

The misdiagnosed mediator:

Estimating the effect of maternal age on
preterm birth risk in the presence of
misclassified gestational hypertension

Kimberly A. H. Webb and Martin T. Wells

Joint Statistical Meetings

August 5, 2025

Mediation analysis

- **Mediation analysis** quantifies the effect of an **exposure (X)** on an **outcome (Y)**, mediated by some **intermediate (M)**.

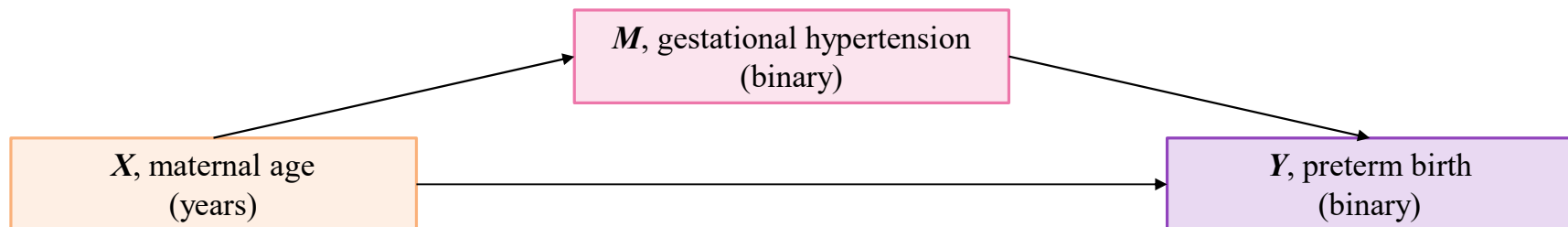
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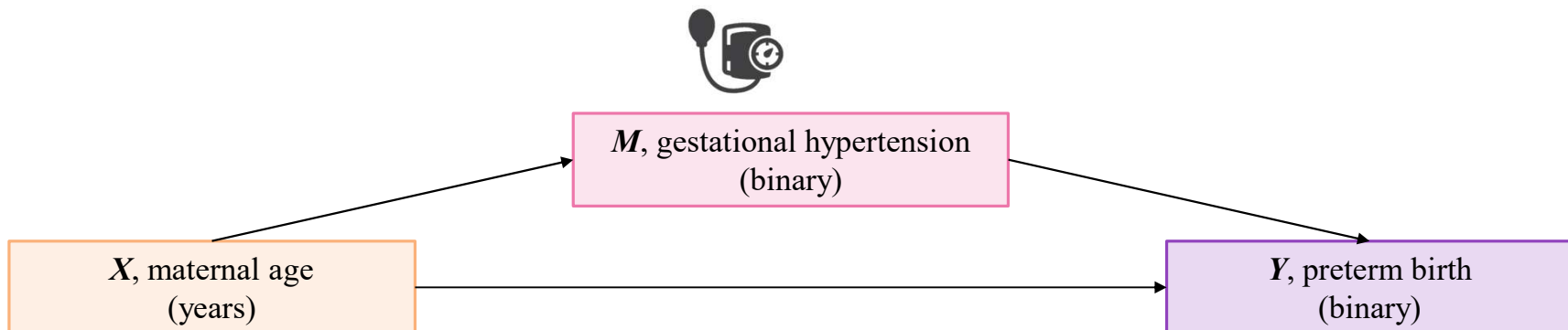
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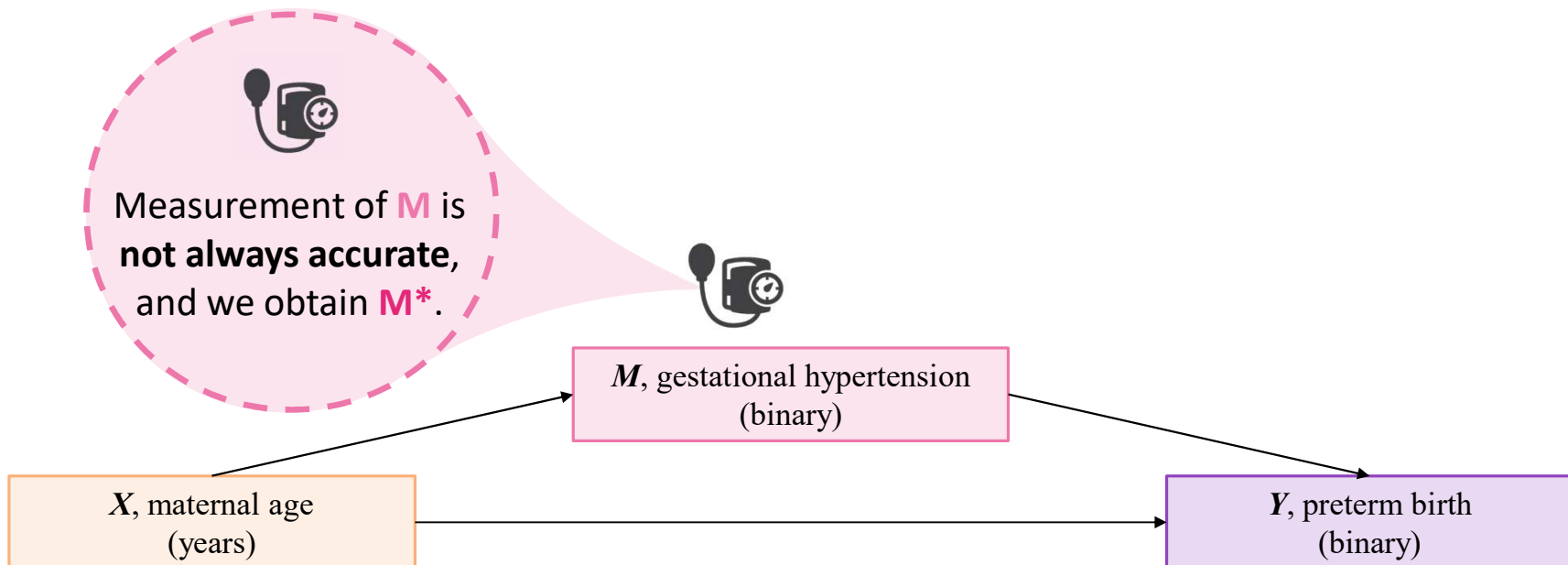
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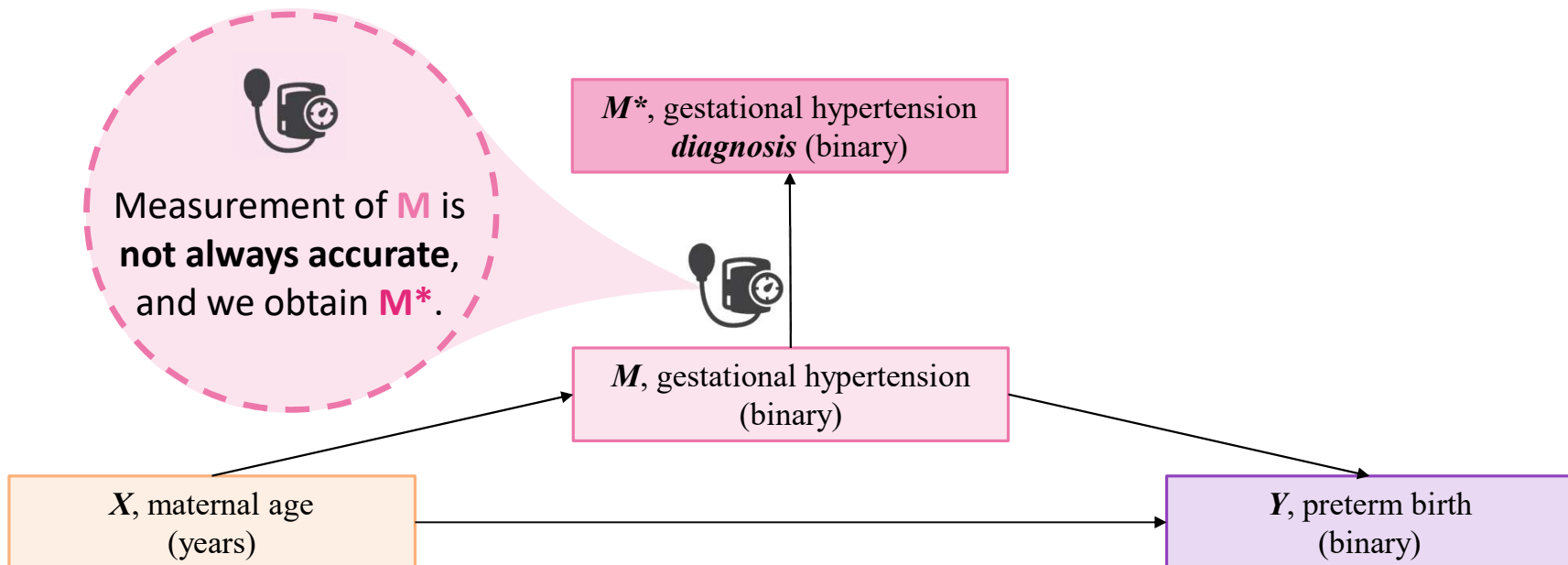
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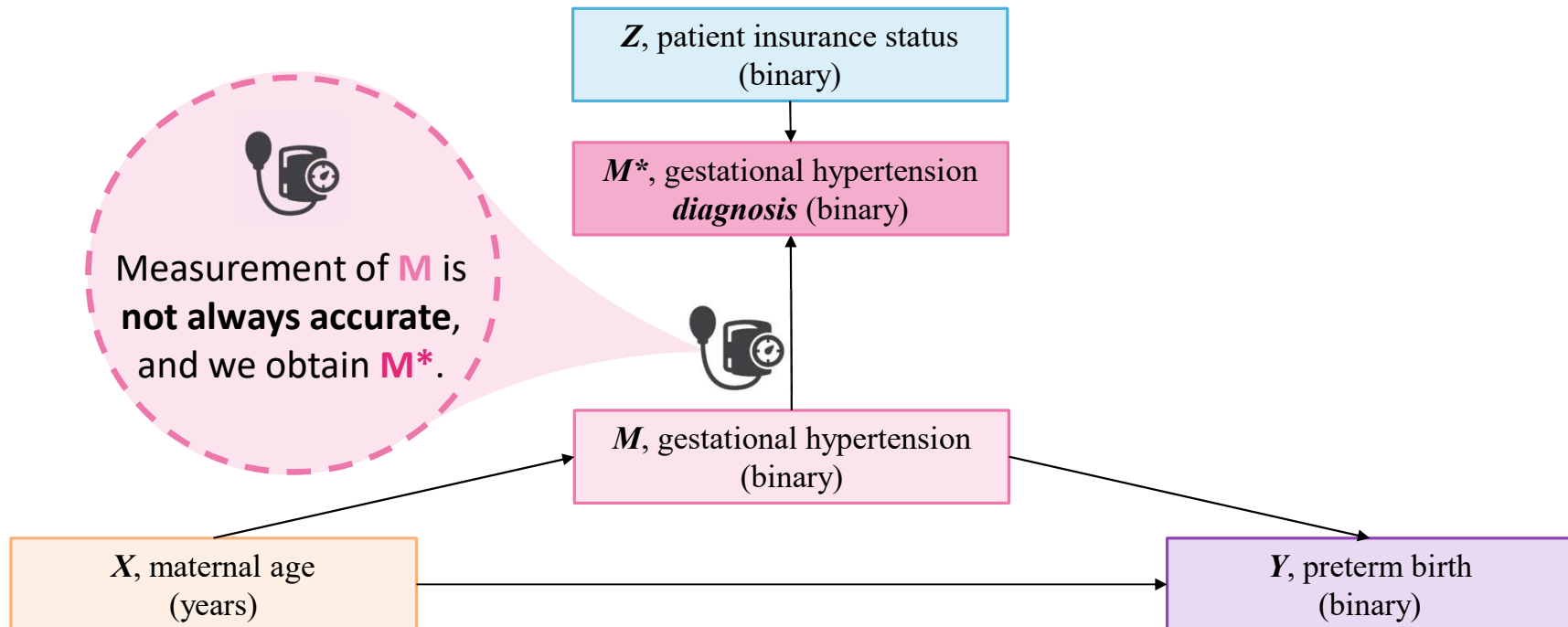
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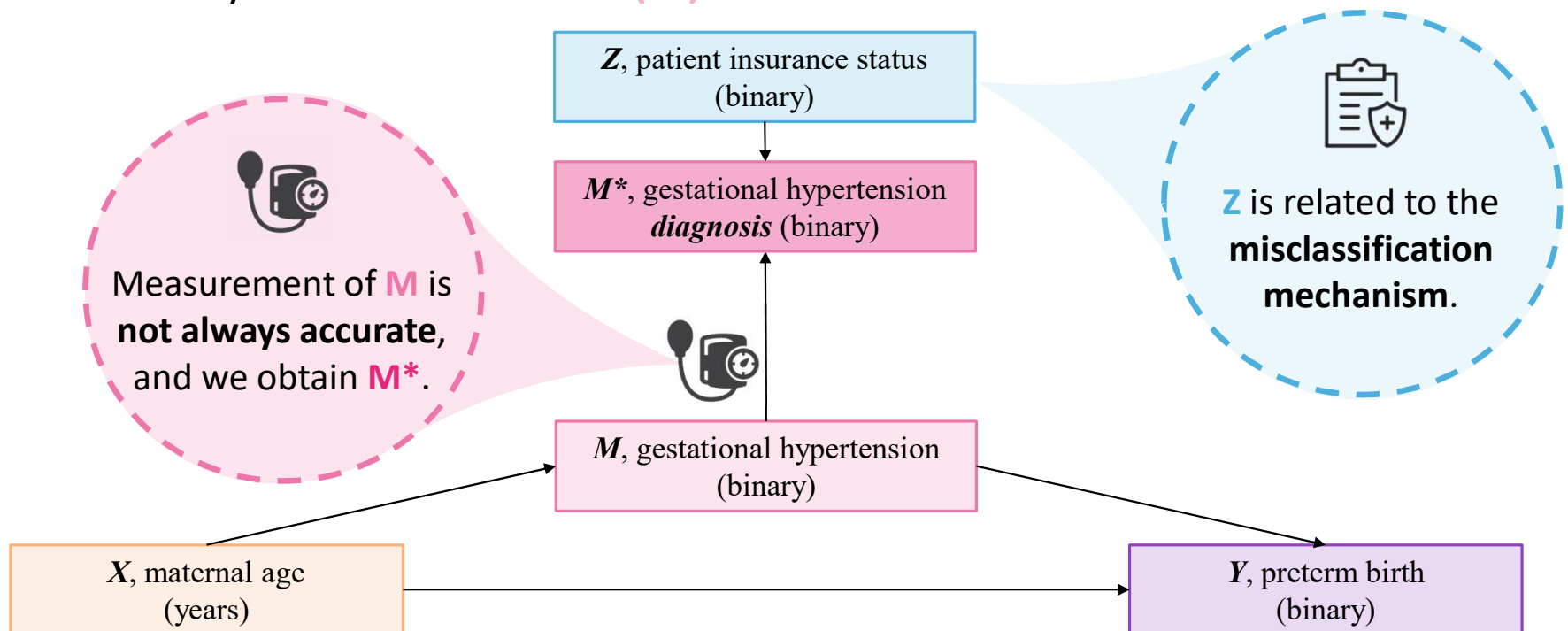
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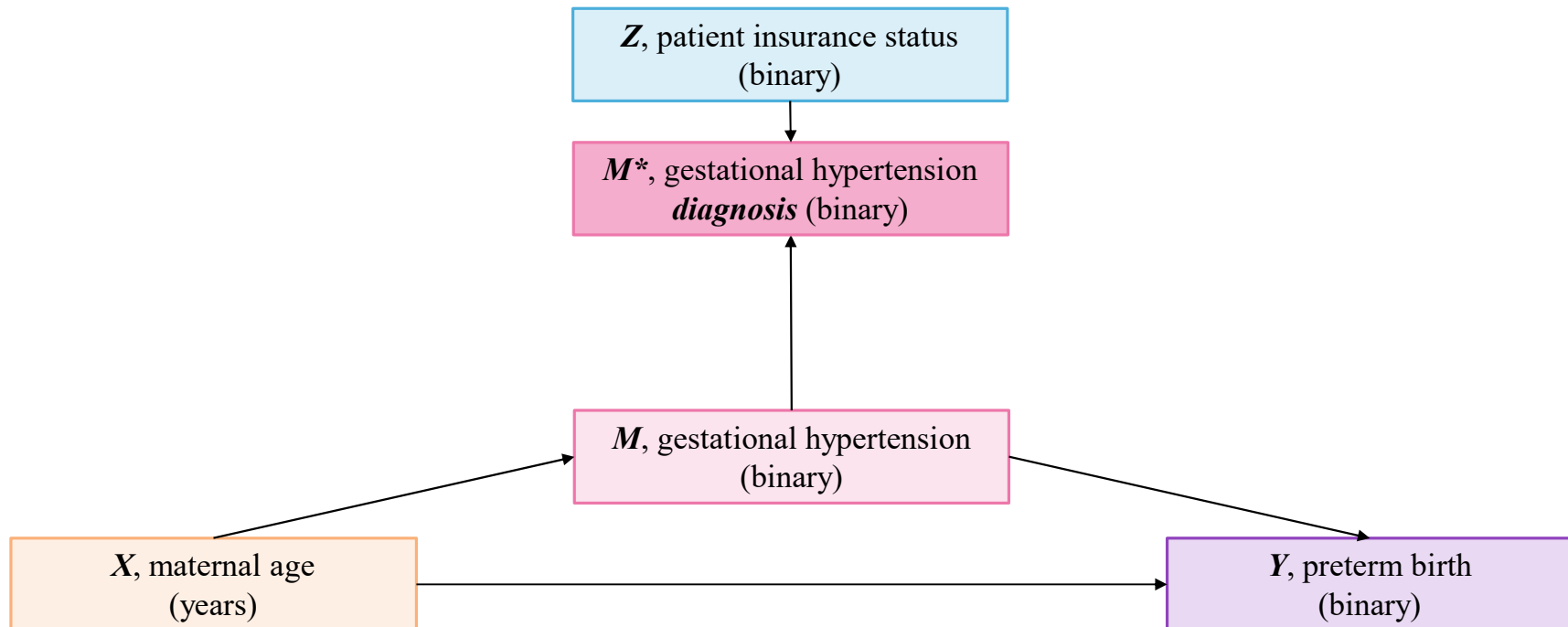


