Compact implementations of pairings

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Outline

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- *Optimized* Tate pairing:

$$\hat{e}(P, Q) : E(\mathbb{F}_q)[l] \times E(\mathbb{F}_q)[l] \mapsto \mu_l$$

$$\mu_l = \text{group of } l\text{th roots of } \mathbb{F}_{q^k}^*$$

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Restrictions

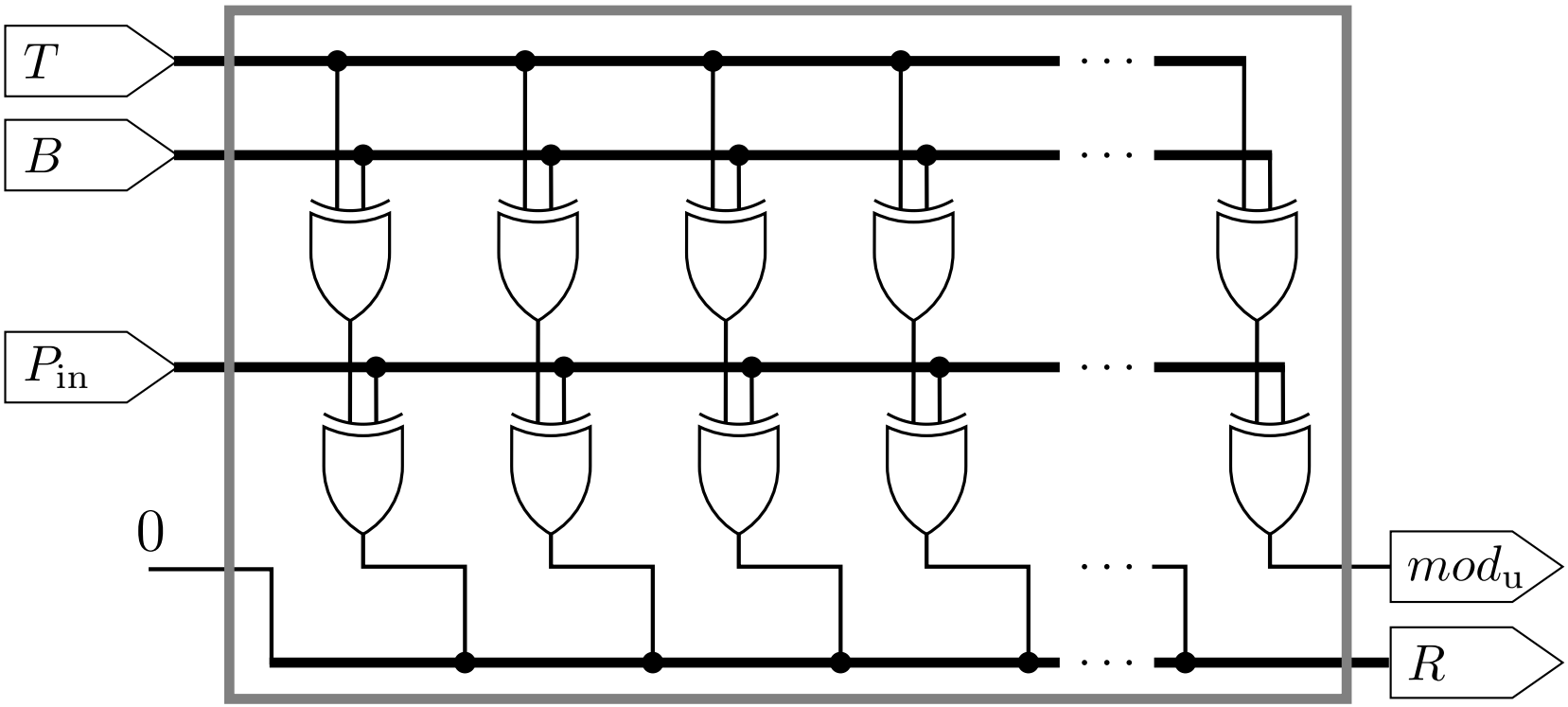
Avoid the use of flip-flops and muxes:

Cell	Area $\left[\frac{\text{gate}}{\text{bit}}\right]$
D flip-flop (reset)	6
D flip-flop (no reset)	5.5
D latch	4.25
3 input MUX	4
2 input XOR	3.75
2 input MUX	2.25
2 input NAND	1
NOT	0.75

