CS 5350/6350: Machine Learining Fall 2017

Homework 2
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September 26, 2017

1 Warm up: Linear Classifiers and Boolean Functions

1. $x_1 \vee \neg x_2 \vee x_3$

$$x_1 - x_2 + x_2 \geq 0$$

2. $(x_1 \vee x_2) \wedge (x_2 \vee x_3)$

$$x_1 + 2x_2 + x_3 \geq 2$$

3. $(x_1 \wedge \neg x_2) \vee x_3$

$$x_1 - x_2 + 2x_3 \geq 1$$

4. x_1 xor x_2 xor x_3
This function is not linearly seperable

5. $\neg x_1 \wedge x_2 \wedge \neg x_3$

$$-x_1 + x_2 - x_3 \geq 1$$

2 Mistake Bound Model of Learning

1.

(a) Since $1 \leq l \leq 80$, the size of concept class $|C| = 80$

(b) The prediciton made by hypothesis of length l on inputs x_1^t and x_2^t can be represented by the equation

$$sgn(2l - |x_1^t| - |x_2^t|)$$

We make a mistake when this prediction does not match $sgn(y^t)$ of the label y^t. Hence we can determine whether mistake has been made using the following inequality:

$$\boxed{y^t(2l - |x_1^t| - |x_2^t|) < 0} \quad (1)$$

(c) Let us assume $l = l_c$ for correct function and $l = l_h$ for current hypothesis function. Let us consider what happens during positive and negative examples:

i. Positive Example:
When there is a mistake on positive example, we have label $y^t = +1$ and equation $2l - |x_1^t| - |x_2^t| < 0$. Therefore we can say that either x_1^t, x_2^t or both were greater than l_h and we correct our hypothesis by setting $l_h = max(x_1^t, x_2^t)$ because the hypothesis will always make mistakes on all values of $l_h < max(x_1^t, x_2^t)$

ii. Negative Example:
When there is a mistake on positive example, we have label $y^t = -1$ and equation $2l - |x_1^t| - |x_2^t| \geq 0$. For negatiive example i.e $y^t = -1$ we must have either x_1^t, x_2^t or both greater than value of l_c for the correct function. l_h is higher than all of these values, therefore we set $l_h = max(x_1^t, x_2^t) - 1$. This will ensure our hypothesis matches the label for current example.

(d) Let the $l = l_h$ for current hypothesis. {..} represents comments in the pseudocode.

Algorithm 1 Mistake driven algorithm to learn correct function $f \in C$

```
Start with l_h = 40 {We set l_h to half of the range of l}
for each example do
  if y^t(2l − |x_1^t| − |x_2^t|) < 0 then
    if y^t == +1 then
      l_h = max(x_1^t, x_2^t) {If mistake is made on positive example}
    else
      l_h = max(x_1^t, x_2^t) − 1 {If mistake is made on negative example}
    end if
  end if
end for
```

On any given dataset, the algorithm will make maximum of m mistakes where

$$m = |40 - l_c|$$

where l_c is value of length l for correct function $f \in C$.

2. In the halving algorithm we predict output which agrees with majority of functions in the current concept class C. If a mistake is made, we remove the functions which predicted the wrong output and repeat the process on next example. This leads to cutting down of experts by atleast half each time a mistake is made. This process stops when we reach down to 1 expert because he has predicted correct output for all the examples. Now if we have M experts in the initial concept class C we will stop the algorithm when we have cut down the pool of experts to these final M functions.
Let $|C| = N$ be the initial size of concept class. When 0 mistakes are made we have size of initial concept class $|C_0| = |C|$. When first mistake is made,

$$|C_1| \leq \frac{1}{2}|C_0|$$

When 2 mistakes are made,

$$|C_2| \leq \frac{1}{2^2}|C_0|$$

. Let n be the number of mistakes for which the size of concept class is cut down to M. Therefore,

$$M = |C_n| \leq \frac{1}{2^n}|C_0|$$

Now in worst case we commit $M = \frac{1}{2^n}|C_0|$ mistakes. But $|C_0| = |C| = N$. Therefore,

$$M = \frac{1}{2^n}N$$

$$\therefore 2^n = \frac{N}{M}$$

$$\therefore n = \log_2\left(\frac{N}{M}\right)$$

$\therefore$ Mistake bound of halving algorithm in case of M experts is $O(\log \frac{N}{M})$

3 The Perceptron Algorithm and its Variants

1. (a) Python3.6 is used to implement the experiment.

