions for $\sum_{t=1}^{T} s_{x,t}^{\text{realized}}$ and $\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}$ can be found in Table 3.

4.3. Analysis of Pricing Results

Figures 1 and 2 present the values obtained for the risk-adjustment term π under different maturities for S-forwards and S-swaps respectively. Firstly, S-forwards generally have a higher risk-adjustment term than S-swaps of the same maturity. For example, under the LC mortality model, the 10-year S-forward risk-adjustment term is 0.52% higher than the 10-year S-swap. This general pattern holds across the various models tested. One possible reason the S-swaps have a smaller risk-adjustment term is that an S-swap involves an annual exchange of cash flows linked to the number of survivors from the reference population at each time period. In contrast, a S-forward involves only a single exchange of cash flows. Second, note that the values of the risk-adjustment term increase as the contract term length increases. This phenomenon is expected since there is more uncertainty in longevity levels when comparing more extended periods to shorter periods. Notice that the risk-adjustment term values for the RH model shows that the premium principles are less spread out compared to the risk-adjustment term values for the other mortality models. If an insurance company aims to have similar risk-adjustment term values for

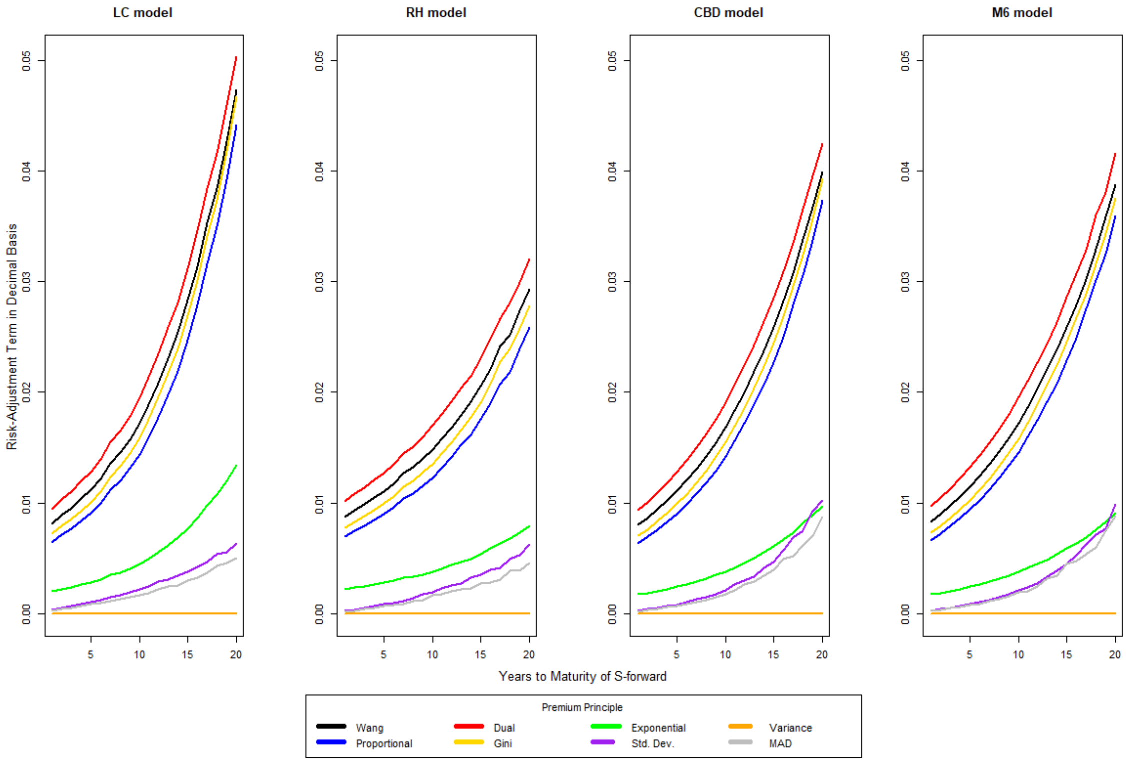

Figure 1. From left to right, the plot of values obtained for the risk-adjustment term π for S-forward under fixed mortality model over different maturities. The plot under the LC model. The plot under the RH model. The plot under the CBD model. The plot under the M6 model.

different premium principles, then the RH model would be a suitable mortality model. The risk-adjustment term values generated under the real-world premium principles are generally lower than those generated from risk-neutral premium principles. The exponential transform produced the lowest risk-adjustment term among the risk-neutral measures. Moreover, a general pattern exists among the risk-adjustment term values. The dual-hazard transform has the largest risk-adjustment term. On the other hand, the variance principle produced the lowest risk-adjustment term. A generalization can be made that the risk-adjustment term generated follows a trend. The highest risk-adjustment term is generated by the dual hazard transform, followed by the Wang transform, the Gini transform, the proportional hazard transform, the standard deviation principle, then finally, the variance principle.

On the other hand, Tables 4 and 5 show that the LC model generated the highest variance and range of risk-adjustment term values across both survivor contracts. For instance, the 10-year S-forward risk-adjustment term range is 3.92% under LC, compared to just 2.06% under RH. This statistic indicates that the LC approach introduces greater uncertainty into mortality projections. This translates into higher and more variable risk-adjustment term values. For a 10-year contract, the S-forward risk-adjustment term is 0.52% larger than the S-swap risk-adjustment term. This gap is the largest across the mortality models, highlighting the LC model's inclination to amplify single-payment contract risk. Across the premium principles, the variance principle produced the lowest and least spread out risk-adjustment term values. However, the dual hazard transform generated the highest risk-adjustment term values with the largest spread between values.