MALU - Addition & Reduction in $\mathbb{F}_{2^m}$

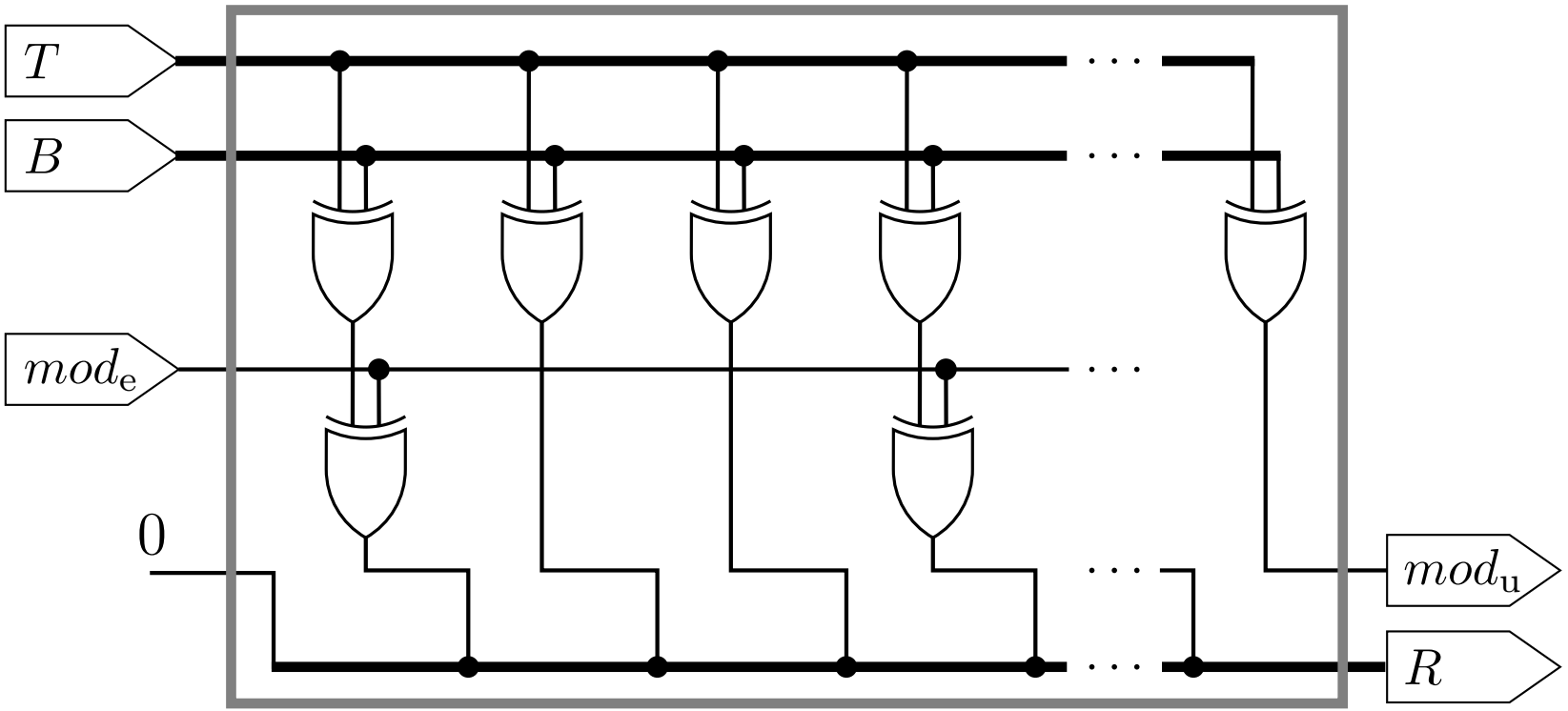
$$R = (T + B \pmod{P_{\text{in}}})_{0:m-2} \ll 1$$