   (b) Each datafile is read and parsed into a numpy array with 68 columns. Each column has appropriate values of the features as in the file. Rest are set to 0.

   (c) I have defined perceptron as a single class with training data, test data, predictions, weights, bias and accuracy as its attributes and train and predict as its methods. The file *perceptron.py* represents the class. Aggressive variants of train and predict methods are also present in the class and called when aggressive flag is set while calling train and predict. Weights are represented by a list of floats with length equal to that of number of columns. Bias is represented as float. When object of perceptron class is created, the weights and bias are initialised randomly between -0.01 and 0.01. There is option of seeding this random number if required.

   (d) Simple perceptron is represented by the class as is. No extra parameters except for data and learning rate are required while calling the train function.
   If dynamic_ lr flag for train function is set, then perceptron will learn with a dynamic learning rate. The learning rate is updated for each example, whether there is a mistake or not.
   While creating object of perceptron object we can pass it margin value which is

used to check if update is to be done for the example or not. Default value of this attribute is 0.
There are extra attributes to store average weights and average bias. These are always updated in each iteration, but the predict function ignores them unless average flag is set, in this case average weights and average bias are used for prediction.
Aggressive perceptron can be trained by passing the aggressive flag as 1 in train and predict function. Here weights and bias are not stored seperately but collection of weights has $n + 1$ weights where n is number of features in data. An column of 1s is appended to x before training and predictions.

(e) I have used a two random functions. One for generation of random weights and bias, and the other for shuffling numpy arrays. Both can be seeded to have output as described in trace file. Seed value can be set by setting the variable in *random_seed.py* file. Just uncomment the lines which are used to set the random seed, in order to seed those functions

(f) It takes around 45-50 seconds to execute the entire program.

2. The number of instances of each label in training set:

$$1 = 4606$$

$$-1 = 3685$$

The number of instances of each label in dev set:

$$1 = 759$$

$$-1 = 623$$

The number of instances of each label in test set:

$$1 = 792$$

$$-1 = 590$$

Let us consider a classifier which predicts the most frequent label. So this classifer will train will always predict the majority label 1.

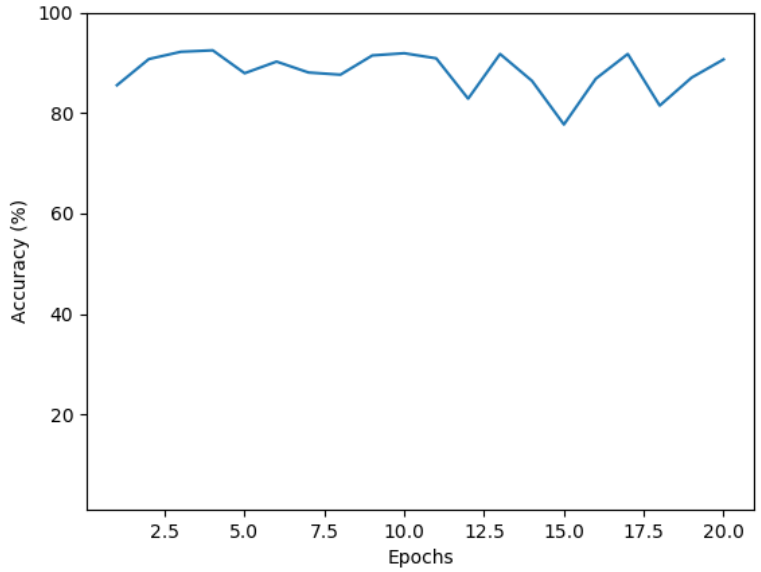
Accuracy on dev set:
$A_{dev} = 759/(759 + 623) * 100 = 54.92\%$

