Article

Uncertainty in Pricing and Risk Measurement of Survivor Contracts

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Abstract: As life expectancy increases, pension plans face growing longevity risk. Standardized longevity-linked derivatives such as survivor contracts allow pension plans to transfer this risk to capital markets. However, more consensus is needed on the appropriate mortality model and premium principle to price these contracts. This paper investigates the impact of the mortality model and premium principle choice on the pricing, risk measurement, and modeling of survivor contracts. We present a framework for evaluating risk measures associated with survivor contracts, specifically survivor forwards and survivor swaps. We analyze how the mortality model and premium principle assumptions affect pricing and risk measures like value-at-risk and expected shortfall. Four mortality models (Lee-Carter, Renshaw-Haberman, Cairns-Blake-Dowd, and M6) and eight premium principles (Wang, proportional, dual, Gini, exponential, standard deviation, variance, median absolute deviation) are considered. The paper compares the dispersion of risk measures by calculating 1- and 2-year 99.5% risk measures using UK male mortality data for a 10-year S-forward and S-swap. We found that S-forwards typically have higher risk-adjustment terms than S-swaps. S-swaps exhibit large differences in risk measures due to multiple cash flow exchanges. When evaluating risk over different time horizons, the choice of premium principles and mortality models influences the valuation.

Keywords: longevity risk management; longevity risk measure; value at risk; expected shortfall; survivor contracts

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1. Introduction

Longevity risk refers to the risk that people live longer relative to expectation or the lifespan assumed in the specification and valuation of insurance policies. Longevity risk poses a significant financial risk to pension and life annuity providers as they are at risk of paying out pensions and annuities for longer than anticipated. In 2013, the estimated potential size of the global longevity risk market for pension liabilities is around USD 60 trillion to USD 80 trillion (Blake et al. 2019). These calculations were based on the accumulated assets of private pension systems, the US social security system, the aggregate liability of the US State retirement system, and the public social security systems in 170 countries.

This longevity risk exposure can strain the finances of providers. Providers seek risk transfer solutions to mitigate this risk, which involve transferring some of the longevity risk to a third party in exchange for a premium payment. Longevity reinsurance is the most common type of longevity risk transfer. However, there is increasing interest in transferring longevity risk to capital markets as reinsurers become concentrated and need some place to lay off their longevity risk exposure. Securitization and longevity risk transfer to the financial market from pension funds to the financial market are becoming increasingly attractive solutions. Pension funds can offload some of their longevity risk exposure by transferring longevity risk through longevity-linked securities to capital markets. Transferring longevity risk to capital markets allows for greater diversification of longevity risk across a broader range of investors rather than concentrating it within a few reinsurers. Securitization also improves liquidity by enabling providers to hedge their exposure through trading standardized longevity-linked securities. An active market for longevity-linked securities increases price transparency and reduces counterparty credit risk compared to reinsurance contracts. Compared to a customized longevity risk transfer, a standardized longevity-linked derivative is more desirable due to cheaper costs and liquidity potential (Coughlan et al. 2011; Lin and Cox 2005). These benefits have led to an emerging market for standardized longevity-linked derivatives. However, the market for longevity-linked derivatives is smaller than typical financial markets, resulting in a slow development for standardized longevity-linked derivatives.

Two primary longevity-linked securities investigated in the literature are the survivor forward and survivor swap (Dowd et al. 2006). A survivor forward (S-forward) is a contract between two parties to exchange an amount proportional to the realized survival rate of a given population for an amount proportional to the fixed survival rate agreed upon by both parties at inception to be payable at a future date.

On the other hand, a survivor swap (S-swap) involves a buyer of the swap paying a pre-arranged fixed level of cash flows to the swap provider in exchange for cash flows linked to the realized mortality experience. A pension of life insurance fund will purchase an S-swap to exchange cash flows linked to a floating mortality rate for a fixed cash flow payment to hedge longevity risk. S-swaps can remove longevity risk without either party needing an upfront payment, allowing pension plans to retain control of the asset allocation (Blake et al. 2019). The first publicly announced S-swap occurred in April 2007 between Swiss Re and UK life office Friend's Provident (MacMinn et al. 2008). Swiss Re paid for an undisclosed premium in exchange for assuming the longevity risk based on Friend's Provident £1.7B book of 78,000 pension annuity contracts written from July 2001 to December 2006. In essence, while a S-forward is a single survival rate exchange at one future time, a S-swap can be described as a portfolio or series of many S-forwards covering all the relevant payment periods. A S-swap specifies a fixed survival rate that the pension fund paying the fixed leg expects for each year. S-swaps provide low transaction costs, great flexibility, and do not require the existence of a liquid life market (Zeddouk and Devolder 2019).

1.1. Pricing Survivor Contracts

This paper examines four mortality models: the Lee-Carter (LC) model, which incorporates historical age-specific mortality rates to forecast future rates, the Renshaw-Haberman (RH) model, which extends the LC model by adding a cohort effect variable, the Cairns, Blake, Dowd (CBD) model, which assumes age, period, and cohort effects are different and randomness exists between years, and the M6 model, which extends the CBD model by incorporating cohort effects. Key differences are that the LC and RH models have a trivial correlation structure between mortality rate changes at different ages while the CBD and M6 models allow for a non-trivial structure with multiple risk factors.

Pricing principles can be broadly categorized into two categories: risk-neutral and real-world. Under the risk-neutral measure, the price of a contract is equal to the expected present value of the cash flows under a distorted version of the real-world probability measure. Meanwhile, real-world valuation principles use the historical probability measure and assume that mortality rates and prices will repeat historical trends. This paper examines five risk-neutral pricing principles: the Wang transform, proportional hazard transform, dual power transform, Gini transform, and exponential transform. The risk-neutral premium principles apply a distortion function to the cumulative distribution function of the risk to produce a risk-adjusted fair value. Risk-neutral pricing relies on risk replication, which is only possible for highly liquid and deeply traded assets. Since the longevity market is immature, there is a lack of liquidity, and risk-neutral pricing methods cannot be used carelessly (Barrieu et al. 2012). Hence, under the real-world measure, the price of an instrument is determined using real-world probabilities derived from historical data. This paper examines three real-world pricing principles: the standard deviation, the variance principle, and the median absolute deviation principle.

1.2. Mortality Model and Premium Principle Uncertainty

The primary problem researchers and practitioners face in standardizing longevity-linked securities is determining appropriate prices ((Bauer et al. 2010), (Denuit et al. 2007), (Lin and Cox 2005), (Tang and Li 2021), (Zeddouk and Devolder 2019)). Pricing longevity-linked securities involves two key components: projecting future mortality rates and selecting an appropriate valuation principle. Projecting future mortality rates for the reference population cohort requires fitting historical mortality data and trends to model and forecast future mortality trends. The projected mortality rates are then used to calculate expected future survival rates that form the basis for valuing the security's cash flows. The second key component in pricing longevity-linked securities is selecting an appropriate premium principle to calculate risk-adjusted premiums. The choice of premium principle, whether risk-neutral or pricing by market expectations (real-world), impacts the valuation. The premium principle dictates the framework for transforming the projected future survival rates and cash flows to arrive at a valuation for longevity-linked securities.

The lack of a universally accepted mortality model and standard premium principle contributes to the difficulty in pricing longevity-linked securities ((Bauer et al. 2010); (Tang and Li 2021); (Wang et al. 2019)). There is currently no single prevailing approach that is universally accepted by market participants for mortality models or premium principles. On the mortality projection front, various models incorporate multiple factors ranging from discrete to continuous stochastic models. The diversity of modeling techniques leads to a need for more consensus on projected mortality rates. Furthermore, there is no standard, consistent premium principle that has been adopted across the industry and agreed upon as the methodology to value longevity-linked cash flows. Practitioners utilize different valuation techniques like risk-neutral pricing, pricing by market expectations using different approaches, or proprietary pricing models. This absence of standardized models and pricing methods results in a lack of transparency and makes pricing processes opaque. It becomes difficult to coherently compare longevity instrument valuations and ensure consistent pricing relationships since the core inputs and pricing frameworks differ.

The primary work in this area by Tang and Li (2021) investigated the impact of different mortality models and premium principles on the pricing of S-forwards and S-swaps using UK mortality data. Tang and Li (2021) compared risk premiums from twelve premium principles calibrated under the Lee-Carter model with cohort effect and Cairns-Blake-Dowd model with cohort effect and quadratic term mortality models. Tang and Li (2021) found that the choice of mortality model has greater influence on risk premiums than the premium principle. Atance et al. (2020) compared different models to predict mortality rates, highlighting the importance of selecting the most effective model for a given location. They found that the Lee-Carter model performed well in predicting life expectancy in European countries and suggested a method to choose the best model for any region. Li et al. (2020) decomposed historical US mortality improvements into age, period, and cohort components, experimenting with various models to find the most robust and explanatory ones. Cairns et al. (2009) compared eight stochastic mortality models to explain improvements in mortality rates in England and Wales and the United States. For higher ages, an extension of the CBD model with a cohort effect fit the England and Wales males' data best, while the RH extension to the LC model with a cohort effect provided the best fit for US males' data. Haberman and Renshaw (2011) compared different mortality models to forecast life expectancy and annuity values for different age groups using mortality data from England & Wales and the USA. The paper investigated how recent model improvements address the shortcomings in Cairns et al. (2009). Dowd et al. (2010) evaluated the forecasting performance of six different stochastic mortality models applied to English & Welsh male mortality data using a backtesting framework. Results indicated that most models perform adequately in backtests, with wider prediction intervals for parameter uncertainty, and show little difference in performance except for the RH model displaying forecast instability. Yang et al. (2015) discussed the importance of accurately predicting mortality rates for longevity-linked products. The authors introduced a method that combined process, parameter, and model errors to improve mortality projections, highlighting the impact of model selection on risk-neutral valuation results. Cairns et al. (2008) discussed various stochastic mortality models proposed, focusing on discrete-time models for statistical modeling and forecasting. Cairns et al. (2008) reviewed the pricing of different financial instruments, including S-swaps, that can be used to hedge against mortality risk in the market. The illiquid and emerging nature of the longevity risk market further complicates determining an appropriate longevity risk premium. Pelsser (2008) argued that the Wang transform is not suitable for pricing such risks in a consistent manner with the market dynamics. Leung et al. (2018) delved into this issue by exploring various pricing approaches using a Bayesian state-space mortality model. Their study aimed to address parameter uncertainty and obtain a distribution of the longevity risk-premium, offering insights into analyzing pricing methods in an illiquid and incomplete longevity market.

The slow development of the longevity-risk transfer market, combined with the market's illiquidity and lack of transparency, needs to be improved in determining what premium principle should be used in pricing longevity-linked securities. Barrieu and Veraart (2016) examined how different factors affect the pricing of a financial instrument called a q-forward contract based on longevity risk. A q-forward contract involves exchanging the realized mortality rate of a population for a predetermined fixed rate. The authors focused on analyzing the impact of model choice for mortality rates, the time window used for estimation, and the pricing method for determining the fixed rate for q-forward contracts. Barrieu et al. (2012) explored advancements in longevity-risk modeling and the challenges faced by the financial and insurance sectors. They discussed concepts for understanding longevity risk and highlighted the need for improved risk assessment and management practices in response to increasing life expectancy. The article also addressed capital markets, such as insurance-linked securities, to transfer longevity risk and emphasized the importance of evolving industry regulations for effective risk management.

1.3. Risk Measurement of Survivor Contracts

Risk measures allow institutions to understand the market risk exposure from their portfolios and determine appropriate capital to buffer against potential losses. Risk measures such as value at risk (VaR) and expected shortfall (ES) have been used in financial markets to estimate how much an investment might lose with a given probability under normal market conditions under a set time period. Frey and McNeil (2002) discussed the problems with using VaR as a risk measure in portfolio credit risk and claimed that ES is a more appropriate risk measure to determine capital allocation. Yamai and Yoshiba (2005) discussed the issues with VaR disregarding losses beyond the VaR level and suggested that ES is a more suitable alternative for scenarios under market stress. One advantage of ES over VaR is that ES considers the size of the worst-case events, whereas VaR only provides a quantile. Boonen (2017) explored the impact on a life annuity insurance company if solvency capital requirements are based on ES instead of VaR. They found that using ES instead of VaR may result in a larger longevity solvency capital requirement and a smaller equity solvency capital requirement. Using ES in stress testing for insurance context is recommended by Wagner (2014) and Sandström (2007). Despite these, VaR is still commonly used in practice because of familiarity and regulatory requirements.

Solvency II is the supervisory framework for insurers and reinsurers in Europe since 2016. A required capital requirement called the solvency capital requirement (SCR) aims to reduce the risk of an insurer being unable to meet claims. The European Insurance and Occupational Pensions Authority (EIOPA) defines the SCR as the 99.5% VaR of the basic own funds over a 1-year period, meaning that enough capital is available to cover the market-consistent losses that may occur over the next year with 99.5% confidence.

