Cooper Union for the Advancement of Science and Art

Designing a Ramjet Engine Using First Principles of Compressible Flow

ME-433: Rocket Science

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1 Motivation

Basic principles of compressible flow are used to design components: inlet (1-2), diffuser (2-3), flameholder (3-3'), combustor (3'-4), converging (4-5) and diverging nozzle (5-6), which are then put together to simulate a ramjet engine. The theoretical performance of the engine is analyzed for inlet mach numbers between 2.75 and 3.25. Analysis is performed for each section in regards to its effect on other components which will be shown later. The placement of each component is shown in Figure 1.1

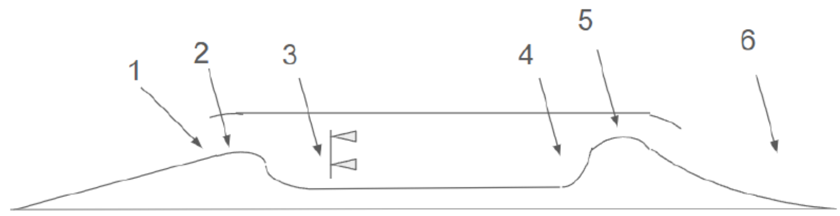


Figure 1.1: Required assembly of jet engine.

2 Assumptions and Design Constraints

Following are the assumptions considered while performing the calculations:

1. Air is considered to be a calorically perfect gas, hence γ, adiabatic constant, stays constant = 1.4.
2. The ramjet is assumed to be flying at an elevation of 55,000 ft.
3. The flow over the jet engine is inviscid while the pressure drag is considered.
4. Air is considered to be an ideal gas. Accordingly, $PV = RT$ is valid.
5. Hydrogen is used as the fuel for the combustor with a lower heating value of 120 $\frac{MJ}{kg}$.
6. Standard 1976 atmospheric model is used to determine the pressure and temperature properties at the given elevation.
7. The fuel in the combustor is assumed to be burnt instantaneously.
8. It is assumed that the engine is made of a typical high temperature alloy - Inconel 625 whose melting point is 1622 K [1]. Since there are methods of cooling the jet engines, the max temperature of the engine is kept under 1800-1900 K.

3 Design of Individual Components

Following are the components designed using basic principles of in compressible flow.

3.1 Inlet

A 2 step inlet is designed to turn the flow from outside of the engine to the diffuser. At every angle, an oblique shock is formed and finally there is a normal shock at the end of the inlet to turn the supersonic flow into subsonic before it enters the diffuser. Otherwise, the diffuser will increase the speed if the flow is supersonic.

Normal shocks occur when they get detached from the body due to a very large turning angle. As the name suggests, the inlet and outlet flow velocities are normal to the shock wave. Across a normal shock, the supersonic flow changes to subsonic flow. Accordingly, $P_2 > P_1$, $T_2 > T_1$, and $\rho_2 > \rho_1$. Using the conservation laws and stagnation conditions shown above, the fluid characteristics after experiencing normal shock can be determined using the relations shown in Equation 1, Equation 2, Equation 3, and Equation 4.

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma - 1}(M_1^2 - 1) \tag{1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_1^2}{2 + (\gamma - 1)M_1^2} \tag{2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} \tag{3}$$

$$M_2 = \frac{1 + \frac{(\gamma-1)}{2}M_1^2}{\gamma M_1^2 - \frac{(\gamma-1)}{2}} \tag{4}$$

Oblique shocks, on the other hand, occur when the supersonic flow is turned into itself. In other words, the flow experiences a re-direction by a giving turning angle θ. An illustration of oblique shock is shown in Figure 3.1, taken from *Modern Compressible Flow With Historical Perspective*. As θ increases from 0 to 90 °, the oblique shock becomes stronger and accordingly, adds more pressure drag into the system.

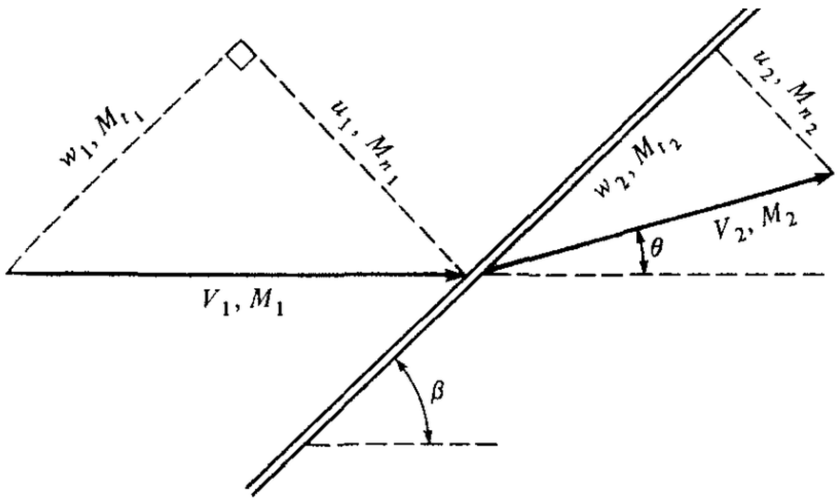


Figure 3.1: Oblique shock due to a surface at an angle θ

The relation between β (shock wave angle), θ (turning angle) and mach number of incoming flow is related using the $\theta-\beta-M$ relation shown in Equation 5. Accordingly, for a given mach number there is a maximum turning angle $\theta_{\max}$.

$$tan(\theta) = 2cot(\beta)[\frac{M_1^2(sin(\beta))^2 - 1}{M_1^2(\gamma + cos(2\beta)) + 2}] \tag{5}$$

To determine the fluid properties, an oblique shock can be treated as a normal shock for its perpendicular components. Accordingly, $M_{1,\text{ normal}} = M_{2,\text{ normal}}$. These components are found using basic trigonometry as shown in Equation 6 and Equation 7. The corresponding the pressure ratio, temperature ratio, and $M_{2,\text{ normal}}$ can be determined using normal shock relations as shown in Equation 1, Equation 3, and Equation 4.

$$M_{1,\text{ normal}} = M_1 sin(\beta) \tag{6}$$

$$M_2 = \frac{M_{2,\text{ normal}}}{sin(\beta - \theta)} \tag{7}$$

Anytime there is an oblique shock wave, $P_2 > P_1$, $T_2 > T_1$, and $M_2 < M_1$. Moreover, oblique shock only happens for supersonic flows.

3.2 Cowl

The cowl is designed in a way that the oblique shock waves from the 2 step inlet intersect at the tip of the cowl. Accordingly, a rough diagram of an inlet along with a cowl is drawn as seen in Figure 3.2 and two triangles are constructed as seen in red lines. Trigonometric relations are derived using both triangles with the parameters known: $\beta_1, \beta_2, \theta_1, \theta_2, \& L$ where L is the length of the first step. It can be seen in the equations the y - coordinate of the tip of the cowl is directly influenced by the length of the first step while second step doesn't change anything.

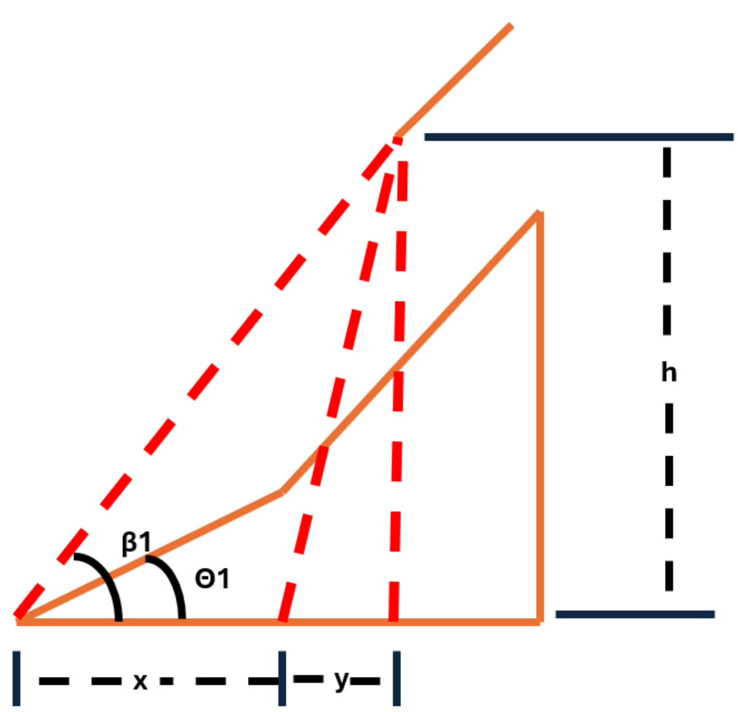


Figure 3.2: Geometric diagram used to determine the height and the starting point of the cowl.