Accuracy on test set:
$A_{test} = 792/(792 + 590) * 100 = 57.30\%$

3. The following results are for when seed in *random_seed.py* file is set to 6. All the accuracies are in %

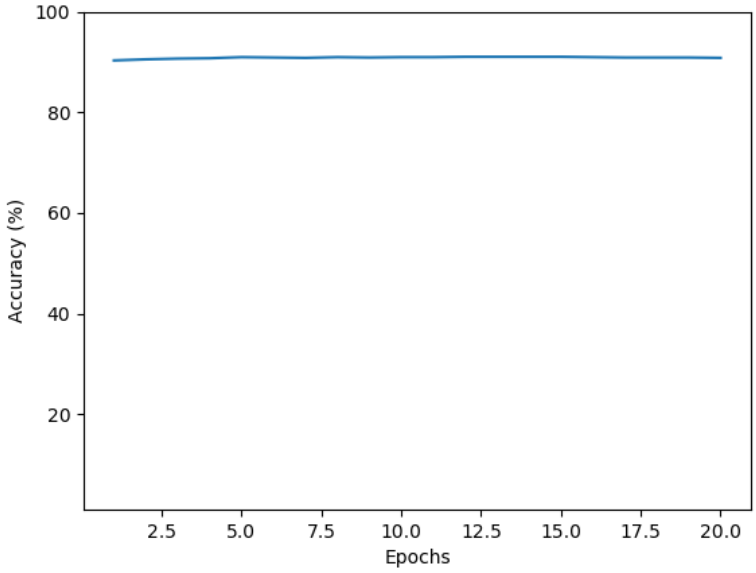
(a) Simple Perceptron

i. Best Hyperparameter: learning rate = 0.1
ii. Cross Validation accuracy for best hyperparameter: 90.7971
iii. Total number of updates performed on training set: 14086
iv. Development set accuracy: 90.6657
v. Test set accuracy: 92.9088
vi. Epoch vs Dev Set Accuracy:



(b) Perceptron with dynamic learning rate

i. Best Hyperparameter: Initial Learning Rate = 1
ii. Cross Validation accuracy for best hyperparameter: 89.2898
iii. Total number of updates performed on training set: 16408
iv. Development set accuracy: 90.8104
v. Test set accuracy: 91.3893
vi. Epoch vs Dev Set Accuracy:



(c) Perceptron with margin

i. Best Hyperparameter:
Intial learning rate = 1
Margin = 0.1

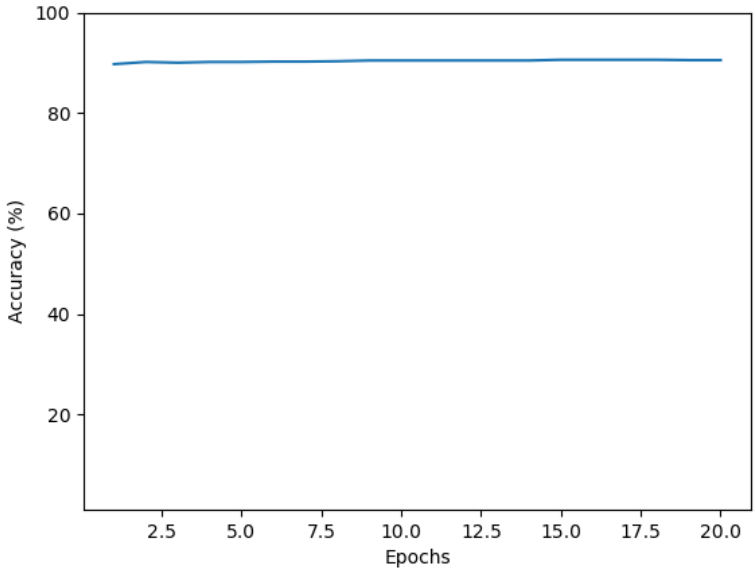
ii. Cross Validation accuracy for best hyperparameter: 91.0869