$$mod_{\text{u}} = (T + B \pmod{P_{\text{in}}})_{m-1}$$

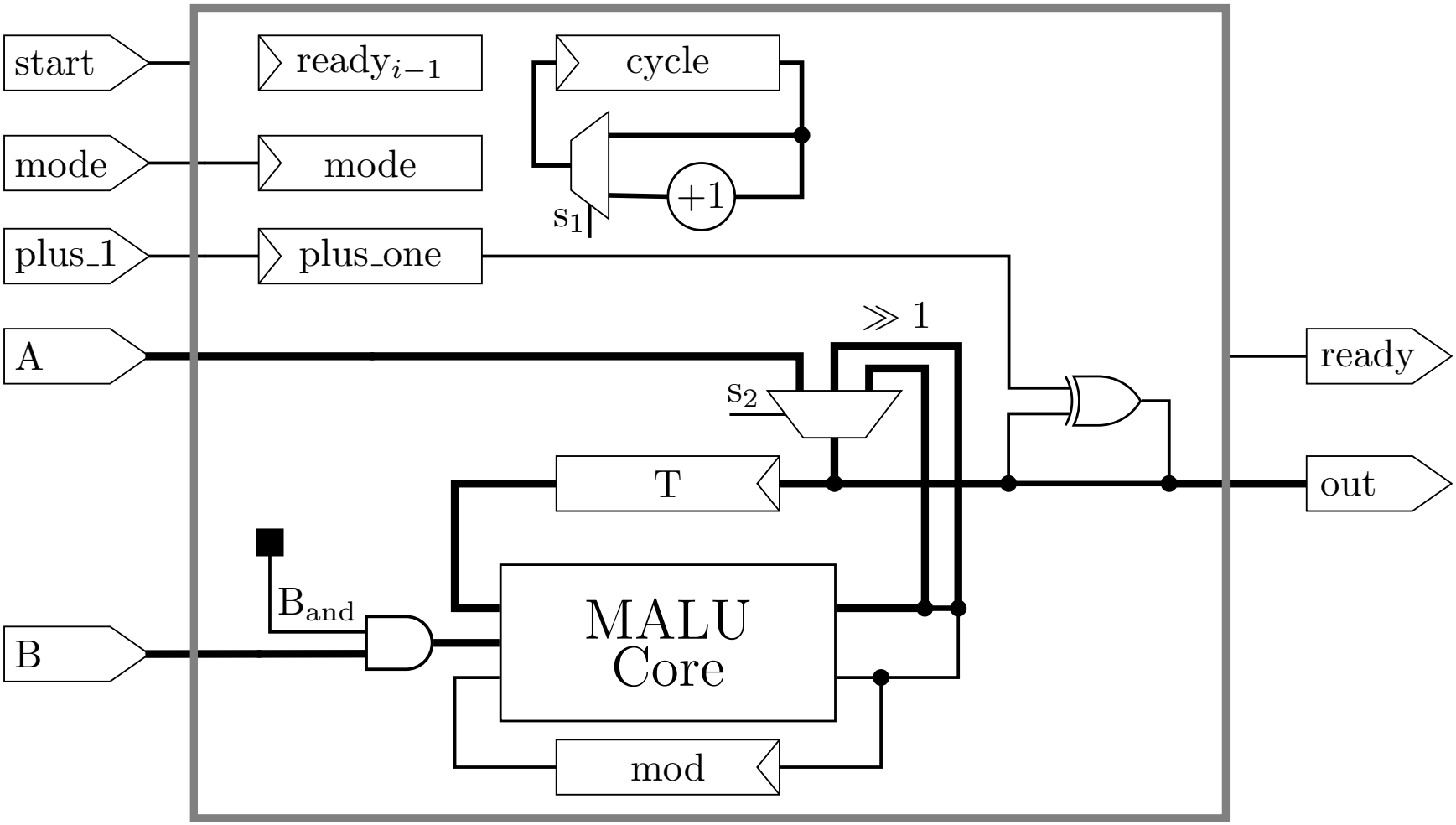


MALU - Addition & Reduction in $\mathbb{F}_{2^m}$

Optimized MALU needs $\Delta = m - (\text{Hamm}(P) - 1)$ less XORs:

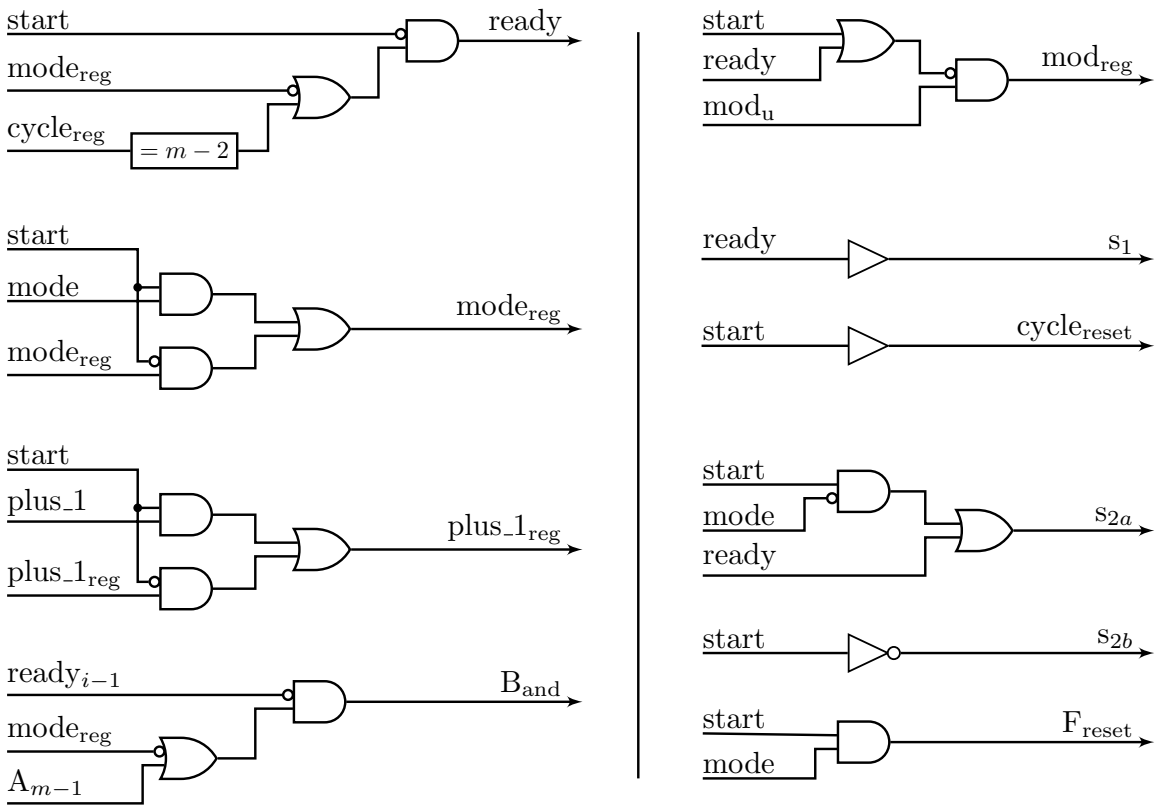


$\mathbb{F}_{2^m}$ Multiplication & Addition



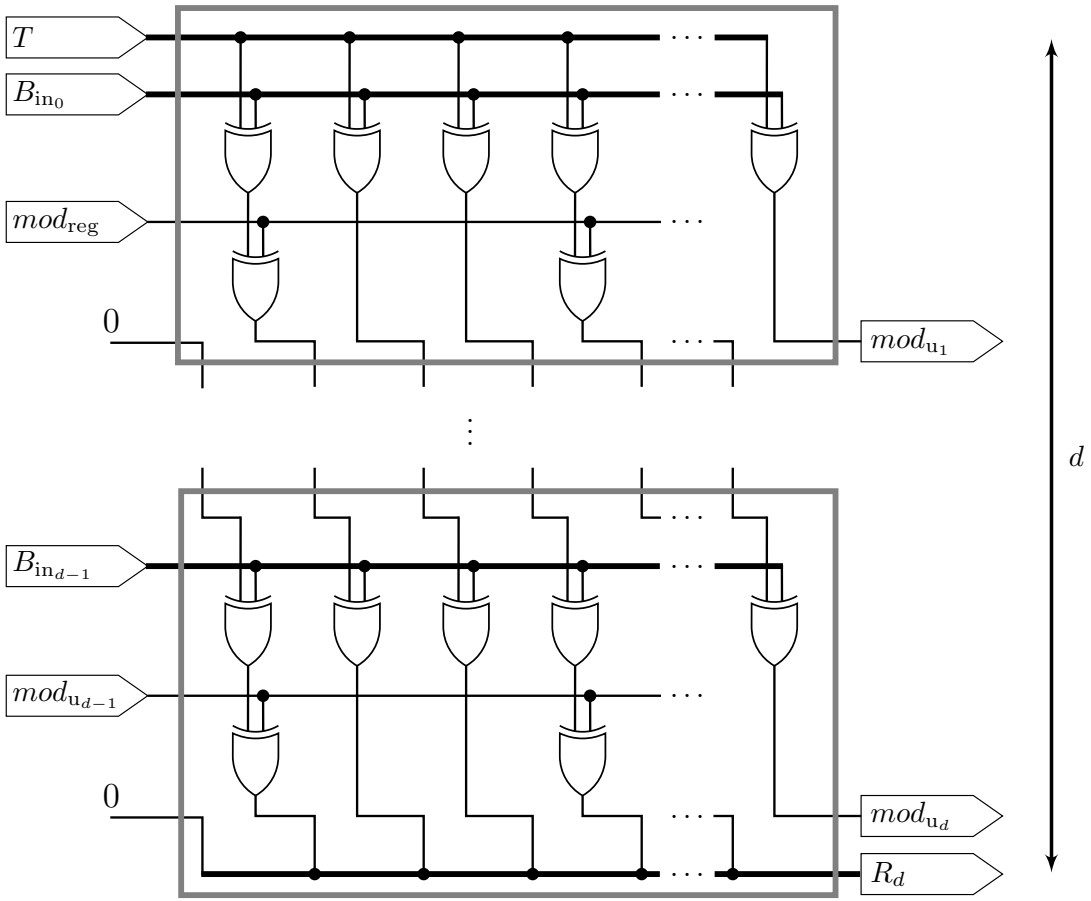
$\mathbb{F}_{2^m}$ Multiplication & Addition

No FSM needed, simple logic:

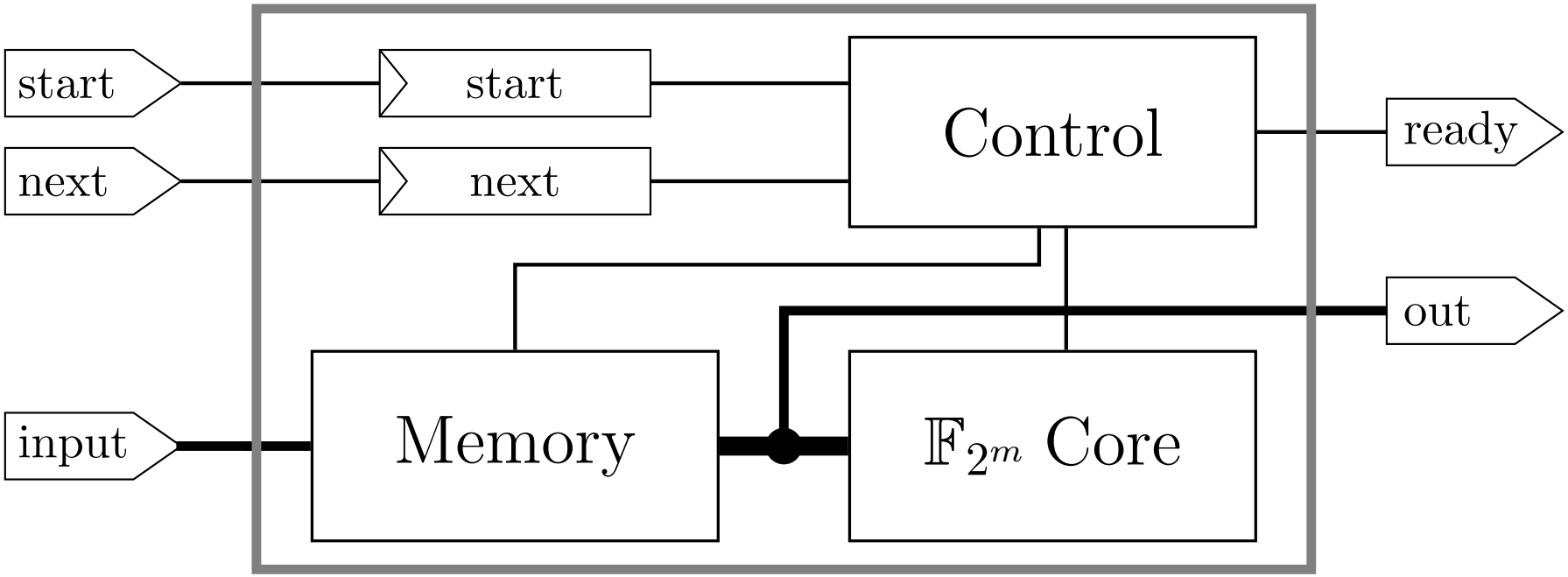


$\mathbb{F}_{2^m}$ Multiplication & Addition

Speed up calculation by daisy-chaining MALUs ($m \bmod d$!):



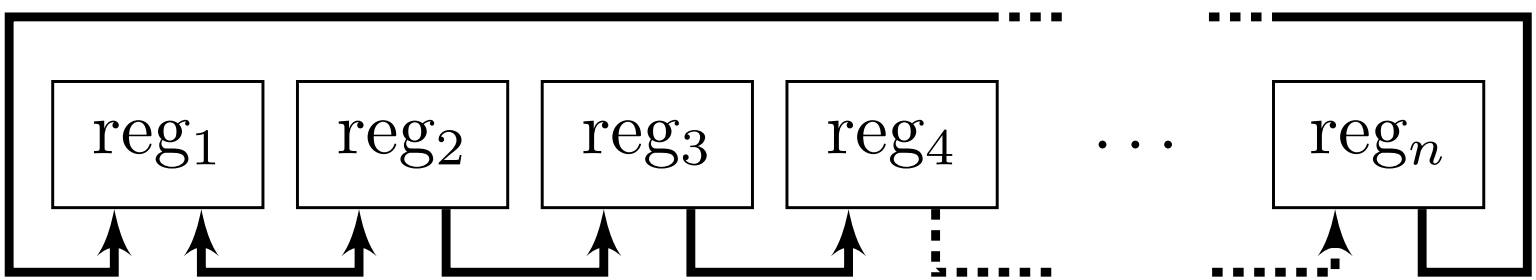
Controller for Miller's algorithm



Memory design

Initial design:

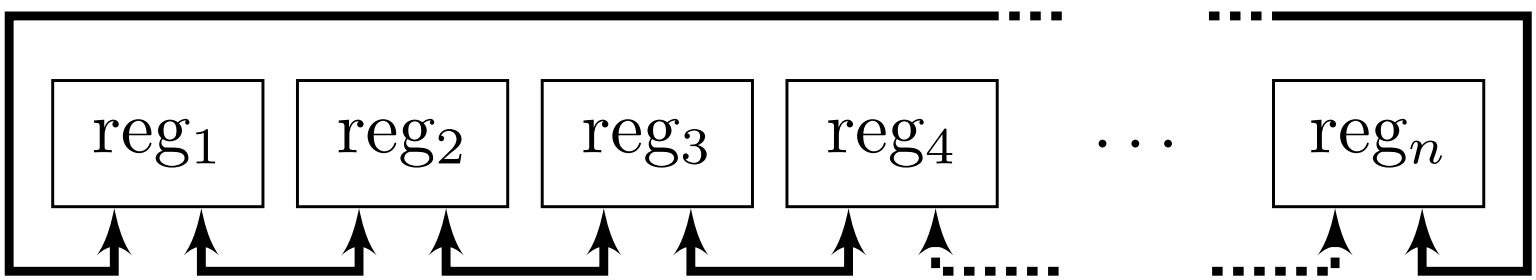
$$\overline{t} = O\left(\frac{n^2}{3}\right) \qquad \overline{w} = O\left(\frac{n^3}{3}\right)$$