Moreover, the RH model generated the lowest mean, range, and variance among the risk-neutral risk-adjustment term values. The RH model having the smallest mean implies that the RH model generates the smallest risk-adjustment term values among the four

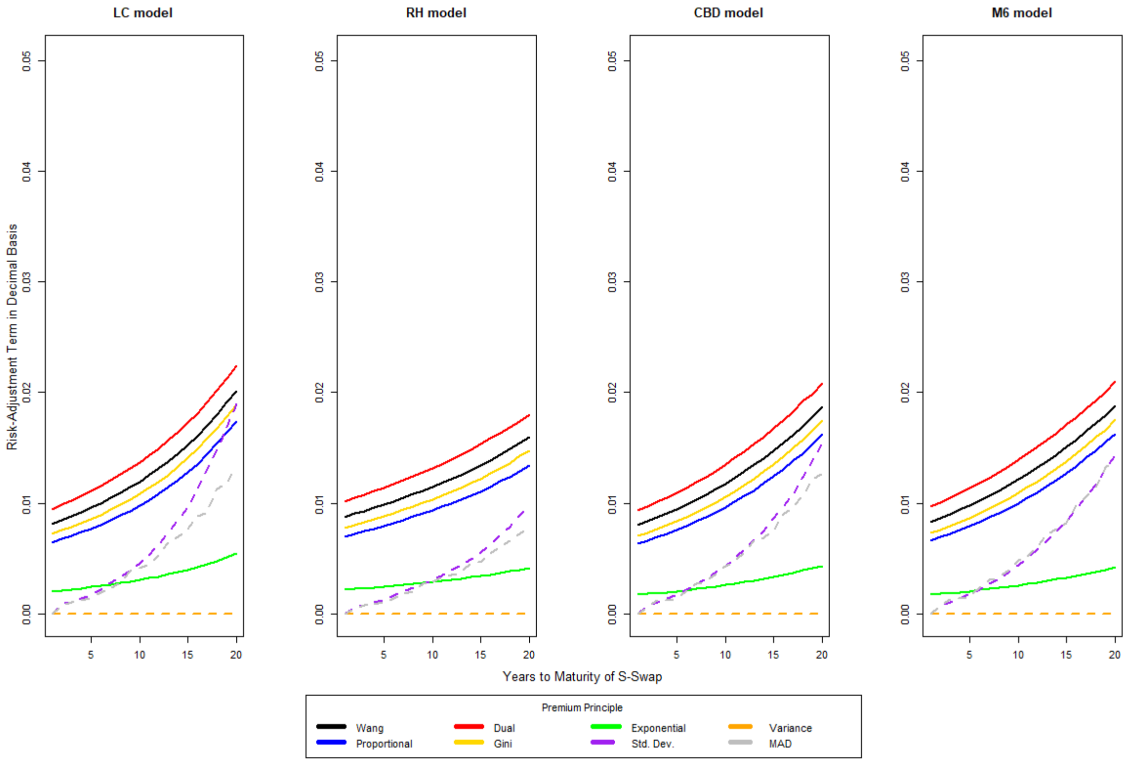

Figure 2. From left to right, the plot of values obtained for the risk-adjustment term π for S-swap under fixed mortality model over different maturities. The plot under LC model. The plot under the RH model. The plot under the CBD model. The plot under the M6 model.

mortality models. Furthermore, having the smallest range and variance suggests that the RH model has the least uncertainty in providing a contract risk-adjustment term.

Table 4. Values for the risk-adjustment parameter π for the LC and RH model for a S-forward

	LC			RH		
	Range	Mean	Variance	Range	Mean	Variance
Wang	3.923e-02	2.165e-02	1.433e-04	2.056e-02	1.671e-02	3.953e-05
Proportional	3.764e-02	1.886e-02	1.299e-04	1.892e-02	1.410e-02	3.293e-05
Dual	4.090e-02	2.397e-02	1.580e-04	2.190e-02	1.884e-02	4.525e-05
Gini	3.955e-02	2.049e-02	1.444e-04	2.006e-02	1.544e-02	3.773e-05
Exponential	1.137e-02	5.833e-03	1.202e-05	5.756e-03	4.344e-03	3.059e-06
Std. Dev.	5.992e-03	2.687e-03	3.285e-06	6.046e-03	2.440e-03	3.204e-06
Variance	6.026e-09	1.634e-09	3.365e-18	5.697e-09	1.438e-09	2.851e-18
MAD	4.737e-03	2.109e-03	2.097e-06	4.442e-03	1.848e-03	1.743e-06

Finally, Tables 6 and 7 show that the CBD model generated higher variance and range of S-forward risk-adjustment term values compared to the M6 model. However, for S-swaps, the risk-adjustment term variance is more similar between CBD and M6. When comparing the differences between S-forward and S-swap premiums, the CBD and M6 models behave more similarly than LC or RH. For 10-year contracts, the S-forward premium exceeds the S-swap by around 0.51% for both CBD and M6. Under the CBD model, the standard deviation premium principle produced the highest variance in risk-adjustment term values for S-forwards, contrasting with other models where the dual hazard premium principle is the highest.

Meanwhile, the M6 model had the lowest mean risk-adjustment term values under the MAD principle. Generally, the CBD and M6 models exhibit a middle-ground behavior

Table 5. Values for the risk-adjustment parameter π for the LC and RH model for a S-swap

	LC			RH		
	Range	Mean	Variance	Range	Mean	Variance
Wang	1.205e-02	1.293e-02	1.360e-05	7.203e-03	1.190e-02	4.908e-06
Proportional	1.092e-02	1.073e-02	1.122e-05	6.453e-03	9.755e-03	3.860e-06
Dual	1.299e-02	1.471e-02	1.585e-05	7.893e-03	1.363e-02	5.878e-06
Gini	1.174e-02	1.181e-02	1.291e-05	6.996e-03	1.078e-02	4.599e-06
Exponential	3.400e-03	3.348e-03	1.074e-06	1.972e-03	3.018e-03	3.641e-07
Std. Dev.	1.903e-02	6.698e-03	3.252e-05	9.747e-03	3.891e-03	8.324e-06
Variance	5.860e-08	1.225e-08	2.882e-16	1.527e-08	3.767e-09	2.085e-17
MAD	1.320e-02	5.233e-03	1.625e-05	7.960e-03	3.372e-03	5.651e-06

between the extremes of LC and RH. The CBD and M6 models strike a balance in the magnitude and spread of risk-adjustment term values generated.