There have been some efforts to create a framework for VaR to estimate longevity risk, but most have focused on analyzing the longevity risk associated with mortality projections or annuity products rather than the uncertainty associated with survivor contracts. Diffouo et al. (2020) quantified longevity risk in annuity products, computing insurer solvency capital within the Solvency II framework using the Hull–White model to assess policyholder mortality, determining optimal product choices for insurers and shareholders, as well as identifying cost-effective and profitable options for policyholders, while examining solvency capital sensitivity to key parameters. Christiansen and Niemeyer (2014) performed a comparative analysis of various mathematical interpretations of the SCR from Solvency II to highlight similarities, differences, and convergence properties, while also proposing a generalized SCR definition extending to future points in time to establish a robust framework for defining the the capital buffer in Solvency II. Devolder and Lebègue (2017) explored how different risk measures affect the solvency capital requirement of a pension fund and demonstrates the impact of the time horizon of long-term guarantee products on this capital. Richards et al. (2014) discussed how expectations of future mortality rates might change over a single year, particularly for annuity portfolios and pension schemes. The authors presented a framework for determining how much longevity liability might change based on new information to assess longevity risk and select appropriate models for management purposes. Richards (2021) presented a framework to assess the risk of mis-estimating actuarial liabilities and developed a methodology for evaluating this risk over a fixed time frame. They also suggested that more parsimonious mortality models tend to have lower mis-estimation risk compared to less parsimonious mortality models. Plat (2011) introduced a stochastic mortality trend model for estimating value at risk and an approximation method for applying the stochastic mortality rates to insurance portfolios. Börger (2010) proposed a standard mortality model framework that insurers can use to approximate regulatory capital requirements based on the VaR framework of Solvency II. Gylys and Šiaulys (2019) analyzed the model-based value at risk associated with mortality in life insurance contracts and compared it to the capital requirements set by the Solvency II standard formula. Pfeifer and Strassburger (2008) discussed the challenges that arise in calculating the aggregated SCR, and proposed calibration adjustments to address skewness in the individual risk distributions.

With the increased interest in utilizing survivor contracts through securitization as a means of transferring longevity risk, there arises a need to establish methods for calculating the SCR of these contracts, particularly in the framework of Solvency II. To effectively gauge the potential risks associated with these contracts, the calculation of metrics such as VaR and ES is important. These metrics not only provide insights into the potential downside risks for both issuers and investors, but also play a role in regulatory compliance efforts. Insurers can ensure they maintain sufficient capital reserves to meet the requirements set forth by Solvency II by accurately quantifying the risk exposures of survivor contracts.

This paper presents a framework for evaluating the VaR and ES associated with survivor contracts. We discuss the uncertainty associated with the choice or mortality model and premium principle on the VaR and ES of survivor contracts and compare the risk measure values obtain under 1- and 2-year horizons. From an asset-liability management perspective, pension and life annuity funds hold portfolios of survivor contracts to hedge longevity risk; each contracted at a fair value at inception. As mortality experience evolves, the contracts may become unfavorable, exposing the hedger to potential future losses. By analyzing risk measures like VaR and ES estimated from the distribution of possible future risk-adjustment terms, the annuity fund can quantify potential downside losses on its survivor contract portfolio at a given confidence level. This informs capital allocation and reserving decisions to withstand adverse longevity experiences in line with regulatory limits.

1.4. Framework Overview

This paper builds upon the literature as follows: first, we extend the work of Tang and Li (2021) by examining not only the pricing of survivor contracts but also by investigating the risk measures and modeling of survivor contracts. Moreover, this paper makes a novel contribution by developing a framework for assessing VaR and ES for survivor contracts like S-forwards and S-swaps. Through Monte Carlo simulation across four mortality models and eight premium principles, our analysis provides insight into how modeling assumptions affect risk measures for survivor contracts. The results reveal which models and principles lead to more conservative estimates of potential losses over different horizons. This highlights the implications of modeling choices for insurers using survivor contracts to hedge longevity exposure and transfer risk. Furthermore, our paper examines the sensitivity of the risk measures for different horizons. Specifically, we analyze the dispersion of the risk measure values obtained for a fixed mortality model and premium principle under different maturities and determine the sensitivity of the risk measure values obtained. Second, we investigate the impact of the choice of four mortality models on the valuation of S-forward and S-swaps. While the paper by Tang and Li (2021) investigated the LC model with cohort effect and CBD model with quadratic terms and cohort effect, we analyzed the LC and CBD model with and without the cohort effect term. Through our analysis of the results from the valuation process, we provide an alternative lens and a point of comparison to the results obtained by Tang and Li (2021). Additionally, we discuss a procedure for calibrating the pricing parameter λ necessary for each premium principle using publicly available data. Third, we adopt a simulation-based approach to pricing and risk measure calculation. We value the contracts using the simulation-based procedure in Boyer and Stentoft (2013). The simulation-based approach is more flexible and allows the pricing of a wide range of longevity-linked securities. Extending the framework used in this paper to price other longevity-linked contracts is possible because of the simulation-based approach. This simulation-based framework is adopted from the finance literature in pricing derivatives. Though it may be possible to derive closed-form solutions for the price of derivatives, closed-form solutions may require restrictive assumptions about the dynamics of the underlying factors. Simulation-based frameworks for pricing are particularly appealing in financial markets because they can be extended to incorporate complex contract features. Furthermore, simulation-based methods are suited for pricing derivatives with multiple risk factors because the computational complexity grows linearly with the number of risk factors.

1.5. Brief Results

The analysis for pricing showed that S-forwards generally had a higher risk-adjustment term than S-swaps of the same maturity across models, with risk-adjustment values increasing with longer contract terms primarily due to increased uncertainty. The RH model yielded a small spread of risk-adjustment term values across principles. Real-world premium principles generally produced lower risk-adjustment term values than risk-neutral ones, with the exponential transform being the lowest. The LC model had the largest values for the risk-adjustment term indicating greater uncertainty and risk, especially for the S-forward, while the RH model implied less uncertainty in estimating contract risk-adjustment terms. The CBD model exhibited a higher variance and range of risk-adjustment term values for S-forwards compared to M6, with CBD and M6 showing similar risk-adjustment term values for similar contracts. Overall, CBD and M6 struck a middle ground between LC and RH in the magnitude and dispersion of risk-adjustment terms, with LC yielding the highest risk premium values. Additionally, real-world premium principle risk-adjustment terms were less dispersed than risk-neutral ones. We found that the choice of mortality model had a greater impact than the premium principle on the value for the risk-adjustment terms.

The analysis of risk measures for both S-forwards and S-swaps demonstrated variations across different models and principles. For S-forwards, the M6 model under the Gini principle shows the highest 1-year 99.5% VaR, indicating conservative estimates. In contrast, the CBD model under the Wang principle showed the lowest VaR, suggesting less conservative estimates. Similarly, the RH mortality model consistently yielded the lowest risk measure values. Real-world premium principles generally led to lower VaR and ES values than risk-neutral premiums. S-swaps exhibit larger differences in VaR due to multiple cash flow exchanges, with specific premium principles and mortality models resulting in variations between 1-year and 2-year risk measures.

It is important to note that we aim not to find the best model and principle, as this varies for every data set. Instead, we aim to understand the impact of model and principle uncertainty. Likewise, this paper does not prescribe a best risk measure but instead seeks to compare VaR and ES quantitatively in the context of S-forwards and S-swaps.

This paper is organized as follows: Section 2 discusses in detail the mortality models and the premium principles, while Section 3 discusses the longevity-linked contracts. Section 4 analyzes the results obtained from valuation. Section 5 sets forth the risk measure framework and examines the results obtained from the risk measures. Section 6 concludes the paper.

2. Mortality Models and Pricing Principles

Mortality models are statistical models that describe how mortality rates and life expectancy change over time in a population. This paper considers four mortality models: LC, RH, CBD, and M6. Premium principles refer to the pricing formulas used to determine the fair price charged for survivor contracts. This paper considers eight premium principles: Wang, proportional, dual, Gini, exponential, standard deviation, variance, and median absolute deviation principle.

2.1. Mortality Models

The LC model (Lee and Carter 1992) expresses the natural logarithm of the central death rate $m_{x,t}$ as

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t, \tag{1}$$

where α_x is the average level of mortality at age x, κ_t is the time-index of mortality, and β_x represents the age sensitivity of mortality to changes in κ_t. We model the mortality index κ_t as a random walk with drift to forecast future mortality values. That is,

$$\kappa_t = \kappa_{t-1} + \theta + u_t, \tag{2}$$

where θ is an estimated drift term, and u_t is a sequence of independent and identically distributed random variables following the standard Gaussian distribution.

Renshaw and Haberman (2006) extend the Lee-Carter model to include the cohort effect. The cohort effect captures the long-term impact of events on people born in different periods and does not change with one's age. The RH model produced a better fit than the LC for mortality data with a prominent cohort effect. The natural logarithm of the central death rate $m_{x,t}$ is given by

$$\ln(m_{x,t}) = \alpha_x + \beta_x \kappa_t + \gamma_{t-x}. \tag{3}$$

To forecast future mortality rates, we model the cohort parameter γ_{t-x} as an AR(1) process,

$$\gamma_{t-x} = a_0 + a_1 \gamma_{t-x-1} + e_t, \tag{4}$$

where a_0 is an estimated drift term, a_1 is the estimated sensitivity of the previous cohort step, and the standard Gaussian error term e_t is assumed to be independent of u_t.

The CBD model (Cairns et al. 2006) is a two-factor parametric mortality model. In contrast to the non-parametric age structure in the LC and RH model, the CBD treats age as a continuous variable that varies linearly with the logit of the force of mortality. The CBD model has two latent factors $\kappa_t^{(1)}$, $\kappa_t^{(2)}$ that allows for more flexibility in capturing the dynamics of mortality changes. Furthermore, the CBD model has the advantage of modeling mortality at higher ages.

The CBD model expresses the logit transform of one-year mortality rates $q_{x,t}$ of a life aged x in year t as

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}). \tag{5}$$

Here, $\kappa_t^{(1)}, \kappa_t^{(2)}$ represent the estimated level and gradient of the mortality curve in year t, and $\bar{x}$ is the mean across the sample age range. The two indices $\kappa_t^{(1)}, \kappa_t^{(2)}$ are modelled by a multivariate random walk with drift,

$$\boldsymbol{K_t} = \boldsymbol{K_{t-1}} + \boldsymbol{\Theta} + \boldsymbol{\epsilon_t}, \tag{6}$$

where $\boldsymbol{K_t} = (\kappa_t^{(1)}, \kappa_t^{(2)})'$, and $\boldsymbol{\Theta}$ is a 2×1 vector which contains two estimated drift coefficients, and the 2×1 error vector $\boldsymbol{\epsilon_t}$ is assumed to follow the standard multivariate Gaussian distribution.

Finally, the M6 model incorporates the cohort parameter into the CBD model. The logit transform of one-year mortality rates $q_{x,t}$ of a life aged x in year t is given by

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_{t-x}. \tag{7}$$

Like the Renshaw-Haberman model, the cohort parameter is modeled as an AR(1) process.

2.2. Premium Principles

Define $V_0[X]$ as the valuation at time 0 of a future liability or cash flow given by the random variable X. Assuming that the loss random variable X is non negative in insurance contexts is usually appropriate. The choice of $V_0[\cdot]$ is equivalent to choosing a valuation principle.

Define the probability density function (pdf) as $f(x)$ and the cumulative distribution function (cdf) as $F(x)$. Define the de-cumulative function $S(x) = 1 - F(x)$. The risk premium is the expectation of the loss random variable X given by,

$$\mathbb{E}[X] = \int_0^\infty x f(x) dx = \int_0^\infty [1 - F(x)] dx = \int_0^\infty S(x) dx. \tag{8}$$

Define $f^*(x), F^*(x), S^*(x), \mathbb{E}^*(x)$ as the risk-neutral pdf, cdf, decumulative function, and expectation of the risk respectively.

This paper considers eight premium principles; the first five are risk-neutral, and the last three are real-world premium principles.

2.2.1. Risk-Neutral Probability Measures

Risk-neutral valuation principles take the expected present value of the cash flows under a distorted version of the real-world prices. The distortion of the real-world valuation into the risk-neutral valuation is parameterized by the pricing parameter λ.

The Wang transform embeds a Gaussian distortion function that returns a distorted cdf (Wang 2002),

$$F^*(x) = \Phi\Big(\Phi^{-1}(F(x)) - \lambda\Big), \quad \lambda \geq 0. \tag{9}$$

Here, $\Phi(\cdot)$ represents the cdf of a standard Gaussian distribution, and $\Phi^{-1}(\cdot)$ is the inverse standard Gaussian cdf. For a given risk X with cdf $F(X)$, the Wang transform produced a risk-adjusted cdf $F^*(X)$. The mean value under $F^*(X)$, denoted as $\mathbb{E}^*[X]$, is the risk-adjusted fair value of X at time T, which will be further discounted to time zero using the risk-free interest rate. One advantage of the Wang transform is that it is reasonably quick to evaluate numerically. (Lin and Cox 2005) used the Wang transform to price mortality bonds by distorting the distribution of mortality rates.

The proportional hazard transform has the advantage of having a simple distortion function of the following form (Wang 1995):

$$F^*(x) = 1 - (1 - F(x))^{1/\lambda}, \quad \lambda \geq 1. \tag{10}$$

The proportional hazard transform is quite sensitive to the choice of λ. (Wang 1995) used the proportional hazard transform to price insurance risk.