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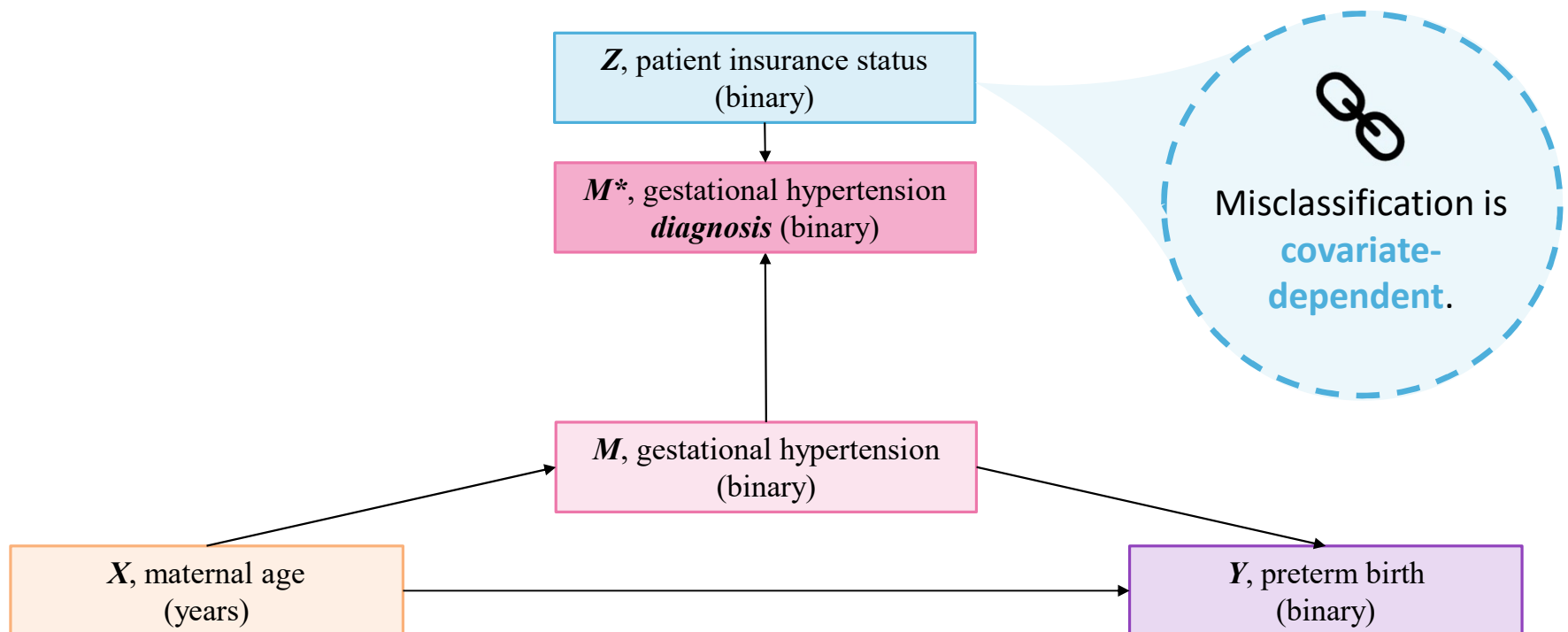
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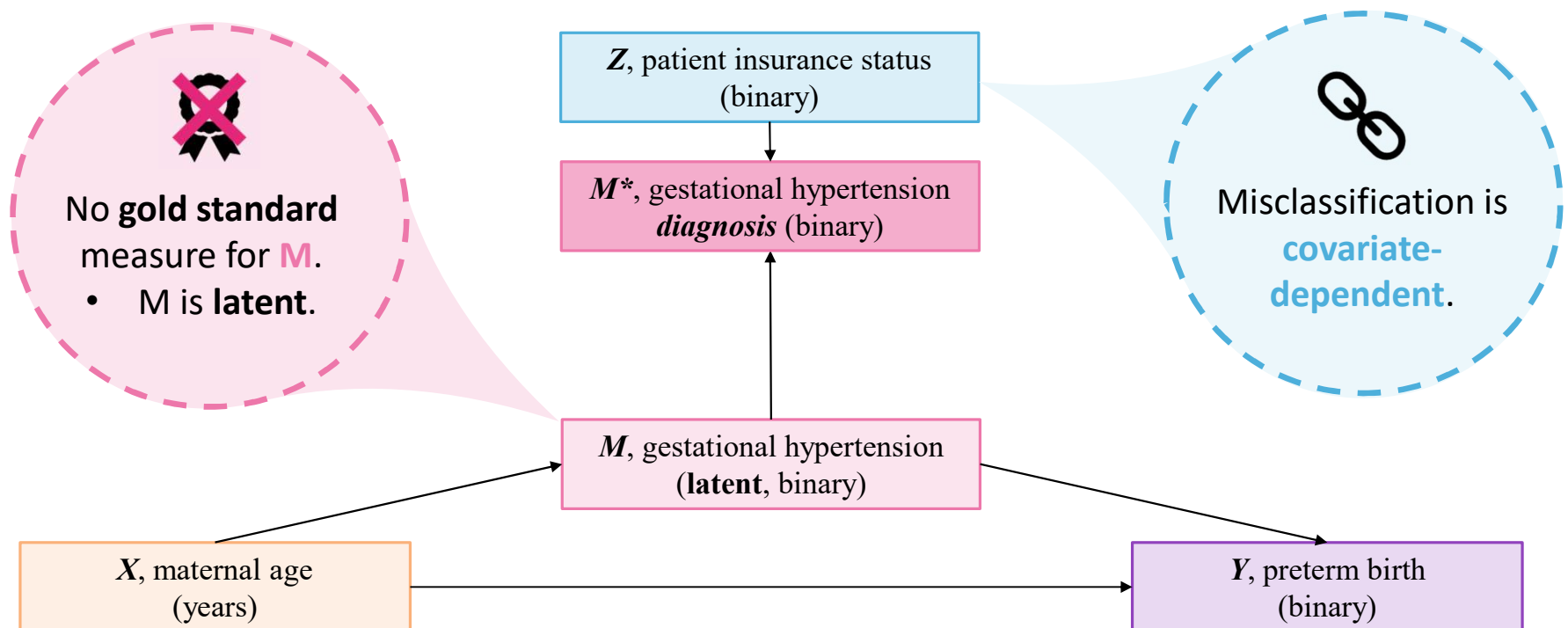
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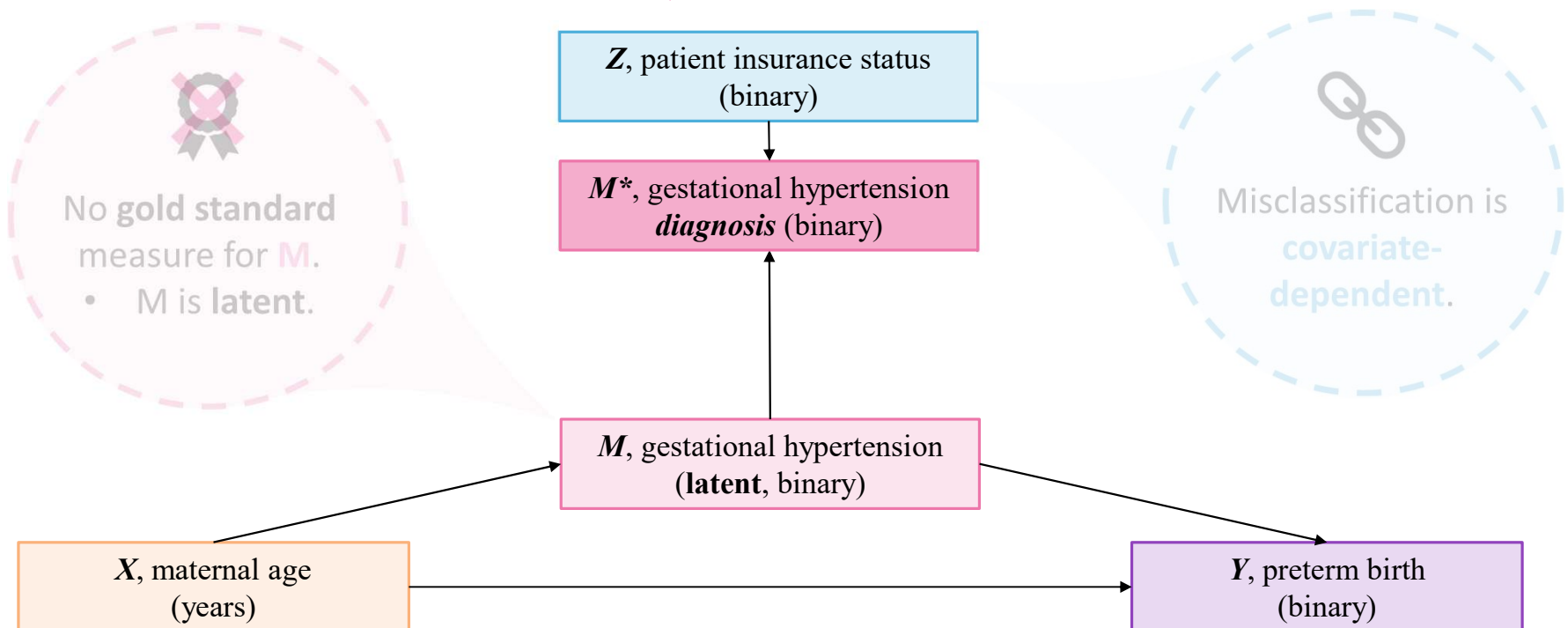
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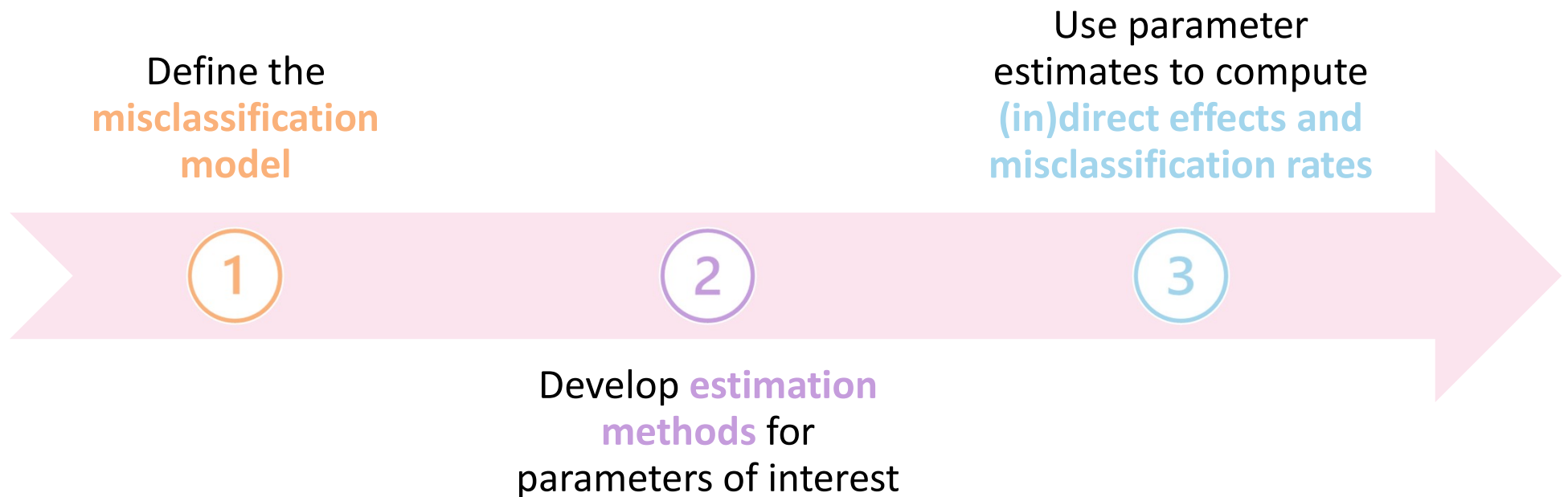
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Ignoring misclassification in M^*