Expression for y, Equation 9, is determined using the two triangles in red. After doing so, law of sines is applied to the smaller obtuse triangle and an expression for x is determined shown in Equation 8. After doing so, the bigger right triangle is used to determine the height and the horizontal distance of the cowl as shown in Equation 10 and Equation 11 respectively.

$$X = \frac{sin(\beta_2 - \theta_1)L}{sin(180 - \beta_2)} \tag{8}$$

$$y = \frac{h}{tan(\beta_2)} \tag{9}$$

$$tan(\beta_1) = \frac{h\tan(\beta_2)}{\alpha tan(\beta_2) + h} \tag{10}$$

$$x + y = (\frac{Lsin(\beta_2 - \theta_1)}{sin(180 - \beta_2}) + \frac{h}{tan(\beta_2)} \tag{11}$$

3.3 Diffuser

The flow in the diffuser is considered to be isentropic while satisfying the conservation of mass and momentum. To determine the properties across the diffuser, small perturbation theory is used to get the differential conservation of mass, momentum (with no body forces) and energy as shown in Equation 12, Equation 13, and Equation 14

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{du}{u} = 0 \tag{12}$$

$$\frac{d\rho}{\rho} + udu = 0 \tag{13}$$

$$dh + udu = 0 \tag{14}$$

Solving these equations together along with the isentropic flow, a differential change between the area and velocity based on mach number can be generated as shown in Equation 15. Accordingly, if M is greater than 1 and dA/A is greater than 1, then du/u is also greater than 1. Accordingly, a diffuser which has a positive area differential with length will increase the flow speed if the mach number is greater than 1. However, if M is less than 1, then the du/u becomes negative; meaning that the flow speed decreases as the subsonic flow moves across the diffuser. To determine the properties of the flow across the diffuser, the mass flow rate is considered to be the same at every point across the diffuser. By doing so, area ratio of the diffuser can be generated using the mach number ratio or vice versa as shown in Equation 16

$$\frac{dA}{A} = (M^2 - 1)\frac{du}{u} \tag{15}$$

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}(\frac{(1 + \frac{\gamma-1}{2}M_1^2)}{(1 + \frac{(\gamma-1)}{2}M_2^2)})^{(\frac{\gamma+1}{2(\gamma-1)})} \tag{16}$$

3.4 Flameholder

The flameholder is modelled to provide an adiabatic pressure drop in the flow stream. The change in pressure is governed by the relation given in Equation 17. It is also assumed that the area profile does not change across the flameholder. In order to determine the temperature and the mach number of the flow after flameholder, conservation of mass flow rate is used. The mass flow rate can be simplified to the basic characteristics of the flow as mentioned in Equation 18. Furthermore, the energy is also conserved as given in Equation 19

$$\frac{\Delta P}{P} = 0.81\gamma M^2 \tag{17}$$

$$\dot{m} = \frac{P}{RT}AM\sqrt{T\gamma R} \tag{18}$$

$$C_pT_1 + \frac{M_1^2\gamma RT_1}{2} = C_pT_2 + \frac{M_2^2\gamma RT_2}{2} \tag{19}$$

Unknown variables are T_2 and M_2. Accordingly, the linear system of equations Equation 19 and Equation 18 are solved using method of substitution to determine expressions for both variables as shown in Equation 20 and Equation 21.

$$T_2 = (2C_PT_1 + M_1^2\gamma RT_1)(\frac{1}{2C_P + M_2^2\gamma R}) \tag{20}$$

$$\frac{P_1M_1}{\sqrt{T_1}} = \frac{P_2M_2\sqrt{2C_P + M_2^2\gamma R}}{\sqrt{2C_PT_1 + M_1^2\gamma RT_1}} \tag{21}$$

3.5 Combustor

To model the combustor, 1D quasi flow with heat addition is used. While the conservation of mass and momentum are same as to regular 1D quasi flow, the conservation of energy now involves and additional term 'q' which refers to the heat addition as shown in Equation 22. The term q refers to the rate of heat added to the incoming flow. It is described as the ratio of the rate of heat provided by the fuel to the mass flow rate of the incoming flow, shown in Equation 23

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{q_2} \tag{22}$$

$$q = \frac{\dot{q}}{\dot{m}} \tag{23}$$

The flow properties across the combustor are determined iteratively by first calculating the the stagnation temperature by using Equation 24 and then finding the stagnation temperature at the M = 1 state. Once all these states are known the relative ratios of the pressure and temperature are determined using equations given in Equation 25 and Equation 26

$$q = C_p(T_{02} - T_{01}) \tag{24}$$

$$\frac{T}{T*} = M^2(\frac{1+\gamma}{1+\gamma M^2})^2 \tag{25}$$

$$\frac{P}{P*} = \frac{1+\gamma}{1+\gamma M^2} \tag{26}$$

For the base design, the combustor section only intakes the specified equivalence ratio. This is changed later when the engine's performance is analyzed for various mach numbers. Accordingly, the equivalence ratio is made variable to provide a constant exit mach number for the combustor. The mass flow rate of the fuel is determined using Equation 27, $\dot{q}$ of fuel using Equation 28, and finally 'q' using Equation 29. For the purposes of the fuel, hydrogen is used and has lower heating value (LHV) of 120 $\frac{MJ}{kg}$. Moreover, the psychometric ratio for hydrogen is 0.0291 which is used to determine the mass flow rate in Equation 27.

$$\dot{m}_{\text{fuel}} = 0.0291\phi\dot{m}_{\text{air}} \tag{27}$$

$$\dot{q} = LHV\dot{m}_{\text{fuel}} \tag{28}$$

$$q = \frac{\dot{q}}{\dot{m}_{\text{air}}} \tag{29}$$

Although it is assumed that the flow burns instantaneously in the combustor, it is still important to determine the length of the combustor which is done by determining the time taken for the fuel to burn using Equation 30. The inlet conditions are used to determine the time constant. To get the length of the combustor, average of the inlet and exit velocities in the combustor is taken and multiplied with τ.

$$\tau = 325p^{-1.6}exp(-0.8\frac{T_0}{1000}) \tag{30}$$

3.6 Converging Nozzle

Similar to the diffuser, isentropic quasi 1D flow relations are used to determine the properties of the flow across the converging nozzle. As it can be seen from Equation 15, if M is less than 1 and dA/A is less than 1, then du/u is greater than 1. This means that as the subsonic flow is constricted due to a negative area differential, the flow gets faster. However, the maximum speed the subsonic flow can attain by constricting the area is the local speed of sound. This is referred to as throat area where mach number is equal to 1. Like the isentropic flow in diffuser, the area ratio and mach number can be solved using Equation 16 if one is known. Since the desired $M_2 = 1$, the corresponding area profile can be derived. The temperature and pressure can be derived from the stagnation parameters since they remain the same throughout the isentropic flow.

3.7 Diverging Nozzle

The diverging nozzle for the ramjet engine is a 2D minimum length nozzle designed using method of characteristics (MOC). To determine the properties of a 2D flow, the flow is considered to be inviscid, compressible, and irrotational. Using the third property velocity potential functions are generated and solved to determine solutions for characteristic lines. These are special lines which can be used to calculate properties along the flow because they have a specific pattern in terms of how their properties evolve. The two lines are called left and right turning characteristic as shown in Figure 3.3. A variable 'K' is a dummy parameter for both the characteristics.