Memory design

Final design:

$$\overline{t} = O\left(\frac{n}{4}\right) \qquad \overline{w} = O\left(n\right)$$



Optimizations

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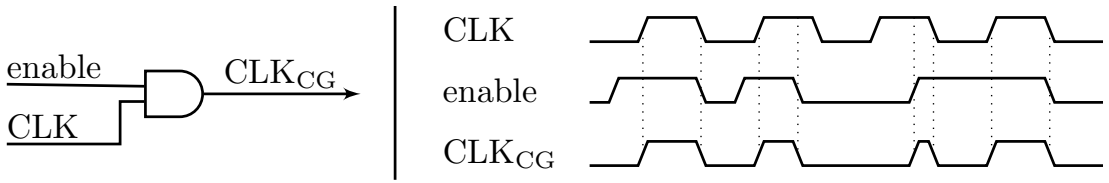

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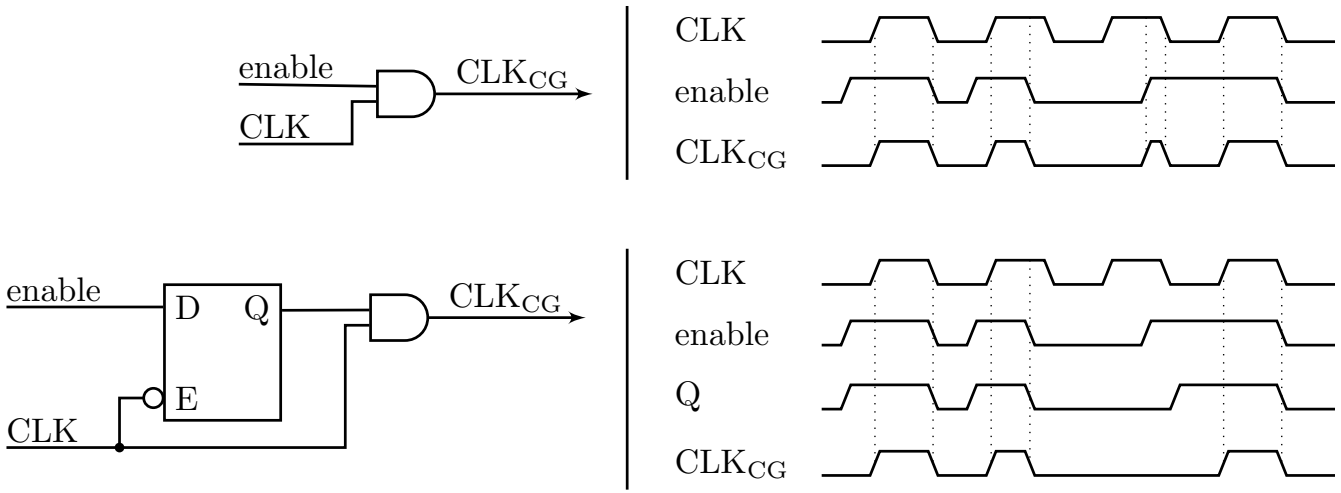

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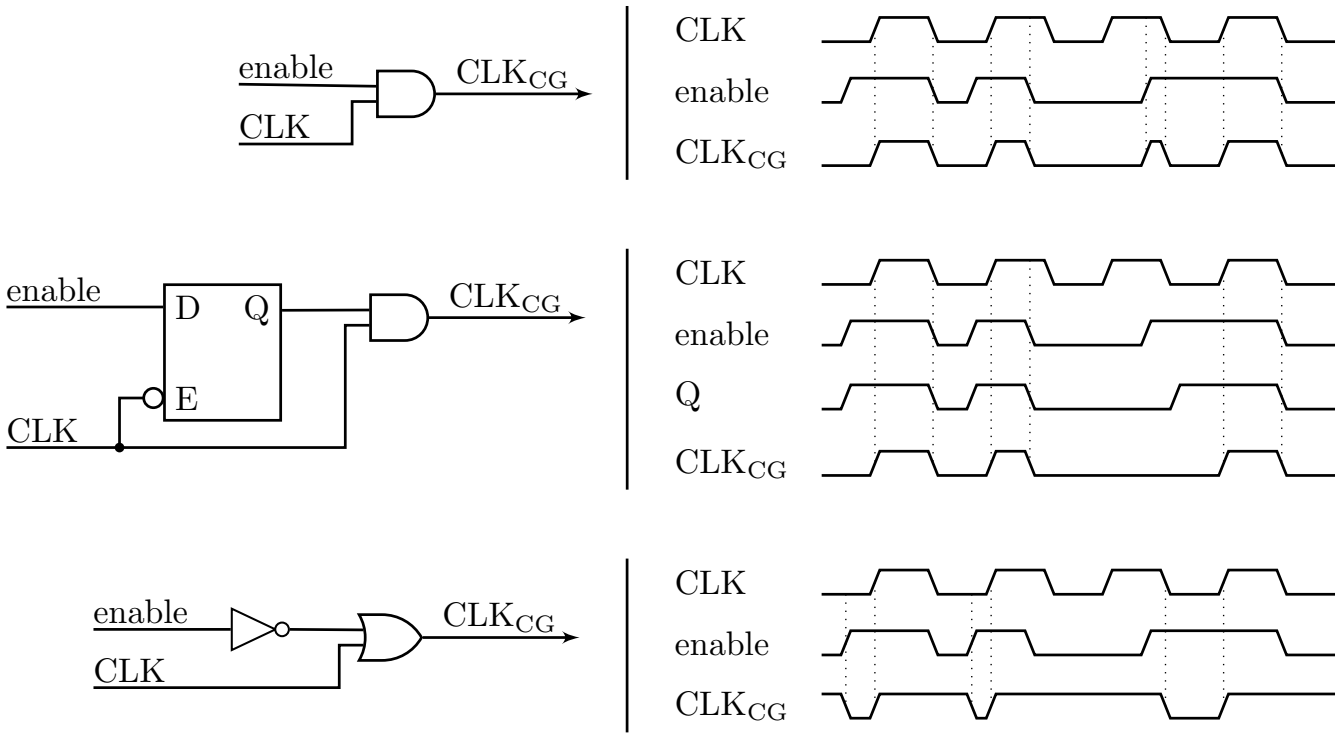