The dual-power transform (Wang 1996) is given by

$$F^*(x) = F(x)^{\lambda}, \quad \lambda \geq 1. \tag{11}$$

The distortion function of the dual power transform is similar to the proportional hazard transform, the difference being the distortion of the de-cumulative distribution function instead of the cumulative distribution function.

The Gini principle (Denneberg 1990) has a risk-adjusted de-cumulative function given by,

$$F^*(x) = 1 - \Big((1+\lambda)(1 - F(x)) - \lambda(1 - F(x))^2\Big), \quad 0 \leq \lambda \leq 1. \tag{12}$$

Lastly, the exponential transform uses weighted probabilities to map the liability denoted by the random variable X from $[0,1]$ onto $[0,1]$. The risk-adjusted de-cumulative function is given by

$$F^*(x) = 1 - \frac{1 - e^{-\lambda(1-F(x))}}{1 - e^{-\lambda}}, \quad \lambda > 0. \tag{13}$$

2.2.2. Real-World Probability Measures

The immaturity of the longevity risk transfer market results in low liquidity for survivor contracts, making it difficult to apply risk-neutral premium principles. Real world premium principles that use historical mortality rates offer an alternative methodology to price survivor contracts.

The price under the standard deviation principle is given by

$$V_0[X] = \mathbb{E}[X] + \lambda \times \mathrm{SD}[X], \quad \lambda > 0. \tag{14}$$

A pure premium is defined as $V_0[X] = \mathbb{E}[X]$. Hence, the standard deviation principle is equal to the pure premium plus a risk-loading term proportional to the standard deviation of the liability.

The price under the variance principle is given by

$$V_0[X] = \mathbb{E}[X] + \lambda \times \mathrm{VAR}[X], \quad \lambda > 0. \tag{15}$$

Like the standard deviation principle, the variance principle is a pure premium plus a risk-loading term proportional to the liability variance.

Finally, the price under the median absolute deviation principle is given by

$$V_0[X] = S^{-1}(0.5) + \lambda \times MAD[X], \quad \lambda > 0. \tag{16}$$

Here, $\mathrm{MAD}[X] = \mathrm{MAD}(|X - S^{-1}(0.5)|)$. Since mean-variance statistics tend to be sensitive to outliers, we include a premium principle that uses the median. The MAD principle is better suited to datasets with small sample sizes and potential outliers.

3. Longevity-Linked Instruments

We assume a pension/annuity fund enters a long position in a survivor contract to hedge longevity risk. In a long position, the fund pays a fixed amount K for floating cash flows linked to a future survival rate S. If survivors exceed expectations, the contract payouts hedge the fund's larger liabilities. If fewer survivors occur, the negative payouts are offset by reduced liabilities. As previously mentioned, this paper considers two survivor contracts, the S-forward and S-swap.

3.1. Survivor Forward

A survivor forward (S-forward) is an agreement between two counterparties to exchange a payment linked to the number of survivors in a reference population at a predetermined future date T. The buyer of an S-forward pays a fixed forward rate K to the seller and receives a floating rate $S(T)$. The forward rate is specified at the contract's start and reflects the expected future longevity level. Assume a notional amount equal to one. The fixed leg K must be determined such that the fair value of the survivor forward at $t = 0$ is zero. Mathematically, this is given by

$$V_0[S(T) - K] = 0. \tag{17}$$

Here, the $V_0[\cdot]$ is a value function, which refers to a valuation principle, and the fixed forward rate is determined such that the S-forward has zero value at the start of the contract.

We follow Boyer and Stentoft (2013) and assume that the fixed leg is the known anticipated one-year survival probability scaled to some unknown constant linear risk-adjustment term π. The concrete formulation for the survival forward is given by

$$V_0[s_{x,T}^{\text{realized}} - (1+\pi)s_{x,T}^{\text{anticipated}}] = 0. \tag{18}$$

Based on the above formulation, the risk-adjustment term under risk-neutral measure is given by

$$\pi = \frac{s_{x,T}^{\text{realized}}}{s_{x,T}^{\text{anticipated}}} - 1. \tag{19}$$

Here, $s_{x,t}^{\text{realized}}$ is the average of simulated one-year survival probabilities that an individual aged $x + T$ is alive at time T under the premium principle considered. The denominator, $s_{x,t}^{\text{anticipated}}$ is the distorted survival probabilities obtained by setting the pricing parameter λ equal to zero.

3.2. Survivor Swap

A survivor swap (S-swap) involves two counterparties exchanging a stream of future cash flow linked to the difference between the floating and fixed rates periodically (i.e., for every $t = 1, 2, \ldots, T$). We assume that the forward rate K is constant for all periods. A S-swap consists of a series of S-forwards with different maturities and can be interpreted as a portfolio of S-forwards. Assume a notional principal is equal to one. Analogous to the S-forward, the fixed leg of a S-swap K is determined such that the S-swap has zero value at the onset of the contract,

$$V_0\left[\sum_{t=1}^{T} S(t) - K\right] = 0. \tag{20}$$

Similar to the S-forward, we assume that the fixed leg for a S-swap is the sum of known anticipated one-year survival probability scaled to some unknown constant linear risk-adjustment term π. The concrete formulation for the survival forward is given by

$$V_0\left[\sum_{t=1}^{T} s_{x,t}^{\text{realized}} - (1+\pi)\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}\right] = 0. \tag{21}$$

Based on the above formulation, the risk-adjustment term under risk-neutral measure is given by

$$\pi = \frac{\sum_{t=1}^{T} s_{x,t}^{\text{realized}}}{\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}} - 1. \tag{22}$$

4. Valuation of Survivor Contracts

This section discusses model calibration to obtain the pricing parameter λ and presents an analysis of the results obtained from pricing survivor contracts.

4.1. Model Calibration

Since mortality-linked derivatives are not publicly traded, there is scant information regarding transaction details. This paper overcomes the model calibration problem by linking the S-forwards and S-swaps to annuity rates.

As of the first quarter of 2011, the annuity rate is level payments of £6,000 per £100,000 funds for a single life aged 66 with level payments. The risk-free rate is assumed to be the 15-year Gilt rate quoted at 2.04% for the first quarter of 2011.

There are no closed-form solutions to the pricing parameter λ for risk-neutral premium measures. Hence, we resort to numerical root-finding algorithms. We apply a Newton-Raphson type of algorithm to obtain the values for the pricing parameter λ. We refer the interested reader to Appendix A for details regarding evaluating the pricing parameter λ

The results obtained for the pricing parameter λ are given in Table 1. The pricing parameters for models of the LC type are larger than those of the RH type. Based on the table, it is implied that the survival rates implied from the LC and RH models require a greater return for the pension fund to take on the risk associated with the survival rate of the reference population of the pension fund. Furthermore, a higher pricing parameter implies that the model assumptions are not aligning well with market expectations, hence the need for a greater correction term. A higher value for the pricing parameter implies that the model might not be as reliable as models with a smaller pricing parameter value, which could lead to greater model uncertainty.

Table 1. Values obtained for the pricing parameter λ

Premium	LC	RH	CBD	M6
Wang	4.373e-01	4.346e-01	3.993e-01	3.906e-01
Proportional	2.300e+00	2.290e+00	2.155e+00	2.125e+00
Dual	1.386e+00	1.383e+00	1.344e+00	1.334e+00
Gini	6.344e-01	6.317e-01	5.951e-01	5.858e-01
Exponential	1.602e+00	1.593e+00	1.479e+00	1.451e+00
Std. Dev.	9.804e-01	9.746e-01	8.971e-01	8.786e-01
Variance	1.586e-04	1.579e-04	1.487e-04	1.460e-04
MAD	7.516e-01	7.491e-01	7.418e-01	7.964e-01

4.2. Pricing Survivor Contracts

Consider a S-forward contract with maturity T. We begin by simulating future mortality scenarios using the fitted mortality model. Define the average simulated value at time t for an individual aged x as $\bar{p}_{x,t}$. We solve for the risk-adjustment term π such that the S-forward has zero value at inception. Mathematically, this is given by Equation 18. The variable of interest is π, which can be evaluated using Equation 19. Expressions for $s_{x,T}^{\text{realized}}$ and $s_{x,T}^{\text{anticipated}}$ can be found in Table 2.

Table 2. Expressions for $s_{x,T}^{\text{realized}}$ and $s_{x,T}^{\text{anticipated}}$ used in pricing S-forwards.

Premium Principle	$s_{x,T}^{\text{realized}}$	$s_{x,T}^{\text{anticipated}}$
Wang	$\text{DF}_T\{1-\Phi[\Phi^{-1}(1-\bar{p}_{x,T})-\lambda]\}$	$\text{DF}_T\{1-\Phi[\Phi^{-1}(1-\bar{p}_{x,T})]\}$
Proportional	$\text{DF}_T(\bar{p}_{x,T})^{\frac{1}{\lambda}}$	$\text{DF}_T\bar{p}_{x,T}$
Dual	$\text{DF}_T\left[1-(1-\bar{p}_{x,T})^{\lambda}\right]$	$\text{DF}_T[1-(1-\bar{p}_{x,T})]$
Gini	$\text{DF}_T\left[(1+\lambda)\bar{p}_{x,T}-\lambda(\bar{p}_{x,T})^2\right]$	$\text{DF}_T(1+\lambda)\bar{p}_{x,T}$
Exponential	$\text{DF}_T\left[\frac{1-e^{-\lambda\bar{p}_{x,t}}}{1-e^{-\lambda}}\right]$	$\text{DF}_T\left[\frac{1-e^{-\bar{p}_{x,t}}}{1-e^{-1}}\right]$
Std. Dev.	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})+\text{STDEV}(\bar{p}_{x,T})]$	$\text{DF}_T\mathbb{E}(\bar{p}_{x,T})$
Variance	$\text{DF}_T[\mathbb{E}(\bar{p}_{x,T})+\text{VAR}(\bar{p}_{x,T})]$	$\text{DF}_T\mathbb{E}(\bar{p}_{x,T})$
MAD	$\text{DF}_T[\text{MEDIAN}(\bar{p}_{x,T})+\lambda\text{MAD}(\bar{p}_{x,T})]$	$\text{DF}_T\text{MEDIAN}(\bar{p}_{x,T})$

On the other hand, consider a S-swap with maturity T that exchanges cash flows annually. Similar to an S-forward, the risk-adjustment term π of the S-swap is determined such that the contract is fair at inception. Mathematically, this is given by Equation 21. The variable of interest is π, which can be evaluated using Equation 22. Expressions for $\sum_{t=1}^{T} s_{x,t}^{\text{realized}}$ and $\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}$ can be found in Table 3.

Table 3. Expressions for $s_{x,t}^{\text{realized}}$ and $\sum_{t=1}^{T} s_{x,t}^{\text{anticipated}}$ used in pricing S-swaps.

Premium Principle	$\sum_{t=1}^{T} s_{x,T}^{\text{realized}}$	$\sum_{t=1}^{T} s_{x,T}^{\text{anticipated}}$
Wang	$\sum_{t=1}^{T}\text{DF}_t\{1-\Phi[\Phi^{-1}(1-\bar{p}_{x,t})-\lambda]\}$	$\sum_{t=1}^{T}\text{DF}_t\{1-\Phi[\Phi^{-1}(1-\bar{p}_{x,t})]\}$
Proportional	$\sum_{t=1}^{T}\text{DF}_t(\bar{p}_{x,t})^{\frac{1}{\lambda}}$	$\sum_{t=1}^{T}\text{DF}_t\bar{p}_{x,t}$
Dual	$\text{DF}_T\left[1-(1-\bar{p}_{x,t})^{\lambda}\right]$	$\text{DF}_T[1-(1-\bar{p}_{x,t})]$
Gini	$\sum_{t=1}^{T}\text{DF}_t(1+\lambda)\bar{p}_{x,t}-\lambda(\bar{p}_{x,t})^2$	$\sum_{t=1}^{T}\text{DF}_t(1+\lambda)\bar{p}_{x,t}$
Exponential	$\sum_{t=1}^{T}\text{DF}_t\frac{1-e^{-\lambda\bar{p}_{x,t}}}{1-e^{-\lambda}}$	$\sum_{t=1}^{T}\text{DF}_t\frac{1-e^{-\bar{p}_{x,t}}}{1-e^{-1}}$
Std. Dev.	$\sum_{t=1}^{T}\text{DF}_t[\mathbb{E}(\bar{p}_{x,t})+\text{SD}(\bar{p}_{x,t})]$	$\sum_{t=1}^{T}\text{DF}_t\mathbb{E}(\bar{p}_{x,t})$
Variance	$\sum_{t=1}^{T}\text{DF}_t[\mathbb{E}(\bar{p}_{x,t})+\text{VAR}(\bar{p}_{x,t})]$	$\sum_{t=1}^{T}\text{DF}_t\mathbb{E}(\bar{p}_{x,t})$
MAD	$\sum_{t=1}^{T}\text{DF}_t[\text{MEDIAN}(\bar{p}_{x,t})+\lambda\text{MAD}(\bar{p}_{x,t})]$	$\sum_{t=1}^{T}\text{DF}_t\text{MEDIAN}(\bar{p}_{x,t})$

4.3. Analysis of Pricing Results

Figure 1 and Figure 2 present the values obtained for the risk-adjustment term π under different maturities for S-forwards and S-swaps respectively. Several interesting observations can be made from Figure 1 and Figure 2. Firstly, S-forwards generally have a higher risk-adjustment term than S-swaps of the same maturity. For example, under the LC mortality model, the 10-year S-forward risk risk-adjustment term is 0.52% higher than the 10-year S-swap. This general pattern holds across the various models tested. One reason the S-swaps have a smaller risk-adjustment term is that an S-swap involves an annual exchange of cash flows linked to the number of survivors from the reference population at each time period. In contrast, a S-forward involves only a single exchange of cash flows. Second, note that the values of the risk-adjustment term increase as the contract term length increases. This phenomenon is expected since there is more uncertainty in longevity levels when comparing more extended periods to shorter periods. Notice that the risk-adjustment term values for the RH model shows that the premium principles are more concentrated compared to the risk-adjustment term values for the other mortality models. If an insurance company aims to have similar risk-adjustment term values for different premium principles, then the RH model would be a good fit for this dataset.

