

Lab One, Part One

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1 Part 1: Foundational Exercises

1.1 1.1 Professional Magic

1.1.1 1.1.1 Type I Error of the test

The type I error rate (i.e. false positive) is the probability of rejecting the null hypothesis whe it is correct. The type I error would be the probability of getting 0 or 6, with the assumption that $p = 0.5$ (null).

This will be the alpha

1.1.2 1.1.2 Power of test given $p = 0.75$

1.2 1.2 Wrong Test, Right Data - Kevin

In the Likert scale, the meaningful distance between the different scale points is not consistent. That is, assuming the Likert scale for the websites survey includes five points from 1 = “Very Unsatisfied” to 5 = “Very Satisfied,” with 3 being “Neutral,” we cannot say that a change from 1 to 3 and from 2 to 4 are equivalent quantifiable changes in opinion ¹. However, in fact the change in quality of experience necessary for a given respondent to go from Very Unsatisfied with one site to Neutral with the other may in fact be considerably less than the change needed to go from Unsatisfied to Satisfied, though these each consist of a difference of two steps. Therefore, though the values produced are numeric, these data violate one of the assumptions for a paired t-test – the use of metric, rather than ordinal, data.

A paired t-test relies on metric data because, like other related tests including the z-test, it is fundamentally a calculation of the difference of means between reference groups. A paired t-test would ask of our survey data: is the mean difference between paired opinion scores different than what we would expect if there were no preference for either website (mean difference within pairs = 0)? Stated otherwise, on average across all respondents, how likely is it that there is really a preference for one site or the other, and how large a preference? However, because of the aforementioned limitation of Likert scale values, we cannot meaningfully parse a mean paired disparity of e.g., +2, because to calculate this requires assuming that non-comparable changes from any one Likert scale point to another are equivalent. The mean of the paired differences is thus meaningless. It is even difficult to trust the directionality of the mean difference across all pairs (respondents like the mobile website more or less than the regular website without regard to how much), as in calculating a mean value purely from the raw Likert scale scores, we may calculate an incorrect value by not correctly taking into account the “weights” of the differences of opinion in, again, Very Unsatisfied and Neutral versus Unsatisfied and Satisfied. It’s possible to conceive of a scenario in which even the sign of the mean difference is therefore incorrect.

It is this last point on directionality that suggests an alternative approach to this analysis. A non-parametric paired sign test allows us to analyze our ordinal data provided the observations are independent and identically distributed. It does not attempt, like the t-test, to quantify the size of the difference in opinion within pairs, if any. Rather, it treats all positive changes in opinion as equivalent, and does likewise with all negative changes. This alternative test has two main drawbacks. First, it does not have the statistical power of a paired t-test. Second, it loses substantial information present in the original survey responses in the form of the exact values within each paired set of responses. However, in doing so, it allows us to avoid the inaccurate mean calculation of the t-test, and focus on a more accurate analysis of a simpler question: do respondents prefer one website over the other? In looking solely at increases or decreases in opinion score, the paired sign test therefore gives us a reasonable expectation of finding such an effect if one is present in the data.

¹At least, not with only five scale points; see, e.g.: Huiping Wu and Shing-On Leung, “Can Likert Scales Be Treated as Interval Scales?—a Simulation Study,” *Journal of Social Service Research* 43, no. 4 (June 2017): pp. 527-532, <https://doi.org/10.1080/01488376.2017.1329775>.

1.3 1.3 Test Assumptions

1.3.1 1.3.1 World Happiness

Scenario: We have two variables: Life.Ladder and Log.GDP.per.Capita, and we want to see whether countries in high GDP per capita are more or less happy than people in countries with low GDP per capita.