The magnitude of the constant 'K' for both left and right turning characteristic is given in Equation 31 and Equation 32. As seen, the properties of point 3 depends upon the properties of 1 and 2. The corresponding location of the third point in the global axes is given the linear system of equations given in Equation 33 and Equation 34. Once the properties of the third point are known, the process can be rolled further to generate more points and design the entire nozzle.

$$\theta_3 - \nu_3 = \theta_2 - \nu_2 = K_{2,\text{plus}} \tag{31}$$

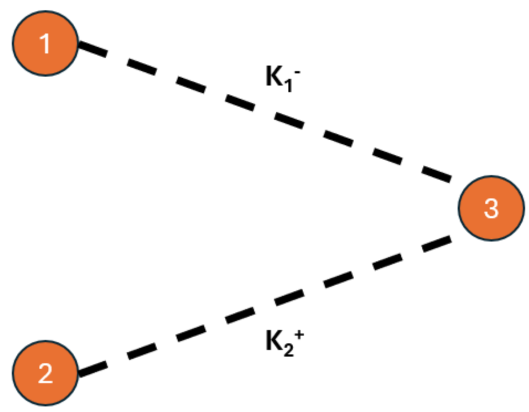


Figure 3.3: Illustration of the left and right turning characteristics used to design a 2D minimum length nozzle.

$$\theta_3 + \nu_3 = \theta_1 + \nu_1 = K_{1,\text{minus}} \tag{32}$$

$$\frac{y_3 - y_2}{x_3 - x_2} = tan(\frac{(\theta_3 + \theta_2) + (\mu_3 + \mu_2)}{2}) \tag{33}$$

$$\frac{y_3 - y_1}{x_3 - x_1} = tan(\frac{(\theta_3 + \theta_1) - (\mu_3 + \mu_1)}{2}) \tag{34}$$

In order to use the method of characteristics to design the nozzle, the points generated are characterized under three categories: wall, centerline, and other points. Description of these points is given as below:

1. Wall: These points generate no reflection or characteristic wave so the θ of the wall equals to that of the incident wave.

2. Centerline: These points have $\theta = 0$ because there is no turning of flow and no reflection. Also, the x coordinate is also set to 0.

3. Others: The properties of these points are generated using left and right turning characteristic that intersect at the point in interest.

Using this methodology, a mesh is generated as shown in Figure 3.4 and the properties: $x, y, \theta, \nu, M, \mu, K^+, K^-$ are determined using the relations above. For temperature and pressure, stagnation parameters are used because the flow is isentropic so they remain the same throughout the flow.

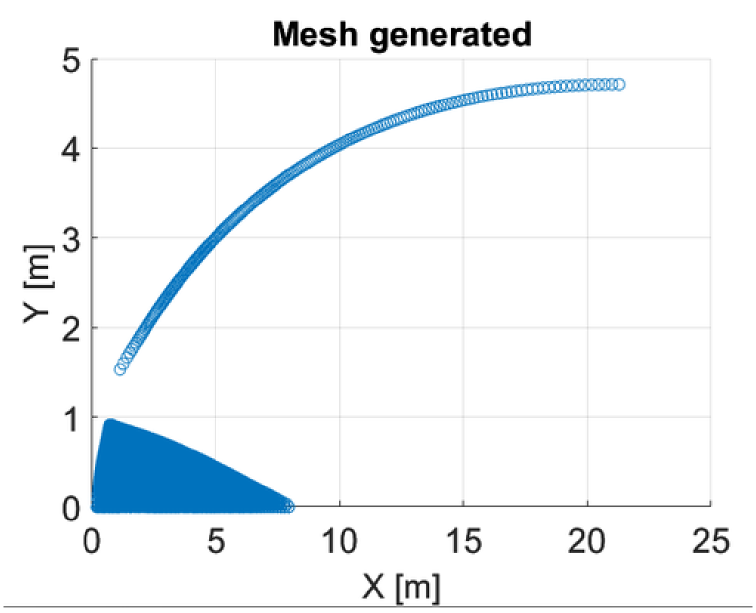


Figure 3.4: Example of the mesh generated for a 2D minimum length nozzle designed using method of characteristics.

3.8 Outside Casing

To finish the design and close the loop, an external casing designed as follows. On the top, the casing runs from the tip of the cowl to the end of the diverging nozzle. On the bottom, the casing runs from the tip of the inlet to the end of the diverging nozzle. An incline is introduced the starting of the casing which runs till an arbitrary x point chosen by eye and then it is flat at the back. Due to being inclined, the casing is a source of drag.

4 Integration of all components

In order to solve for the properties of the flow throughout the engine, the temperature, pressure, and other important characteristics are passed across each component. To make the understanding easier, each component is modelled as a different function in MATLAB and the "ramjet_main.m" file integrates all the components together in the following way:

1. To start off the process, inlet temperature and pressure properties are determined using the 1976 standard atmospheric model. The corresponding file used is "atmosphere.m". It takes in the elevation and returns the corresponding temperature (T1) and pressure (P1).
2. T1, P1, and M1 are passed into the inlet functions which passes the flow through a 2 step inlet, hence 2 oblique shocks, and finally a normal shock to turn the supersonic flow into subsonic. The function also designs the cowl and finally returns the inlet area, inlet drag, T2, P2, M2 $\theta_1, \& \theta_2$.
3. The inlet area along with the flow characteristics are passed into the diffuser which designs the area profile based on the specified exit number. The horizontal design of the diffuser is done by eye glance and the only constraint considered is to prevent the flow from turning a lot. Accordingly, the length of the diffuser is 4 times the exit area of the diffuser. Finally, the diffuser returns the flow properties along with the exit area.
4. Next, the flow properties are passed to the flameholder function which provides an adiabatic pressure loss as discussed before. The flow conditions are returned.
5. Next, the flow properties are passed through combustor which iterates over the ϕ to provide a specified exit mach number. It returns the mass flow rate of the fuel and the flow conditions.
6. Next, the flow properties are passed through the converging nozzle which determines the area profile to get the exit mach number to be 1. Like the diffuser, the length of the converging nozzle is 4 times the exit area. The throat area and the flow conditions at the exit are returned.
7. Finally, the flow properties are passed through the diverging nozzle which designs the 2D minimum length nozzle based on method of characteristics as discussed before. The function returns the exit area of the nozzle along with the flow properties.

5 Post Processing

After putting all the components together, the performance of the engine is determined using the following metrics:

5.1 Thrust generated

The maximum thrust generated by the engine is determined using conservation of momentum on the control volume which runs along with casing of the engine. The inlet of the diffuser is considered as the entrance for the flow and the end of the diverging nozzle is considered as the exit point. By performing conservation of momentum, Equation 35 is derived. For design purposes the contribution of $\dot{m}_{\text{fuel}}$ is considered to be 0 accordingly, the mass flow rate of the fuel remains consistent.

$$\text{Thrust} = (\dot{m_a}ir + \dot{m_f}uel)v_{\text{exit}} - \dot{m_a}irv_{\text{in}} + P_{\text{exit}}A_{\text{exit}} - P_{\text{in}}A_{\text{in}} \tag{35}$$

5.2 Inviscid drag

The inviscid drag comes from the casing of the engine as well as the inlet because that is considered as part of the control volume. To determine the drag for all 4 of the inclined surfaces (two surfaces for the casing and two for the 2 step inlet), Equation 36 is used. The pressures are calculated using the oblique shock relations, the width is fixed as 1 m and the length is calculated using euclidean distance formulas. Accordingly, the actual thrust generated is maximum thrust calculated in Equation 35 minus the drag.

$$drag = \sum_{i=1}^{4} P_i L_i sin(\theta_i) \tag{36}$$

5.3 Other key characteristics

Besides the thrust, other important characteristics are also determined to analyze the performance of the engine.