The risk-adjustment term values generated under the real-world measure are generally lower than those generated from risk-neutral measures. The exponential transform generally produced the lowest risk-adjustment term among the risk-neutral measures. Moreover, A general pattern exists among the risk-adjustment term values. The dual-hazard transform has the largest risk-adjustment term. On the other hand, the variance principle produced the lowest risk-adjustment term. A generalization can be made that the risk-adjustment term generated follows a trend. The highest risk-adjustment term is generated by the dual hazard transform, followed by the Wang transform, the Gini transform, the proportional hazard transform, the standard deviation principle, then finally, the variance principle.

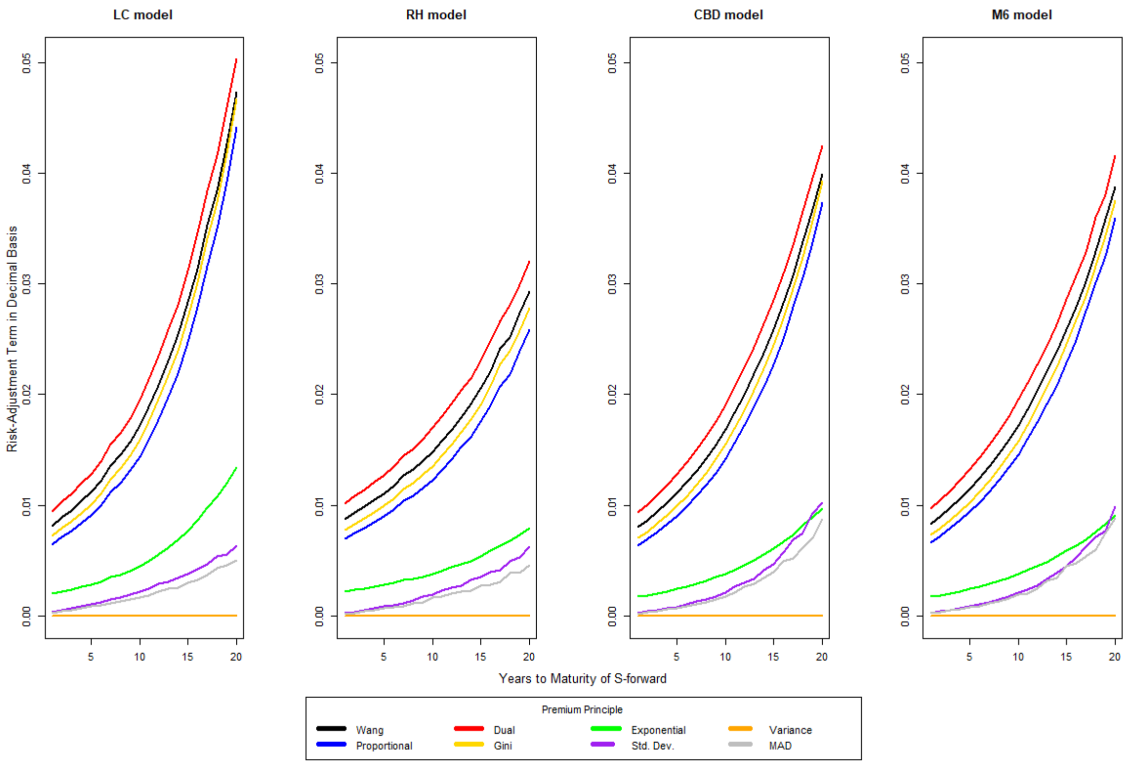

Figure 1. From left to right, the plot of values obtained for the risk-adjustment term π for S-forward under fixed mortality model over different maturities. (**a**) The plot under LC model. (**b**) The plot under the RH model. (**c**) The plot under the CBD model. (**d**) The plot under the M6 model.

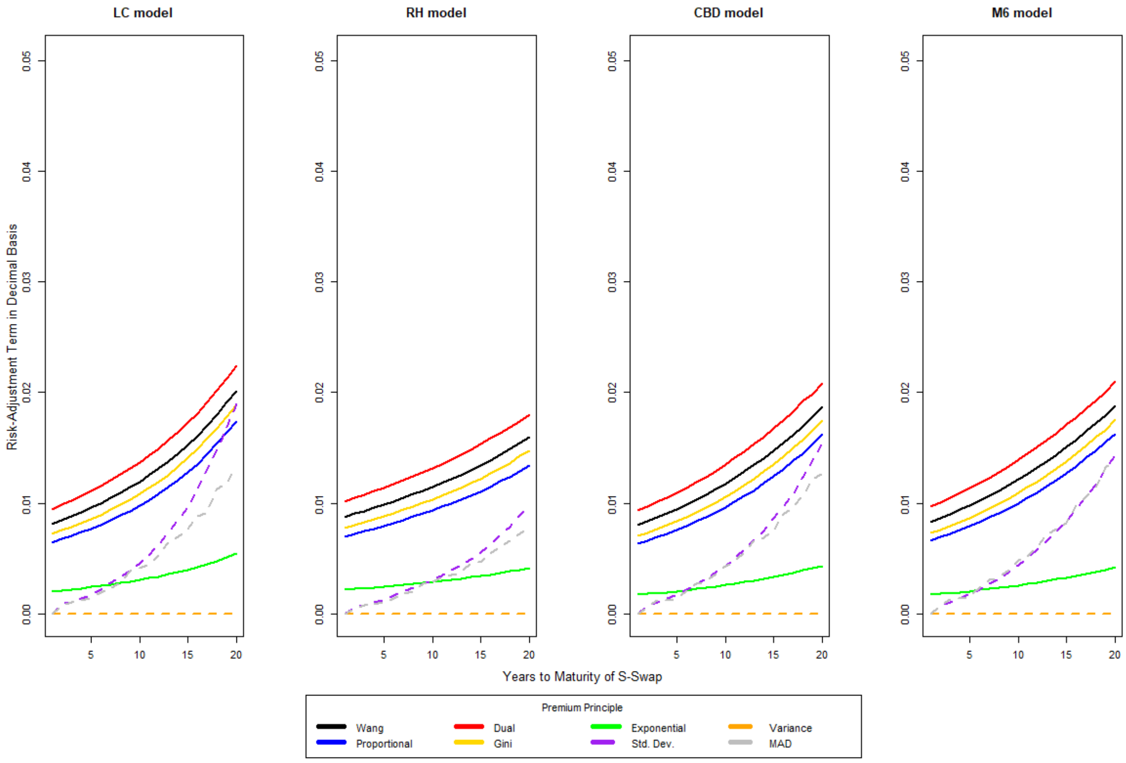

Figure 2. From left to right, the plot of values obtained for the risk-adjustment term π for S-swap under fixed mortality model over different maturities. (**a**) The plot under LC model. (**b**) The plot under the RH model. (**c**) The plot under the CBD model. (**d**) The plot under the M6 model.

On the other hand, Table 4 and Table 5 show that the LC model generates the highest variance and range of risk-adjustment term values across both instruments. For instance, the 10-year S-forward risk-adjustment term range is 3.92% under LC, compared to just 2.06% under RH. This statistic indicates that the LC approach introduces greater uncertainty into longevity projections. This translates into higher and more variable risk-adjustment term values. For a 10-year contract, the S-forward risk-adjustment term is 0.52% larger than the S-swap risk-adjustment term. This gap is the largest across the mortality models, highlighting the LC model's propensity to amplify single-payment instrument risk. Across the premium principles, the variance principle still produces the lowest and least dispersed risk-adjustment term values. However, the dual hazard transform generates the highest risk-adjustment term values with the widest spread.

Moreover, the RH model generated the lowest mean, range, and variance among the risk-neutral risk-adjustment term values. The RH model having the smallest mean implies that the RH model generates the smallest risk-adjustment term values among the four mortality models. Furthermore, having the smallest range and variance suggests that the RH model has the least uncertainty in providing a contract risk-adjustment term. The variance in risk premiums across different pricing principles is smallest under RH model.

Table 4. Values for the risk-adjustment parameter π for the LC and RH model for a survivor forward

	LC			RH		
	Range	Mean	Variance	Range	Mean	Variance
Wang	3.923e-02	2.165e-02	1.433e-04	2.056e-02	1.671e-02	3.953e-05
Proportional	3.764e-02	1.886e-02	1.299e-04	1.892e-02	1.410e-02	3.293e-05
Dual	4.090e-02	2.397e-02	1.580e-04	2.190e-02	1.884e-02	4.525e-05
Gini	3.955e-02	2.049e-02	1.444e-04	2.006e-02	1.544e-02	3.773e-05
Exponential	1.137e-02	5.833e-03	1.202e-05	5.756e-03	4.344e-03	3.059e-06
Std. Dev.	5.992e-03	2.687e-03	3.285e-06	6.046e-03	2.440e-03	3.204e-06
Variance	6.026e-09	1.634e-09	3.365e-18	5.697e-09	1.438e-09	2.851e-18
MAD	4.737e-03	2.109e-03	2.097e-06	4.442e-03	1.848e-03	1.743e-06

Table 5. Values for the risk-adjustment parameter π for the LC and RH model for a survivor swap

	LC			RH		
	Range	Mean	Variance	Range	Mean	Variance
Wang	1.205e-02	1.293e-02	1.360e-05	7.203e-03	1.190e-02	4.908e-06
Proportional	1.092e-02	1.073e-02	1.122e-05	6.453e-03	9.755e-03	3.860e-06
Dual	1.299e-02	1.471e-02	1.585e-05	7.893e-03	1.363e-02	5.878e-06
Gini	1.174e-02	1.181e-02	1.291e-05	6.996e-03	1.078e-02	4.599e-06
Exponential	3.400e-03	3.348e-03	1.074e-06	1.972e-03	3.018e-03	3.641e-07
Std. Dev.	1.903e-02	6.698e-03	3.252e-05	9.747e-03	3.891e-03	8.324e-06
Variance	5.860e-08	1.225e-08	2.882e-16	1.527e-08	3.767e-09	2.085e-17
MAD	1.320e-02	5.233e-03	1.625e-05	7.960e-03	3.372e-03	5.651e-06

Finally, Table 6 and Table 7 show that the CBD model generates higher variance and range of S-forward premiums compared to the M6 model. However, for S-swaps, the premium variance is more similar between CBD and M6. When comparing the differences between S-forward and S-swap premiums, the CBD and M6 models behave more similarly than LC or RH. For 10-year contracts, the S-forward premium exceeds the S-swap by around 0.51% for both CBD and M6. Under the CBD model, the standard deviation premium principle produces the highest variance premiums for S-forwards, contrasting with other models where the dual hazard is highest.

Meanwhile, the M6 model sees the lowest mean premiums under the mean absolute deviation principle. Generally, the CBD and M6 models exhibit a middle-ground behavior between the extremes of LC and RH for variance, range, and internal consistency of longevity risk pricing. They strike a balance in the magnitude and dispersion of premiums generated.

Table 6. Values for the risk-adjustment parameter π for the CBD and M6 model for a survivor forward

	CBD			M6		
	Range	Mean	Variance	Range	Mean	Variance
Wang	3.189e-02	1.991e-02	9.686e-05	3.043e-02	1.995e-02	8.764e-05
Proportional	3.097e-02	1.737e-02	8.902e-05	2.926e-02	1.743e-02	8.032e-05
Dual	3.312e-02	2.209e-02	1.056e-04	3.189e-02	2.215e-02	9.586e-05
Gini	3.230e-02	1.868e-02	9.733e-05	3.023e-02	1.867e-02	8.664e-05
Exponential	8.020e-03	4.635e-03	6.128e-06	7.370e-03	4.454e-03	5.069e-06
Std. Dev	9.967e-03	3.381e-03	9.359e-06	9.579e-03	3.172e-03	7.736e-06
Variance	1.938e-08	3.544e-09	2.812e-17	1.799e-08	3.333e-09	2.482e-17
MAD	8.572e-03	2.786e-03	6.182e-06	8.591e-03	2.862e-03	6.341e-06

Table 7. Values for the risk-adjustment parameter π for the CBD and M6 model for a survivor swap

	CBD			M6		
	Range	Mean	Variance	Range	Mean	Variance
Wang	1.069e-02	1.250e-02	1.092e-05	1.048e-02	1.281e-02	1.058e-05
Proportional	9.850e-03	1.039e-02	9.240e-06	9.614e-03	1.068e-02	8.925e-06
Dual	1.150e-02	1.425e-02	1.279e-05	1.129e-02	1.459e-02	1.222e-05
Gini	1.043e-02	1.133e-02	1.050e-05	1.029e-02	1.160e-02	1.004e-05
Exponential	2.592e-03	2.794e-03	6.540e-07	2.469e-03	2.755e-03	5.815e-07
Std. Dev.	1.537e-02	5.833e-03	2.141e-05	1.441e-02	5.685e-03	1.896e-05
Variance	4.174e-08	9.742e-09	1.529e-16	3.847e-08	9.286e-09	1.302e-16
MAD	1.273e-02	5.282e-03	1.570e-05	1.388e-02	5.796e-03	1.842e-05

Regarding the risk-adjustment term statistics for the S-forward (Tables 4 and 6), the results suggest that the LC model produces the highest risk-adjustment term values, followed by the M6, the CBD, and the RH model. Furthermore, the risk-adjustment term values obtained for the real-world risk-adjustment term values are less dispersed than those obtained from the risk-neutral risk-adjustment term values.