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$$c = 21681 + 4322 + 2998 \cdot \left\lceil \frac{m}{d} \right\rceil$$

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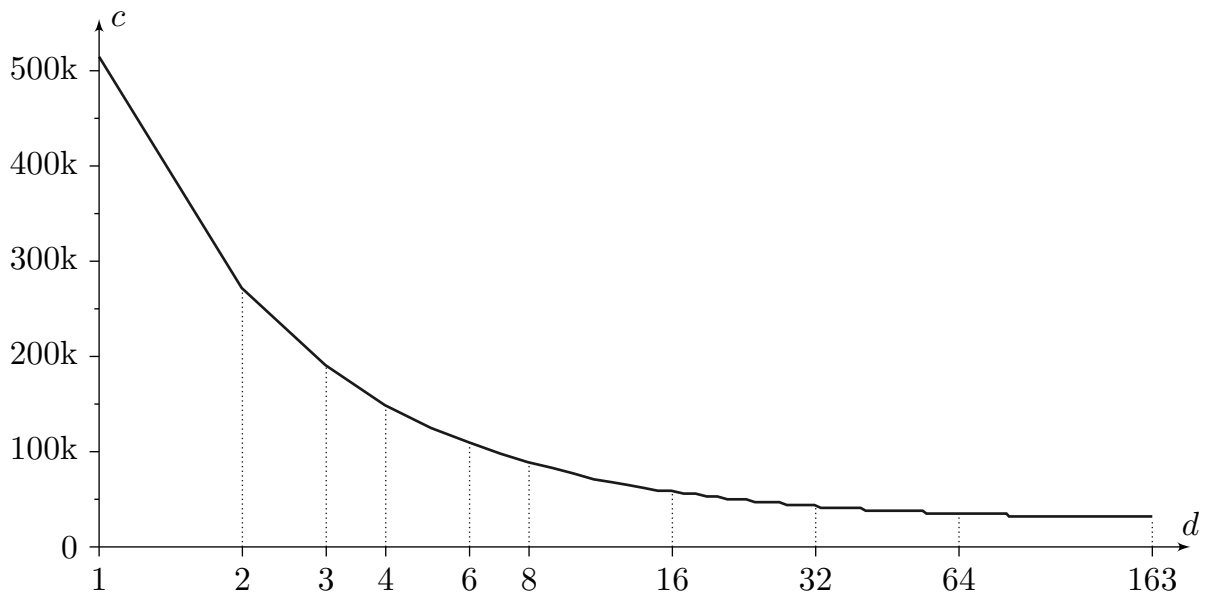
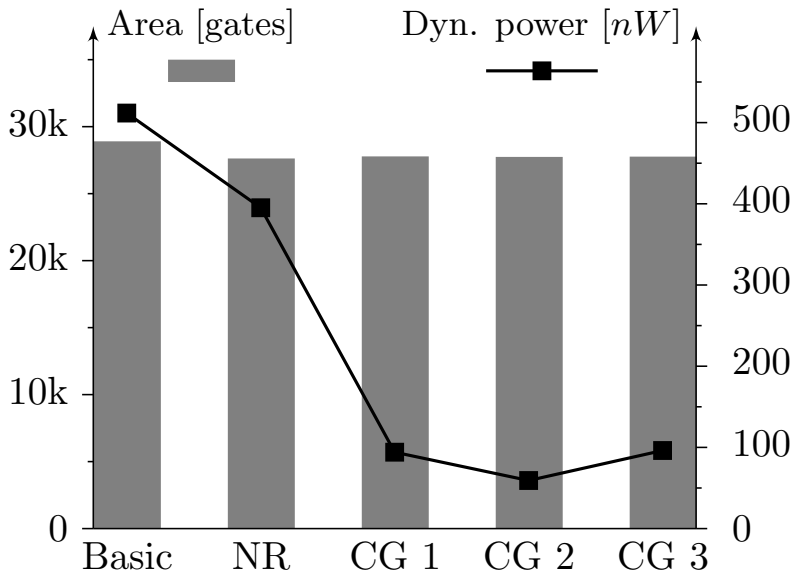