5.3.1 Specific Thrust

Specific thrust is the thrust per unit air mass flow rate of a jet engine as shown in Equation 37. Low specific thrust engines tend to be more efficient of propellant (at subsonic speeds), but also have a lower effective exhaust velocity and lower maximum airspeed. High specific thrust engines are mostly used for supersonic speeds, and high specific thrust engines can achieve hypersonic speeds.

$$\text{Specific Thrust} = \frac{\text{Thrust}}{\dot{m}_{\text{air}}} \tag{37}$$

5.3.2 Specific Impulse

Specific impulse (usually abbreviated I_{sp}) is a measure of how efficiently a reaction mass engine, such as a rocket using propellant or a jet engine using fuel, generates thrust. A propulsion system with a higher specific impulse uses the mass of the propellant more efficiently. In the case of a jet engine, this means less propellant needed for a given δv so that the vehicle attached to the engine can more efficiently gain altitude and velocity.

$$I_{\text{sp}} = \frac{\text{Thrust}}{g\dot{m}_{\text{fuel}}} \tag{38}$$

5.3.3 Specific Fuel Consumption

The specific fuel consumption, denoted as C_j, measures the weight flow rate of fuel used for each unit of thrust produced and is a major figure of merit for engines. This variable is dependent upon the fuel flow rate for the actual thrust level produced and is often quoted by the manufacturer as evaluated at the static or takeoff thrust level,

$$\text{Specific Fuel Consumption} = \frac{\dot{m}_{\text{fuel}}}{\text{Thrust}} \tag{39}$$

6 Dummy Engine For Comparison & Other Characteristics

Lockheed's D21 Engine, Marquardt XRJ43-MA-20 shown in Figure 6.1, is considered as a comparison for HAWK 2.0 to down size the components and also check numbers. Also, parameters such as mass and fuel flow rate into engines are found online to make HAWK 2.0 more compatible with an actual ramjet.

Figure 6.1: Marquardt XRJ43-MA-20

Following are the important parameters derived to design HAWK 2.0.

1. XRJ43 produces 6.6 kN of thrust [3], 2.69 m in length and 0.72 m in diameter. [4].
2. A typical mass flow rate into a ramjet engine is about 45.3 kg/sec.[5]
3. A typical fuel flow rate into a ramjet engine is about 0.9 kg/sec. [5]

7 HAWK 1.0

The first engine developed by the Cooper Hawks Team is called HAWKS 1.0 as shown in Figure 7.1. Since this is the first iteration, a very basic crude design is put together to validate the accuracy and the precision of the code in integrating all of the components together. The engine is 30 m in length and 10 m in width. Compared to the Lockheed D21's engine, HAWK 1.0 is much more massive. Moreover, this engine is designed strictly for incoming flow with mach number of 3.25.

7.1 Design Parameters

The magnitude of the variable parameters used for the design of HAWK 1.0 are given in Table 1

7.2 Characteristics

After performing the post analysis, the following characteristics are derived. To summarize the few, HAWK 1.0 generates 98 kN of thrust for incoming mach number of 3.25 only (the only analyzed case). The inlet area is 1.24 m^2 and the exit area of the engine as a whole is 9.9 m. It has a air flow rate of 345 kg/sec and fuel flow rate of 2.01 kg/sec.

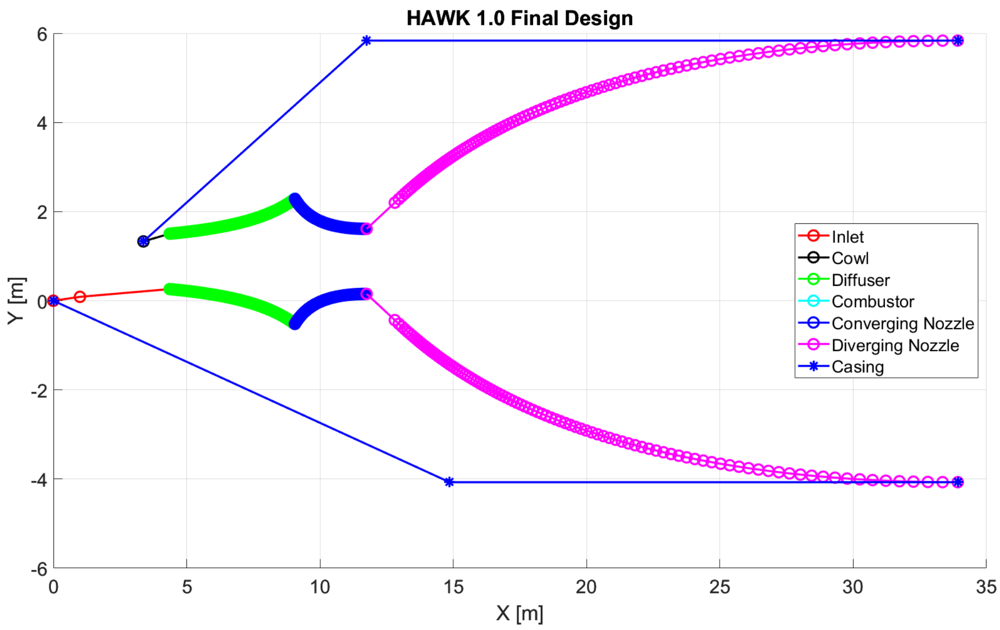


Figure 7.1: HAWK 1.0 Final Design

Parameter	Magnitude
Length of the individual steps in the 2 step inlet	1 m
Angles of the individual steps in the 2 step inlet	5, 10 degree
Diffuser exit mach number	0.2
Equivalence ratio in the combustor	0.2
Exit Mach Number	3.5

Table 1: Magnitude of the variable characteristics used to design HAWK 1.0

Parameter	Magnitude
Engine length	33.92 m
Engine Width	9.9 m
Thrust generated	98 kN
Specific impulse	5000 sec
Specific thrust	285.4 m/sec
Specific fuel consumption	5.03 e-5

Hawk 1.0 Characteristics

Parameter	Magnitude
Inlet area	1.24 m^2
Mass flow rate of air	345 kg/sec
Diffuser inlet mach	0.55
Diffuser exit area	2.8 m^2
Diffuser length	4.68 m
Combustor length	0.0096 m

Hawk 1.0 Characteristics (continued)

Parameter	Magnitude
Fuel flow rate	2.01 kg/sec
Combustor exit mach number	0.32
Converging Nozzle length	0.031 m
Throat area	1.4597
Diverging nozzle length	22.16 m
Diverging nozzle exit area	9.9 m

Hawk 1.0 Characteristics (continued)

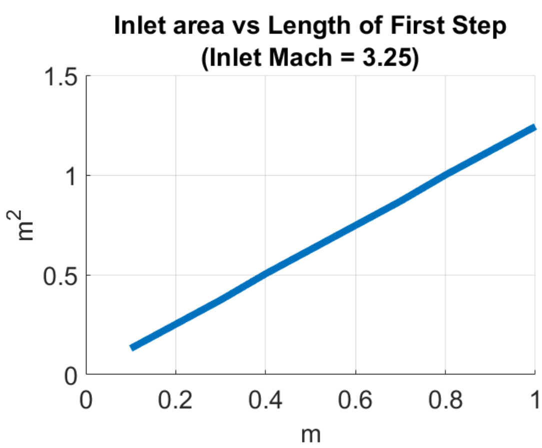
Table 2: Comparison of Hawk 1.0 Characteristics

8 Optimization

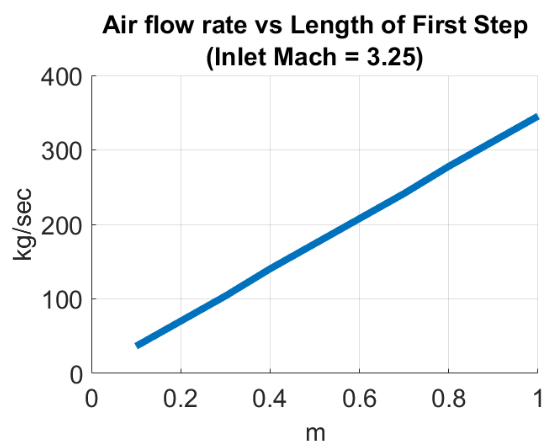
Clearly, HAWK 1.0 is way too powerful and huge to be actually used in the industry. Accordingly, an optimization study is performed where the variable parameters shown above are varied and their effect on the engine as a whole is discussed. Each of the following section iterates over one of the variable parameters while keeping the other variables fixed. As the optimization study continues, the suitable value chosen from the previous optimization study is chosen as the fixed parameter for the next one.

