

## Task 4.6

Условие:

$$\begin{aligned}
 x_1 - 8x_2 &\leq 10 \\
 x_1 + x_2 &\geq 6 \\
 x_1 &\geq 2 \\
 x_2 &\leq 5 \\
 x_{1,2} &\geq 0 \\
 f = 10x_1 - 6x_2 &\rightarrow \max
 \end{aligned} \tag{1}$$

Каноническая форма:	
$x_1 - 8x_2 + y_1 = 10$  $x_1 + x_2 - y_2 = 6$ $x_1 - y_3 = 2$ $x_2 + y_4 = 5$ $x_{1,2} \geq 0; y_j \geq 0$ $f = 10x_1 - 6x_2 \rightarrow \max$	$  \begin{pmatrix} 1 & -8 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \\ 2 \\ 5 \end{pmatrix}  $ $  (10 \quad -6) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \max  $
Стандартная форма:	
$x_1 - 8x_2 \leq 10$  $-x_1 - x_2 \leq -6$ $-x_1 \leq -2$ $x_2 \leq 5$ $x_{1,2} \geq 0$ $f = 10x_1 - 6x_2 \rightarrow \max$	$  \begin{pmatrix} 1 & -8 \\ -1 & -1 \\ -1 & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 10 \\ -6 \\ -2 \\ 5 \end{pmatrix}  $ $  (10 \quad -6) \times \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow \max  $

## 1. Метод штрафа

$$\begin{aligned}
 x_1 - 8x_2 + y_1 &= 10 \\
 x_1 + x_2 - y_2 + R_1 &= 6 \\
 x_1 - y_3 + R_2 &= 2 \\
 x_2 + y_4 &= 5 \\
 x_{1,2} &\geq 0; y_j \geq 0 \\
 R_{1,2} &\geq 0 \\
 F &= 10x_1 - 6x_2 - M * R_1 - M * R_2 \rightarrow \max
 \end{aligned} \tag{2}$$

бп	f	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.	
f	1	-10	6	0	0	0	0	30	30	0	
	1	-70	-24	0	30	30	0	0	0	-240	
	0	-70	0	0	0	70	0	0	0	-140	
y1	0	1	-8	1	0	0	0	0	0	10	10
	0	1	0	0	0	-1	0	0	0	1	2
r1	0	1	1	0	-1	0	0	1	0	6	6
	0	1	0	0	0	-1	0	0	0	1	2
r2	0	1	0	0	0	-1	0	0	1	2	2
	0	1	0	0	0	-1	0	0	1	2	2
y4	0	0	1	0	0	0	1	0	0	5	#DIV/0!
	0	0	0	0	0	0	0	0	0	0	

  

бп	f	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.	
f	1	0	-24	0	30	-40	0	0	70	-100	
	0	0	-40	0	40	-40	0	-40	40	-160	
y1	0	0	-8	1	0	1	0	0	-1	8	8
	0	0	1	0	-1	1	0	1	-1	-1	4
r1	0	0	1	0	-1	1	0	1	-1	4	4
	0	0	1	0	-1	1	0	1	-1	4	4
x1	0	1	0	0	0	-1	0	0	1	2	-2
	0	0	-1	0	1	-1	0	-1	1	-4	
y4	0	0	1	0	0	0	1	0	0	5	#DIV/0!
	0	0	0	0	0	0	0	0	0	0	

бн	f	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.		
f	1	0	16	0	-10	0	0	40	30	60		
	0	0	90	-10	-10	0	0	10	0	-40		
y1	0	0	-9	1	1	0	0	-1	0	4	4	
	0	0	-9	1	1	0	0	-1	0	4		
y3	0	0	1	0	-1	1	0	1	-1	4	-4	
	0	0	9	-1	-1	0	0	1	0	-4		
x1	0	1	1	0	-1	0	0	1	0	6	-6	
	0	0	9	-1	-1	0	0	1	0	-4		
y4	0	0	1	0	0	0	1	0	0	5	#DIV/0!	
	0	0	0	0	0	0	0	0	0	0		
бн	f	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.		
f	1	0	-74	10	0	0	0	30	30	100		
	0	0	-74	0	0	0	0	-74	0	0	-370	
y2	0	0	-9	1	1	0	0	-1	0	4	-0,44444	
	0	0	-9	0	0	0	0	-9	0	0	-45	
y3	0	0	-8	1	0	1	0	0	-1	8	-1	
	0	0	-8	0	0	0	0	-8	0	0	-40	
x1	0	1	-8	1	0	0	0	0	0	10	-1,25	
	0	0	-8	0	0	0	0	-8	0	0	-40	
y4	0	0	1	0	0	0	1	0	0	5	5	
	0	0	1	0	0	0	0	1	0	0	5	
бн	f	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.		
f	1	0	0	10	0	0	74	30	30	470		
y2	0	0	0	1	1	0	9	-1	0	49		
y3	0	0	0	1	0	1	8	0	-1	48		
x1	0	1	0	1	0	0	8	0	0	50		
x2	0	0	1	0	0	0	1	0	0	5		
				x1	50	f	470	<div></div>				
				x2	5							
				Constraint	10	<=	10					
					55	>=	6					
					x1	>=	2					
					x2	<=	5					