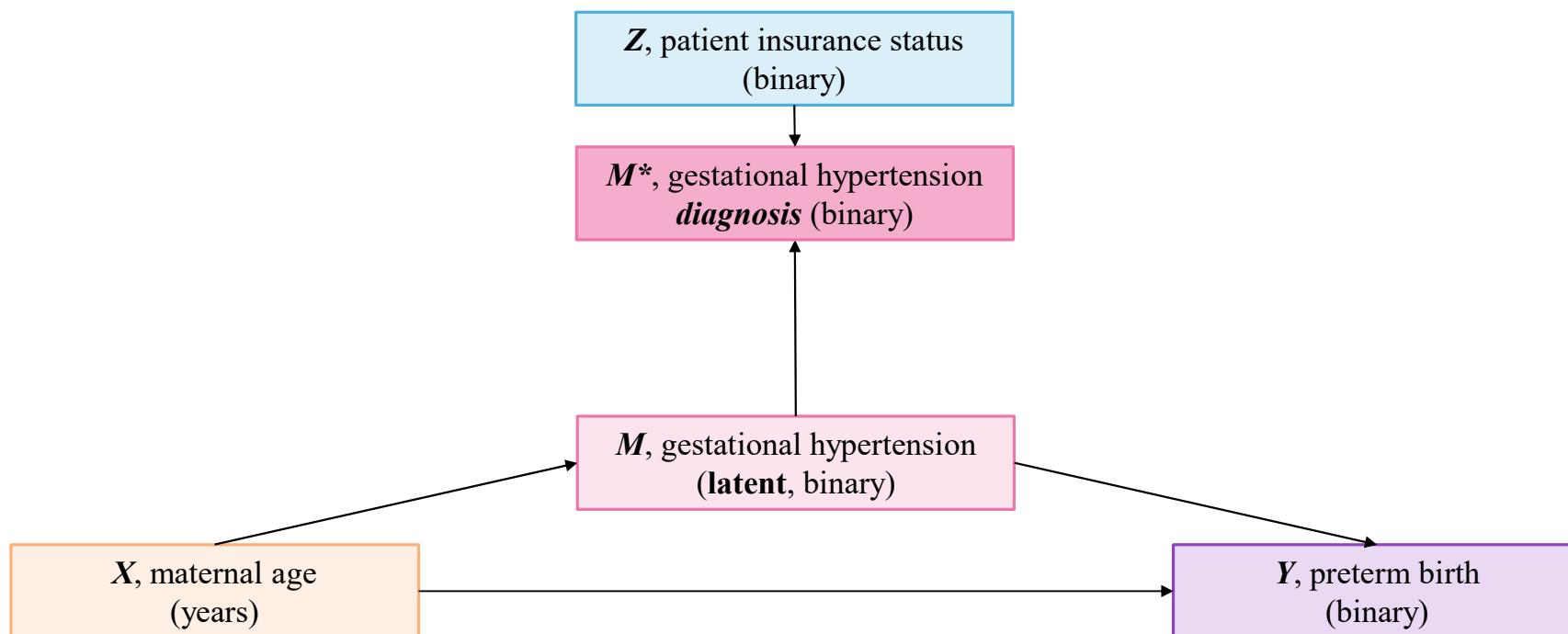
Bias in parameter and effect estimates



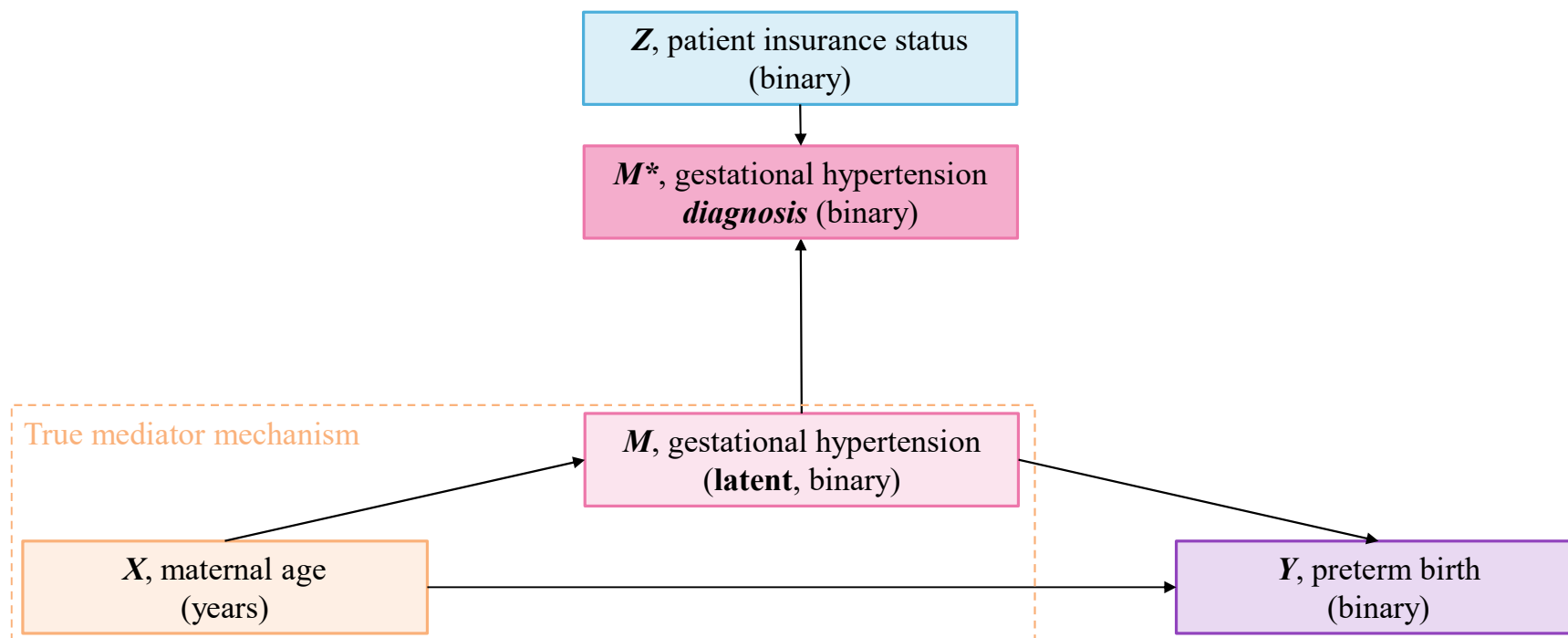
Analysis Plan



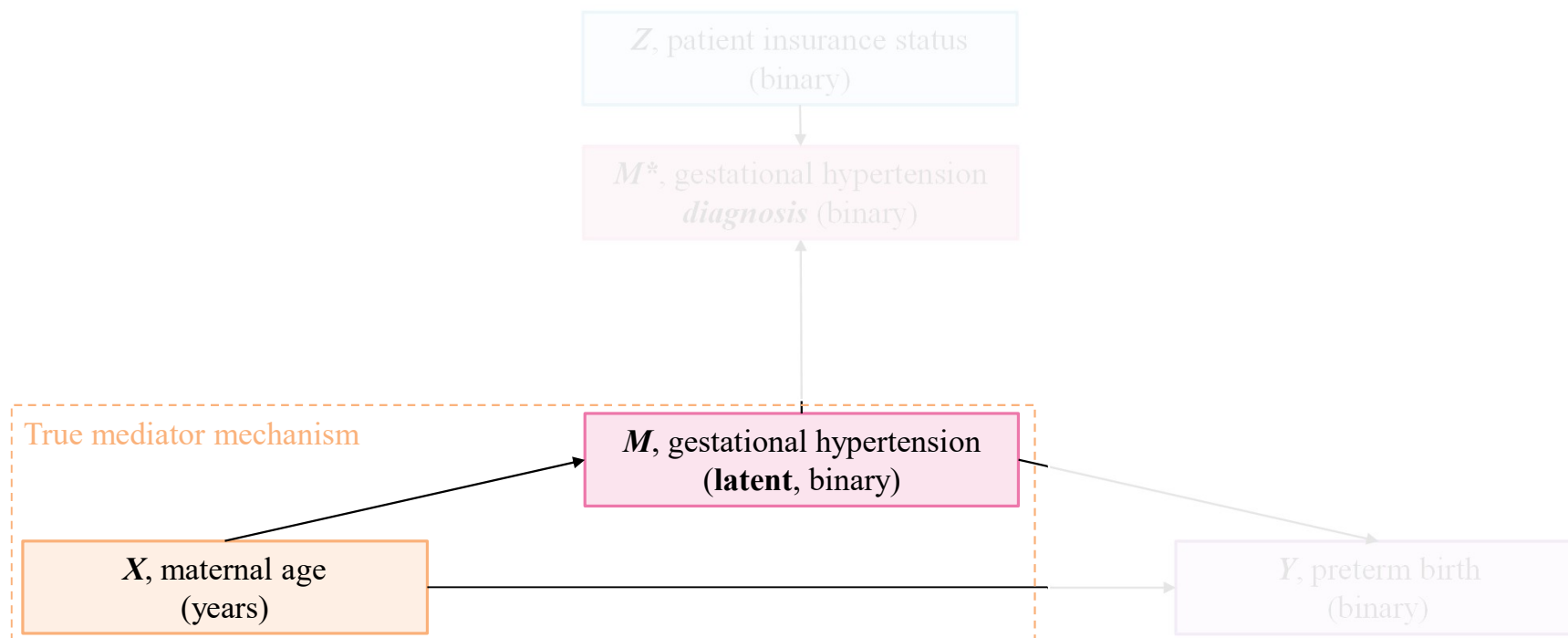
Our model accounts for misdiagnosis in M



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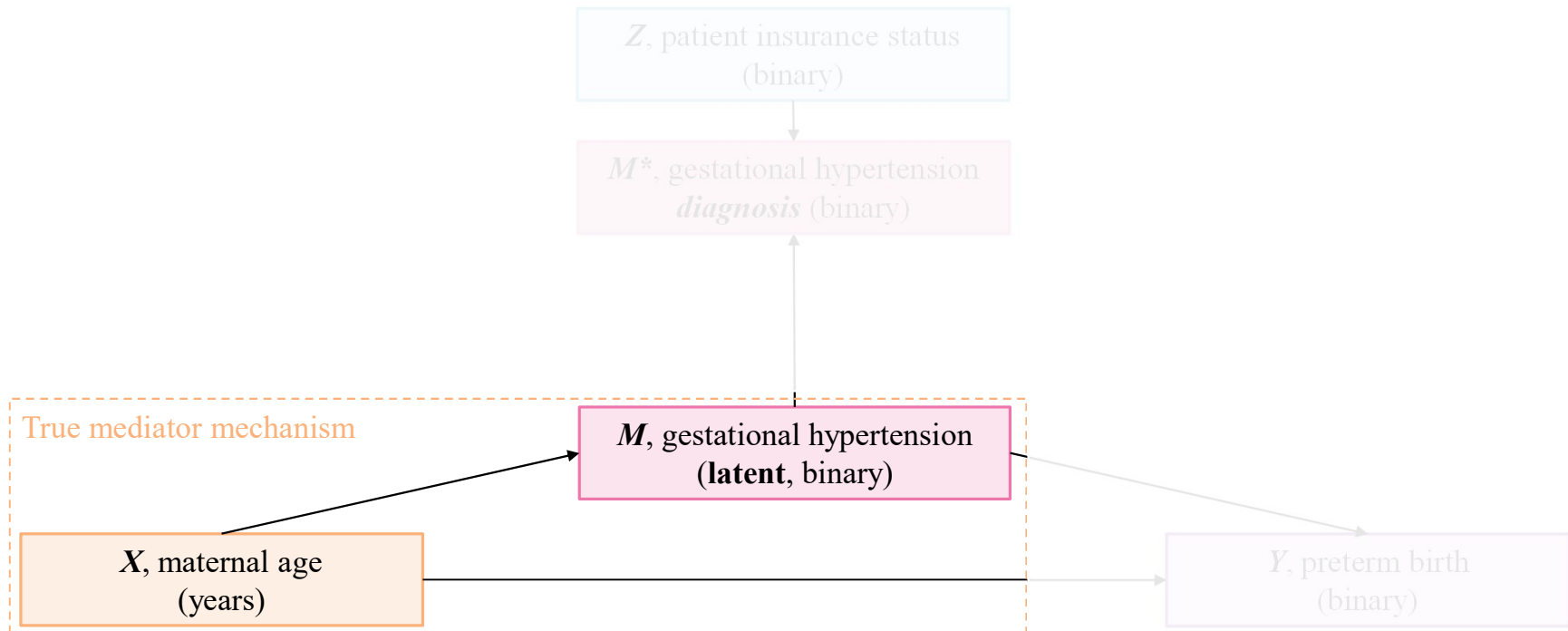