8.1 Length of inlet's first step

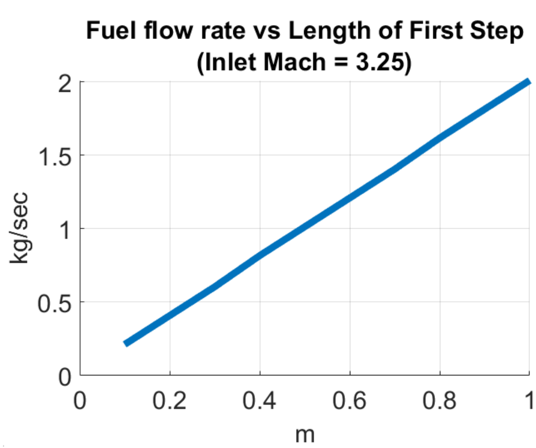
The first optimization routine is performed on the length of the first step of the 2 step inlet. Only the first step is considered because the length of the second step does not affect position of the cowl as explained before. Since the inlet area is determined based off the cowl height, it won't change if the length step is altered. For the analysis, $\phi = 0.2, M_{\text{exit}} - 3.5$, exit mach of diffuser = 0.2 and combustor exit mach = 0.32. The length of the first step is varied from 0.1 to 1 m and the results are shown in Figure 8.1.



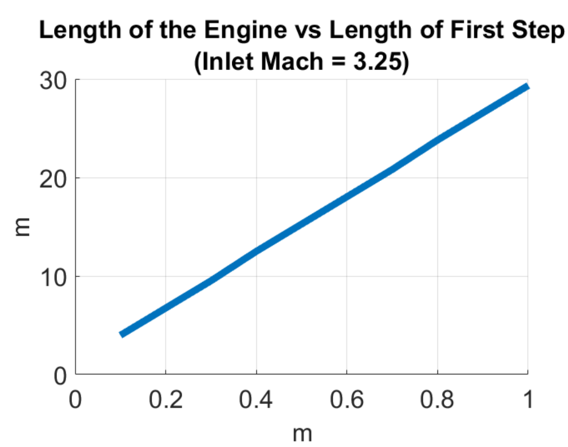
(a) Inlet Area vs Length of First Step (Inlet Mach = 3.25)



(b) Air Flow Rate vs Length of First Step (Inlet Mach = 3.25)



(c) Fuel Flow Rate vs Length of First Step (Inlet Mach = 3.25)



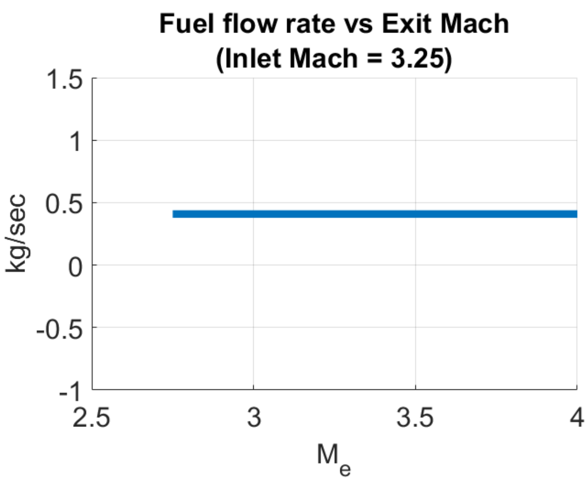
(d) Length of Engine vs Length of First Step (Inlet Mach = 3.25)

Figure 8.1: Effect of length of first step on other characteristics and components

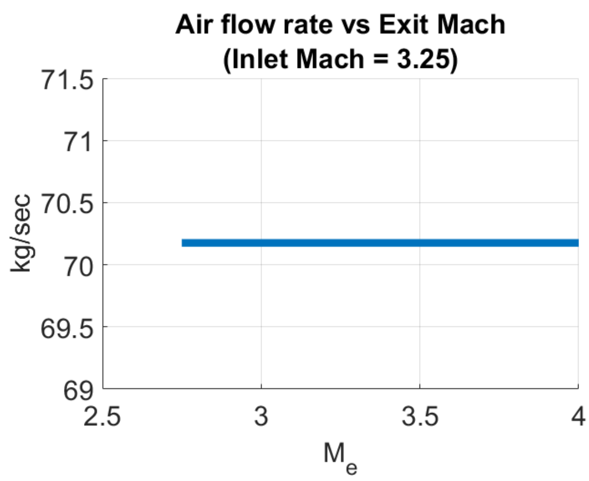
As expected, the inlet area and the mass flow rate of the air increases linearly with respect to the length. To generate similar characteristics as the Marquardt XRJ43-MA-20 engine, it can be seen that inlet length of 0.2 m should be enough to provide an air flow of about 50 kg/sec. By doing so, the fuel flow rate also goes down as necessitated and also brings down the length of the engine because every component needs to process much less air flow now. Accordingly, the length of the first step is set to 0.2 m.

8.2 Exit Mach number

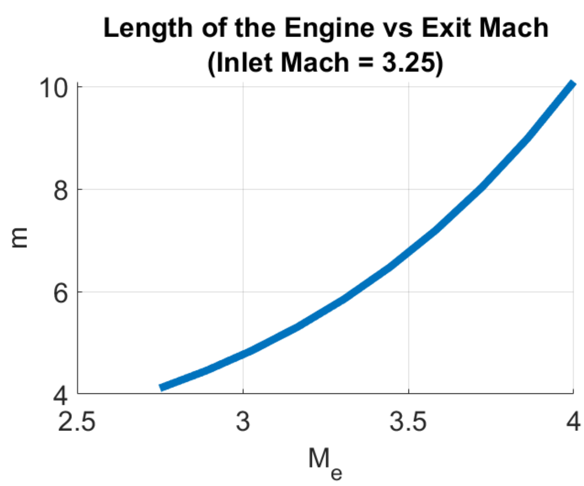
Next, the exit mach number is optimized. Since the exit mach number does not change the inlet or the corresponding parts, the fuel flow rate and the air flow rate are constant from the previous optimization step. The results are shown in Figure 8.2As expected the length of the engine decreases because the smaller exit mach number requires a smaller nozzle. To minimize the length further, exit mach number of 3 is set.



(a) Fuel Flow Rate vs Exit Mach (Inlet Mach = 3.25)



(b) Air Flow Rate vs Exit Mach (Inlet Mach = 3.25)

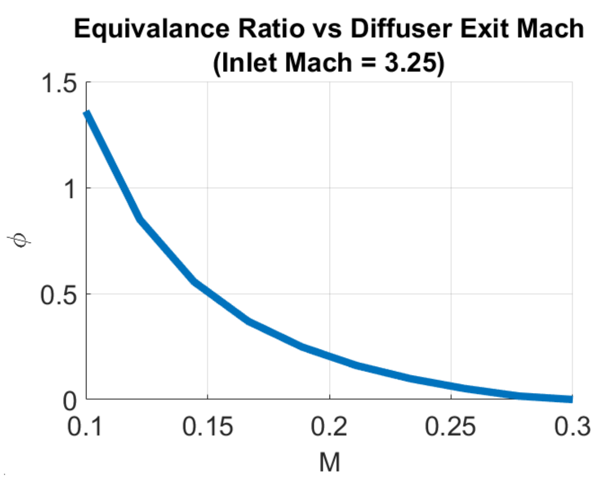


(c) Length of the Engine vs Exit Mach (Inlet Mach = 3.25)

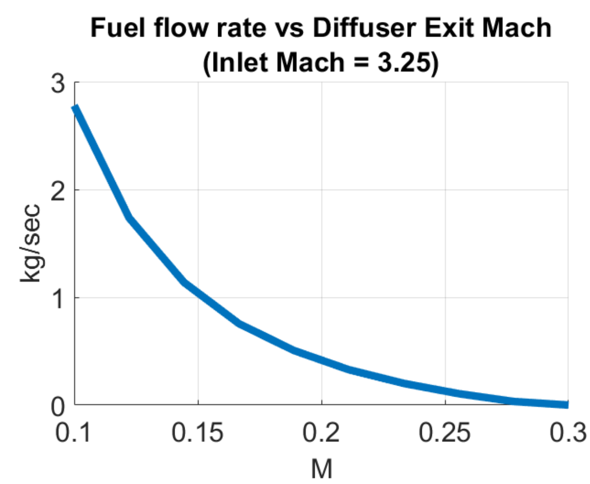
Figure 8.2: Effect of exit mach number on other characteristics and components