Proposed test: Two Sample t-Test

Test Assumptions: 1. Metric variables 2. Random variables are independent and identically distributed (hereby referred to as i.i.d.) 3. Normalcy of random variables

Both of the variables continuous numeric and metric variables, thus satisfying the first condition. They are also independent and identically distributed based on the fact that the respondents were asked to rank the

Life.Ladder	Log.GDP.per.capita
Min. :2.375	Min. : 6.966
1st Qu.:4.971	1st Qu.: 8.827
Median :5.768	Median : 9.669
Mean :5.678	Mean : 9.584
3rd Qu.:6.428	3rd Qu.:10.527
Max. :7.889	Max. :11.648
	NA's :13

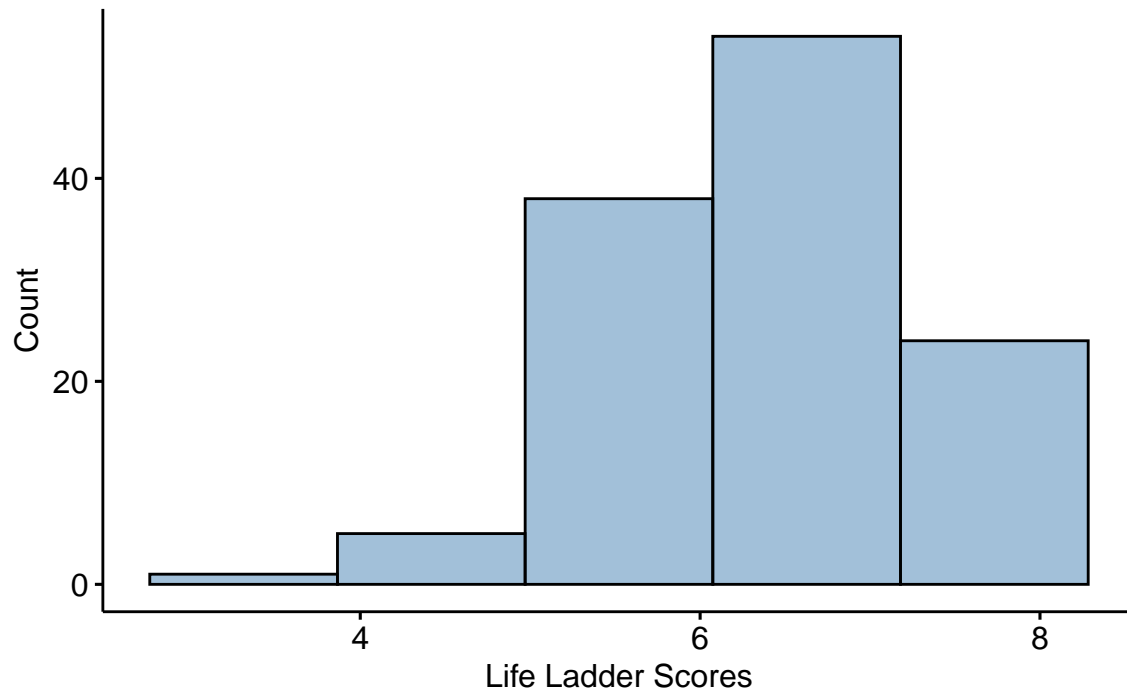
Summary: Life Ladder Score and GDP per Capitas > Sample Mean

Life.Ladder	Log.GDP.per.capita
Min. :3.471	Min. : 9.583
1st Qu.:5.917	1st Qu.: 9.993
Median :6.291	Median :10.483
Mean :6.349	Mean :10.415
3rd Qu.:7.027	3rd Qu.:10.768
Max. :7.889	Max. :11.648