$$f = -10y_1 - 74y_4 - 30R_1 - 30R_2 + 470 \quad (3)$$

$$x_1 = 50; x_2 = 5; y_2 = 49; y_3 = 48; y_1 = 0; y_4 = -45$$

## 2. Двухэтапный метод

$$\begin{aligned}
 x_1 - 8x_2 + y_1 &= 10 \\
 x_1 + x_2 - y_2 + R_1 &= 6 \\
 x_1 - y_3 + R_2 &= 2 \\
 x_2 + y_4 &= 5 \\
 x_{1,2} &\geq 0; y_j \geq 0; R_{1,2} \geq 0 \\
 f &= 10x_1 - 6x_2 \rightarrow \max \\
 R &= R_1 + R_2 \rightarrow \min
 \end{aligned} \tag{4}$$

### Первый этап

бп	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.	
R	0	0	0	0	0	0	-1	-1	0	
	2	1	0	-1	-1	0	0	0	8	4
	2	0	0	0	-2	0	0	2	4	
y1	1	-8	1	0	0	0	0	0	10	10
	1	0	0	0	-1	0	0	0	1	2
r1	1	1	0	-1	0	0	1	0	6	6
	1	0	0	0	-1	0	0	0	1	2
r2	1	0	0	0	-1	0	0	1	2	2
	1	0	0	0	-1	0	0	1	2	
y4	0	1	0	0	0	1	0	0	5	#DIV/0!
	0	0	0	0	0	0	0	0	0	

  

бп	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.	
R	0	1	0	-1	1	0	0	-2	4	
	0	1	0	-1	1	0	0	1	-1	4
y1	0	-8	1	0	1	0	0	-1	8	-1
	0	-8	0	8	-8	0	-8	8	-32	
r1	0	1	0	-1	1	0	1	-1	4	4
	0	1	0	-1	1	0	1	-1	4	
x1	1	0	0	0	-1	0	0	1	2	#DIV/0!
	0	0	0	0	0	0	0	0	0	
y4	0	1	0	0	0	1	0	0	5	5
	0	1	0	-1	1	0	1	-1	4	

## Второй этап

бп	x1	x2	y1	y2	y3	y4	r1	r2	св. чл.
R	0	0	0	0	0	0	-1	-1	0
y1	0	0	1	-8	9	0	8	-9	40
x2	0	1	0	-1	1	0	1	-1	4
x1	1	0	0	0	-1	0	0	1	2
y4	0	0	0	1	-1	1	-1	1	1
бп	x1	x2	y1	y2	y3	y4	св. чл.		
f	-10	6	0	0	0	0	0		
	0	0	0	6	-16	0	-4		
	0	-16	0	16	-16	0	-64		
y1	0	0	1	-8	9	0	40	4,444444	
	0	9	0	-9	9	0	36		
x2	0	1	0	-1	1	0	4	4	
	0	1	0	-1	1	0	4		
x1	1	0	0	0	-1	0	2	-2	
	0	-1	0	1	-1	0	-4		
y4	0	0	0	1	-1	1	1	-1	
	0	-1	0	1	-1	0	-4		
бп	x1	x2	y1	y2	y3	y4	св. чл.		
f	0	16	0	-10	0	0	60		
	0	90	-10	-10	0	0	-40		
y1	0	-9	1	1	0	0	4	4	
	0	-9	1	1	0	0	4		
y3	0	1	0	-1	1	0	4	-4	
	0	9	-1	-1	0	0	-4		
x1	1	1	0	-1	0	0	6	-6	
	0	9	-1	-1	0	0	-4		
y4	0	1	0	0	0	1	5	#DIV/0!	
	0	0	0	0	0	0	0		
бп	x1	x2	y1	y2	y3	y4	св. чл.		
f	0	-74	10	0	0	0	100		
	0	-74	0	0	0	-74	-370		
y2	0	-9	1	1	0	0	4	-0,44444	
	0	-9	0	0	0	-9	-45		
y3	0	-8	1	0	1	0	8	-1	
	0	-8	0	0	0	-8	-40		
x1	1	-8	1	0	0	0	10	-1,25	
	0	-8	0	0	0	-8	-40		
y4	0	1	0	0	0	1	5	5	
	0	1	0	0	0	1	5		