8.3 Diffuser exit mach number

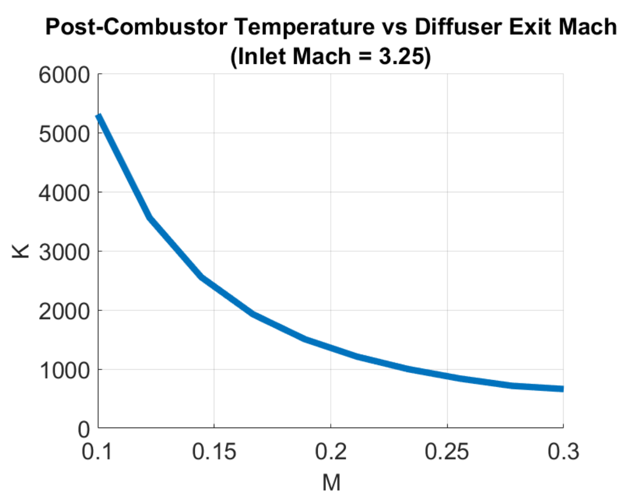
For the next optimization step, the diffuser exit mach number is varied from 0.1 to 0.3 while keeping other properties constant. The exit mach of the combustor is set to 0.32 and other parameters are copied from the previous optimization steps. The results are shown in Figure 8.3. As expected, the equivalence ratio and the fuel flow required decrease as the diffuser exit mach number increases because it gets closer to the required exit mach of the combustor (0.32). However, the key thing to notice is the post combustor temperature which goes extremely high as the diffuser exit mach decreases. As mentioned, the max desired temperature is around 1800-1900 K which requires the diffuser exit mach to be greater than 0.17. Accordingly, 0.2 is set for the diffuser exit mach.



(a) Equivalance Ratio vs Diffuser Exit Mach (Inlet Mach = 3.25)



(b) Fuel Flow Rate vs Diffuser Exit Mach (Inlet Mach = 3.25)

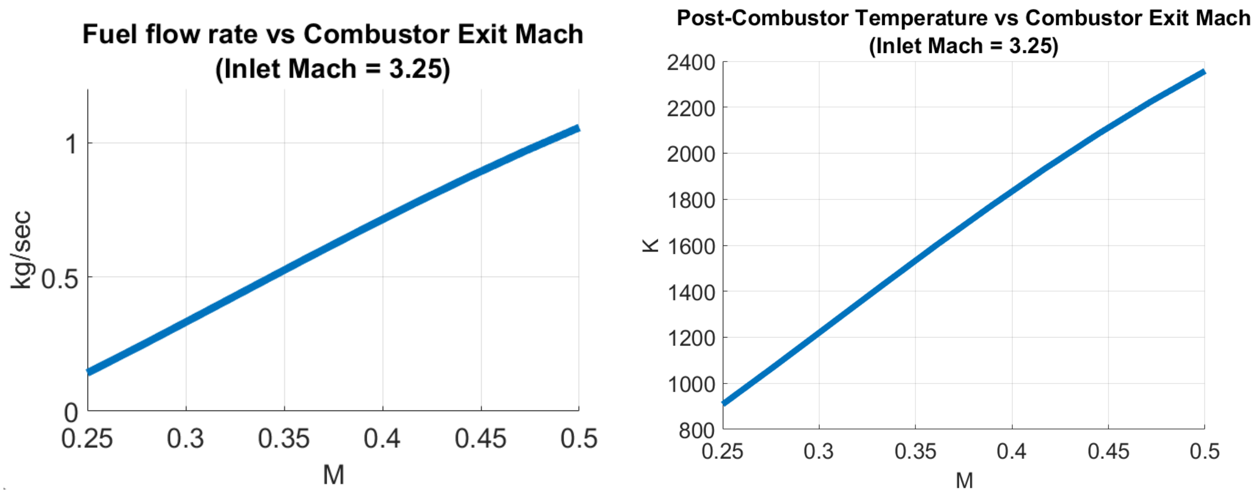


(c) Post-Combuster Temperature vs Diffuser Exit Mach (Inlet Mach = 3.25)

Figure 8.3: Effect of length of first step on other characteristics and components

8.4 Combustor Exit Mach number

For the final optimization, the exit mach number of the combustor is varied from 0.25 to 0.4 while carrying the optimized parameters from the previous steps. The key results are shown in Figure 8.4. As expected, the fuel flow rate increases as the exit mach number of the combustor increases because it requires more heat to become faster. The important thing to to look is the post combustor temperature. Since we want the max temperature to be in the range of 1800 - 1900 K, the exit mach number satisfies the that condition. It is decided to not change the combustor exit mach to allow less consumption of fuel.



(a) Fuel Flow Rate vs Combustor Exit mach (Inlet Mach = 3.25)

(b) Post-Combustor Temperature vs Combustor Exit mach (Inlet Mach = 3.25)

Figure 8.4: Effect of converging nozzle inlet mach number on other characteristics and components

9 HAWK 2.0

Using all the parameters generated from the optimization scheme as ball park numbers for the variable parameters, HAWK 2.0 is designed as shown in Figure 9.1. The engine is 2.72 m in length and 0.56 m in width which is very similar to the Lockheed's D21 Engine, Marquardt XRJ43-MA-20.

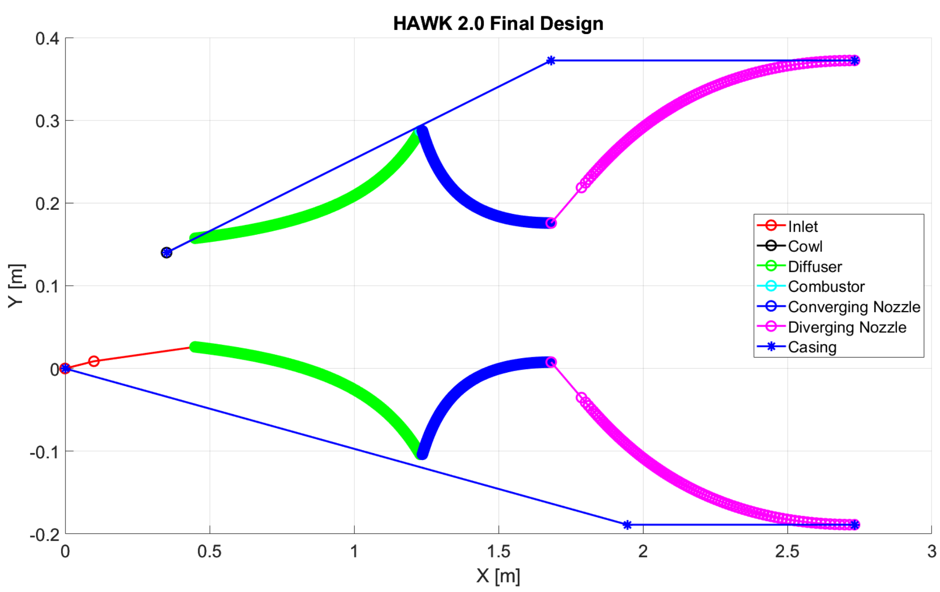


Figure 9.1: HAWK 2.0 Final Design

9.1 Design Parameters

After trial and error, the final magnitudes of the variable parameters determined are shown in Table 3.

Parameter	Magnitude
Length of the individual steps in the 2 step inlet	0.1 m
Angles of the individual steps in the 2 step inlet	5, 10 degree
Diffuser exit mach number	0.15
Equivalence ratio in the combustor	0.3
Exit Mach Number	2.75

Table 3: Magnitude of the variable characteristics used to design HAWK 2.0

9.2 Comparison between HAWK 2.0 designed for incoming M = 2.75 and 3.25

The final optimization performed to design HAWK 2.0 is to determine mach number of the incoming flow. Accordingly, the respective area profiles for the diffuser, converging nozzle, and other important characteristics are saved as .mat files for both 2.75 and 3.25 mach numbers and their performance is analyzed over the range of mach numbers in between. As seen in Figure 9.2, the temperature of the engine with incoming mach number of 2.75 gets above 1900 K which is set as the maximum. Accordingly, that design is rejected and HAWK 2.0 with incoming mach number of 3.25 is selected as the final design.

Despite this, the inlet area used for HAWK 2.0 is from the design using 2.75 as the incoming mach number. Since the cowl is not adjustable, it is necessary to make sure that no oblique shocks go into the engine. Accordingly, the cowl is fixed and designed in a way that the shocks from the inlet will intersect at the tip. However, shocks from incoming mach numbers greater than 2.75 will intersect somewhere outside the cowl which is inefficient but unavoidable.

