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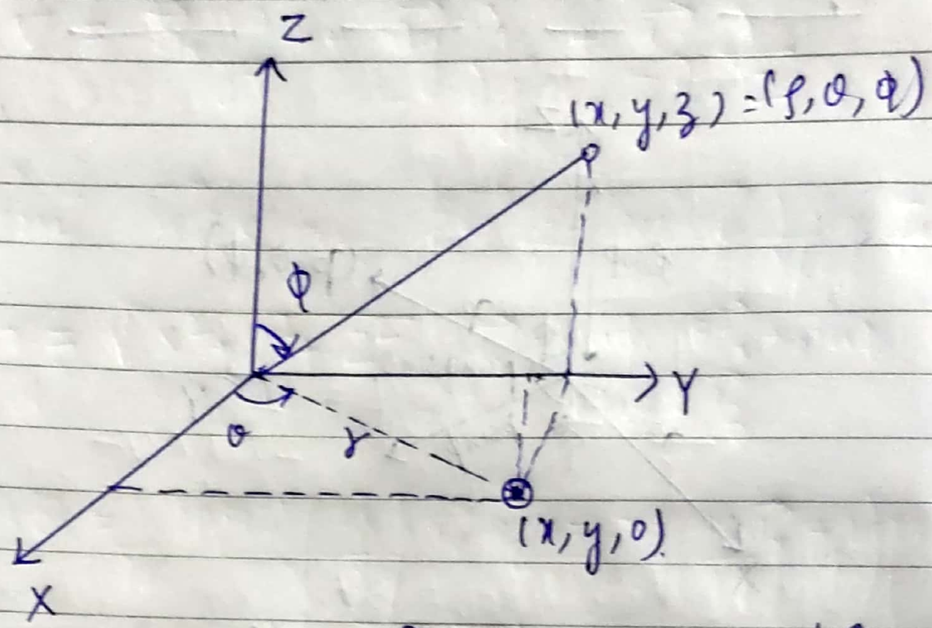
IEC2019053

AWP- assignment

Ans-1 (i) **Cartesian to spherical coordinates**

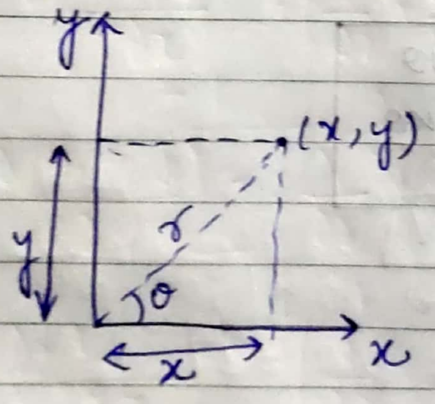


Here ρ = Distance from origin

ϕ = Angle from z-axis.

θ = Angle from x-axis

Deriving θ from x, y plane



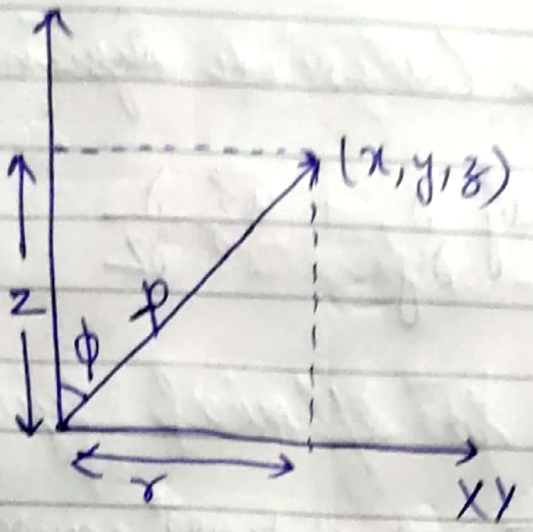
$r = \sqrt{x^2+y^2}$ (pythagoras's) —①

using trignometric identity

$\tan\theta = \frac{y}{x}$

$$\boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right)}$$

Deriving ρ and ϕ from Z-XY plane.



$\rho = r^2 + Z^2$ (pythagora theorem)

From eq ① we know $r^2 = x^2 + y^2$

so $\rho^2 = x^2 + y^2 + z^2$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

By using trignometric identities.

$$\tan\phi = r/Z = \frac{\sqrt{x^2+y^2}}{Z}$$

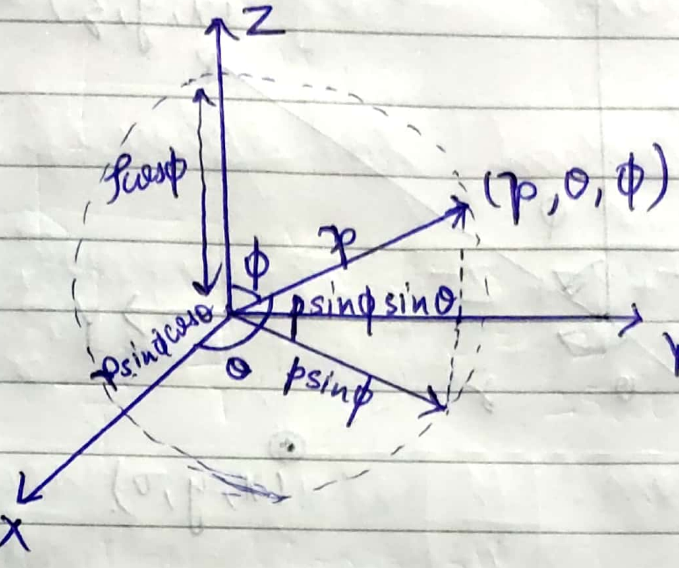
$$\boxed{\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{Z}\right)}$$