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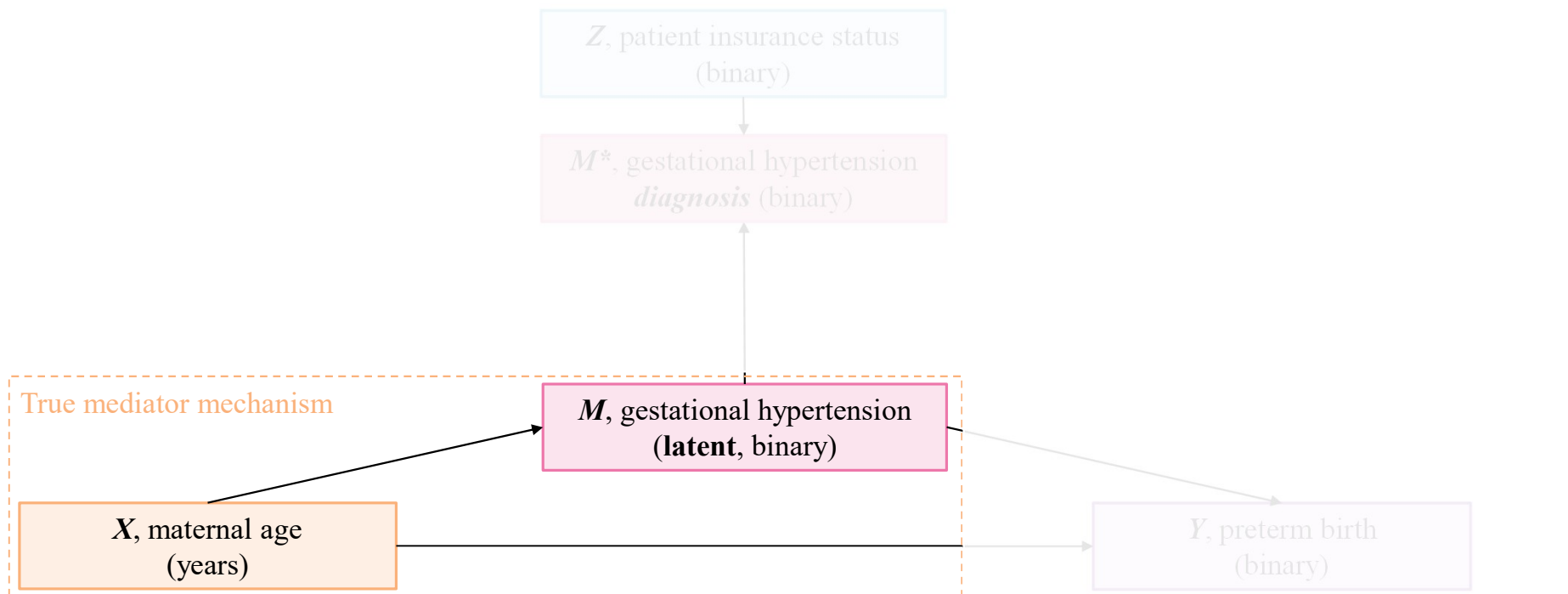


True mediator mechanism: $\text{logit}\{P(M = 1|X, C; \beta)\} = \beta_0 + \beta_X X + \beta_C C$

