

# Compact implementations of pairings

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# Outline

1 Pairings

2 Implementation

3 Results

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- *Optimized* Tate pairing:

$$\hat{e}(P, Q) : E(\mathbb{F}_q)[l] \times E(\mathbb{F}_q)[l] \mapsto \mu_l$$

$$\mu_l = \text{group of } l\text{th roots of } \mathbb{F}_{q^k}^*$$

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# Restrictions

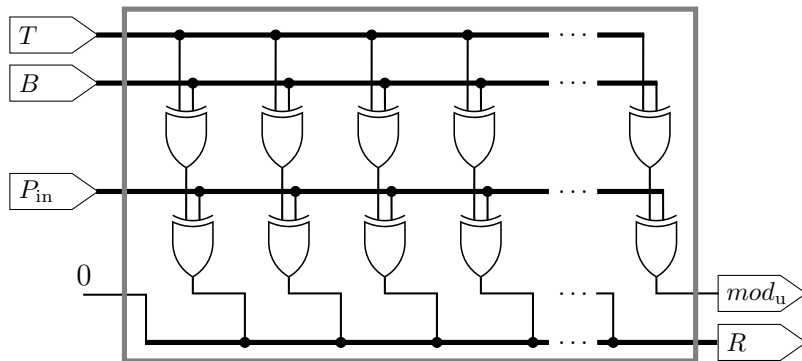
Avoid the use of flip-flops and muxes:

| Cell                   | Area $\left[ \frac{\text{gate}}{\text{bit}} \right]$ |
|------------------------|--|
| D flip-flop (reset)    | 6  |
| D flip-flop (no reset) | 5.5  |
| D latch                | 4.25   |
| 3 input MUX            | 4  |
| 2 input XOR            | 3.75   |
| 2 input MUX            | 2.25   |
| 2 input NAND           | 1  |
| NOT                    | 0.75   |

# MALU - Addition & Reduction in $\mathbb{F}_{2^m}$

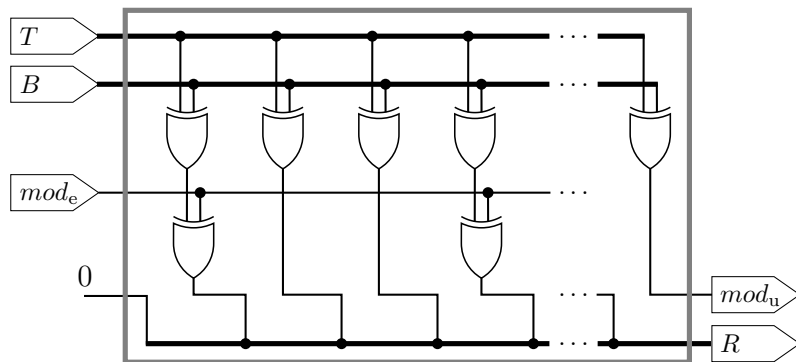
$$R = (T + B \pmod{P_{\text{in}}})_{0:m-2} \ll 1$$

$$\text{mod}_u = (T + B \pmod{P_{\text{in}}})_{m-1}$$

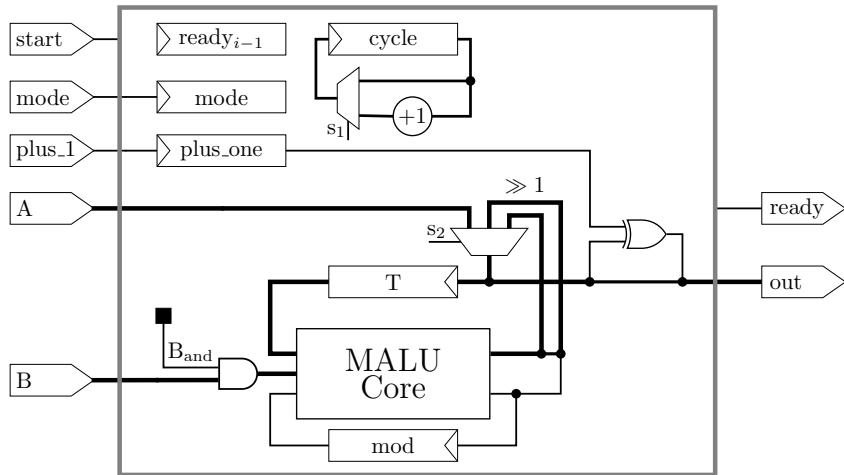


# MALU - Addition & Reduction in $\mathbb{F}_{2^m}$

Optimized MALU needs  $\Delta = m - (\text{Hamm}(P) - 1)$  less XORs:



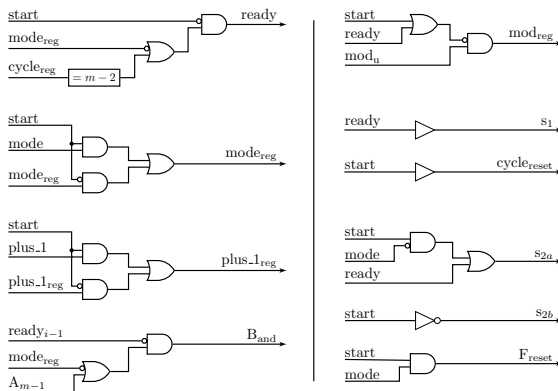
# $\mathbb{F}_{2^m}$ Multiplication & Addition





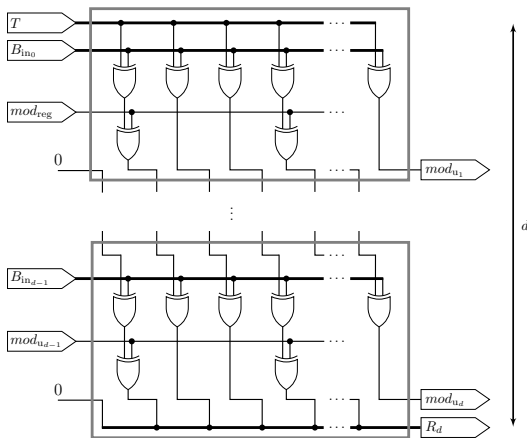
# $\mathbb{F}_{2^m}$ Multiplication & Addition

No FSM needed, simple logic:

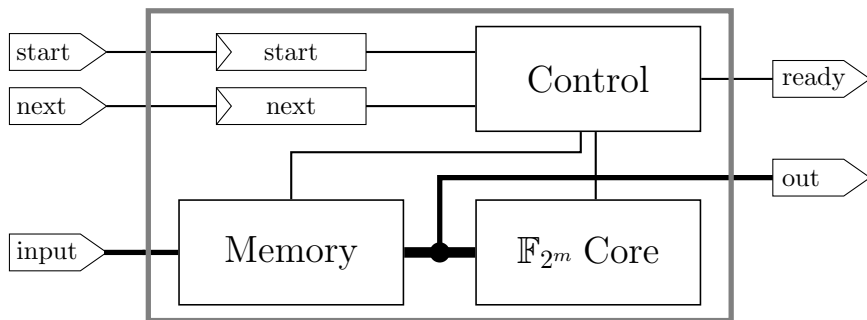


# $\mathbb{F}_{2^m}$ Multiplication & Addition

Speed up calculation by daisy-chaining MALUs ( $m \bmod d!$ ):



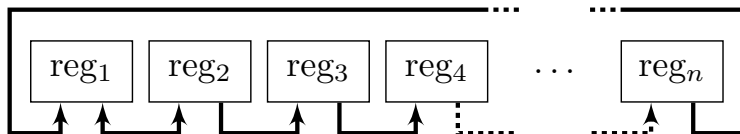
# Controller for Miller's algorithm



# Memory design

Initial design:

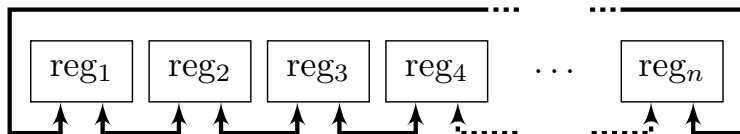
$$\bar{t} = O\left(\frac{n^2}{3}\right) \quad \bar{w} = O\left(\frac{n^3}{3}\right)$$



# Memory design

Final design:

$$\bar{t} = O\left(\frac{n}{4}\right) \quad \bar{w} = O(n)$$



# Optimizations

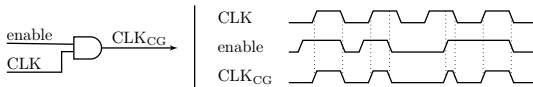
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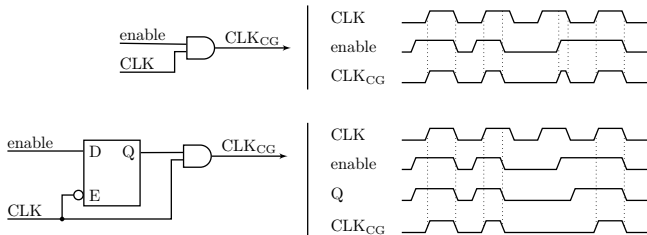
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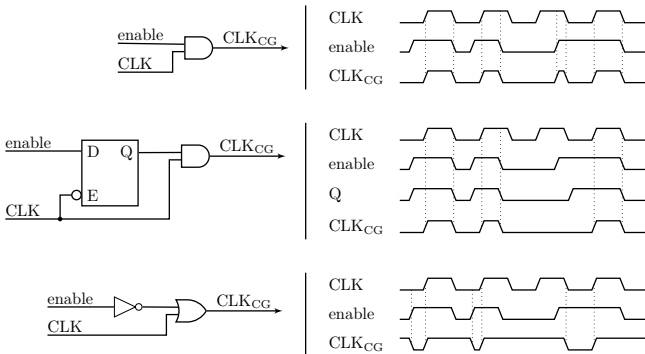
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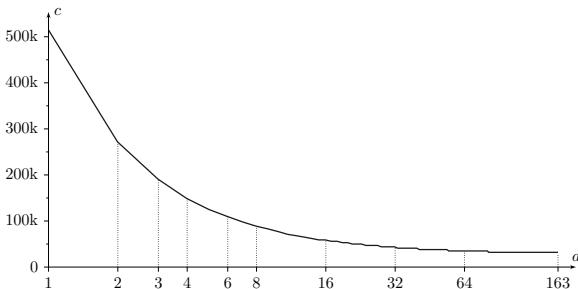
- FSM with 553 states
- Total n° of clock cycles  $c$  for one pairing:

$$c = 21681 + 4322 + 2998 \cdot \left\lceil \frac{m}{d} \right\rceil$$

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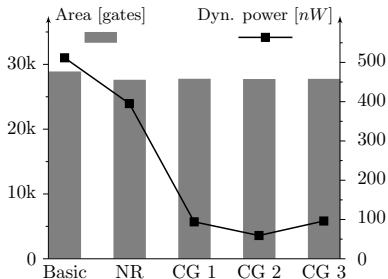
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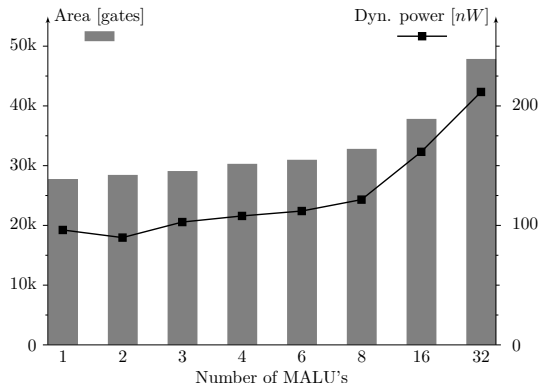
# Synthesis

| Implementation | Area [gates] |     | Power @ 10 kHz [ <i>nW</i> ] |     |         |     |
|----------------|--------------|-----|------------------------------|-----|---------|-----|
|                |              |     | Dynamic                      |     | Leakage |     |
| Basic          | 28 876       |     | 512                          |     | 117     |     |
| No Reset       | 27 596       | 96% | 395                          | 77% | 107     | 92% |
| CG 1           | 27 751       | 96% | 94                           | 18% | 109     | 94% |
| CG 2           | 27 713       | 96% | 59                           | 12% | 102     | 88% |
| CG 3           | 27 734       | 96% | 96                           | 19% | 110     | 94% |



# Synthesis - Continued

| Component               | Opp. [gates] |      |
|-------------------------|--------------|------|
| MALU                    | 458          | 1.7% |
| $\mathbb{F}_{2^m}$ core |              |      |
| Logic                   | 783          | 2.8% |
| Registers               | 962          | 3.5% |
| Controller              |              |      |
| Logic                   | 13 044       | 47%  |
| Registers               | 12 487       | 45%  |
| Total                   | 27 734       | 100% |





# Comparison

|                                 | This work              |                        | Beuchat<br><i>et al.</i> |
|---------------------------------|------------------------|------------------------|--------------------------|
|                                 | 1 MALU                 | 2 MALUs                |                          |
| Field                           | $\mathbb{F}_{2^{163}}$ | $\mathbb{F}_{2^{163}}$ | $\mathbb{F}_{3^{97}}$    |
| Pairing                         | Tate                   | Tate                   | $\eta_T$                 |
| Security [bit]                  | 652                    | 652                    | 922                      |
| Technology [ $\mu m$ ]          | 0.13                   | 0.13                   | 0.18                     |
| Area [gates]                    | 27 430                 | 28 155                 | 193 765                  |
| $f$ [MHz]                       | 10.3                   | 5.44                   | 200                      |
| Calc. time [ $\mu s$ ]          | $50 \cdot 10^3$        | $50 \cdot 10^3$        | 46.7                     |
| Power [ $mW$ ]                  | $98.3 \cdot 10^{-3}$   | $48.6 \cdot 10^{-3}$   | 672                      |
| Efficiency [ $\frac{nJ}{bit}$ ] | 7.54                   | 3.73                   | 34.0                     |

$$\text{Efficiency} = \frac{\text{power} \times \text{calc. time}}{\text{security}}$$

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- Example with 3 MALUs:

Area =  $29\text{k}$  gates

$f = 9.70\text{ Mhz}$

Power =  $100\text{ }\mu\text{A}$

Time =  $19.6\text{ ms}$

# The end

Questions?