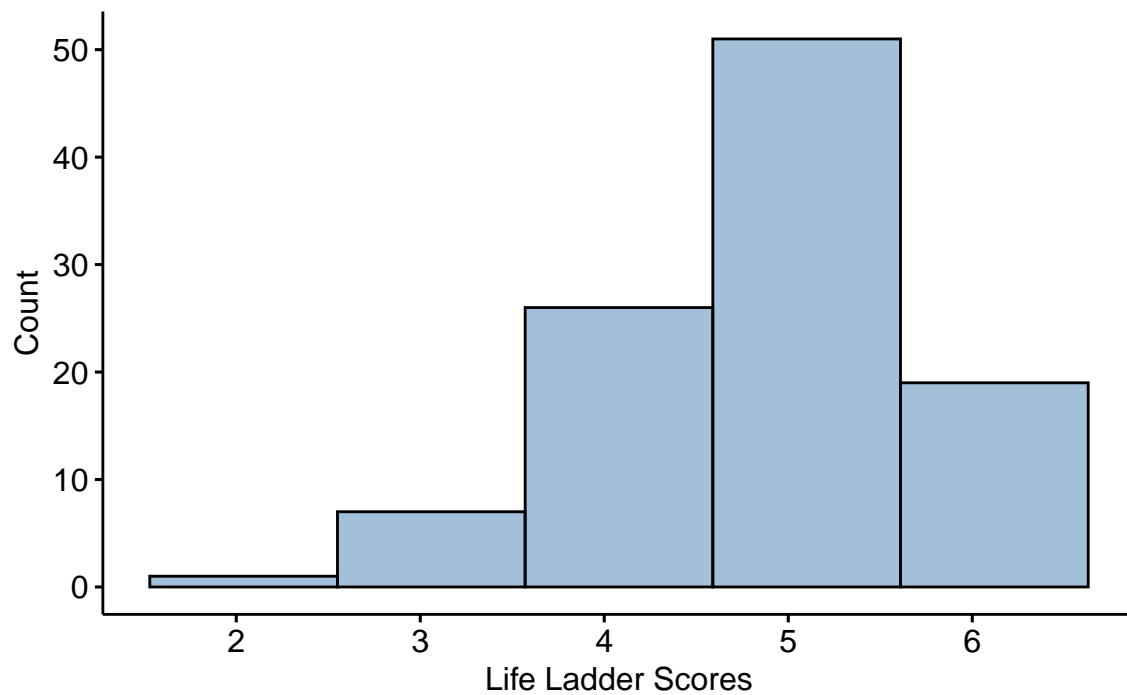
Summary: Life Ladder Score and GDP per Capitas < Sample Mean

Life.Ladder	Log.GDP.per.capita
Min. :2.375	Min. :6.966
1st Qu.:4.433	1st Qu.:8.100
Median :4.992	Median :8.592
Mean :4.919	Mean :8.609
3rd Qu.:5.463	3rd Qu.:9.234
Max. :6.455	Max. :9.575

Life Ladder Scores: High GDP per Capita Countries



Life Ladder Scores: Low GDP per Capita Countries



Shapiro-Wilk normality test

data: low_gdp_cap\$Life.Ladder
W = 0.97549, p-value = 0.05055

$p > 0.05$ (barely) \rightarrow normal-ish

```
shapiro.test(low_gdp_cap$Log.GDP.per.capita)
```

Shapiro-Wilk normality test

```
data: low_gdp_cap$Log.GDP.per.capita  
W = 0.93951, p-value = 0.0001319
```

[#https://www.datanovia.com/en/lessons/normality-test-in-r/#check-normality-in-r](https://www.datanovia.com/en/lessons/normality-test-in-r/#check-normality-in-r)

$p < 0.05 \rightarrow$ not normal

Shapiro-Wilk normality test

```
data: high_gdp_cap$Life.Ladder  
W = 0.97281, p-value = 0.01434
```

$p < 0.05 \rightarrow$ not normal

```
shapiro.test(high_gdp_cap$Log.GDP.per.capita)
```

Shapiro-Wilk normality test

```
data: high_gdp_cap$Log.GDP.per.capita  
W = 0.96977, p-value = 0.00764
```

[#https://www.datanovia.com/en/lessons/normality-test-in-r/#check-normality-in-r](https://www.datanovia.com/en/lessons/normality-test-in-r/#check-normality-in-r)

$p < 0.05 \rightarrow$ not normal

Conduct the test? NO

1.3.2 1.3.2 Legislators

Scenario: We want to test whether Democratic or Republican senators are older, with two variables party and age (age needs to be calculated from DOB).