Table 6. Values for the risk-adjustment parameter π for the CBD and M6 model for a S-forward

	CBD			M6		
	Range	Mean	Variance	Range	Mean	Variance
Wang	3.189e-02	1.991e-02	9.686e-05	3.043e-02	1.995e-02	8.764e-05
Proportional	3.097e-02	1.737e-02	8.902e-05	2.926e-02	1.743e-02	8.032e-05
Dual	3.312e-02	2.209e-02	1.056e-04	3.189e-02	2.215e-02	9.586e-05
Gini	3.230e-02	1.868e-02	9.733e-05	3.023e-02	1.867e-02	8.664e-05
Exponential	8.020e-03	4.635e-03	6.128e-06	7.370e-03	4.454e-03	5.069e-06
Std. Dev	9.967e-03	3.381e-03	9.359e-06	9.579e-03	3.172e-03	7.736e-06
Variance	1.938e-08	3.544e-09	2.812e-17	1.799e-08	3.333e-09	2.482e-17
MAD	8.572e-03	2.786e-03	6.182e-06	8.591e-03	2.862e-03	6.341e-06

Table 7. Values for the risk-adjustment parameter π for the CBD and M6 model for a S-swap

	CBD			M6		
	Range	Mean	Variance	Range	Mean	Variance
Wang	1.069e-02	1.250e-02	1.092e-05	1.048e-02	1.281e-02	1.058e-05
Proportional	9.850e-03	1.039e-02	9.240e-06	9.614e-03	1.068e-02	8.925e-06
Dual	1.150e-02	1.425e-02	1.279e-05	1.129e-02	1.459e-02	1.222e-05
Gini	1.043e-02	1.133e-02	1.050e-05	1.029e-02	1.160e-02	1.004e-05
Exponential	2.592e-03	2.794e-03	6.540e-07	2.469e-03	2.755e-03	5.815e-07
Std. Dev.	1.537e-02	5.833e-03	2.141e-05	1.441e-02	5.685e-03	1.896e-05
Variance	4.174e-08	9.742e-09	1.529e-16	3.847e-08	9.286e-09	1.302e-16
MAD	1.273e-02	5.282e-03	1.570e-05	1.388e-02	5.796e-03	1.842e-05

Regarding the risk-adjustment term statistics for the S-forward (Tables 4 and 6), the results suggested that the LC model produced the highest risk-adjustment term values, followed by the M6, the CBD, and the RH model. Furthermore, the risk-adjustment term values obtained for the real-world risk-adjustment term values are closer to one another than those obtained from the risk-neutral risk-adjustment term values.

On the other hand, the risk-adjustment statistics for the S-swap (Tables 5 and 7) suggested that the LC model generated the highest risk-adjustment term values, followed by the M6 model, CBD model, and the RH model. Among the real-world risk-adjustment term values for the S-swap, the spread between the CBD and M6 models is less pronounced. However, the difference between the risk-adjustment term values generated by the LC and RH models is more pronounced. This suggests that for S-swaps, adding the cohort effect term to the CBD and M6 models do not seem to greatly affect the risk-adjustment term values obtained.

4.3.1. Comparison with Previous Works

Comparing our results to that of Tang and Li (2021), we observed the following. In their paper, the authors found that S-forwards have higher risk premiums than S-swaps of the same maturity. We obtained similar results, which can be seen in Figures 1 and 2. One explanation for this is that S-swaps involve multiple cash flow exchanges over its horizon compared to the single payment structure of an S-forward. Second, the authors found that within the risk-neutral premium principles, the risk premiums obtained are very similar across the nine premiums. In contrast, real-world premiums produce higher risk premiums than risk-neutral ones. Our results show that four of the five risk-neutral premiums are close to one another, except the exponential transform with a lower risk-adjustment term value than other risk-neutral premiums. Furthermore, our results generally show that real-world premiums generate a lower risk adjustment term compared to risk-neutral premiums. Lastly, the authors found that the choice of mortality model has a bigger impact on risk premiums than the choice of premium principle. This results of show that the RH model tends to produce higher premiums than the M6 model with quadratic terms. Our results indicate that the LC model produced the largest values for the risk-adjustment term, while the RH model generated the smallest values. We obtained similar observations that the magnitude of the risk-adjustment term values generated from one mortality model versus another is substantially different compared to fixing a mortality model and varying the premium principle. This suggests that the choice of mortality model has a greater impact on the implied risk-adjustment term when compared to the premium principle.

5. Risk Measure Framework

This section presents the VaR and ES of S-forwards and S-swaps evaluated by eight premium principles and four mortality models. The VaR with confidence level $\alpha \in (0,1)$ is the smallest number x such that the probability that the loss X exceeds x is no larger than $(1-\alpha)$. Formally, this is given by

$$\mathrm{VaR}_{\alpha}(X) = \inf\{x \in \mathbb{R} : \mathbb{P}(X > x) \leq 1 - \alpha\}. \tag{23}$$

The ES (also called conditional VaR) is the conditional expectation of loss given that the loss is beyond the VaR level and is given by

$$\mathrm{ES}_{\alpha}(X) = \mathbb{E}[X | X \geq \mathrm{VaR}_{\alpha}(X)] \tag{24}$$

The ES indicates the average loss when the loss exceeds the VaR level. We introduce a Monte Carlo approach to calculating the risk measures. Suppose an insurer is interested in the maximum amount expected to be lost for some survivor contract after some time n at a pre-defined confidence level α. Here, T is the contract time to maturity, and n is the years already accrued by the hedger. A framework for evaluating a n-year VaR and ES is as follows:

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1. First, select a dataset covering ages x_L to x_U, and running from years y_L to y_U.
2. Next, select a mortality model and fit it to the dataset. This gives fitted values for $\ln(m_{x,t})$ or $\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right)$, where x is the age in years and t is the calendar year. Here, $m_{x,t}$ is the central force of mortality and $q_{x,t}$ is the initial force of mortality.
3. Use the mortality model in Step 2 to simulate sample paths and evaluate the risk-adjustment term π_m for a survivor contract with tenor m using the method outlined in Sections 3.1 and 3.2.
4. For the same mortality model parameters in Step 2, generate scenarios for the remaining liability of the insurer after n years using the same π_m as in Step 3 resulting in a distribution for possible remaining liability values.
5. Using the distribution obtained in Step 4, calculate the desired risk measure.