1

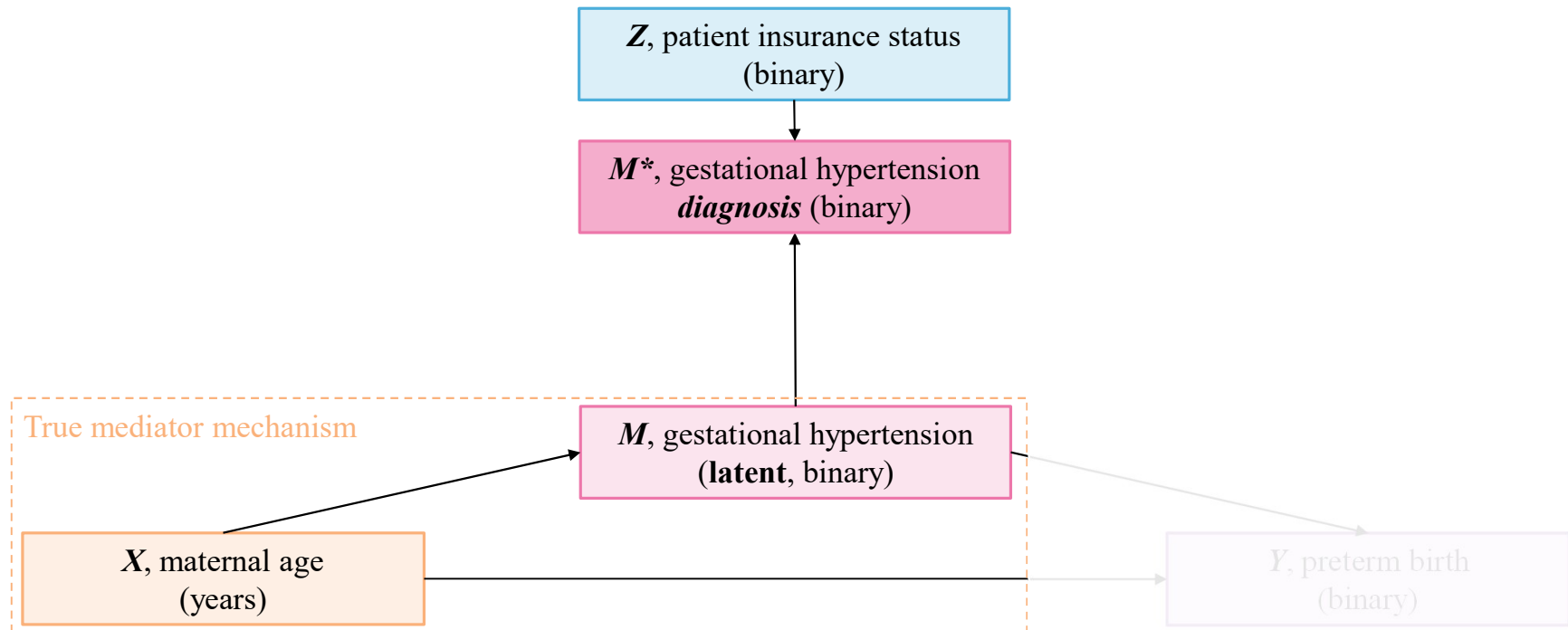


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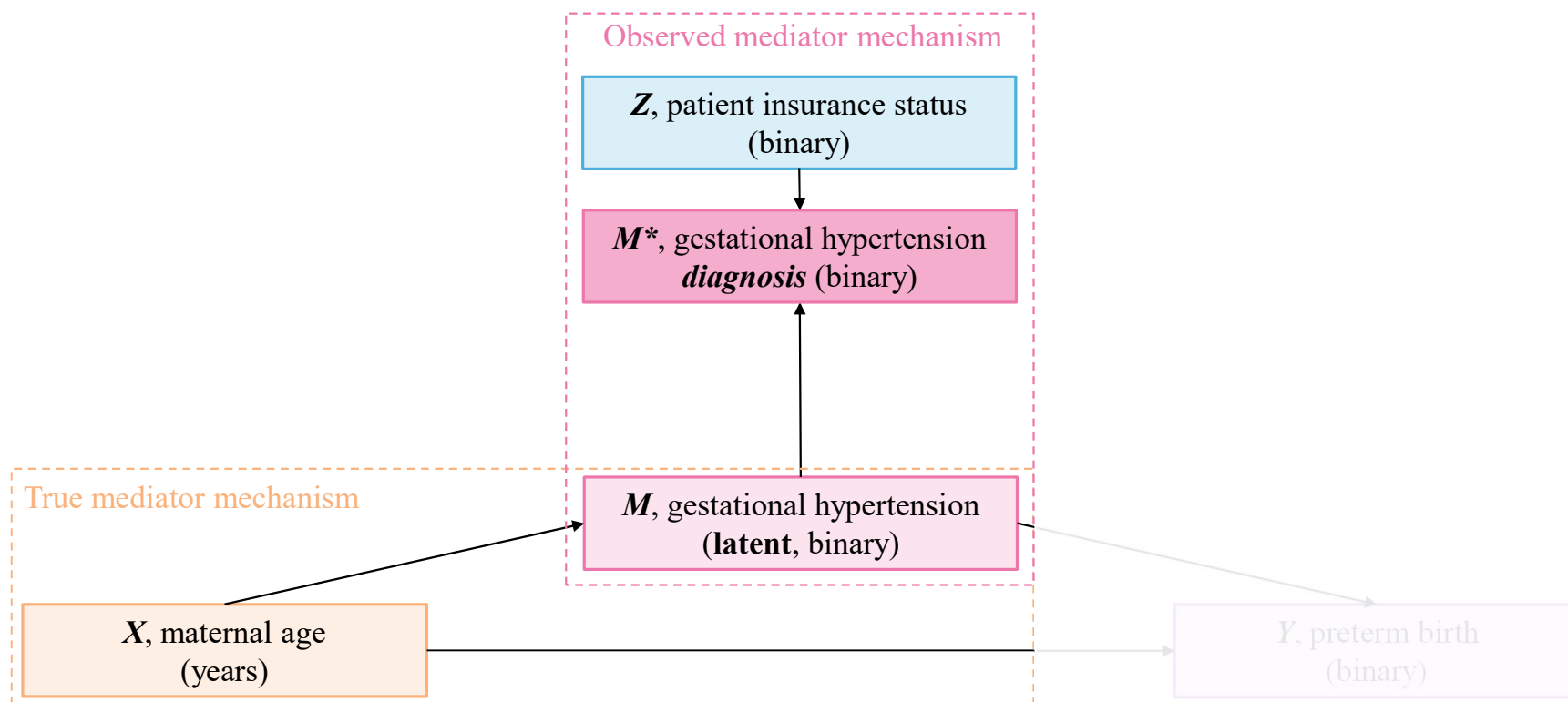
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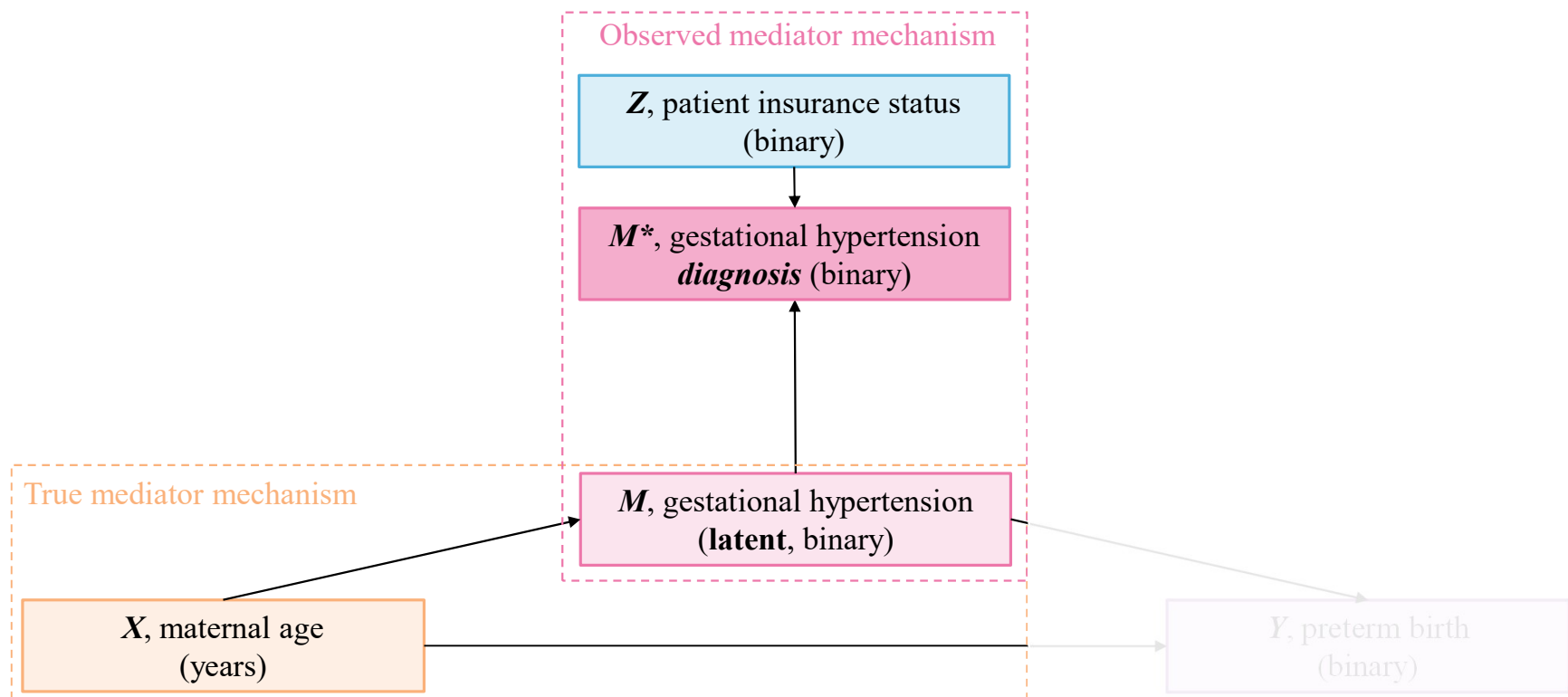