Proposed test: Wilcoxon Rank Sum Test

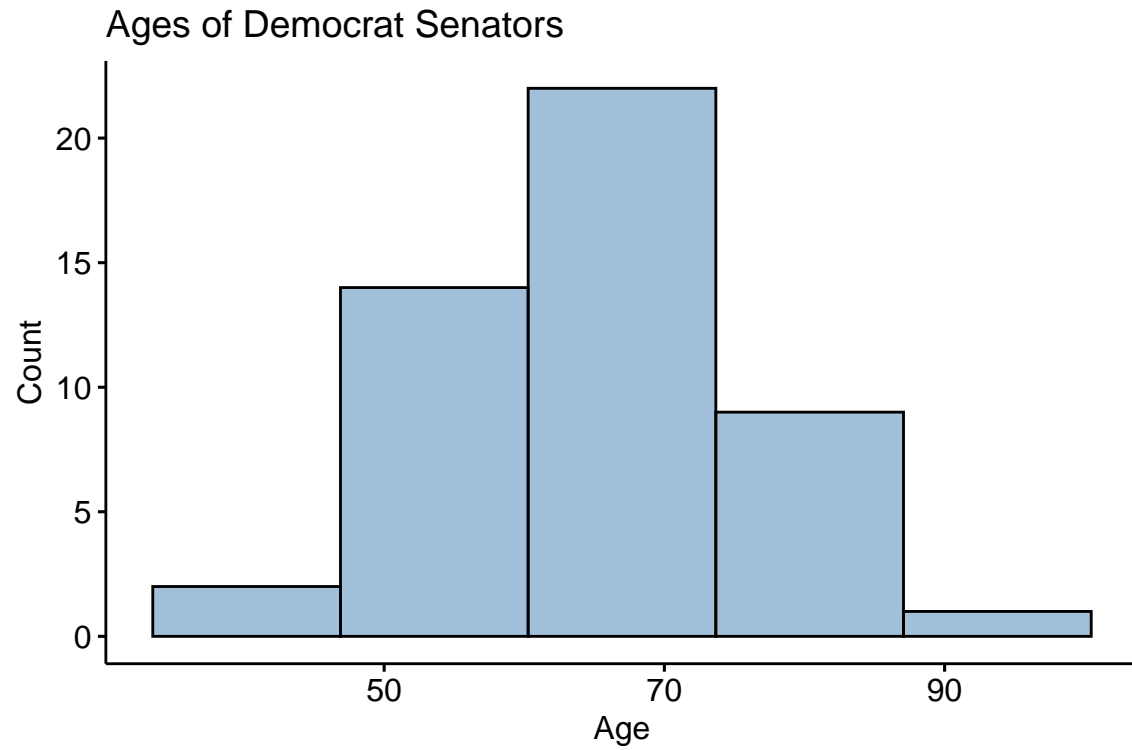
Test Assumptions: 1. Ordinal variables 2. i.i.d. 3. Same shape and spread of the two variables

Summary: Democrat Senators

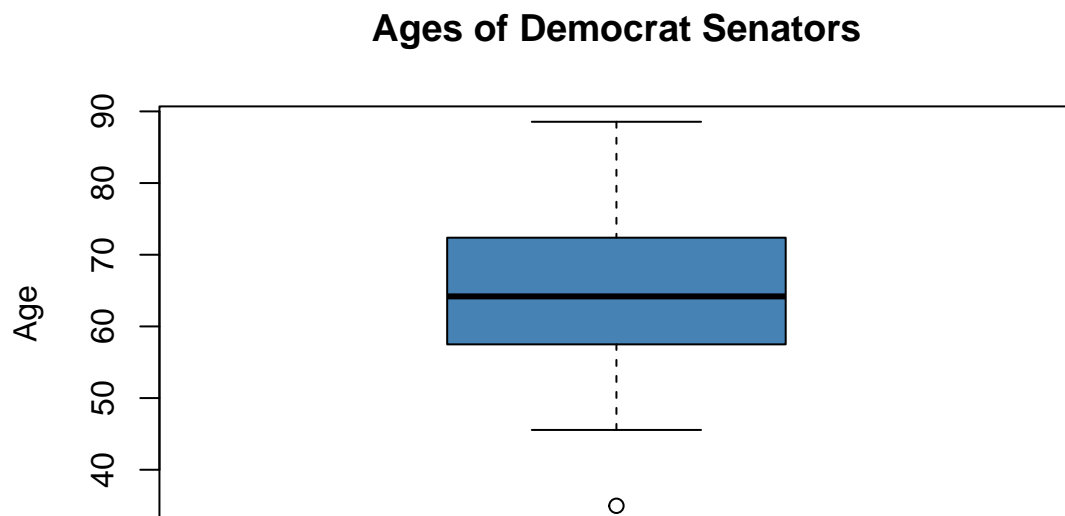
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
34.97	57.66	64.19	64.17	72.28	88.55