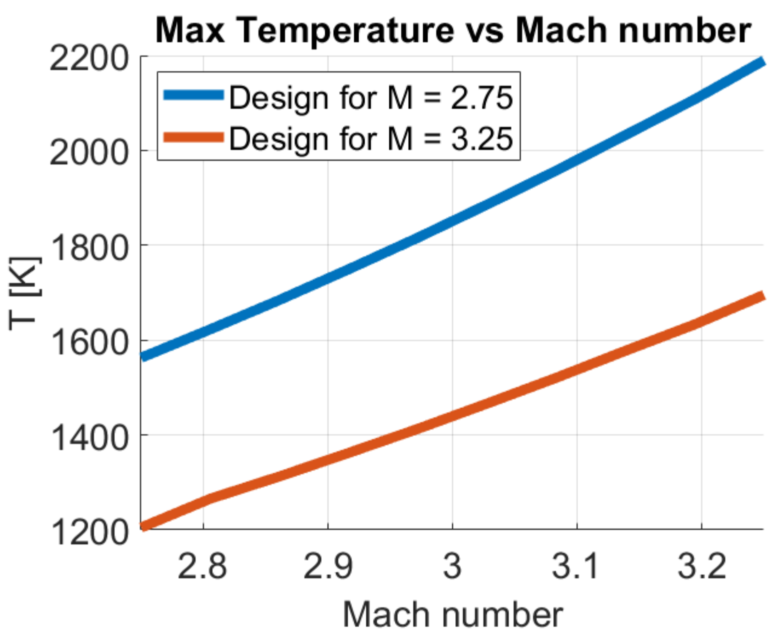


Figure 9.2: Max temperature of HAWK 2.0 designed for 2.75 and 3.25 incoming mach numbers.

9.3 Characteristics

Some of the key characteristics of the engine are shown in Table 4. Also plots of the key results about HAWK 2.0 as they vary for different mach numbers are shown in Figure 9.4

Parameter	**Magnitude**
Engine length	2.72 m
Engine Width	.56 m
Max Thrust generated	11 kN
Max Specific thrust	560 m/sec
Specific fuel consumption	1.5 e-5 sec/m

Hawk 2.0 Characteristics

Parameter	**Magnitude**
Inlet area	0.1613 m^2
Max mass flow rate of air	45 kg/sec
Max diffuser exit mach	0.55
Diffuser exit area	0.39 m^2
Diffuser length	0.78 m
Combustor length	0.0078 m

Hawk 2.0 Characteristics (continued)

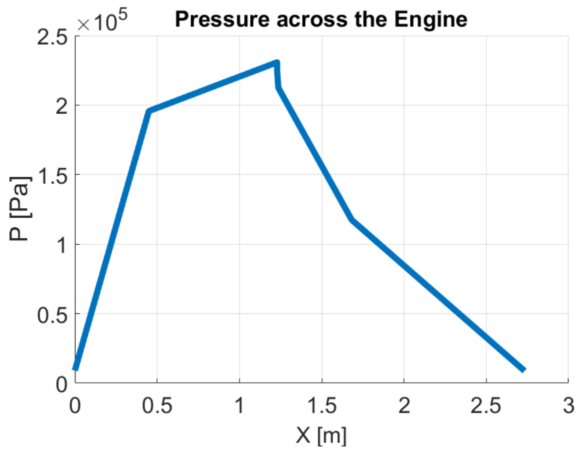
Parameter	**Magnitude**
Max fuel flow rate	0.32 kg/sec
Combustor exit mach number	0.26
Converging Nozzle length	0.44 m
Throat area	0.16 m^2
Diverging nozzle length	1.049 m
Diverging nozzle exit area	0.56 m^2

Hawk 2.0 Characteristics (continued)

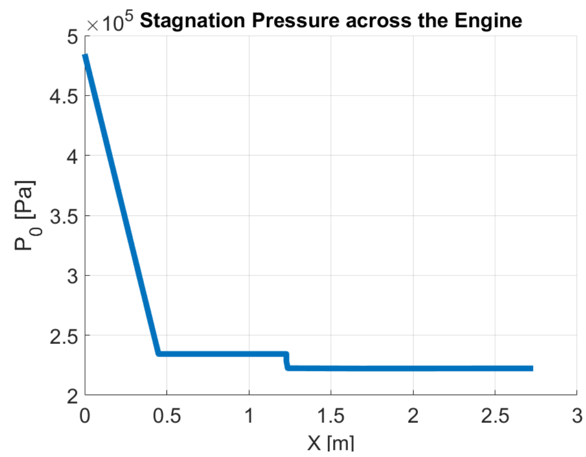
Table 4: Important HAWK 2.0 Characteristics

In order to ensure that the analysis is correct, the fluid properties are plotted across the axial length of the engine for the incoming mach number of 3.25, shown in Figure 9.3. Starting off with stagnation conditions, they are constant across the isentropic components: diffuser, converging and diverging nozzle. For stagnation pressure, the drop happens after the inlet because of the shocks. The stagnation temperature, on the other hand, remains constant across the shock as it should be. It does increase at the combustor though because it is not isentropic and also external heat is being added to the flow.

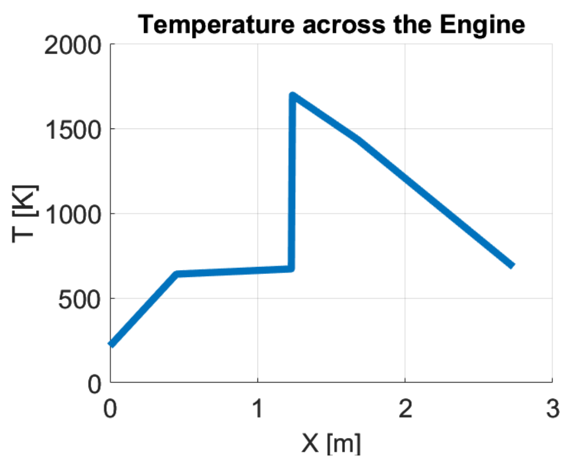
The static temperature increases across the shocks, increases slightly across the diffuser because the flow is slowing down, jumps to the maximum after combustor, and then drops due to the increasing speed of the flow. The pressure follows a similar profile. The entropy change with respect to the inlet is also constant across the isentropic devices which ensures the accuracy of the code.



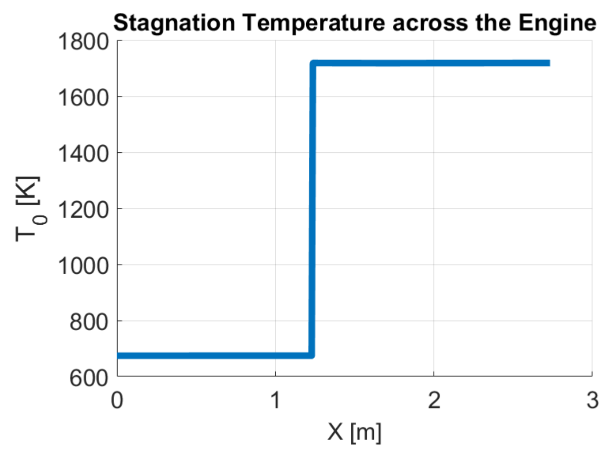
(a) Pressure across the engine)



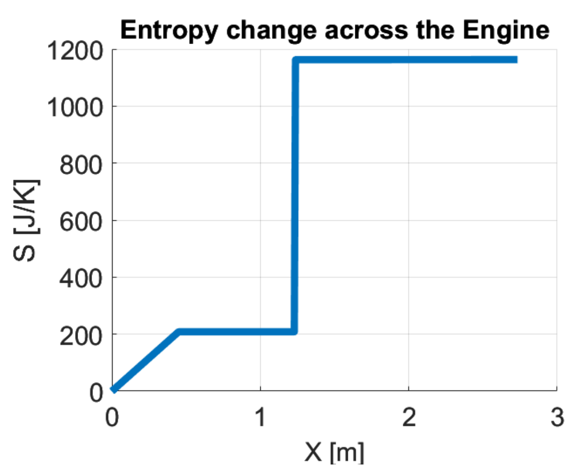
(b) Stagnation pressure across the engine



(c) Temperature across the engine



(d) Stagnation temperature across the engine



(e) Entropy change (with respect to the inlet conditions) across the engine

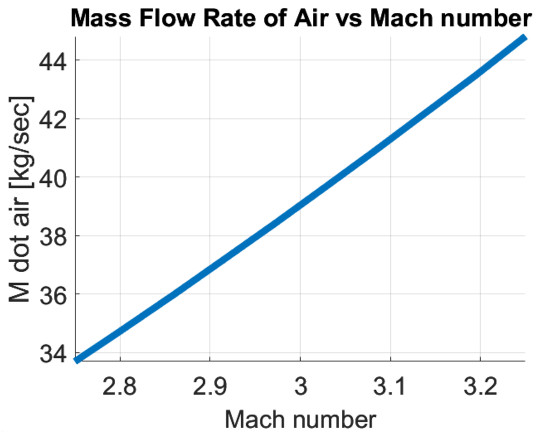
Figure 9.3: Change of the air flow properties as a function of the axial length. (For incoming M = 3.25)

Some interesting things to note about the performance of the engine across various mach numbers.