iii. Total number of updates performed on training set: 73105

iv. Development set accuracy: 90.521

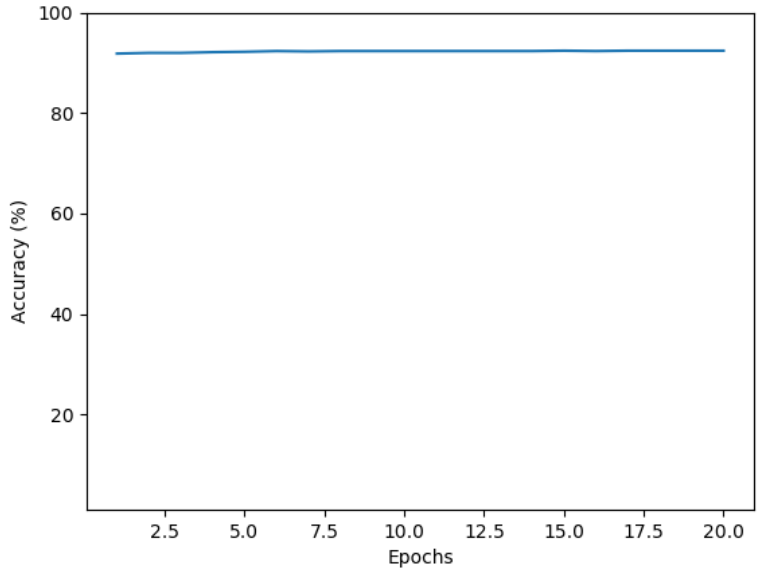
v. Test set accuracy: 92.2576

vi. Epoch vs Dev Set Accuracy:



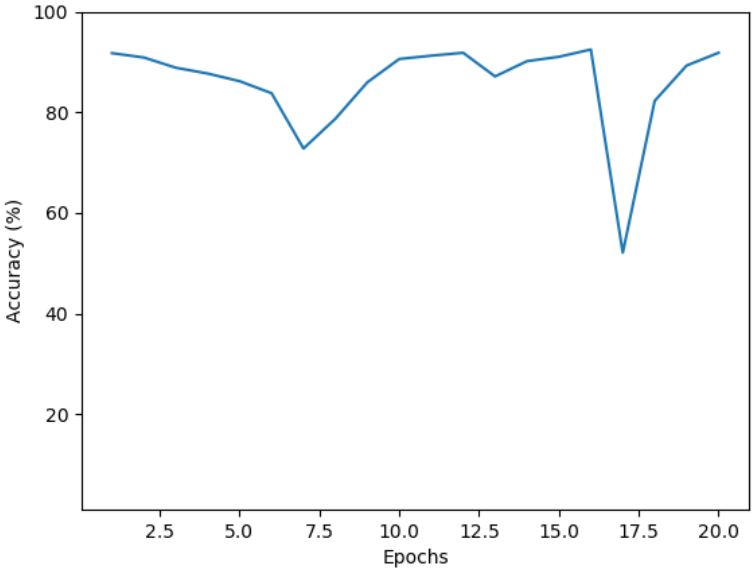
(d) Averaged Perceptron

i. Best Hyperparameter: Learning Rate = 0.01
ii. Cross Validation accuracy for best hyperparameter: 93.5835
iii. Total number of updates performed on training set: 14027
iv. Development set accuracy: 92.4023
v. Test set accuracy: 92.9812
vi. Epoch vs Dev Set Accuracy:



(e) Aggressive Perceptron with margin

i. Best Hyperparameter: Margin = 0.1
ii. Cross Validation accuracy for best hyperparameter: 88.2397
iii. Total number of updates performed on training set: 63574
iv. Development set accuracy: 91.8234
v. Test set accuracy: 93.1983
vi. Epoch vs Dev Set Accuracy:


100
80
60
40
20
Accuracy (%)
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Epochs