On the other hand, the risk-adjustment statistics for the S-swap (Tables 5 and 7) suggest that the LC model generates the highest risk-adjustment term values, followed by the M6 model, CBD model, and the RH model. Among the real-world risk-adjustment term values for the S-swap, the dispersion between the CBD and M6 models is less pronounced. However, the difference between the risk-adjustment term values generated by the LC and RH models is more pronounced. This suggests that for a S-swap, adding the cohort effect term to models of the CBD type does not seem to affect the risk-adjustment term.

4.3.1. Comparison with Previous Works

We compare our results to that of Tang and Li (2021). In their paper, the authors found that S-forwards have higher risk premiums than S-swaps of the same maturity. We obtained similar results, which can be seen in Figures 1 and 2. One explanation for this is that S-swaps involve multiple cash flow exchanges over its horizon compared to the single payment structure of an S-forward. Second, the authors found that within the risk-neutral premium principles, the risk premiums obtained are very similar across the nine premiums. In contrast, real-world premiums produce somewhat higher risk premiums than risk-neutral ones. Our results show that four of the five risk-neutral premiums are close to one another, except the exponential transform with a lower risk-adjustment term value than other risk-neutral premiums. Furthermore, our results generally show that real-world premiums generate a lower risk adjustment term compared to risk-neutral premiums. Lastly, the authors found that the choice of mortality model has a bigger impact on risk premiums than the choice of premium principle. This results of show that the RH model tends to produce higher premiums than the M6 model with quadratic terms. Our results indicate that the LC model produced the largest values for the risk-adjustment term, while the RH model generated the smallest values. We obtained similar observations that the magnitude of the risk-adjustment term values generated from one mortality model versus another is substantially different compared to fixing a mortality model and varying the premium principle. This suggests that the choice of mortality model has a greater impact on the implied risk-adjustment term when compared to the premium principle.

5. Risk Measure Framework

This section presents the VaR and ES of S-forwards and S-swaps evaluated by eight premium principles and four mortality models.

The VaR with confidence level $\alpha \in (0,1)$ is the smallest number x such that the probability that the loss X exceeds x is no larger than $(1-\alpha)$. Formally, this is given by

$$\mathrm{VaR}_\alpha(X) = \inf\{x \in \mathbb{R} : \mathbb{P}(X > x) \leq 1 - \alpha\}. \tag{23}$$

The ES (also called conditional VaR) is the conditional expectation of loss given that the loss is beyond the VaR level and is given by

$$\text{ES}_\alpha(X) = \mathbb{E}[X|X \geq \text{VaR}_\alpha(X)] \tag{24}$$

The ES indicates the average loss when the loss exceeds the VaR level.

We introduce a Monte Carlo approach to calculating the risk measures. Suppose a hedger is interested in the maximum amount expected to be lost for some survivor contract after some time n at a pre-defined confidence level α. Here, T is the contract time to maturity, and n is the years already accrued by the hedger. A framework for evaluating a n-year VaR and ES is as follows:

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1. First, select a dataset covering ages x_L to x_U and running from years y_L to y_U.
2. Next, select a mortality model and fit it to the dataset. This gives fitted values for $\ln(m_{x,t})$ or $\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right)$, where x is the age in years and t is the calendar year. Here, $m_{x,t}$ is the central force of mortality and $q_{x,t}$ is the initial force of mortality
3. Use the mortality model in Step 2 to simulate sample paths and evaluate the risk-adjustment term π_m for a survivor contract with tenor m using the method outlined in Sections 3.1 and 3.2.
4. For the same mortality model parameters in Step 2, generate scenarios for the remaining liability of the insurer after n years using the same π_m as in Step 3 resulting in a distribution for possible remaining liability values.
5. Using the distribution obtained in Step 4, calculate the desired risk measure.

We simulate 5,000 future mortality scenarios. The premium principles are calibrated using market annuity quotations with a starting age of 66. Each calibrated pricing principle is then applied to simulate forward survivor rates and evaluate the risk-adjustment term π using Equations (19) and (22). For the same contract, fix π and evaluate the remaining liability.

For example, consider a 10-year S-forward indexed on an individual aged 66 at inception that exchanges a single cash flow at maturity. The distribution of the liability after one year given by

$$V[s_{75,9}^{\text{realized}} - (1 + \pi_{\text{10Y, S-Forward}})s_{75,9}^{\text{anticipated}}]. \tag{25}$$

Here, $\pi_{\text{10Y, S-forward}}$ is the risk-adjustment term obtained previously for a ten-year S-forward under a specific premium principle.

Consider a 10-year S-swap that exchanges cash flows yearly indexed on the cohort of people aged 66. the distribution of the remaining liability after one year of the contract has passed can be obtained by simulating possible survivor rates after one year. The distribution of S-swap risk-adjustment terms after a year is the sum of simulated survivor rates of the cohort of people aged 67 up to the cohort of people aged 75. Mathematically,

$$V\left[\sum_{t=2}^{10} s_{66,t}^{\text{realized}} - (1 + \pi_{\text{10Y, S-Swap}})\sum_{t=2}^{10} s_{66,t}^{\text{anticipated}}\right]. \tag{26}$$

Here, $s_{66,t}$ refers to the survival probability of an individual aged $66 + t$ at time t

5.1. Risk Measures for S-forwards and S-swaps

Using the framework previously discussed, the values for the risk measures of the S-forwards can be seen in Figure 3. The solid bars represent the VaR, while the transparent bars represent the ES. Specific values obtained can be found in Appendix B. Under the Gini principle, the M6 model produced the highest 1-year 99.5% VaR value overall. Meanwhile, the CBD model under the Wang principle generated the smallest VaR value. Since the Gini principle under the M6 model has the largest VaR value, this model configuration is the most conservative in estimating potential losses. In contrast, the CBD model under the Wang principle is the least conservative value for possible losses.

The Gini principle under the CBD model produced the highest 1-year 99.5% ES value overall. Meanwhile, the Wang principle under the CBD model generated the smallest ES value. Since the Gini principle under CBD had the largest ES value, this model configuration is the most conservative in estimating potential losses for the 1-year ES of a 10-year S-forward. In contrast, the Wang principle under CBD had the least conservative ES value.

Notice that the standard deviation principle has the highest VaR and ES values after the Gini principle. Furthermore, the VaR and ES have similar values to one another. This means that the expected losses are minimal, given that the VaR is exceeded under the standard deviation principle. On the other hand, the Gini and MAD principles have large ES values compared to their corresponding VaR. Given this, the Gini and MAD principles capture the uncertainty associated with unfavorable outcomes in longevity risk.

The RH mortality model produced the lowest 1-year VaR and ES for a 10-year S-forward among the Gini, standard deviation, variance, and MAD principles. This means that the RH model consistently generated the smallest risk measure value among the four mortality models and could undervalue the potential losses.

Interestingly, risk-neutral premium principles generally have lower VaR and ES values than real-world premium principles, with the Gini principle being an exception. This means that insurers must be careful when using risk-neutral premium principles to price survivor contracts since these capture less potential risk compared to real-world premiums.

The spread between 1-year 99.5% VaR values obtained is the largest under the CBD model, while the smallest spread obtained is the smallest under the M6 model for the 10-year S-forward. On the other hand, the spread between 1-year 99.5% ES values obtained is the largest under the M6 model, while the smallest is also under the RH model.

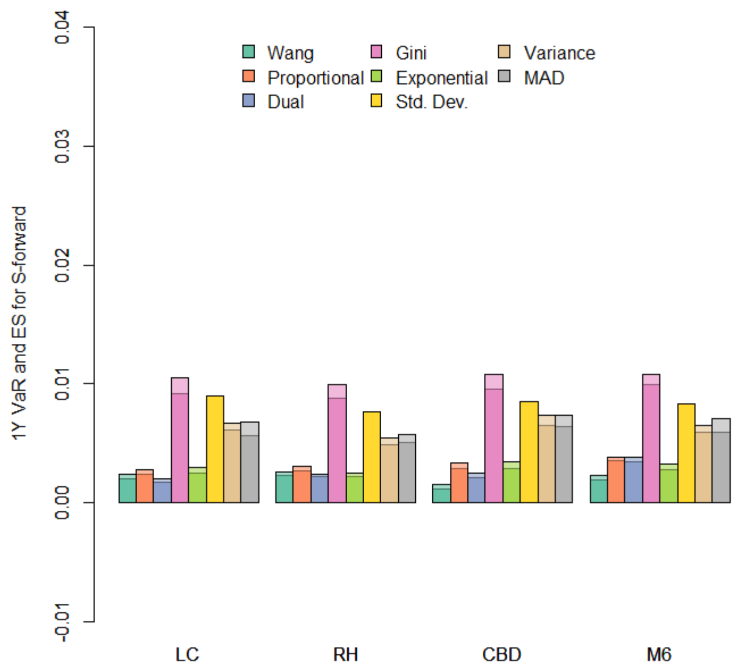

Figure 3. 1-year 99.5% VaR and ES for S-forwards. The solid color represents the VaR, while the transparent color represents the ES.

For the survivor swap, values for the risk measures can be seen in Figure 4. The M6 model under the MAD principle produced the highest 1-year 99.5% VaR value overall. Meanwhile, the LC model under the variance principle generated the smallest VaR value. Since the MAD principle under the M6 model has the largest VaR value, this model configuration is the most conservative in estimating potential losses. In contrast, the CBD model under the variance principle is the least conservative value for potential losses. Furthermore, the CBD model produced the smallest VaR and ES under all risk-neutral principles.

Notice that The MAD principle consistently produced the largest 1-year 99.5% VaR and ES values for the 10-year S-swap. On the other hand, the variance principle generated the lowest VaR and ES values.

If we compare the spread between 1-year 99.5% VaR values, the M6 model has the greatest spread, while the RH model has the smallest spread. Likewise, the M6 model has the greatest spread between the 1-year 99.5% ES values obtained, and the RH model has the smallest spread.

While the real-world premium principles produced ES values similar to the VaR, the risk-neutral premium principles differ between the VaR and ES. Notably, the ES values for S-swaps are much larger compared to their corresponding VaR when compared with S-forwards. Similar to the risk measures for the S-forward, the risk-neutral premium principles have smaller VaR and ES compared to the real-world premium principles.

Notice that the VaR and ES of the real-world premium principles have similar values for the S-swap. This means that if a loss is larger than the VaR threshold, those losses tend to be close in magnitude to the VaR estimate.

Comparing Figure 3 and Figure 4, the VaR and ES values are higher for S-swaps versus S-forwards overall. This indicates that S-swaps have a greater potential for losses. The exception is the variance principle, which has a lower VaR for S-swaps. The variance principle is omitted from the graph since it generated a negligible value for VaR and ES.

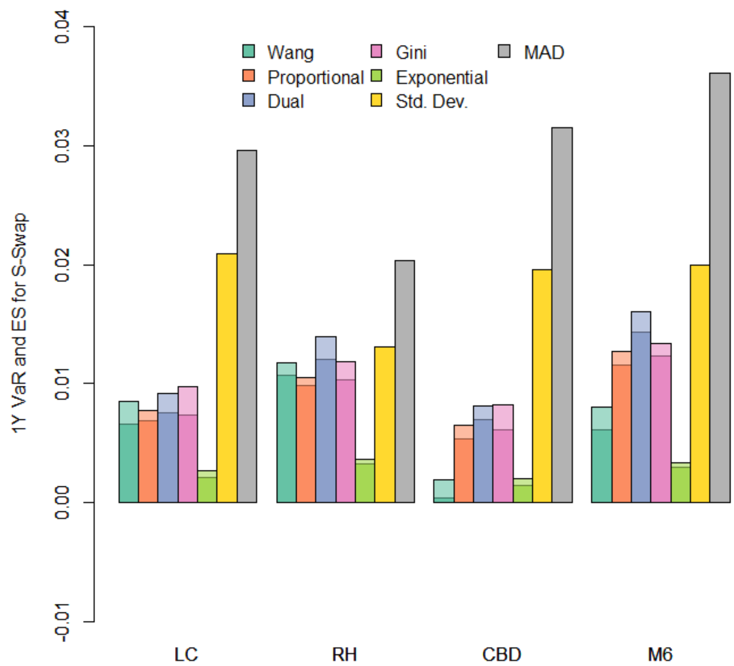

Figure 4. 1-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

When assessing risk, it is common to calculate risk measures for different time horizons, such as 1-year and 2-year horizons. It is of interest to institutions to compare the relative distances between the values of risk measures for different mortality models and premium principles. We discuss the risk measure values obtained for 2-year risk measures and compare the range of values with 1-year risk measures to characterize the uncertainty under specific model assumptions.