We simulate 5,000 future mortality scenarios. The premium principles are calibrated using market annuity quotations with a starting age of 66. Each calibrated pricing principle is then applied to simulate forward survivor rates and evaluate the risk-adjustment term π using Equations (19) and (22). For the same contract, fix π and evaluate the remaining liability.

For example, consider a 10-year S-forward indexed on an individual aged 66 at inception that exchanges a single cash flow at maturity. The distribution of the liability after one year given by

$$V[s_{75,9}^{\text{realized}} - (1+\pi_{\text{10Y, S-Forward}})s_{75,9}^{\text{anticipated}}]. \tag{25}$$

Here, $\pi_{\text{10Y, S-forward}}$ is the risk-adjustment term obtained previously for a ten-year S-forward under a specific premium principle. Furthermore, $s_{75,9}^{\text{realized}}$ is the average of the distorted simulated one-year survival probabilities that an individual aged 75 is alive 9 years from inception under the premium principle considered with the calibrated pricing parameter λ, and $s_{75,9}^{\text{anticipated}}$ is obtained similar to $s_{75,9}^{\text{anticipated}}$, but the pricing parameter λ is set to either zero or one.

Consider a 10-year S-swap that exchanges cash flows yearly indexed on the cohort of people aged 66. The distribution of the remaining liability after one year of the contract has passed can be obtained by simulating possible survivor rates after one year. The distribution of S-swap risk-adjustment terms after a year is the sum of simulated survivor rates of the cohort of people aged 67 up to the cohort of people aged 75. Mathematically,

$$V\left[\sum_{t=2}^{10} s_{66,t}^{\text{realized}} - (1+\pi_{\text{10Y, S-Swap}})\sum_{t=2}^{10} s_{66,t}^{\text{anticipated}}\right]. \tag{26}$$

Here, $s_{66,t}$ refers to the survival probability of an individual aged $66+t$ at time t

5.1. Risk Measures for S-forwards and S-swaps

Using the previously discussed framework, Figure 3 displays the risk measures for the survivor contracts. Solid bars denote VaR, while transparent bars denote ES. Detailed values are provided in Appendix B.

First, we discuss the results for the S-forward. Among the various combinations of mortality models and premium principles, the M6 model under the Gini principle yielded the highest 1-year 99.5% VaR and ES values for the S-forward. In contrast, the CBD model under the Wang principle yielded the lowest VaR and ES values. Consequently, the M6 model under the Gini principle is the most conservative, while the CBD model under the Wang principle is the least conservative in estimating potential losses. After the Gini transform, the standard deviation principle ranks next regarding the highest VaR and ES values among all mortality models. Notably, real-world premium principles produced higher VaR and ES values than risk-neutral principles. For a fixed mortality model and risk measure, the spread between the risk measure values is widest under the CBD model and narrowest under the RH model. When VaR and ES values are compared, the Gini, variance, and MAD principles exhibit large ES values relative to VaR, indicating that the VaR does not capture the tail risk associated with losses larger than the VaR for the S-forward.

Next, we discuss the results for the S-swap. The most distinguishing difference between the S-forward and S-swap is that S-swaps generally have larger risk measure values. Notably, the MAD principle produced the largest risk measure values, followed by the standard deviation principle. Notice that the standard deviation and MAD principle for the S-swap have similar VaR and ES values indicating that losses exceeding the VaR are not much larger in magnitude. The results for the variance principle were omitted because the values were negligibly small, which was likely a result of the structure of the variance premium principle. Given that the three real-world premium principles have similar VaR

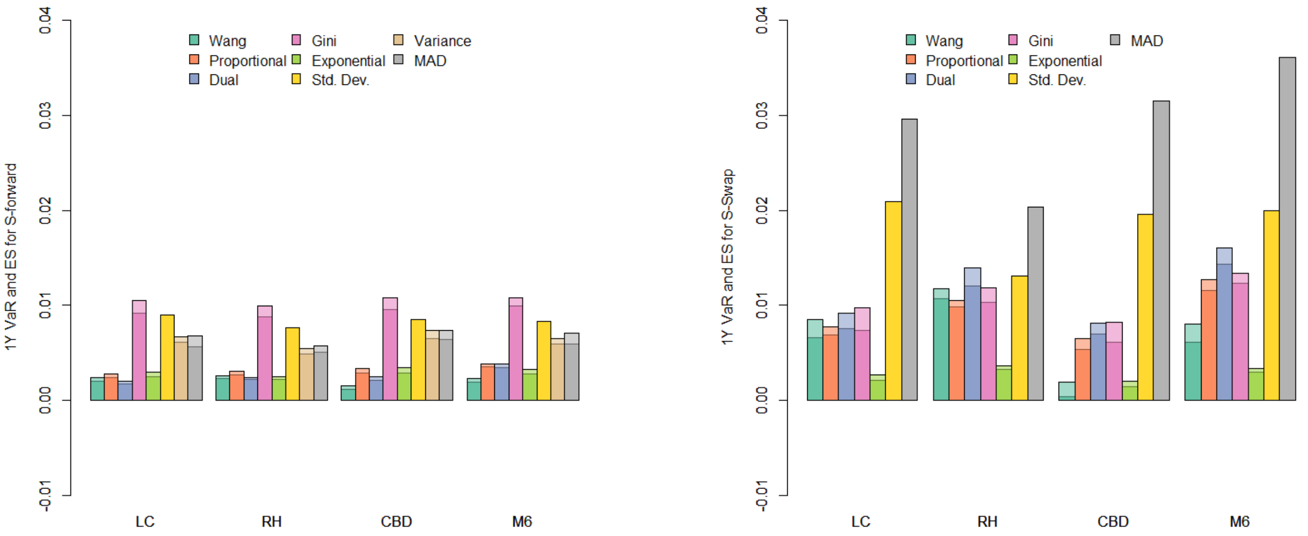

Figure 3. From left to right, the plot of values obtained for a 1-year 99.5% VaR and ES for S-forwards. the plot of values obtained for a 1-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