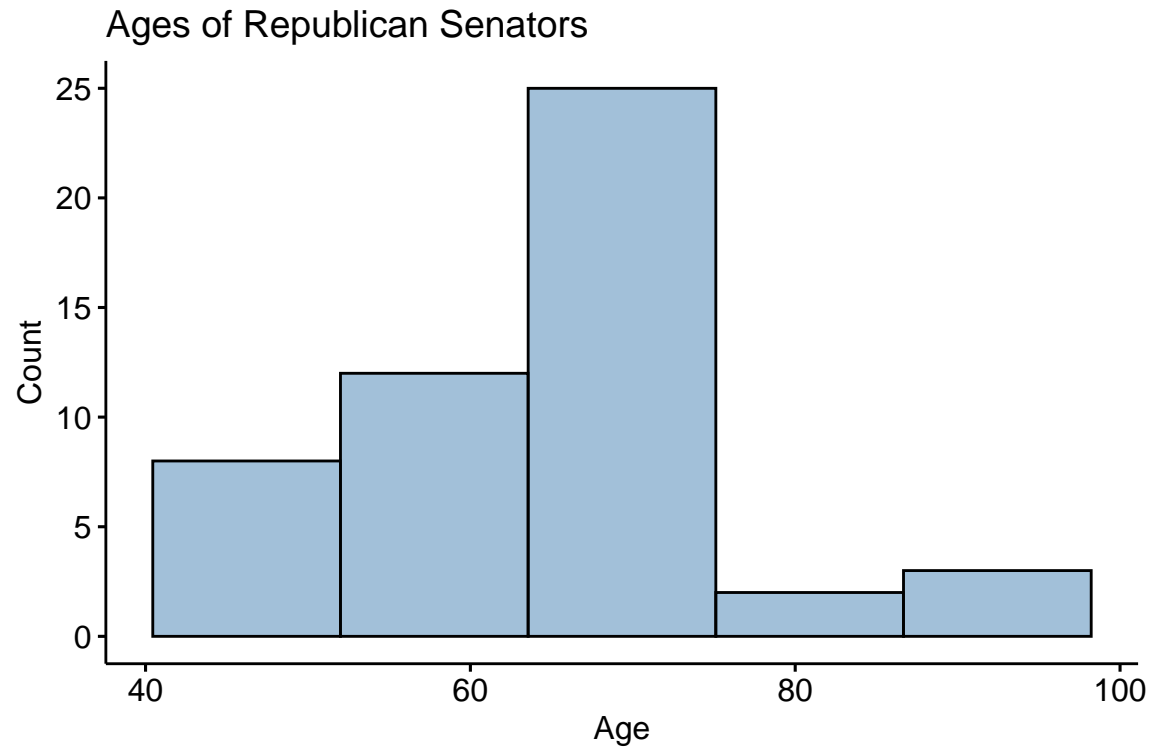
Summary: Republican Senators

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
42.09	59.62	66.34	64.81	69.88	88.31

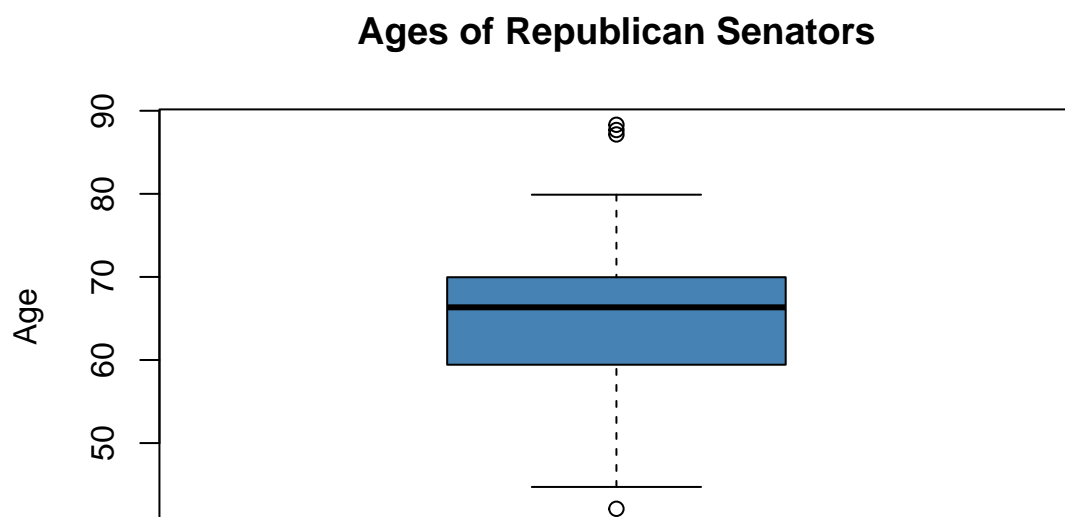


```
boxplot(x=dem_sen$age, data = dem_sen,  
        main="Ages of Democrat Senators",  
        ylab="Age",  
        col = 'steelblue')
```





```
boxplot(x=rep_sen$age, data = rep_sen,  
        main="Ages of Republican Senators",  
        ylab="Age",  
        col = 'steelblue')
```