бп	x1	x2	y1	y2	y3	y4	св. чл.
f	0	0	10	0	0	74	470
y1	0	0	1	1	0	9	49
y3	0	0	1	0	1	8	48
x1	1	0	1	0	0	8	50
x2	0	1	0	0	0	1	5

$$f = -10y_1 - 74y_4 + 470 \quad (5)$$

$$x_1 = 50; x_2 = 5; y_2 = 49; y_3 = 48; y_1 = 0; y_4 = -45$$

## Task 5.6

### Task 2.6

Условие:

$$\begin{aligned} 2x_1 - x_2 &\leq 8 \\ -x_1 + x_2 &\leq 1 \\ -x_1 + x_2 &\geq -1 \\ f &= -5x_1 - 2x_2 \rightarrow \min \end{aligned} \quad (6)$$

Каноническая форма

$$\begin{aligned} 2x_1 - x_2 + y_1 &= 8 \rightarrow \lambda_1 \\ -x_1 + x_2 + y_2 &= 1 \rightarrow \lambda_2 \\ x_1 - x_2 + y_3 &= 1 \rightarrow \lambda_3 \\ f &= -5x_1 - 2x_2 \rightarrow \min \end{aligned} \quad (7)$$

Составим двойственную задачу

$$\begin{aligned} \Phi &= 8\lambda_1 + \lambda_2 + \lambda_3 \rightarrow \max \\ x_1 : 2\lambda_1 - \lambda_2 + \lambda_3 &\leq -5 \\ x_2 : -\lambda_1 + \lambda_2 - \lambda_3 &\leq -2 \\ y_1 : \lambda_1 &\leq 0 \\ y_2 : \lambda_2 &\leq 0 \\ y_3 : \lambda_3 &\leq 0 \end{aligned} \quad (8)$$

Целевая функция в оптимуме имеет вид:

$$f = -65 + 9y_2 + 7y_1 \quad (9)$$

Решение

$$\begin{aligned} -7 &= \lambda_1 - 0 \Rightarrow \lambda_1 = -7 \\ -9 &= \lambda_2 - 0 \Rightarrow \lambda_2 = -9 \\ 0 &= \lambda_3 - 0 \Rightarrow \lambda_3 = 0 \end{aligned} \quad (10)$$

Проверка результата

		<b>f</b>	<b>-65</b>
<b>l1</b>	<b>-7</b>		
<b>l2</b>	<b>-9</b>		
<b>l3</b>	<b>0</b>		
<b>Constraint</b>	<b>-5</b>	<b>&lt;=</b>	<b>-5</b>
	<b>-2</b>	<b>&lt;=</b>	<b>-2</b>

## Task 4.6

Условие:

$$\begin{aligned}
 x_1 - 8x_2 &\leq 10 \\
 x_1 + x_2 &\geq 6 \\
 x_1 &\geq 2 \\
 x_2 &\leq 5 \\
 f = 10x_1 - 6x_2 &\rightarrow \max
 \end{aligned} \tag{11}$$

Каноническая форма

$$\begin{aligned}
 x_1 - 8x_2 + y_1 &= 10 \rightarrow \lambda_1 \\
 x_1 + x_2 - y_2 &= 6 \rightarrow \lambda_2 \\
 x_1 - y_3 &= 2 \rightarrow \lambda_3 \\
 x_2 + y_4 &= 5 \rightarrow \lambda_4 \\
 f = 10x_1 - 6x_2 &\rightarrow \max
 \end{aligned} \tag{12}$$

Целевая функция в оптимуме имеет вид:

$$f = 470 - 10y_1 - 74y_4 \tag{13}$$

Составим двойственную задачу

$$\begin{aligned}
 \Phi &= 10\lambda_1 + 6\lambda_2 + 2\lambda_3 + 5\lambda_4 \rightarrow \min \\
 x_1 : \lambda_1 + \lambda_2 + \lambda_3 &\geq 10 \\
 x_2 : -8\lambda_1 + \lambda_2 + \lambda_4 &\geq -6 \\
 y_1 : \lambda_1 &\geq 0 \\
 y_2 : \lambda_2 &\leq 0 \\
 y_3 : \lambda_3 &\leq 0 \\
 y_4 : \lambda_4 &\geq 0
 \end{aligned} \tag{14}$$

$$\tag{15}$$

Решение

$$\begin{aligned}10 &= \lambda_1 - 0 = \lambda_1 \\0 &= \lambda_2 - 0 = \lambda_2 \\0 &= \lambda_3 - 0 = \lambda_3 \\74 &= \lambda_4 - 0 = \lambda_4\end{aligned}\tag{16}$$