and ES values, this implies that real-world premium principles for an S-swap capture the tail risk associated with losses exceeding the VaR. One explanation for this could be the way real-world premium principles are defined using historical survival rates, resulting in the contract values converging to a singular value after multiple cash flow exchanges. On the other hand, risk-neutral premium principles generally have larger ES values when compared to the VaR for the risk-neutral premium principles. One explanation for this is the way risk-neutral premium principles apply a distortion function to the survival rates, resulting in a modified or risk-corrected survival rate. Since a distortion function modifies the survival rates, risk-neutral premium principles result in vastly different risk-adjusted survival rates. The additive effect of multiple cash flows from the S-swap resulted in a fat-tailed distribution for possible contract values for risk-neutral principles, hence the larger ES values than VaR values. Lastly, we discuss the spread between premium principles values for a fixed mortality model and risk measure. For the S-swap, the M6 model had the largest spread between premium principles for both VaR and ES values, while the RH model had the smallest spread. When assessing risk, it is common to calculate risk measures for different time horizons, such as 1-year and 2-year horizons. It is of interest to institutions to compare the relative distances between the values of risk measures for different mortality models and premium principles. We discuss the risk measure values obtained for 2-year risk measures and compare the range of values with 1-year risk measures to characterize the uncertainty under specific model assumptions. The 2-year risk measure values and their figures can be seen in Appendix A1. Figure 4 presents the difference between 1- and 2-year 99.5% VaR, while Figure 5 presents the difference between 1- and 2-year 99.5% ES for the S-forward.

First, we discuss the difference between the 1-and 2-year VaR for both survivor contracts. While the RH model had the smallest spread between the premium principles for the 1-year risk measures, it had the largest spread among the difference between the 1-and 2-year VaR for both S-forward and S-swap. On the other hand, the LC model had the smallest spread between the 1-and 2-year VaR for both S-forward and S-swap. Note that risk-neutral premium principles generally have a greater difference between 1- and 2-year VaR for both the S-forward and S-swap. We omit the standard deviation principle for both survivor contracts since the differences were negligibly small. It is generally the case that

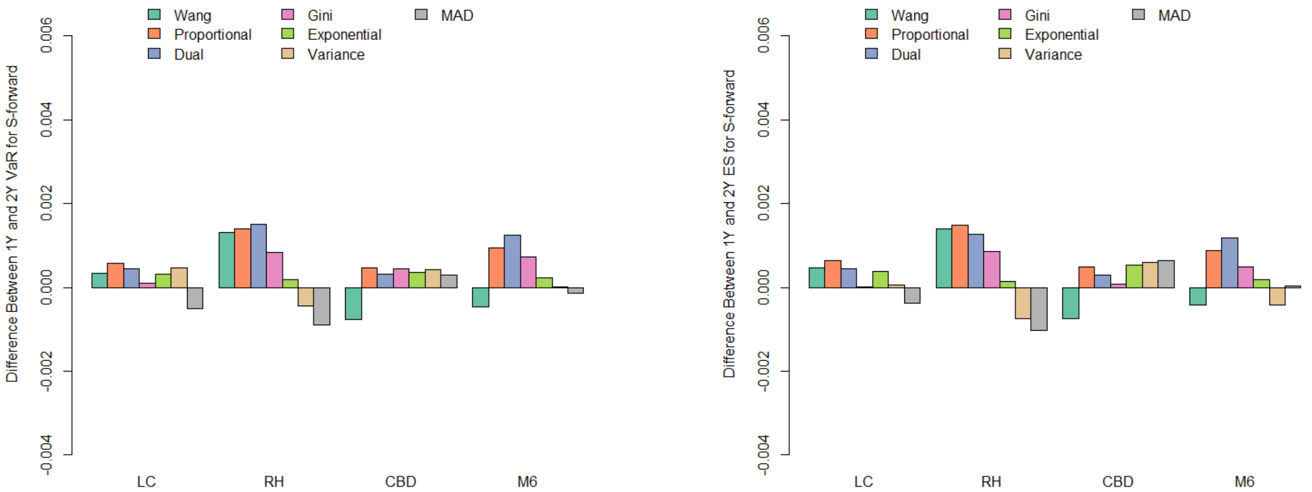

Figure 4. From left to right, the plot of values obtained for a 1-year 99.5% VaR and ES for S-forwards. the plot of values obtained for a 1-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

the 1-year VaR is larger than the 2-year VaR. Suppose that we are interested in the 1-year VaR of a ten-year S-forward for an individual aged 66. This calculation would involve the simulated survival rate for an individual aged 75, nine years from today. On the other hand, the 2-year VaR for the same S-forward would involve the simulated survival rate for the same individual eight years from today. Since life expectancy exhibits an increasing trend, the probability of survival of an individual nine years from today is generally larger than the same probability of survival eight years from today. Because of this phenomenon, the 1-year VaR is generally larger than the 2-year VaR for both survivor contracts.

Moreover, we discuss the difference between the 1- and 2-year ES for both survivor contracts. Since the ES measures the average losses that exceed the VaR, the difference between the 1- and 2-year ES is expected to be much larger compared to the VaR. Note that while the MAD principle had a small difference between the 1- and 2-year VaR, the results for the difference between the ES are much larger. While other risk-neutral premium principles tend to hover around the same level of difference, the exponential transform had the smallest difference among 1- and 2-year ES values for both the S-forward and S-swap. The standard deviation and variance principle were omitted since the difference between their 1- and 2-year ES was negligibly small.