The 2-year risk measure values and their figures can be seen in Appendix A1 and Appendix A2.

Figure 5 presents the difference between 1- and 2-year 99.5% VaR, while Figure 6 presents the difference between 1- and 2-year 99.5% ES for the S-forward.

Notice that the RH model has the largest spread among the differences between the S-forward's 1-year and 2-year risk measures. This means that the risk measures under the RH model had great variations for the S-forward. The largest VaR and ES values can be seen under the risk-neutral premium principles, with the exponential transform being an exception. After the RH model, the second largest spread between 1- and 2-year VaR is under the M6 mortality model, followed by the CBD model, and last is the LC model. This means the LC model is the most consistent in providing similar VaR and ES values for the S-forward.

The largest difference can be found in the RH model and dual transform. Notice that while the dual transform generated the largest difference under the RH and M6 models, it does not generate the largest difference under the LC and CBD models. The largest difference under the LC model is under the proportional transform, while the largest difference for the CBD model is under the Wang transform.

Notably, while most 1-year risk VaRs are larger than their 2-year counterparts, some 2-year VaRs are larger than their 1-year counterpart. This is the case for the MAD principle among the mortality models besides the CBD model.

Since the LC model has the smallest spread between 1- and 2-year VaR, the risks under the LC model are consistent over the two time horizons. This implies that the LC model will generate similar VaR values regardless of the premium principle. Similar to the spread between the differences of 1- and 2-year VaR, the spread between the differences in ES is the largest for the RH model, followed by the M6 model, CBD model, and LC model.

Under the Gini principle, the RH model produced the greatest difference between 1- and 2-year ES values. On the other hand, the Gini principle under the LC model had the smallest difference.

The largest difference under the LC model can be found under the proportional transform, while the smallest difference is under the Gini transform. For the RH model, the largest difference is under the proportional transform, while the smallest difference is under the exponential transform. The CBD model's largest difference is under the MAD principle, while the smallest is under the Gini transform. The M6 model's largest differences are under the dual transform, while the smallest difference is under the MAD principle.

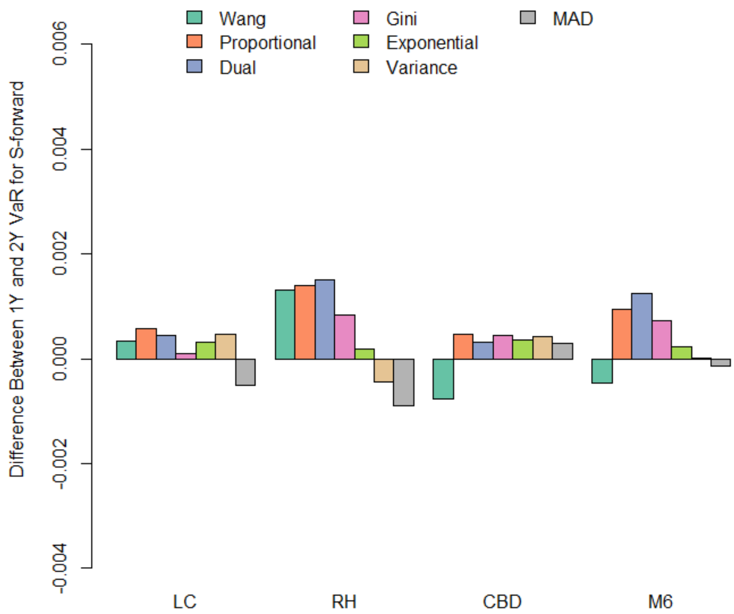

Figure 5. The difference between the 1-year and 2-year VaR for the S-forward.

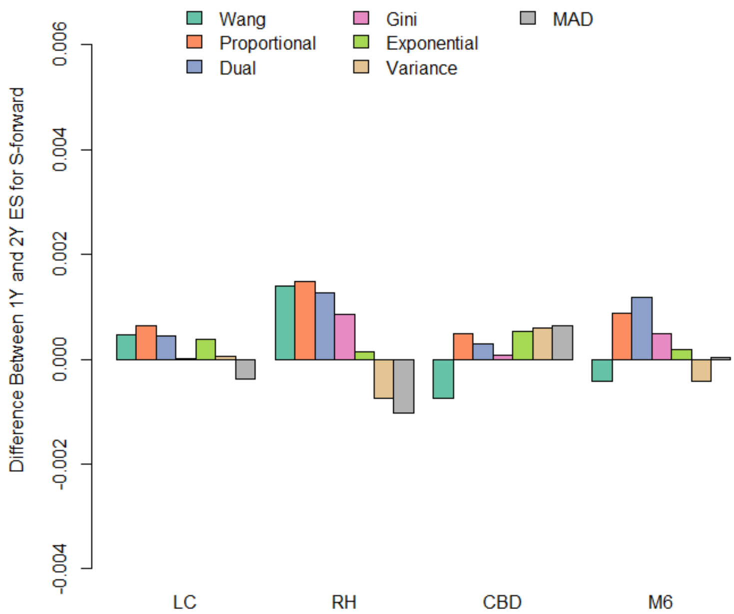

Figure 6. The difference between the 1-year and 2-year ES for the S-forward.

The difference between the 1- and 2-year VaR for the S-swap can be seen in Figure 7, while the difference for the ES can be seen in Figure 8. 717 718

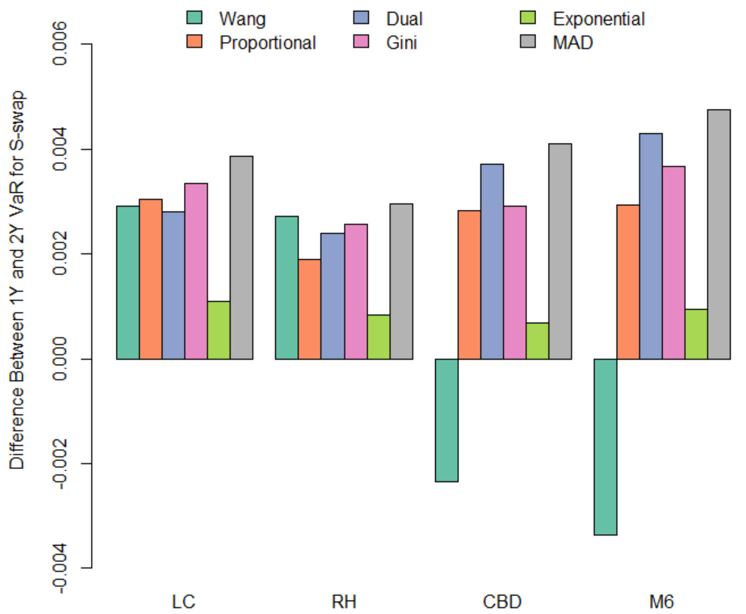

Figure 7. The difference between the 1-year and 2-year VaR for the S-swap.

Notice that the differences between 1- and 2-year VaR are larger for the S-swap compared to the S-forward. This is because of the multiple cash flow exchanges associated with an S-swap compared to an S-forward's single cash flow exchange.

The Wang transform is the only premium principle considered where the 2-year VaR and ES are greater than the 1-year VaR and ES under the CBD and M6 model. Generally, the 1-year risk measures are larger than their 2-year counterparts.

The exponential transform generated the smallest difference between 1- and 2-year risk measures for all mortality models among the VaR and ES. The MAD principle generated the largest difference between the 1- and 2-year VaR. Notice that the dual transform has a larger difference under the CBD and M6 models.

The standard deviation and variance principles had negligible differences between the 1- and 2-year risk measures; hence, the two premium principles were omitted from the figures. As for the differences between the 1- and 2-year ES for the S-swap, the largest difference can be seen under the Gini transform for the LC, the dual transform for the RH, and the MAD principle under the M6 model. There is greater variation in the ES values compared to the VaR values. For example, the difference under the Gini principle is noticeably larger under the LC model than the ES values obtained under other mortality models. A similar spike occurs under the dual transform for the M6 model.

The largest spread between the differences of the 1- and 2-year ES for the S-swap occured under the M6 model, while the smallest spread was under the RH model.

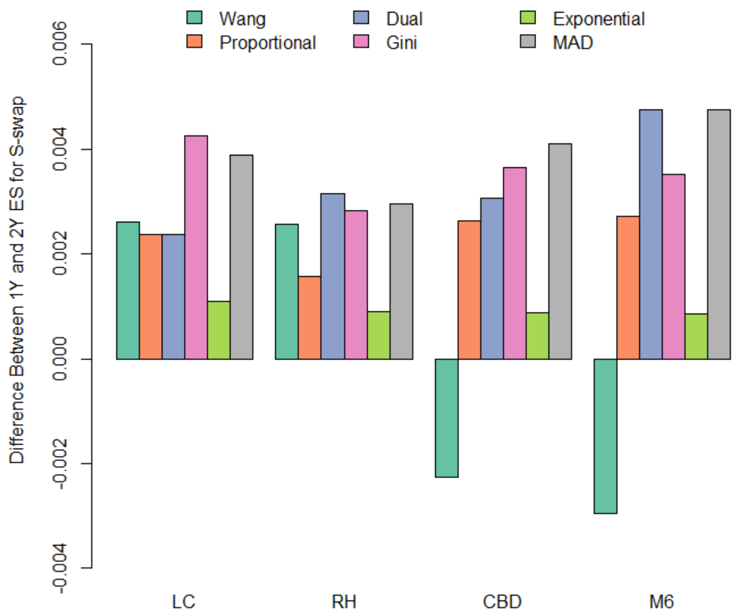

Figure 8. The difference between the 1-year and 2-year ES for the S-swap.

6. Conclusion

This paper investigated the impact of the choice of mortality model and premium principle on pricing, risk measure, and modeling of S-forward and S-swaps. We developed a framework to evaluate the VaR and ES for S-forward and S-swaps through Monte Carlo simulation. We also provided insight into how modeling choices impact potential loss estimates over various horizons. Finally, we analyzed the sensitivity of risk measures to different maturity lengths and premium principles to highlight the implications for insurers hedging longevity risk.

To be specific, we analyzed the impact of four mortality models (LC, RH, CBD, and M6), along with eight premium principles (Wang, proportional hazard, dual power, Gini, exponential transforms, standard deviation, variance, and median absolute deviation principles).

We adopted a flexible simulation-based framework to overcome restrictive assumptions and enable the future extension to price other longevity-linked securities. The analysis included a discussion of 1-year and 2-year VaR and ES for 10-year S-forwards and S-swaps under four mortality models and eight premium principles.

The analysis for pricing showed that S-forwards generally have a higher risk-adjustment term than S-swaps of the same maturity across models. Risk-adjustment values increase with longer contract terms due to greater longevity uncertainty. The RH model produces more concentrated values across principles. Real-world measures generate lower values than risk-neutral measures, with exponential transform being the lowest overall. A trend exists from dual-hazard transform having the highest values to variance principle as the lowest. The LC model produces the greatest variance and range of values, introducing greater uncertainty into projections and amplifying risk, especially for single-payment instruments. In contrast, the RH model generates the lowest mean, range, and variance of risk-adjustment values across principles, implying less uncertainty in estimating contract risk premiums.

The CBD model produced a higher variance and range of risk-adjustment term values for S-forwards than the M6, but the models behave more similarly for S-swaps. The CBD and M6 models show closer alignment in premium differences between S-forwards and S-swaps than the LC or RH. Overall, CBD and M6 exhibit a middle ground between LC and RH in the magnitude and dispersion of risk premiums generated. Regarding models, LC gives the highest risk premium values, followed by M6, CBD, and RH for both contracts. We notice that real-world premium principle values are less dispersed than risk-neutral ones. The choice of mortality model has a bigger impact than the premium principle regarding the value for the risk-adjustment term.

The analysis of risk measures for S-forwards revealed variations across different models and principles. Notably, the M6 model under the Gini principle exhibited the highest 1-year 99.5% VaR, indicating conservative estimates of potential losses. In contrast, the CBD model under the Wang principle showed the lowest VaR, suggesting less conservative estimates. Similarly, the Gini principle under the CBD model produced the highest 1-year 99.5% ES, reflecting a conservative approach, whereas the Wang principle under CBD resulted in the lowest ES. The RH mortality model consistently generated the lowest risk measure values among the tested mortality models, potentially undervaluing risk. Risk-neutral premium principles generally led to lower VaR and ES values than real-world premiums, except for the Gini principle. Furthermore, the spread between VaR values was widest under the CBD model and narrowest under the RH model. Similarly, the spread between ES values was widest under the CBD model and narrowest under the RH model for a 10-year S-forward.