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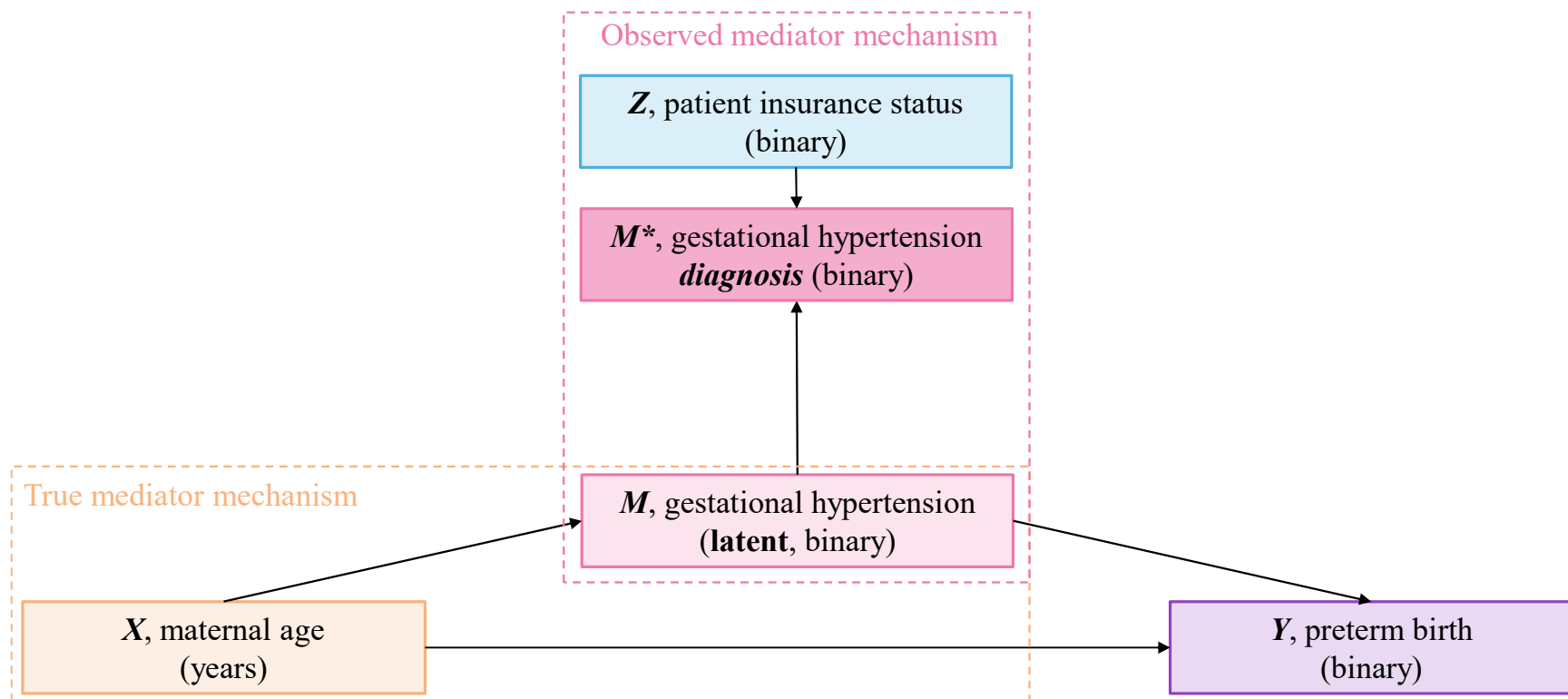
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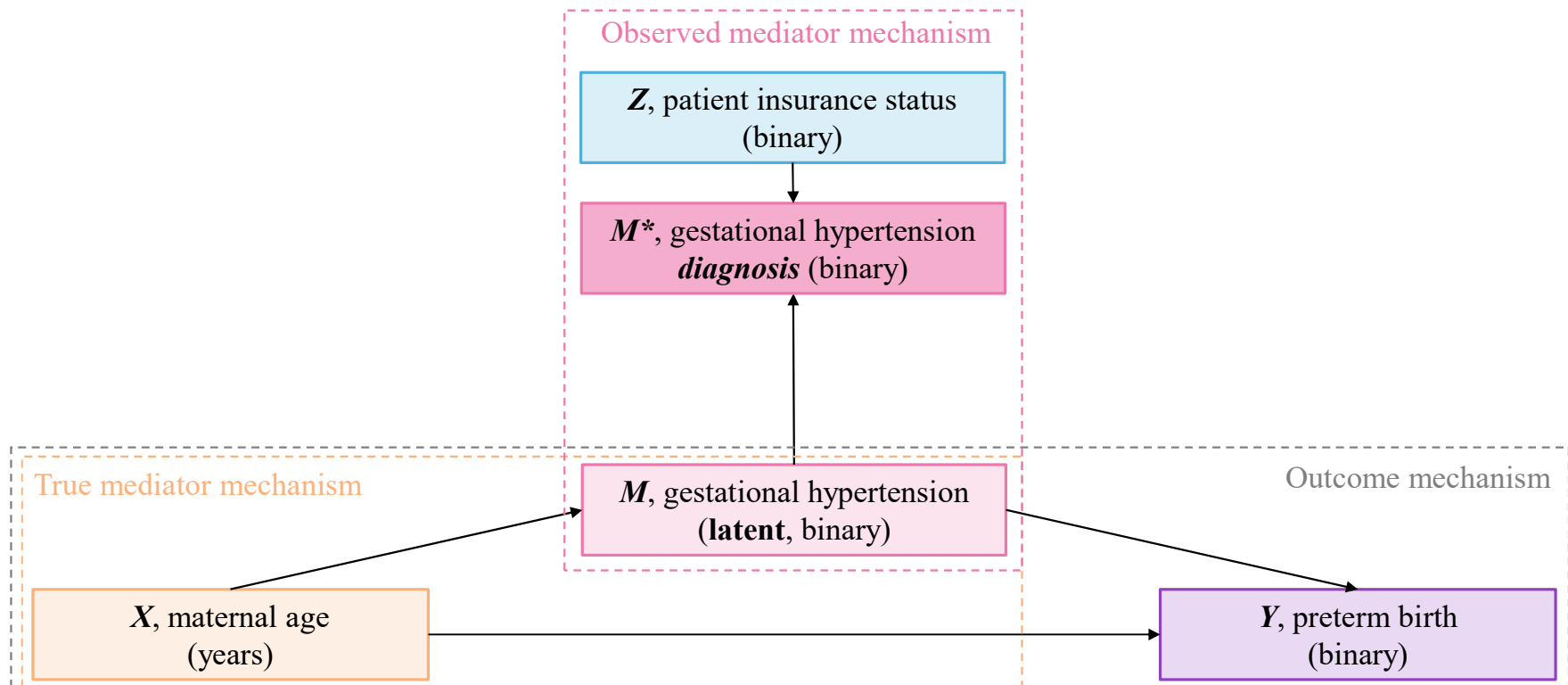
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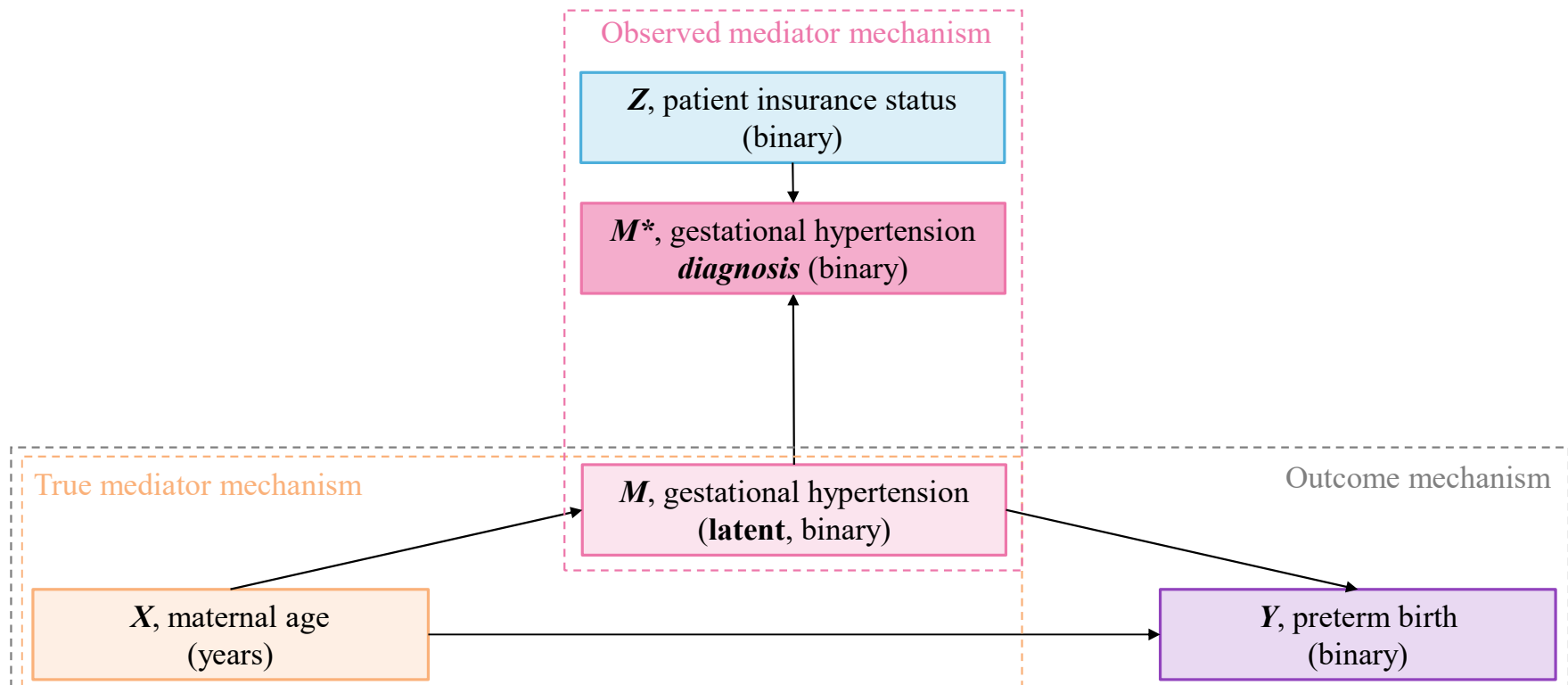