6. Conclusion

This paper investigated the impact of the choice of mortality model and premium principle on pricing, risk measurement, and modeling of S-forward and S-swaps. We developed a framework to evaluate the VaR and ES for S-forward and S-swaps through Monte Carlo simulation. We also provided insight into how modeling choices impact potential loss estimates over various horizons. Finally, we analyzed the sensitivity of risk measures to different maturity lengths and premium principles to highlight the implications for insurers hedging longevity risk. To be specific, we analyzed the impact of four mortality models (LC, RH, CBD, and M6), along with eight premium principles (Wang, proportional hazard, dual power, Gini, exponential transforms, standard deviation, variance, and median absolute deviation principles). We adopted a flexible simulation-based framework to overcome restrictive assumptions and enable the future extension to price other longevity-

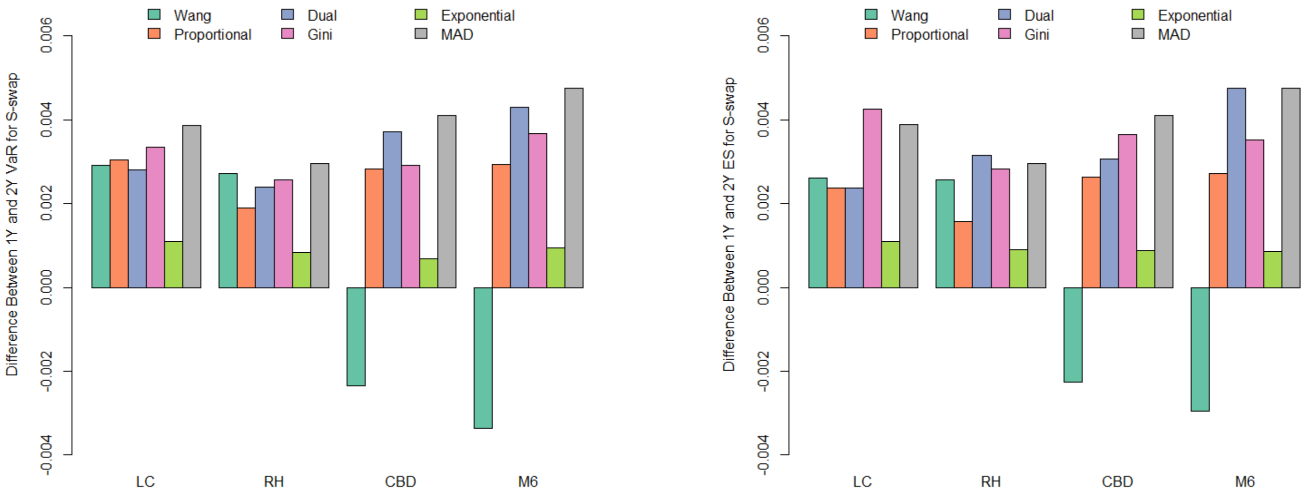

Figure 5. From left to right, the plot of values obtained for a 1-year 99.5% VaR for S-swap. the plot of values obtained for a 1-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

linked securities. The analysis discussed 1-year and 2-year VaR and ES for 10-year S-forwards and S-swaps under four mortality models and eight premium principles.

The analysis for pricing showed that S-forwards generally have a higher risk-adjustment term compared to S-swaps of the same maturity across models. Risk-adjustment term values increase with longer contract terms due to greater uncertainty in mortality rates. The RH model produced risk-adjustment term values that are the closest to one another across principles. Real-world premium principles generate lower risk-adjustment term values than risk-neutral premium principles, with exponential transform being the lowest overall. A trend exists from dual-hazard transform having the highest risk-adjustment term values to the variance principle having the lowest. The LC model produced the greatest variance and range of risk-adjustment term values, introducing greater uncertainty into projections and amplifying risk, especially for the S-forward. In contrast, the RH model generated the lowest mean, range, and variance of risk-adjustment values across principles, implying less uncertainty in estimating risk-adjustment term values. The CBD model produced a large variance and range of risk-adjustment term values for S-forwards compared to the M6, but the models behave more similarly for S-swaps. The CBD and M6 models showed closer alignment in premium differences between S-forwards and S-swaps than the LC or RH. Overall, CBD and M6 exhibit a middle ground between LC and RH in the magnitude and dispersion of risk premiums generated. Regarding mortality models, the LC gives the highest risk-adjustment term values, followed by M6, CBD, and RH for both survivor contracts. We notice that real-world premium principle values were less spread out than risk-neutral ones. Lastly, the choice of mortality model has a bigger impact than the premium principle regarding the value for the risk-adjustment term.

The analysis of the risk measures revealed that risk-neutral premium principles generally produce lower risk measure values compared to the real-world premium principles. We note that the difference between the VaR and ES is generally smaller for S-forwards compared to the risk measures for the S-swap for risk-neutral premium principles. This highlights the additive effect of multiple cash flows on the risk measure values. Despite the ES being smaller for S-forwards than S-swaps, the spread between VaR and ES is large. This implies that insurers should consider employing ES in their risk measurement to incorporate adverse longevity experiences into their risk measurement. On the other hand, real-world premium principles for the S-swap have similar ES values, implying that VaR is a sufficient risk measure for capital allocation purposes under real-world premium principles. Furthermore, the MAD principle for the S-swap serves as an upper bound for risk measure values under all the mortality models considered. Likewise, the Gini transform for the S-forward is also an upper bound for risk measure values. These premium principles provide a conservative estimate for the risk measurement of their respective survivor contracts. Lastly, the spread between risk measure values obtained for various principles was the smallest for a fixed mortality model under the RH model for both survivor contracts. This implies that the RH model is a suitable mortality model for risk measurement purposes if the insurer would like to have similar risk measure values under various premium principles. The analysis between 1- and 2-year risk measures revealed that 1-year risk measures are generally larger than 2-year risk measures for survivor contracts. This trend is a result of increasing life expectancy, which results in a higher probability of survival over longer time horizons. Likewise, we found that the difference between 1- and 2-year ES is much larger than the differences for the VaR. This is because ES measures the average losses exceeding the VaR. While ES captures more risk compared to VaR for the same time horizon, there is greater uncertainty when comparing ES over multiple time horizons compared to VaR. While Solvency II calculates the SCR as the 99.5% VaR over a 1-year horizon, our analysis highlights that insurers may benefit from evaluating the ES of their survivor contracts to inform their capital allocation and risk management decisions.

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Appendix A. Calibration of Pricing Parameter λ

Define the following notation:

- $\psi := 6,000$ is the annuity payment
- $\xi := 100,000$ is annuity fund value
- $r := 2.04\%$ is the risk-free rate
- $DF_t := e^{-rt}$ is the discount factor at time t.