As for the S-swap, the M6 model under the MAD principle exhibited the highest 1-year 99.5% VaR, indicating a conservative estimation of potential losses. In contrast, the LC model under the variance principle showed the lowest VaR, suggesting less conservative estimates. The MAD principle consistently produced the largest VaR and ES values for a 10-year S-swap, whereas the variance principle generated the lowest values. Real-world premium principles yielded ES values similar to their VaR counterparts, indicating close magnitudes of possible losses exceeding the VaR threshold.

When evaluating risk across various time horizons, institutions commonly computed risk measures for both 1-year and 2-year periods to assess relative differences. Comparisons of these measures aided in characterizing uncertainty within specific model frameworks. The RH model exhibited the widest disparity between 1-year and 2-year risk measures for S-forwards, indicating variability in potential losses. Risk-neutral premium principles generally yielded larger VaR and ES values over a 1-year horizon than a 2-year horizon. The LC model generated the smallest spread between 1-year and 2-year VaR, suggesting consistency across time horizons. Similarly, differences in ES values followed a similar trend, with the RH model exhibiting the widest spread and the LC model maintaining the smallest spread.

Unlike S-forwards, S-swaps exhibited larger differences in VaR due to multiple cash flow exchanges. Under the Wang transform, 2-year VaR and ES exceeded 1-year values for CBD and M6 models, contrasting with general trends where 1-year risk measures surpassed their 2-year counterparts. The exponential transform consistently yielded the smallest differences across mortality models for VaR and ES, while the MAD principle showed the largest discrepancies in VaR. Notably, ES differences varied more than VaR. The M6 model displayed the widest spread between 1- and 2-year ES differences, while the RH model exhibited the narrowest spread. Specific premium principles and mortality models resulted in differences between 1-year and 2-year risk measures, underscoring the importance of understanding model behavior over different time frames.

In conclusion, this analysis illustrates that longevity risk quantification is highly sensitive to modeling assumptions, including both the choice of mortality model and premium principle. For pricing, the LC model introduces the greatest uncertainty while RH is most stable. Overall, this analysis highlighted the uncertainty in accurately quantifying longevity risk. Proper risk management requires careful sensitivity analysis when using these instruments to account for model risk. This analysis provides insights into longevity risk transfers and the need for robust modeling approaches.

Appendix A. Calibration of Pricing Parameter λ

Since mortality-linked derivatives are not publicly traded, there is scant information regarding transaction details. This thesis overcomes the model calibration problem by linking the S-forwards and S-swaps to annuity rates.

The fitted historical mortality rates end in 2011; hence, we use 2012 annuity rates. As of the first quarter of 2011, the annuity rate is level payments of £6,000 per £100,000 funds for a single life aged 65 with level payments. The risk-free rate is assumed to be the 15-year Gilt rate quoted at 2.04% for the first quarter of 2011.

There are no closed-form solutions to the pricing parameter λ for risk-neutral premium measures. Hence, we resort to numerical root-finding algorithms. We apply a Newton-Raphson type of algorithm to obtain the value of λ.

Define the following parameters and notation:

- $\psi := 6,000$ is the annuity payment
- $\xi := 100,000$ is annuity value
- $r := 2.04\%$ is the risk-free rate
- DF_t is the discount factor at time t.

Suppose the premium principle is the Wang transform, then we solve the following equation for λ:

$$\xi = \sum_{t=1}^{T} \psi DF_t [1 - \Phi(\Phi^{-1}(1 - p_{x,t}) - \lambda)]. \tag{A1}$$

To pose the equation above as a root-finding problem, we find λ using a Newton-Raphson-type algorithm such that the sum of the squared errors is minimized:

$$\left\{ \sum_{t=1}^{T} \psi DF_t [1 - \Phi(\Phi^{-1}(1 - p_{x,t}) - \lambda)] - \xi \right\}^2 = 0. \tag{A2}$$

A summary of the minization problem for each premium principle is given in Table A1

Premium	Minimize
Wang	$\left\{ \sum_{t=1}^{T} \psi DF_t [1 - \Phi(\Phi^{-1}(1 - p_{x,t}) - \lambda)] - \xi \right\}^2 = 0$
Proportional	$\left\{ \sum_{t=1}^{T} \psi DF_t (p_{x,t})^{\lambda} - \xi \right\}^2 = 0$
Dual	$\left\{ \sum_{t=1}^{T} \psi DF_t [1 - (1 - (p_{x,t})^{\lambda})] - \xi \right\}^2 = 0$
Gini	$\left\{ \sum_{t=1}^{T} \psi DF_t [(1 + \lambda) p_{x,t} - \lambda (p_{x,t})^2] - \xi \right\}^2 = 0$
Exponential	$\left\{ \sum_{t=1}^{T} \psi DF_t \left[\frac{1 - e^{-\lambda p_{x,t}}}{1 - e^{-\lambda}} \right] - \xi \right\}^2 = 0$

Table A1. Risk-neutral premium principles and their associated minimization problem to obtain pricing parameter λ

For the real-world premium principles, standard deviation, variance, MAD principle, closed-form expressions for λ can be derived. Suppose that the liability X is a Bernoulli random variable such that

$$X = \begin{cases} \sum_{t=1}^{T} DF_t \psi, & \text{person is alive} \\ 0, & \text{person is not alive.} \end{cases} \tag{A3}$$

In other words, the liability of the pension/ life annuity fund is the present value of the payment stream if the person is alive and zero if the person is not.

Since the probability that a person aged x is alive at time t is given by $p_{x,t}$, taking the expectation and variance of the Bernoulli random variable yields,

$$\mathbb{E}[X] = \sum_{t=1}^{T} DF_t \psi p_{x,t} \tag{A4}$$

$$\text{VAR}[X] = \sum_{t=1}^{T} (DF_t \psi p_{x,t})[DF_t \psi (1 - p_{x,t})]. \tag{A5}$$

Suppose the premium is the standard deviation principle, then

$$\xi = \sum_{t=1}^{T} DF_t \psi p_{x,t} + \lambda \sqrt{\sum_{t=1}^{T} (DF_t \psi p_{x,t})[DF_t \psi (1 - p_{x,t})]}. \tag{A6}$$

Solving for λ yields,

$$\lambda = \frac{\xi - \sum_{t=1}^{T} DF_t \psi p_{x,t}}{\sqrt{\sum_{t=1}^{T} (DF_t \psi p_{x,t})[DF_t \psi (1 - p_{x,t})]}}. \tag{A7}$$

Analogous to the standard deviation principle, if the premium is the variance principle, then λ is given by,

$$\lambda = \frac{\xi - \sum_{t=1}^{T} DF_t \psi p_{x,t}}{\sum_{t=1}^{T} (DF_t \psi p_{x,t})[DF_t \psi (1 - p_{x,t})]}. \tag{A8}$$

If the premium is the MAD principle, then λ is given by,

$$\lambda = \frac{\xi - \sum_{t=1}^{T} DF_t \psi \text{MEDIAN}(p_{x,t})}{\sum_{t=1}^{T} DF_t \psi \text{MAD}(p_{x,t})}. \tag{A9}$$

Here, $\text{MAD}(p_{x,t}) = \text{MEDIAN}(|p_{x,t} - S^{-1}(0.5)|)$.

Appendix B. 1-year Risk Measures for Survivor Contracts 856

Table A2. 1-year 99.5% risk measures for survivor forwards

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	1.973e-03	2.276e-03	1.170e-03	1.881e-03
	ES	2.403e-03	2.592e-03	1.560e-03	2.324e-03
Proportional	VaR	2.354e-03	2.702e-03	2.823e-03	3.485e-03
	ES	2.794e-03	3.066e-03	3.314e-03	3.858e-03
Dual	VaR	1.712e-03	2.232e-03	2.114e-03	3.422e-03
	ES	2.033e-03	2.380e-03	2.472e-03	3.841e-03
Gini	VaR	9.128e-03	8.754e-03	9.561e-03	9.920e-03
	ES	1.047e-02	9.937e-03	1.076e-02	1.075e-02
Exponential	VaR	2.500e-03	2.172e-03	2.868e-03	2.782e-03
	ES	2.914e-03	2.501e-03	3.445e-03	3.237e-03
Std. Dev.	VaR	8.943e-03	7.649e-03	8.482e-03	8.347e-03
	ES	8.944e-03	7.650e-03	8.483e-03	8.348e-03
Variance	VaR	6.101e-03	4.891e-03	6.473e-03	5.918e-03
	ES	6.663e-03	5.481e-03	7.340e-03	6.478e-03
MAD	VaR	5.597e-03	5.038e-03	6.377e-03	5.903e-03
	ES	6.737e-03	5.765e-03	7.321e-03	7.066e-03

Table A3. 1-year 99.5% risk measures for survivor swap

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	6.556e-03	1.071e-02	3.599e-04	6.092e-03
	ES	8.489e-03	1.175e-02	1.872e-03	8.012e-03
Proportional	VaR	6.832e-03	9.870e-03	5.321e-03	1.152e-02
	ES	7.731e-03	1.049e-02	6.517e-03	1.271e-02
Dual	VaR	7.553e-03	1.205e-02	6.947e-03	1.429e-02
	ES	9.126e-03	1.395e-02	8.104e-03	1.605e-02
Gini	VaR	7.370e-03	1.026e-02	6.102e-03	1.228e-02
	ES	9.753e-03	1.182e-02	8.195e-03	1.340e-02
Exponential	VaR	2.081e-03	3.201e-03	1.443e-03	2.979e-03
	ES	2.641e-03	3.599e-03	1.960e-03	3.303e-03
Std. Dev.	VaR	2.088e-02	1.303e-02	1.956e-02	1.995e-02
	ES	2.088e-02	1.304e-02	1.956e-02	1.995e-02
Variance	VaR	1.517e-08	4.541e-09	1.747e-08	2.066e-08
	ES	1.518e-08	4.544e-09	1.747e-08	2.067e-08
MAD	VaR	2.963e-02	2.035e-02	3.149e-02	3.611e-02
	ES	2.964e-02	2.035e-02	3.150e-02	3.613e-02

Appendix C. 2-year Risk Measures for Survivor Contracts 857

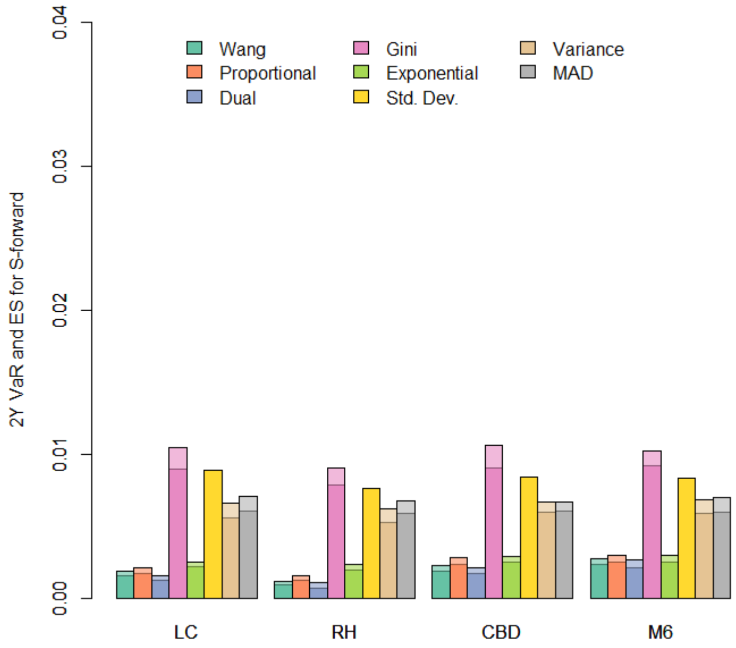

Figure A1. 2-year 99.5% VaR and ES for S-forwards. The solid color represents the VaR, while the transparent color represents the ES.

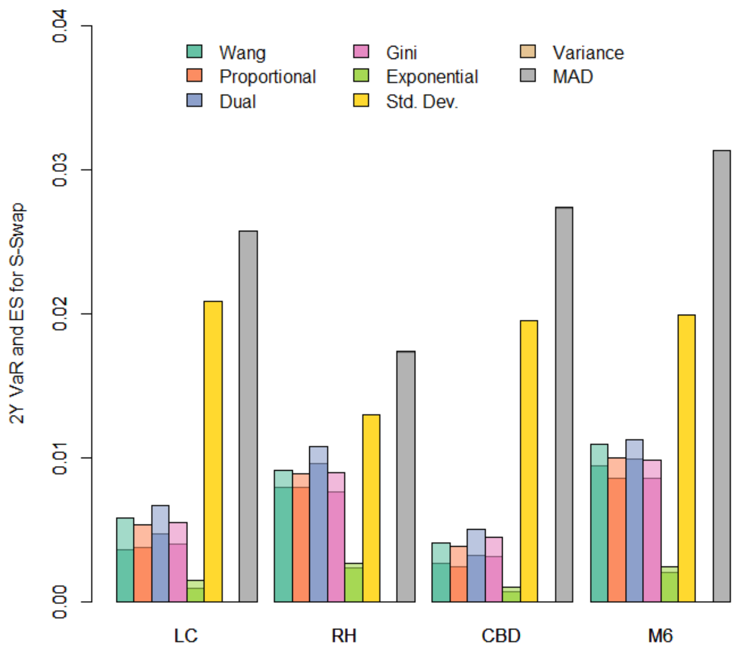

Figure A2. 2-year 99.5% VaR and ES for S-swap. The solid color represents the VaR, while the transparent color represents the ES.