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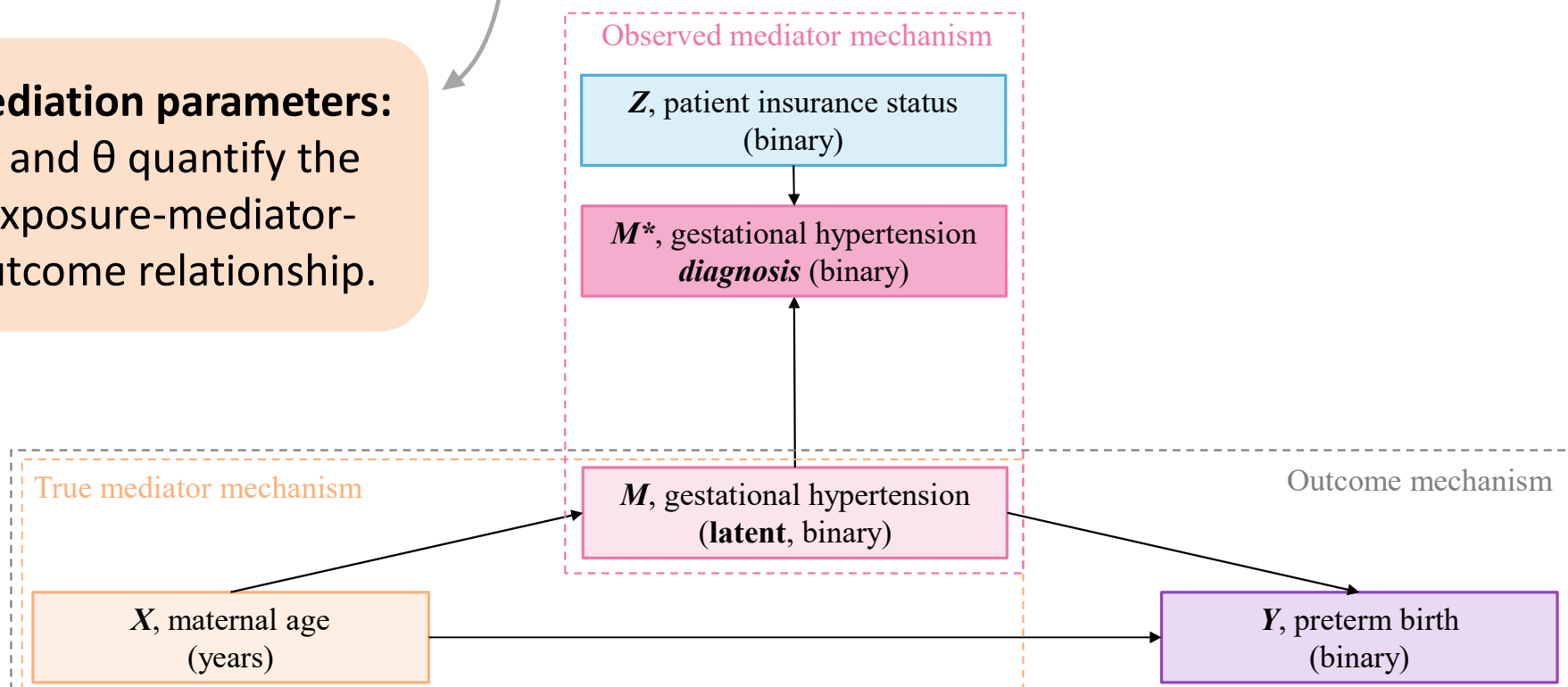
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Mediation parameters:
 β and θ quantify the
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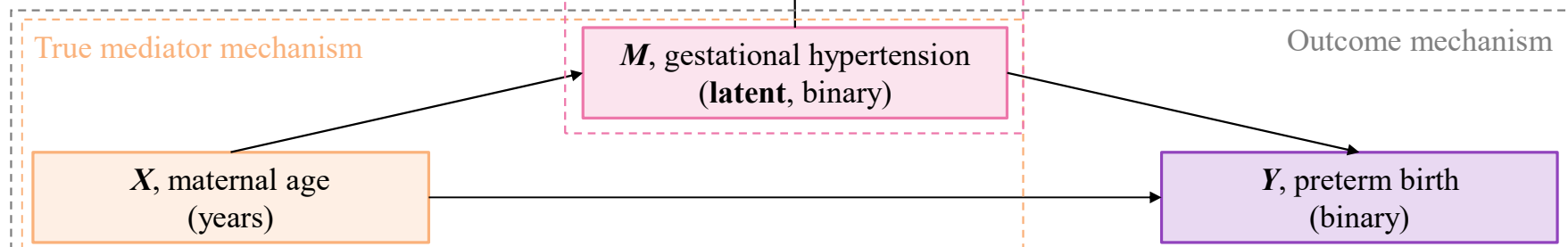
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Mediation parameters:
 β and θ quantify the exposure-mediator-outcome relationship.

Misclassification parameters: γ
quantifies the effect of Z on misclassification rates



We developed 3 estimation methods

True mediator mechanism: $\text{logit}\{P(M = 1|X, C; \beta)\} = \beta_0 + \beta_X X + \beta_C C$

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#2: Predictive value weighting

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Key point: We can use **COMBO** to estimate subject-level sensitivity and specificity, and then plug these values into existing misclassification correction procedures.

- Existing procedures relied on *known* sensitivity and specificity.

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Complete data log-likelihood:

$$\begin{aligned} \ell_{\text{complete}}(\beta, \gamma, \theta; X, C, Z, Y) \\ = \sum_{i=1}^N \left[\ell_{Y|X, M, C}(\theta; X_i, M_i, C_i, Y_i) + \sum_{j=1}^2 m_{ij} \log\{\pi_{ij}\} + \sum_{j=1}^2 \sum_{\ell=1}^2 m_{ij} m_{i\ell}^* \log\{\pi_{i\ell j}^*\} \right] \end{aligned}$$

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Outcome

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$P(M_i = j)$
 \uparrow
 $I(M_i = j)$

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Maximization Step

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$$w_{ij} = P(M_i = j|M_i^*, X_i, C_i, Z_i, Y_i)$$

$$= \sum_{\ell=1}^2 \frac{m_{i\ell}^* \pi_{i\ell}^* \pi_{ij} E[Y_i|X_i, M_i = j, C_i, \theta^{(t)}]}{\sum_{k=1}^2 \pi_{i\ell k}^* \pi_{ik} E[Y_i|X_i, M_i = k, C_i, \theta^{(t)}]}$$

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$$Q = \sum_{i=1}^N \left[\sum_{j=1}^2 \ell_{Y|X,M,C}(\theta; X_i, M_i = w_{ij}, C_i, Y_i) \right. \\ \left. + \sum_{j=1}^2 w_{ij} \log\{\pi_{ij}\} + \sum_{j=1}^2 \sum_{\ell=1}^2 w_{ij} m_{i\ell}^* \log\{\pi_{i\ell}^*\} \right]$$

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Expectation Step

Apply label switching correction
from Webb and Wells (2023)

Maximization Step

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$$= \sum_{\ell=1}^2 \frac{m_{i\ell}^* \pi_{i\ell}^* \pi_{ij} E[Y_i|X_i, M_i = j, C_i, \theta^{(t)}]}{\sum_{k=1}^2 \pi_{i\ell k}^* \pi_{ik} E[Y_i|X_i, M_i = k, C_i, \theta^{(t)}]}$$

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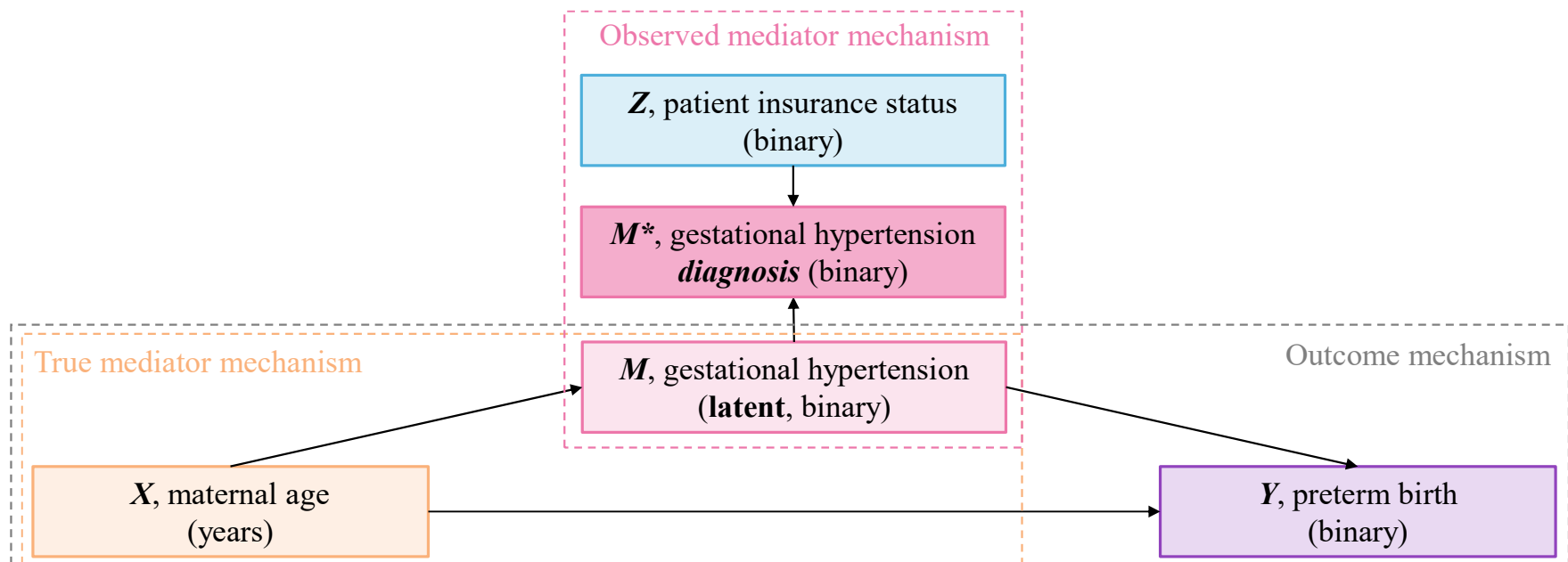
- Use the resulting bias-corrected parameter estimates to compute **(in)direct effects** for a change from \tilde{x} to x :

$$OR^{NDE} \cong \frac{\exp(\theta_X x) \{1 + \exp(\theta_M + \theta_{XM} x + \beta_0 + \beta_X \tilde{x} + \beta_C c)\}}{\exp(\theta_X \tilde{x}) \{1 + \exp(\theta_M + \theta_{XM} \tilde{x} + \beta_0 + \beta_X \tilde{x} + \beta_C c)\}}$$

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We applied our methods to a preterm birth study

? Does **gestational hypertension** mediate the association between **maternal age** and **preterm birth**, after accounting for potential **misdiagnosis of gestational hypertension** based on **patient insurance status**?