Suppose the premium principle is the Wang transform, then we solve the following equation for λ:

$$\xi = \sum_{t=1}^{T} \psi DF_t[1 - \Phi(\Phi^{-1}(1 - p_{x,t}) - \lambda)]. \tag{A1}$$

To pose the equation above as a root-finding problem, we find λ using a Newton-Raphson-type algorithm such that the sum of the squared errors is minimized:

$$\left\{ \sum_{t=1}^{T} \psi DF_t[1 - \Phi(\Phi^{-1}(1 - p_{x,t}) - \lambda)] - \xi \right\}^2 = 0. \tag{A2}$$

A summary of the minization problem for each premium principle is given in Table A1

Table A1. Risk-neutral premium principles and their associated minimization problem to obtain pricing parameter λ

Premium	Minimize
Wang	$\left\{\sum_{t=1}^{T} \psi DF_t[1-\Phi(\Phi^{-1}(1-p_{x,t})-\lambda)]-\xi\right\}^2=0$
Proportional	$\left\{\sum_{t=1}^{T} \psi DF_t(p_{x,t})^{\lambda}-\xi\right\}^2=0$
Dual	$\left\{\sum_{t=1}^{T} \psi DF_t[1-(1-(p_{x,t})^{\lambda})]-\xi\right\}^2=0$
Gini	$\left\{\sum_{t=1}^{T} \psi DF_t[(1+\lambda)p_{x,t}-\lambda(p_{x,t})^2]-\xi\right\}^2=0$
Exponential	$\left\{\sum_{t=1}^{T} \psi DF_t\left[\frac{1-e^{-\lambda p_{x,t}}}{1-e^{-\lambda}}\right]-\xi\right\}^2=0$

For the real-world premium principles, standard deviation, variance, MAD principle, closed-form expressions for λ can be derived. Suppose that the liability X is a Bernoulli random variable such that

$$X=\begin{cases}\sum_{t=1}^{T} DF_t\psi, & \text{person is alive at time } t\\ 0, & \text{otherwise.}\end{cases} \tag{A3}$$

In other words, the liability of the pension/ life annuity fund is the present value of the payment stream if the person is alive and zero if the person is not. Since the probability that a person aged x is alive at time t is given by $p_{x,t}$, taking the expectation and variance of the Bernoulli random variable yields,

$$\mathbb{E}[X]=\sum_{t=1}^{T} DF_t\psi p_{x,t} \tag{A4}$$

$$\text{VAR}[X]=\sum_{t=1}^{T}(DF_t\psi p_{x,t})[DF_t\psi(1-p_{x,t})]. \tag{A5}$$

Suppose the premium is the standard deviation principle, then

$$\xi=\sum_{t=1}^{T} DF_t\psi p_{x,t}+\lambda\sqrt{\sum_{t=1}^{T}(DF_t\psi p_{x,t})[DF_t\psi(1-p_{x,t})]}. \tag{A6}$$

Solving for λ yields,

$$\lambda=\frac{\xi-\sum_{t=1}^{T} DF_t\psi p_{x,t}}{\sqrt{\sum_{t=1}^{T}(DF_t\psi p_{x,t})[DF_t\psi(1-p_{x,t})]}}. \tag{A7}$$

Analogous to the standard deviation principle, if the premium is the variance principle, then λ is given by,

$$\lambda=\frac{\xi-\sum_{t=1}^{T} DF_t\psi p_{x,t}}{\sum_{t=1}^{T}(DF_t\psi p_{x,t})[DF_t\psi(1-p_{x,t})]}. \tag{A8}$$

If the premium is the MAD principle, then λ is given by, 658

$$\lambda = \frac{\xi - \sum_{t=1}^{T} DF_t \psi \text{MEDIAN}(p_{x,t})}{\sum_{t=1}^{T} DF_t \psi \text{MAD}(p_{x,t})}. \tag{A9}$$

Appendix B. 1-year Risk Measures for Survivor Contracts 659

Table A2. 1-year 99.5% risk measures for survivor forwards

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	1.973e-03	2.276e-03	1.170e-03	1.881e-03
	ES	2.403e-03	2.592e-03	1.560e-03	2.324e-03
Proportional	VaR	2.354e-03	2.702e-03	2.823e-03	3.485e-03
	ES	2.794e-03	3.066e-03	3.314e-03	3.858e-03
Dual	VaR	1.712e-03	2.232e-03	2.114e-03	3.422e-03
	ES	2.033e-03	2.380e-03	2.472e-03	3.841e-03
Gini	VaR	9.128e-03	8.754e-03	9.561e-03	9.920e-03
	ES	1.047e-02	9.937e-03	1.076e-02	1.075e-02
Exponential	VaR	2.500e-03	2.172e-03	2.868e-03	2.782e-03
	ES	2.914e-03	2.501e-03	3.445e-03	3.237e-03
Std. Dev.	VaR	8.943e-03	7.649e-03	8.482e-03	8.347e-03
	ES	8.944e-03	7.650e-03	8.483e-03	8.348e-03
Variance	VaR	6.101e-03	4.891e-03	6.473e-03	5.918e-03
	ES	6.663e-03	5.481e-03	7.340e-03	6.478e-03
MAD	VaR	5.597e-03	5.038e-03	6.377e-03	5.903e-03
	ES	6.737e-03	5.765e-03	7.321e-03	7.066e-03