Table A4. 2-year 99.5% risk measures for survivor forwards

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	1.627e-03	9.546e-04	1.946e-03	2.355e-03
	ES	1.946e-03	1.193e-03	2.300e-03	2.753e-03
Proportional	VaR	1.783e-03	1.301e-03	2.360e-03	2.549e-03
	ES	2.158e-03	1.579e-03	2.827e-03	2.988e-03
Dual	VaR	1.272e-03	7.229e-04	1.789e-03	2.182e-03
	ES	1.583e-03	1.109e-03	2.187e-03	2.664e-03
Gini	VaR	9.027e-03	7.919e-03	9.107e-03	9.198e-03
	ES	1.047e-02	9.081e-03	1.067e-02	1.027e-02
Exponential	VaR	2.194e-03	1.979e-03	2.519e-03	2.551e-03
	ES	2.542e-03	2.350e-03	2.905e-03	3.042e-03
Std. Dev.	VaR	8.943e-03	7.649e-03	8.482e-03	8.347e-03
	ES	8.944e-03	7.650e-03	8.483e-03	8.348e-03
Variance	VaR	5.629e-03	5.330e-03	6.046e-03	5.901e-03
	ES	6.607e-03	6.233e-03	6.749e-03	6.897e-03
MAD	VaR	6.097e-03	5.931e-03	6.084e-03	6.039e-03
	ES	7.112e-03	6.785e-03	6.688e-03	7.041e-03

Table A5. 2-year 99.5% risk measures for survivor swap

Premium	Risk Measure	LC	RH	CBD	M6
Wang	VaR	3.635e-03	7.988e-03	2.698e-03	9.454e-03
	ES	5.874e-03	9.192e-03	4.141e-03	1.096e-02
Proportional	VaR	3.781e-03	7.965e-03	2.502e-03	8.582e-03
	ES	5.359e-03	8.911e-03	3.880e-03	9.999e-03
Dual	VaR	4.750e-03	9.653e-03	3.228e-03	9.982e-03
	ES	6.744e-03	1.080e-02	5.046e-03	1.130e-02
Gini	VaR	4.028e-03	7.692e-03	3.195e-03	8.616e-03
	ES	5.496e-03	9.007e-03	4.547e-03	9.881e-03
Exponential	VaR	9.965e-04	2.366e-03	7.598e-04	2.045e-03
	ES	1.555e-03	2.705e-03	1.083e-03	2.439e-03
Std. Dev.	VaR	2.088e-02	1.303e-02	1.956e-02	1.995e-02
	ES	2.088e-02	1.304e-02	1.956e-02	1.995e-02
Variance	VaR	1.230e-08	3.189e-09	1.495e-08	1.773e-08
	ES	1.231e-08	3.192e-09	1.496e-08	1.774e-08
MAD	VaR	2.575e-02	1.738e-02	2.739e-02	3.135e-02
	ES	2.577e-02	1.739e-02	2.740e-02	3.136e-02

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References

Atance, David, Ana Debón, and Eliseo Navarro. 2020. A comparison of forecasting mortality models using resampling methods. *Mathematics 8*(9). doi.org/10.3390/math8091550.

Barrieu, Pauline, Harry Bensusan, Nicole El Karoui, Caroline Hillairet, Stéphane Loisel, Claudia Ravanelli, and Yahia Salhi. 2012. Understanding, modelling and managing longevity risk: key issues and main challenges. *Scandinavian Actuarial Journal 2012*(3), 203–231. doi.org/10.1080/03461238.2010.511034,.

Barrieu, Pauline M. and Luitgard A.M. Veraart. 2016. Pricing q-forward contracts: an evaluation of estimation window and pricing method under different mortality models. *Scandinavian Actuarial Journal 2016*(2), 146–166. doi.org/10.1080/03461238.2014.916228.

Bauer, Daniel, Matthias Börger, and Jochen Ruß. 2010. On the pricing of longevity-linked securities. *Insurance: Mathematics and Economics 46*(1), 139–149. doi.org/10.1016/j.insmatheco.2009.06.005.

Blake, D., A. J. G. Cairns, K. Dowd, and A. R. Kessler. 2019. Still living with mortality: the longevity risk transfer market after one decade. *British Actuarial Journal 24*, 1–80. doi.org/10.1017/S1357321718000314.

Boonen, Tim J. 2017, Dec. Solvency ii solvency capital requirement for life insurance companies based on expected shortfall. *European Actuarial Journal 7*(2), 405–434. doi.org/10.1007/s13385-017-0160-4.

Börger, Matthias. 2010, Oct. Deterministic shock vs. stochastic value-at-risk — an analysis of the solvency ii standard model approach to longevity risk. *Blätter der DGVFM 31*(2), 225–259. doi.org/10.1007/s11857-010-0125-z.

Boyer, M. Martin and Lars Stentoft. 2013. If we can simulate it, we can insure it: An application to longevity risk management. *Insurance: Mathematics and Economics 52*(1), 35–45. doi.org/10.1016/j.insmatheco.2012.10.003.

Cairns, Andrew J. G., David Blake, and Kevin Dowd. 2006. A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *Journal of Risk and Insurance 73*(4), 687–718. doi.org/10.1111/j.1539-6975.2006.00195.x.

Cairns, Andrew J. G., David Blake, and Kevin Dowd. 2008. Modelling and management of mortality risk: a review. *Scandinavian Actuarial Journal 2008*(2-3), 79–113. doi.org/10.1080/03461230802173608.

Cairns, Andrew J. G., David Blake, Kevin Dowd, Guy D. Coughlan, David Epstein, Alen Ong, and Igor Balevich. 2009. A quantitative comparison of stochastic mortality models using data from england and wales and the united states. *North American Actuarial Journal 13*(1), 1–35. doi.org/10.1080/10920277.2009.10597538.

Christiansen, Marcus C. and Andreas Niemeyer. 2014. Fundamental definition of the solvency capital requirement in solvency ii. *ASTIN Bulletin 44*(3), 501–533. doi.org/10.1017/asb.2014.10.

Coughlan, Guy D., Marwa Khalaf-Allah PhD, Yijing Ye, Sumit Kumar, Andrew J. G. Cairns, David Blake, and Kevin Dowd. 2011. Longevity hedging 101. *North American Actuarial Journal 15*(2), 150–176. doi.org/10.1080/10920277.2011.10597615.

Denneberg, Dieter. 1990. Premium calculation: Why standard deviation should be replaced by absolute deviation. *ASTIN Bulletin: The Journal of the IAA 20*(2), 181–190. doi.org/10.2143/AST.20.2.2005441.

Denuit, Michel, Pierre Devolder, and Anne-Cécile Goderniaux. 2007. Securitization of longevity risk: Pricing survivor bonds with wang transform in the lee-carter framework. *The Journal of Risk and Insurance 74*(1), 87–113. http://www.jstor.org/stable/4138426.

Devolder, Pierre and Adrien Lebègue. 2017. Iterated var or cte measures: A false good idea? *Scandinavian Actuarial Journal 2017*(4), 287–318. doi.org/10.1080/03461238.2015.1126343.

Diffouo, Ngugnie, Pauline Milaure, and Pierre Devolder. 2020. Longevity risk measurement of life annuity products. *Risks 8*(1). doi.org/10.3390/risks8010031.

Dowd, Kevin, David Blake, Andrew J. G. Cairns, and Paul Dawson. 2006. Survivor swaps. *The Journal of Risk and Insurance 73*(1), 1–17. http://www.jstor.org/stable/3519967.

Dowd, Kevin, Andrew J. G. Cairns, and David Blake. 2010. Backtesting stochastic mortality models. *North American Actuarial Journal 14*(3), 281–298. doi.org/10.1080/10920277.2010.10597592.

Frey, Rüdiger and Alexander J. McNeil. 2002. Var and expected shortfall in portfolios of dependent credit risks: Conceptual and practical insights. *Journal of Banking & Finance 26*(7), 1317–1334. doi.org/10.1016/S0378-4266(02)00265-0.

Gylys, Rokas and Jonas Šiaulys. 2019. Revisiting calibration of the solvency ii standard formula for mortality risk: Does the standard stress scenario provide an adequate approximation of value-at-risk? *Risks 7*(2), 58. doi.org/10.3390/risks7020058.

Haberman, Steven and Arthur Renshaw. 2011. A comparative study of parametric mortality projection models. *Insurance: Mathematics and Economics 48*(1), 35–55. doi.org/10.1016/j.insmatheco.2010.09.003.

Lee, Ronald D. and Lawrence R. Carter. 1992. Modeling and forecasting u. s. mortality. *Journal of the American Statistical Association 87*(419), 659–671. http://www.jstor.org/stable/2290201.

Leung, Melvern, Man Chung Fung, and Colin O'Hare. 2018. A comparative study of pricing approaches for longevity instruments. *Insurance: Mathematics and Economics 82*, 95–116. doi.org/10.1016/j.insmatheco.2018.06.010.

Li, Johnny, Rui Zhou, and Yanxin Liu. 2020. Drivers of mortality dynamics identifying age period cohort components of historical u.s. mortality improvements. *North American Actuarial Journal 24*(2), 228–250. doi.org/10.1080/10920277.2020.1716808.

Lin, Yijia and Samuel H. Cox. 2005. Securitization of mortality risks in life annuities. *The Journal of Risk and Insurance 72*(2), 227–252. http://www.jstor.org/stable/3519949.

MacMinn, Richard, Jennifer Wang, and David Blake. 2008. Longevity risk and capital markets: The 2007-2008 update. *Asia-Pacific Journal of Risk and Insurance 3*(1), 1–5. doi.org/10.2202/2153-3792.1026.

Pelsser, Antoon. 2008. On the applicability of the wang transform for pricing financial risks. *ASTIN Bulletin 38*(1), 171–181. doi.org/10.2143/AST.38.1.2030409, https://doi.org/10.2143/AST.38.1.2030409.

Pfeifer, Dietmar and Doreen Strassburger. 2008. Solvency ii: stability problems with the scr aggregation formula. *Scandinavian Actuarial Journal 2008*(1), 61–77. doi.org/10.1080/03461230701766825.
Plat, Richard. 2011. One-year value-at-risk for longevity and mortality. *Insurance: Mathematics and Economics 49*(3), 462–470. doi.org/10.1016/j.insmatheco.2011.07.002.
Renshaw, A.E. and S. Haberman. 2006. A cohort-based extension to the lee–carter model for mortality reduction factors. *Insurance: Mathematics and Economics 38*(3), 556–570. doi.org/10.1016/j.insmatheco.2005.12.001.
Richards, Stephen J. 2021. A value-at-risk approach to mis-estimation risk. *British Actuarial Journal 26*, e13. doi.org/10.1017/S1357321721000131.
Richards, S. J., I. D. Currie, and G. P. Ritchie. 2014. A value-at-risk framework for longevity trend risk. *British Actuarial Journal 19*(1), 116–139. doi.org/10.1017/S1357321712000451.
Sandström, Arne. 2007. Solvency ii: Calibration for skewness. *Scandinavian Actuarial Journal 2007*(2), 126–134. doi.org/10.1080/03461230701250481.
Tang, Sixian and Jackie Li. 2021. Market pricing of longevity-linked securities. *Scandinavian Actuarial Journal 2021*(5), 408–436. doi.org/10.1080/03461238.2020.1852105.
Wagner, Joel. 2014, Jan. A note on the appropriate choice of risk measures in the solvency assessment of insurance companies. *The Journal of Risk Finance 15*(2), 110–130. doi.org/10.1108/JRF-11-2013-0082.
Wang, Shaun. 1995. Insurance pricing and increased limits ratemaking by proportional hazards transforms. *Insurance: Mathematics and Economics 17*(1), 43–54. doi.org/10.1016/0167-6687(95)00010-P.
Wang, Shaun. 1996. Premium calculation by transforming the layer premium density. *ASTIN Bulletin: The Journal of the IAA 26*(1), 71–92. doi.org/10.2143/AST.26.1.563234.
Wang, Shaun. 2002. A universal framework for pricing financial and insurance risks. *ASTIN Bulletin: The Journal of the IAA 32*(2), 213–234. doi.org/10.2143/AST.32.2.1027.
Wang, Yige, Nan Zhang, Zhuo Jin, and Tin Long Ho. 2019. Pricing longevity-linked derivatives using a stochastic mortality model. *Communications in Statistics - Theory and Methods 48*(24), 5923–5942. doi.org/10.1080/03610926.2018.1563171.
Yamai, Yasuhiro and Toshinao Yoshiba. 2005. Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking & Finance 29*(4), 997–1015. doi.org/10.1016/j.jbankfin.2004.08.010.
Yang, Bowen, Jackie Li, and Uditha Balasooriya. 2015. Using bootstrapping to incorporate model error for risk-neutral pricing of longevity risk. *Insurance: Mathematics and Economics 62*, 16–27. doi.org/10.1016/j.insmatheco.2015.02.004.
Zeddouk, Fadoua and Pierre Devolder. 2019. Pricing of longevity derivatives and cost of capital. *Risks 7*(2), 41. doi.org/10.3390/risks7020041.