Con-

duct the test? Maybe yes?

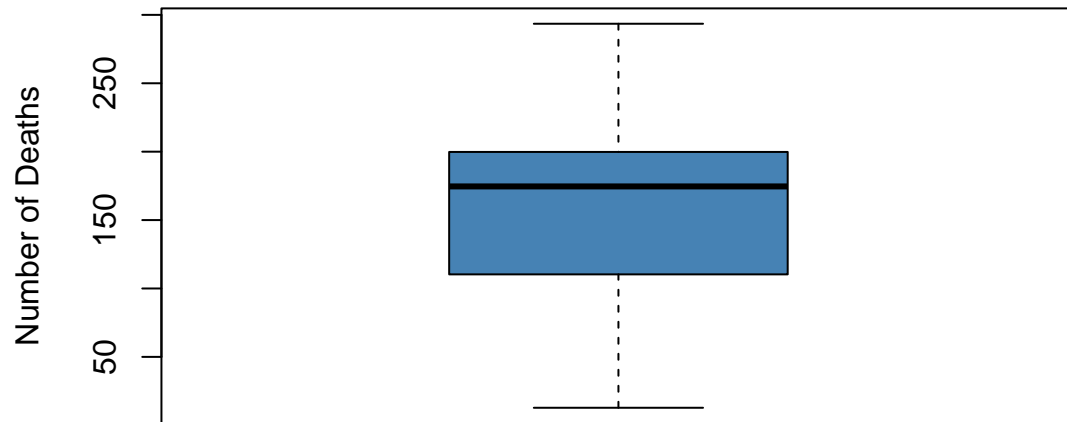
1.3.3 1.3.3 Wine and Health

Scenario: We want to test whether these countries have more deaths from heart disease or liver disease.

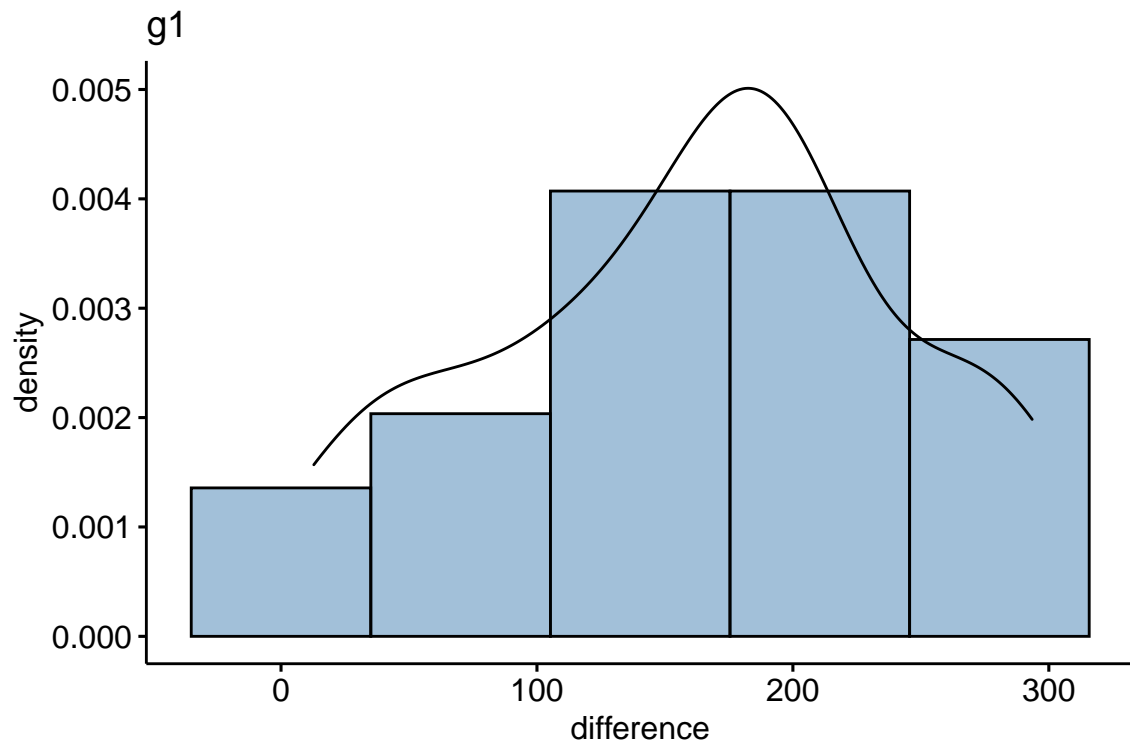
Proposed test: Wilcoxon Signed Rank Test