Table A3. 1-year 99.5% risk measures for survivor swap

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	6.556e-03	1.071e-02	3.599e-04	6.092e-03
	ES	8.489e-03	1.175e-02	1.872e-03	8.012e-03
Proportional	VaR	6.832e-03	9.870e-03	5.321e-03	1.152e-02
	ES	7.731e-03	1.049e-02	6.517e-03	1.271e-02
Dual	VaR	7.553e-03	1.205e-02	6.947e-03	1.429e-02
	ES	9.126e-03	1.395e-02	8.104e-03	1.605e-02
Gini	VaR	7.370e-03	1.026e-02	6.102e-03	1.228e-02
	ES	9.753e-03	1.182e-02	8.195e-03	1.340e-02
Exponential	VaR	2.081e-03	3.201e-03	1.443e-03	2.979e-03
	ES	2.641e-03	3.599e-03	1.960e-03	3.303e-03
Std. Dev.	VaR	2.088e-02	1.303e-02	1.956e-02	1.995e-02
	ES	2.088e-02	1.304e-02	1.956e-02	1.995e-02
Variance	VaR	1.517e-08	4.541e-09	1.747e-08	2.066e-08
	ES	1.518e-08	4.544e-09	1.747e-08	2.067e-08
MAD	VaR	2.963e-02	2.035e-02	3.149e-02	3.611e-02
	ES	2.964e-02	2.035e-02	3.150e-02	3.613e-02

Appendix C. 2-year Risk Measures for Survivor Contracts

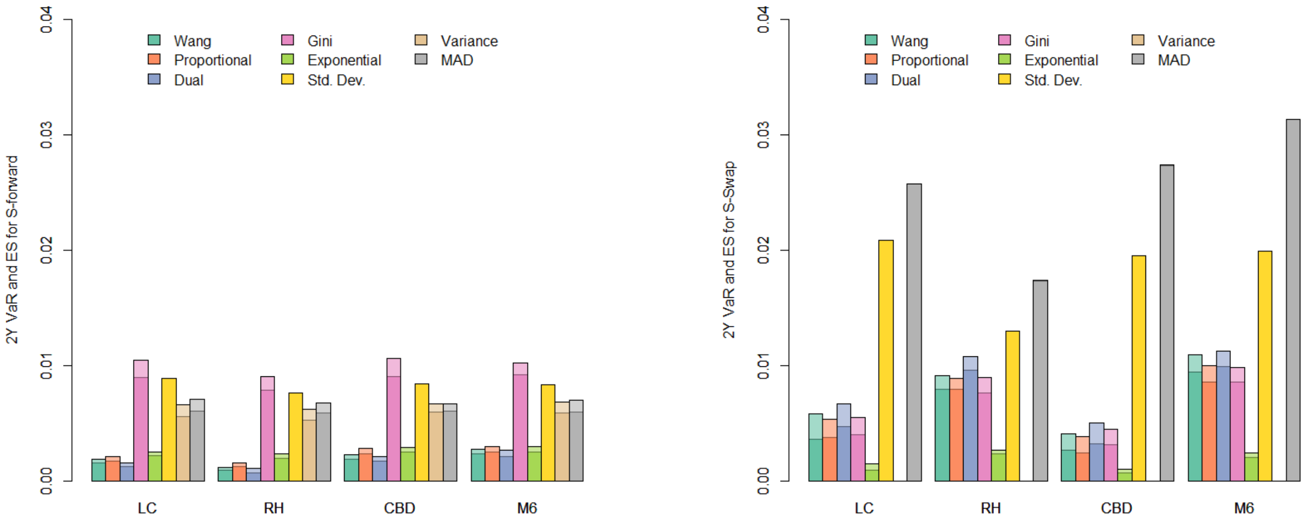

Figure A1. From left to right, the plot of values obtained for a 1-year 99.5% VaR and ES for S-forwards. the plot of values obtained for a 1-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

Table A4. 2-year 99.5% risk measures for survivor forwards

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	1.627e-03	9.546e-04	1.946e-03	2.355e-03
	ES	1.946e-03	1.193e-03	2.300e-03	2.753e-03
Proportional	VaR	1.783e-03	1.301e-03	2.360e-03	2.549e-03
	ES	2.158e-03	1.579e-03	2.827e-03	2.988e-03
Dual	VaR	1.272e-03	7.229e-04	1.789e-03	2.182e-03
	ES	1.583e-03	1.109e-03	2.187e-03	2.664e-03
Gini	VaR	9.027e-03	7.919e-03	9.107e-03	9.198e-03
	ES	1.047e-02	9.081e-03	1.067e-02	1.027e-02
Exponential	VaR	2.194e-03	1.979e-03	2.519e-03	2.551e-03
	ES	2.542e-03	2.350e-03	2.905e-03	3.042e-03
Std. Dev.	VaR	8.943e-03	7.649e-03	8.482e-03	8.347e-03
	ES	8.944e-03	7.650e-03	8.483e-03	8.348e-03
Variance	VaR	5.629e-03	5.330e-03	6.046e-03	5.901e-03
	ES	6.607e-03	6.233e-03	6.749e-03	6.897e-03
MAD	VaR	6.097e-03	5.931e-03	6.084e-03	6.039e-03
	ES	7.112e-03	6.785e-03	6.688e-03	7.041e-03

Table A5. 2-year 99.5% risk measures for survivor swap

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	3.635e-03	7.988e-03	2.698e-03	9.454e-03
	ES	5.874e-03	9.192e-03	4.141e-03	1.096e-02
Proportional	VaR	3.781e-03	7.965e-03	2.502e-03	8.582e-03
	ES	5.359e-03	8.911e-03	3.880e-03	9.999e-03
Dual	VaR	4.750e-03	9.653e-03	3.228e-03	9.982e-03
	ES	6.744e-03	1.080e-02	5.046e-03	1.130e-02
Gini	VaR	4.028e-03	7.692e-03	3.195e-03	8.616e-03
	ES	5.496e-03	9.007e-03	4.547e-03	9.881e-03
Exponential	VaR	9.965e-04	2.366e-03	7.598e-04	2.045e-03
	ES	1.555e-03	2.705e-03	1.083e-03	2.439e-03
Std. Dev.	VaR	2.088e-02	1.303e-02	1.956e-02	1.995e-02
	ES	2.088e-02	1.304e-02	1.956e-02	1.995e-02
Variance	VaR	1.230e-08	3.189e-09	1.495e-08	1.773e-08
	ES	1.231e-08	3.192e-09	1.496e-08	1.774e-08
MAD	VaR	2.575e-02	1.738e-02	2.739e-02	3.135e-02
	ES	2.577e-02	1.739e-02	2.740e-02	3.136e-02

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