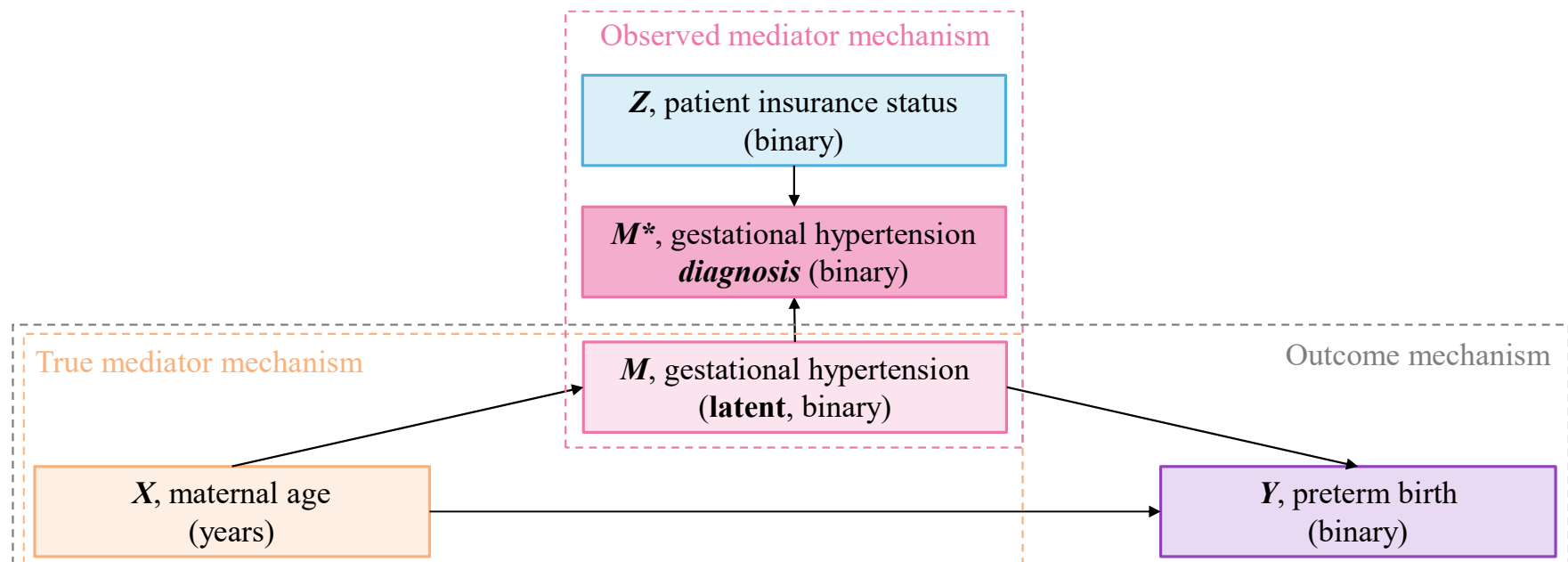


We applied our methods to a preterm birth study



Data: National Vital Statistics System

- Provides demographic and health data for all births in a year in the US.
- Random subsample from calendar year 2021, **N = 20,000**.

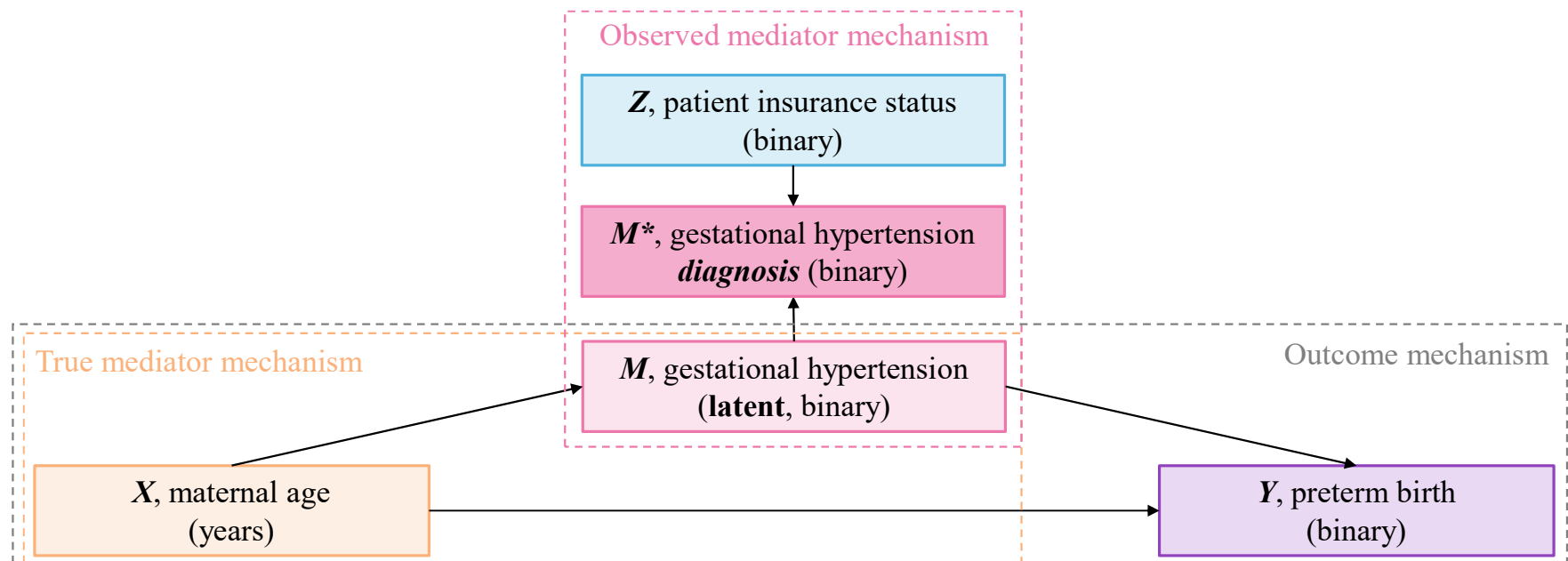


We applied our methods to a preterm birth study

True mediator mechanism: $M \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI}$

Observed mediator mechanism: $M^* \mid M \sim \text{Race} + Z$

Outcome mechanism: $Y \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI} + M + M * X$



Results change when we account for misdiagnosis

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	EM Algorithm		Naïve Analysis	
	Est.	SE	Est.	SE
β_X				
$\gamma_{Z, G=1}$				
$\gamma_{Z, G=2}$				
θ_X				
θ_M				
θ_{XM}				

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Association between **age & gestational hypertension** is unchanged after accounting for misdiagnosis.

	EM Algorithm		Naïve Analysis	
	Est.	SE	Est.	SE
β_X	0.10	0.04	0.08	0.03
$\gamma_{Z, G=1}$				
$\gamma_{Z, G=2}$				
θ_X				
θ_M				
θ_{XM}				

Results change when we account for misdiagnosis

True mediator mechanism: $M \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI}$

Observed mediator mechanism: $M^* \mid M \sim \text{Race} + Z$

Outcome mechanism: $Y \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI} + M + M * X$

Association between **age & gestational hypertension** is unchanged after accounting for misdiagnosis.

Association between **gestational hypertension & preterm birth** strengthens.

	EM Algorithm		Naïve Analysis	
	Est.	SE	Est.	SE
β_X	0.10	0.04	0.08	0.03
$\gamma_{Z, G=1}$				
$\gamma_{Z, G=2}$				
θ_X	0.02	0.05	0.10	0.03
θ_M	1.19	0.17	0.88	0.06
θ_{XM}	0.19	0.09	0.06	0.06

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**age & gestational
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unchanged after
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misdiagnosis.

Association between
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preterm birth**
strengthens.

	EM Algorithm		Naïve Analysis	
	Est.	SE	Est.	SE
β_X	0.10	0.04	0.08	0.03
$\gamma_{Z, G=1}$	-1.01	0.40	-	-
$\gamma_{Z, G=2}$	2.09	8.81	-	-
θ_X	0.02	0.05	0.10	0.03
θ_M	1.19	0.17	0.88	0.06
θ_{XM}	0.19	0.09	0.06	0.06

Use γ estimates to
compute **sensitivity
and specificity**.

M is measured with perfect specificity and low sensitivity

True mediator mechanism: $M \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI}$

Observed mediator mechanism: $M^* \mid M \sim \text{Race} + Z$

Outcome mechanism: $Y \sim X + \text{Race} + \text{Education} + \text{Parity} + \text{Smoking} + \text{BMI} + M + M * X$

	Estimated Specificity $P(\text{no } M^* \mid \text{no } M)$	Estimated Sensitivity $P(M^* \mid M)$
Insured	99.9%	43.1%
Self-Pay	99.4%	21.7%

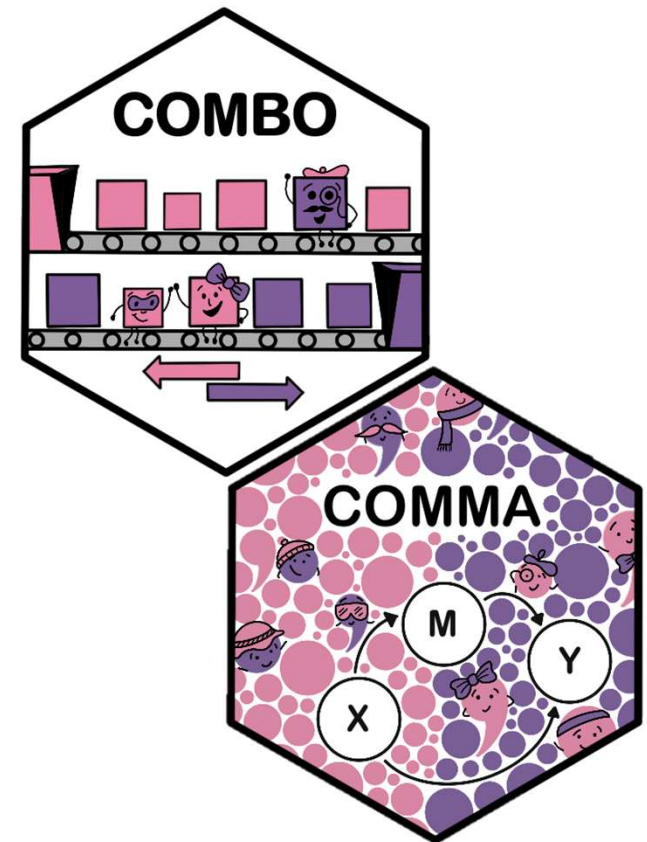


Key takeaways

- Developed new methods for handling misclassified binary mediator variables.
- Computed (in)direct effects using bias-corrected parameter estimates.
- Quantified gestational hypertension misdiagnosis rates based on insurance status.

Software

- Estimation methods for **misclassified outcomes** are available in the *COMBO* R Package on CRAN.
 - **C**orrecting **M**isclassified **B**inary **O**utcomes
- Estimation methods for **misclassified mediators** are available in the *COMMA* R Package on CRAN.
 - **C**orrecting **M**isclassified **M**ediation **A**alysis



Thank you!

Kimberly A. H. Webb

kimberlywebb@pitt.edu

kimhwebb.com —————> My “webb-site” ☺

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paper



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Statistics and Data Science