Test Assumptions: 1. Metric Variables 2. i.i.d. 3. Paired data 4. Difference is symmetric

Difference between Deaths from Heart Disease – Liver Diseases



```
gghistogram(wine_data, main = 'g1', x = 'difference', y = '..density..', fill= 'steelblue', bins = 5, a
```



Con-

duct the test? Not sure

First box whisker done with advice from https://www.youtube.com/watch?v=Y4-wAT4SNM4&ab_channel=Dr.ToddGrande go to around 6 min

Second chart done with advice from <https://www.datanovia.com/en/lessons/wilcoxon-test-in-r/>

1.3.4 1.3.4 Attitudes Toward the Religion

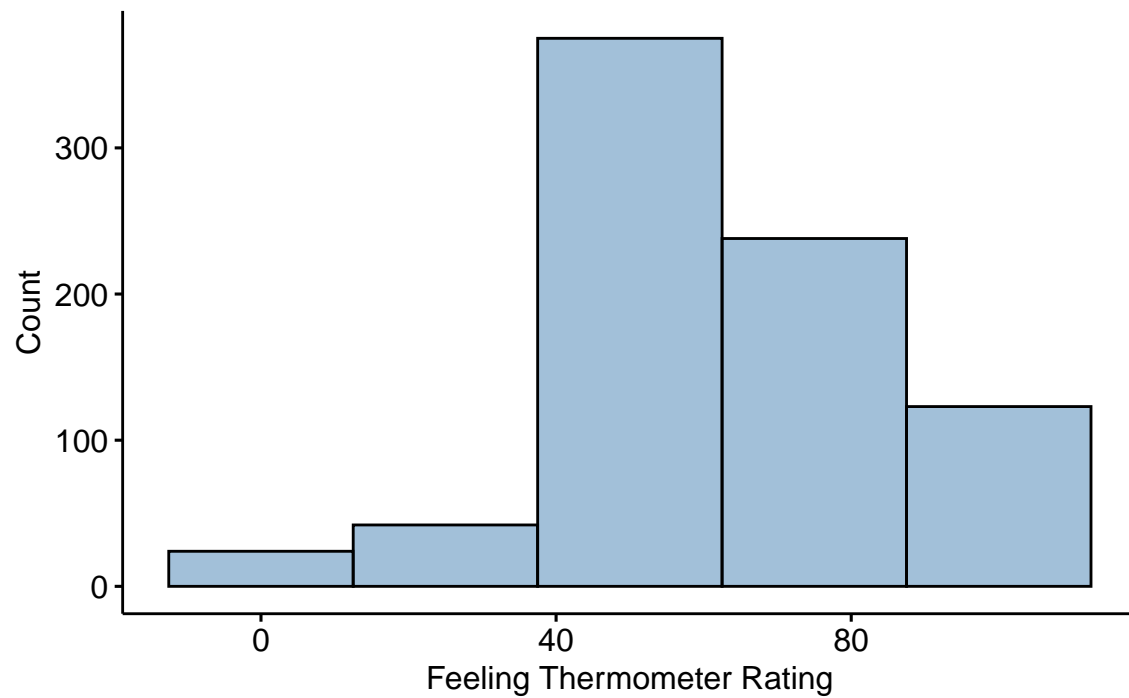
Scenario: We would like to know whether the U.S. population feels more positive towards Protestants or Catholics.