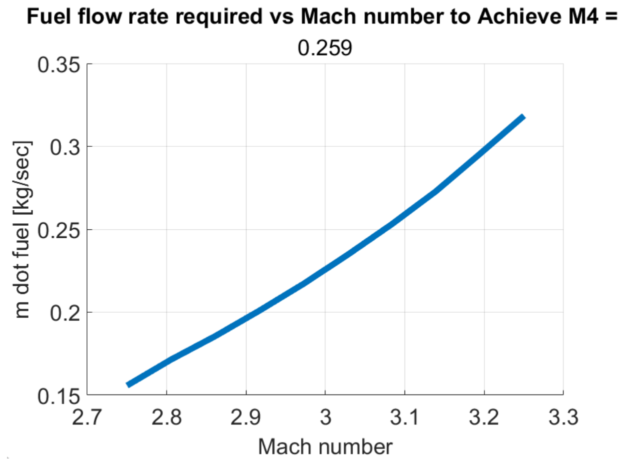
1. The time required to burn the fuel decrease as the mach number increases. Although it seems wrong, it makes sense because time taken to burn fuel is dependent on the decaying exponential of pressure and temperature. Since both temperature and pressure increase as the flow gets faster, the time taken to burn the fuel decreases.

2. As the flow becomes faster, more fuel is required to reach the specified combustor exit mach number. This also makes sense since the mass flow rate of the incoming flow increases which increases the heat requirement.

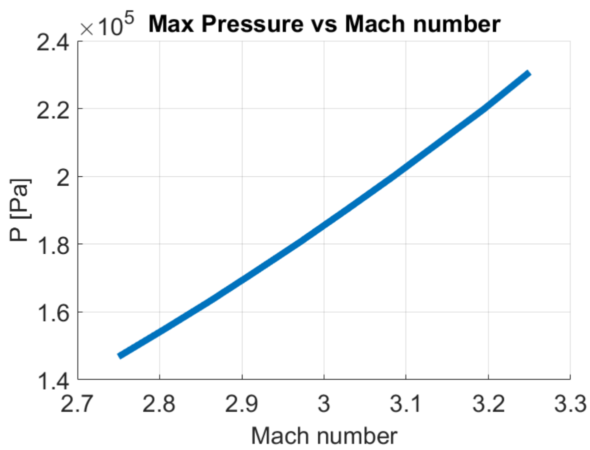
3. As required, the max temperature is below 1700 K which is good.



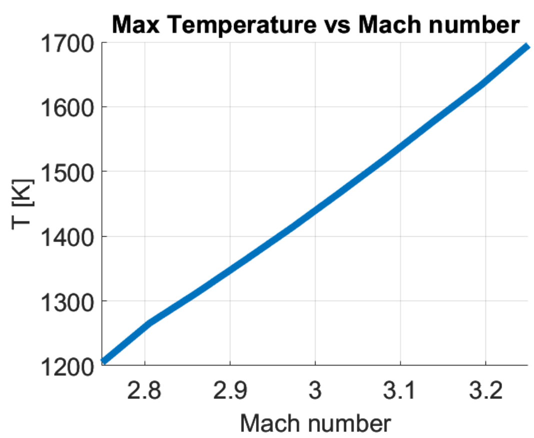
(a) Air flow rate vs Mach number)



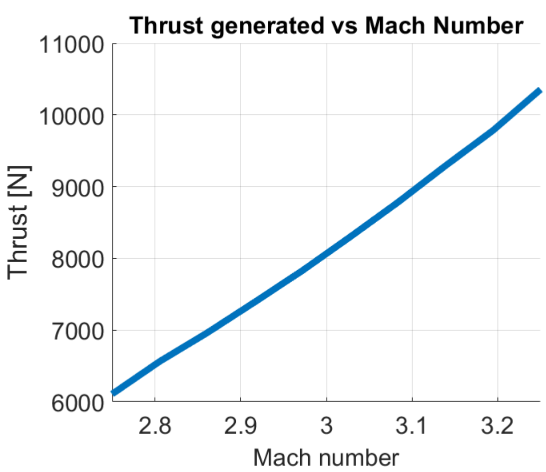
(b) Fuel flow rate vs Mach number



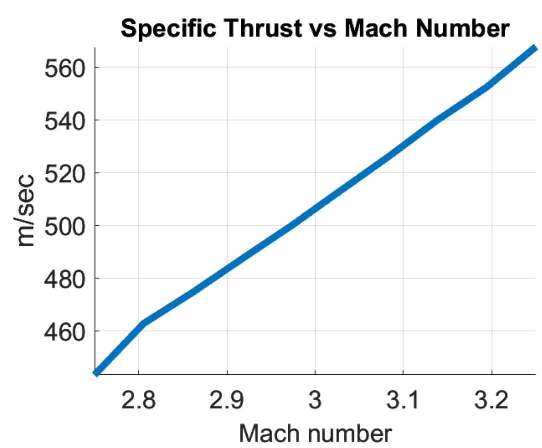
(c) Max pressure vs Mach number)



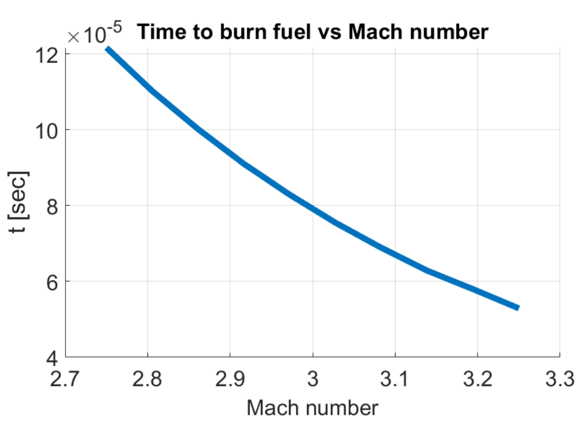
(d) Max temperature vs Mach number



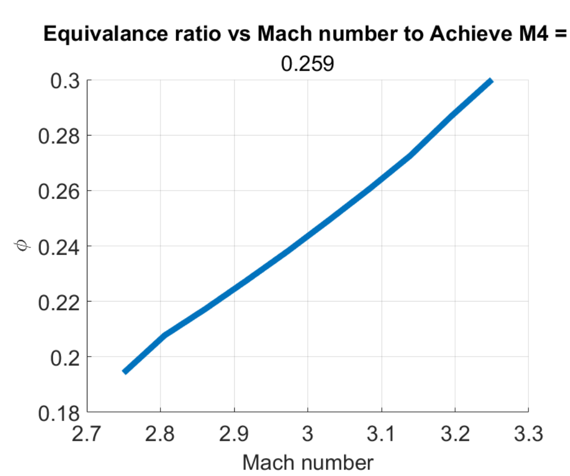
(e) Thrust generated vs Mach number



(f) Specific thrust vs Mach number



(g) Time to burn fuel vs Mach number



(h) Equivalence ratio vs Mach number

Figure 9.4: Effect of converging nozzle inlet mach number on other characteristics and components

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