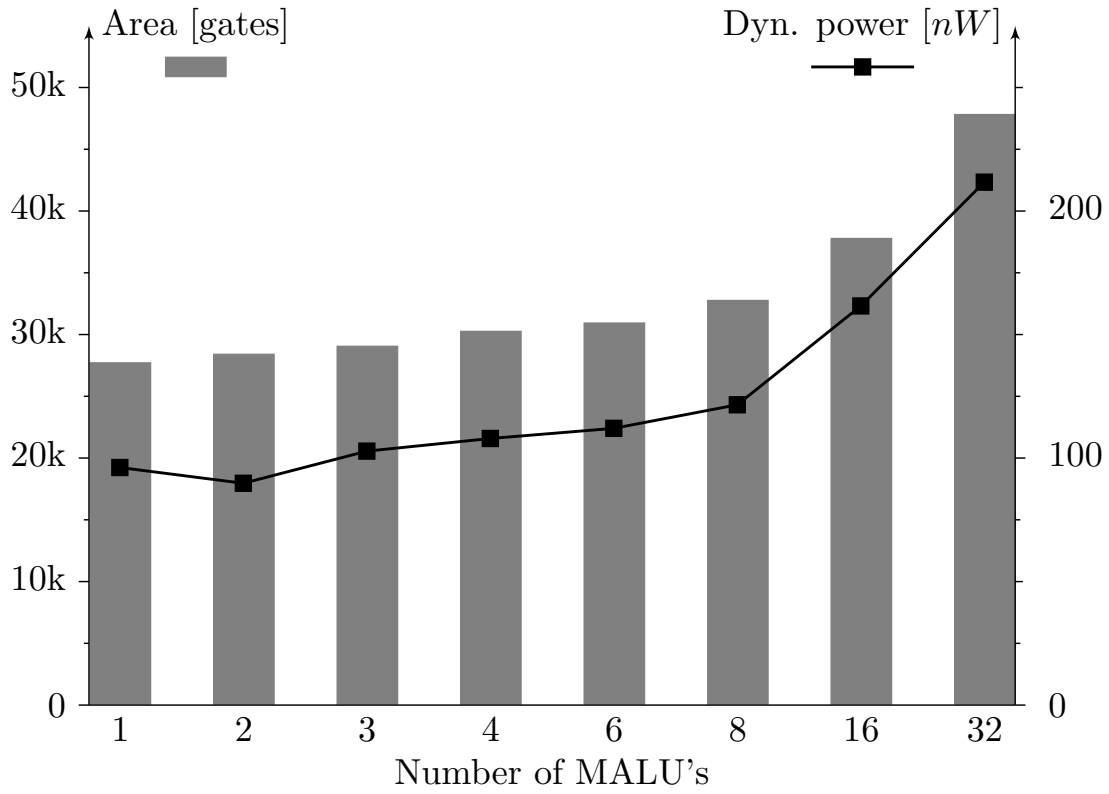
Synthesis

Implementation	Area [gates]		Power @ 10 kHz $[nW]$			
			Dynamic		Leakage	
Basic	28 876		512		117	
No Reset	27 596	96%	395	77%	107	92%
CG 1	27 751	96%	94	18%	109	94%
CG 2	27 713	96%	59	12%	102	88%
CG 3	27 734	96%	96	19%	110	94%



Synthesis - Continued

Component	Opp. [gates]	
MALU	458	1.7%
$\mathbb{F}_{2m}$ core		
Logic	783	2.8%
Registers	962	3.5%
Controller		
Logic	13 044	47%
Registers	12 487	45%
Total	27 734	100%



Comparison

	This work		Beuchat *et al.*
	1 MALU	2 MALUs	
Field	$\mathbb{F}_{2^{163}}$	$\mathbb{F}_{2^{163}}$	$\mathbb{F}_{3^{97}}$
Pairing	Tate	Tate	η_T
Security [bit]	652	652	922
Technology $[\mu m]$	0.13	0.13	0.18
Area [gates]	27 430	28 155	193 765
f [MHz]	10.3	5.44	200
Calc. time $[\mu s]$	$50 \cdot 10^3$	$50 \cdot 10^3$	46.7
Power $[mW]$	$98.3 \cdot 10^{-3}$	$48.6 \cdot 10^{-3}$	672
Efficiency $\left[\frac{nJ}{\text{bit}}\right]$	7.54	3.73	34.0

$$\text{Efficiency} = \frac{\text{power} \times \text{calc. time}}{\text{bits security}}$$

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- Example with 3 MALUs:

Area $= 29$k gates	$f = 9.70$ Mhz
Power $= 100$ μA	Time $= 19.6$ ms

The end

Questions?