Proposed Test: Paired t-Test

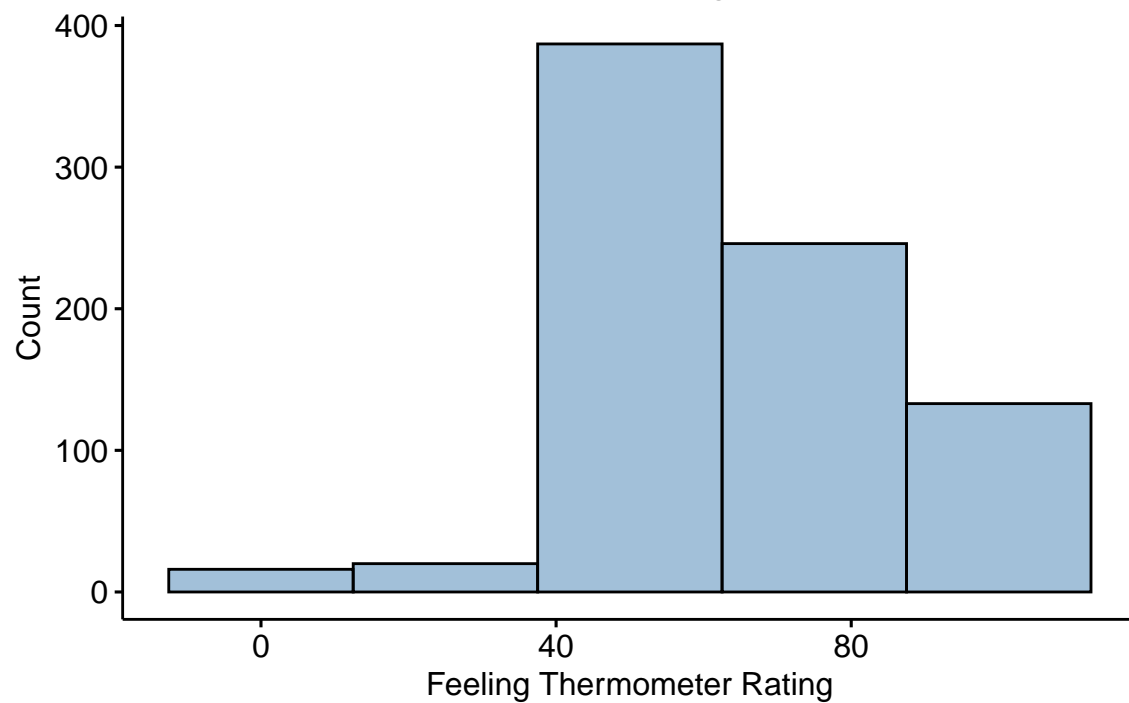
Test Assumptions: 1. Metric Variables 2. i.i.d. 3. Paired 4. Normalcy

cathtemp		prottemp	
Min.	: 0.00	Min.	: 0.00
1st Qu.	: 50.00	1st Qu.	: 50.00
Median	: 60.00	Median	: 60.00
Mean	: 63.16	Mean	: 65.56
3rd Qu.	: 85.00	3rd Qu.	: 85.00
Max.	:100.00	Max.	:100.00

Distribution of US Population Feelings Towards Catholics



Distribution of US Population Feelings Towards Protestants



Shapiro-Wilk normality test

```
data: rel_data$cathtemp  
W = 0.93377, p-value < 2.2e-16
```

p-value < 0.05 -> not normal

Shapiro-Wilk normality test

```
data: rel_data$prottemp  
W = 0.89479, p-value < 2.2e-16
```

p-value < 0.05 -> not normal

Conduct the test? No

Note on Shapiro wilks test The Shapiro-Wilk test is a statistical test used to check if a continuous variable follows a normal distribution. The null hypothesis (H0) states that the variable is normally distributed, and the alternative hypothesis (H1) states that the variable is NOT normally distributed. So after running this test:

If $p \leq 0.05$: then the null hypothesis can be rejected (i.e. the variable is NOT normally distributed). If $p > 0.05$: then the null hypothesis cannot be rejected (i.e. the variable MAY BE normally distributed).

source: <https://quantifyinghealth.com/report-shapiro-wilk-test/>