Compact implementations of pairings

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***Abstract*—The recent discovery of the constructive use of pairings in cryptography has opened up a wealth of new research options into identity-based encryption. In this paper, we will investigate the possible use of pairings in constrained environments. The focus will be on an small, energy efficient ASIC implementation of an accelerator for the Tate pairing over a supersingular curve.**

The results are encouraging for further research. It is possible to obtain an implementation of less than 30k gates. Furthermore, energy efficiency improvements over twenty times compared to other published designs are possible.

***Index Terms*—Identity-based cryptography, elliptic curve cryptography, Tate pairing, hardware accelerator, ASIC.**

I. Introduction

EVER since Shamir's proposal [1] in '86, there's been an interest in identity-based cryptography. Particulary Boneh and Franklin's [2] discovery of the constructive use of pairings for identity-based encryption has helped spur on new research into possible applications and implementations.

Multitudes of protocols have seen the light, however, untill recently the lack of efficient hardware accelerators for the computationally expensive pairings was always kind of a showstopper towards implementing them. Thus most of the published implementations have a focus on speed. Implementations for area- and/or power-constrainted devices were either deemed infeasible or just not interesting enough.

In 2007 Oliveira *et al.* introduced their TinyTate [3] implementation to the world. 2008 saw the light of TinyPBC [4] and NanoECC [5] from the same authors. All three papers present implementations of pairings (either the Tate or η_T) on the AT128Mega microchip of a Mica node [6], designed for deeply embedded networks. Thus it was proven that pairings were indeed feasible for use in constrained environments, such as sensor networks.

In this paper, we will investigate the feasability of a hardware accelerator for the Tate pairing in constrained environments. In Section II necessary parameters will be defined and we will take a look at the pairing arithmetic. Section III consits of a concise overview of the implementation's hardware. Finally, results from ASIC synthesis will be presented in Section IV and from these a conclussion will be drawn in Section V.

II. Parameters and arithmetic for the Tate pairing

As mentioned, we will be creating an accelerator for the Tate pairing. Recently, variants on that pairing have been published, such as the η_T [7] and Ate [8] pairings. However, seeing as they are very recent discoveries, we felt it more appropriate to focus on the better known Tate pairing.

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A. Definition of the Tate pairing

The Tate pairing $e(P,Q)$ is defined as a mapping from two additive groups $\mathbb{G}_1, \mathbb{G}_2$ to a multiplicative group $\mathbb{G}_T$. To be suitable for use in cryptography, it should have the following three properties:

- Well-defined:
$$e(\mathcal{O},Q) = 1 \,\forall\, Q \in \mathbb{G}_2$$
$$e(P,\mathcal{O}) = 1 \,\forall\, P \in \mathbb{G}_1.$$
- Non-degenerate:
$$\forall\, P \in \mathbb{G}_1,\ \exists\, Q \in \mathbb{G}_2 \text{ for which } e(P,Q) \neq 1.$$
- Bilinear: $\forall\, P_1, P_2, P \in \mathbb{G}_1$ and $\forall\, Q_1, Q_2, Q \in \mathbb{G}_2$:
$$e(P_1 + P_2, Q) \equiv e(P_1, Q) \cdot e(P_2, Q)$$
$$e(P, Q_1 + Q_2) \equiv e(P, Q_1) \cdot e(P, Q_2).$$

The point $\mathcal{O}$ is the point at infinity on the elliptic curve E over which the pairing is defined.

Instead of calculating the Tate pairing as proposed by Miller in '86 [9], we will be using an optimized version of Miller's algorithm as proposed by Barreto *et al.* [10]. The Tate pairing is then a mapping:

$$\hat{e}(P,Q) : E(\mathbb{F}_q)[l] \times E(\mathbb{F}_q)[l] \mapsto \mathbb{F}^*_{q^k}/(\mathbb{F}^*_{q^k})^l.$$

The notation $E(\mathbb{F}_q)[l]$ meaning the group of points $P \in E(\mathbb{F}_q)$ for which $lP = \mathcal{O}$. The result of the pairing is an element of the equivalence group $\mathbb{F}^*_{q^k}/(\mathbb{F}^*_{q^k})^l$, in which two elements $a \equiv b$ iif $a = bc^l$ with $c \in \mathbb{F}^*_{q^k}$. To eliminate this ambiguity, we will elevate the result of the pairing to the power $\frac{q^k-1}{l}$, the result of which will be an lth root of unity μ_l.

B. Parameters

Before we can take a look at the arithmetic behind the Tate pairing computation, some parameters need to be set. First and foremost, the elliptic curve and the field over which it is defined need to be defined. Due to the simplicity of it's arithmetic, we choose a field $\mathbb{F}_{2^m}$. We are then forced to use the curve [10]:

$$E(\mathbb{F}_{2^m}) : y^3 + y = x^3 + x + b,$$

with $b \in \{0,1\}$. We also define [11]:

$$\delta = \begin{cases} b & m \equiv 1,7 \pmod 8 \\ 1-b & m \equiv 3,5 \pmod 8 \end{cases}$$
$$\nu = (-1)^\delta$$

The value of b set to whatever value maximizes the order of the curve:

$$\#E(\mathbb{F}_{2^m}) = 2^m + \nu\sqrt{2^{m+1}} + 1.$$

So, before the value of b can be decided on, m is to be set. We also define $l = \#E$.

Considering that the final implementation should be as small as possible, we settle on $m = 163$, which, according to [12], should still provide reasonable security. If necessary, the hardware which will be proposed in Section III can easily be adapted to larger fields. From [13] the reduction polynomial is chosen to be

$$R = z^{163} + z^7 + z^6 + z^3 + 1.$$

Now that's been decided on these parameters, we can see that b needs to equal one.

The type of supersingular curve that's being used has an embedding degree $k = 4$. The result of the Tate pairing will thus be an element in $\mathbb{F}^*_{2^{4m}}$. We define this field by means of tower extensions [14]:

$$\mathbb{F}_{2^{2m}} \cong \mathbb{F}_{2^m}[x]/\left(x^2 + x + 1\right)$$
$$\mathbb{F}_{2^{4m}} \cong \mathbb{F}_{2^{2m}}[y]/\left(y^2 + (x+1)y + 1\right)$$

C. Arithmetic

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III. Hardware implementation

IV. Results

V. Conclussion

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