So for given catesian co-ordinate, eq. spherical coordinate comes out to be.

$$(\rho, \theta, \phi) \equiv \left(\sqrt{x^2+y^2+Z^2},\ \tan^{-1}\left(\frac{y}{x}\right),\ \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{Z}\right)\right)$$

Ans1

ii) Spherical to Cartesian Coordinate



$\rho \rightarrow$ Radius of sphere

Taking components of $\vec{\rho}$ as a vector on x, y and z we get.

$$\boxed{\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$

Ans2 Del operator in Cartesian System

$$\nabla = \vec{a_x}\frac{\partial}{\partial x} + \vec{a_y}\frac{\partial}{\partial y} + \vec{a_z}\frac{\partial}{\partial z}$$

From cartesian to spherical System

$r = \sqrt{x^2+y^2+z^2}$ $\quad \vec{a_r} = \sin\phi\cos\theta\,\vec{a_x} + \sin\phi\sin\theta\,\vec{a_y} + \cos\phi\,\vec{a_z}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ $\quad \vec{a_\theta} = -\sin\theta\,\vec{a_x} + \cos\theta\,\vec{a_y}$

$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)$ $\quad \vec{a_\phi} = \cos\phi\cos\theta\,\vec{a_x} + \cos\phi\sin\theta\,\vec{a_y} - \sin\theta\,\vec{a_y}$

Applying chain rule

$$\frac{\partial(r,\theta,\phi)}{\partial x} = \left(\frac{\partial}{\partial r}\times\frac{\partial r}{\partial x}\right) + \left(\frac{\partial}{\partial \theta}\times\frac{\partial \theta}{\partial x}\right) + \left(\frac{\partial}{\partial \phi}\times\frac{\partial \phi}{\partial x}\right)$$

$$\frac{\partial(r,\theta,\phi)}{\partial y} = \left(\frac{\partial}{\partial r}\times\frac{\partial r}{\partial y}\right) + \left(\frac{\partial}{\partial \theta}\times\frac{\partial \theta}{\partial y}\right) + \left(\frac{\partial}{\partial \phi}\times\frac{\partial \phi}{\partial y}\right)$$

$$\frac{\partial(r,\theta,\phi)}{\partial z} = \left(\frac{\partial}{\partial r}\times\frac{\partial r}{\partial z}\right) + \left(\frac{\partial}{\partial \theta}\times\frac{\partial \theta}{\partial z}\right) + \left(\frac{\partial}{\partial \phi}\times\frac{\partial \phi}{\partial z}\right)$$

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Calculating ~~desired~~ required derivatives:-

$$\frac{\partial r}{\partial x} = \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x} = \frac{x}{\sqrt{x^2+y^2+z^2}} = \cos\theta\sin\phi$$

$$\frac{\partial\theta}{\partial x} = \frac{\partial}{\partial x}\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = \frac{-y}{x^2+y^2} = \frac{-r\sin\phi\sin\theta}{r^2\sin^2\phi} = -\frac{\sin\theta}{\sin\phi}$$

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\left(\tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)\right) = \frac{xz}{(x^2+y^2+z^2)\sqrt{x^2+y^2}}$$

$$= \frac{r\sin\phi\cos\theta\cos\phi}{r\sin\phi}$$

$$\frac{\partial r}{\partial y} = \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial y} = \sin\theta\sin\phi$$

$$\frac{\partial\theta}{\partial y} = \frac{\partial}{\partial y}\left(\tan^{-1}\left(\frac{y}{x}\right)\right) = \cos\phi\sin\theta$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}\left(\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}\right) = \frac{\cos\theta}{r\sin\phi}$$

$$\frac{\partial r}{\partial z} = \frac{\partial}{\partial z}\left(\sqrt{x^2+y^2+z^2}\right) = \cos\phi$$

$$\frac{\partial\theta}{\partial z} = \frac{\partial}{\partial z}\left(\tan^{-1}\frac{y}{x}\right) = 0$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial}{\partial z}\left(\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}\right) = \frac{-1}{r\sin\phi}$$

Substituting all derivatives in above eq 3

$$* \quad \boxed{\nabla = \vec{a_r}\frac{\partial}{\partial r} + \vec{a_\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \vec{a_\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}} \quad *$$

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Ans3 Curl of a vector is defined as

$$\vec{\nabla} \times \vec{A}$$

In spherical co-ordinates

$$\vec{\nabla} = \vec{a_r}\frac{\partial}{\partial r} + \left(\frac{1}{r}\right)\vec{a_\theta}\frac{\partial}{\partial \theta} + \vec{a_\phi}\frac{\partial}{\partial \phi}\left(\frac{1}{r\sin\theta}\right)$$

$$\vec{A} = \vec{a_r}A_r + \vec{a_\theta}A_\theta + \vec{a_\phi}A_\phi$$

$$\vec{\nabla}\times\vec{A} = \left(\frac{\partial}{\partial r}\vec{a_r} + \frac{1}{r}\left(\frac{\partial}{\partial \theta}\right)\vec{a_\theta} + \left(\frac{1}{r\sin\theta}\right)\left(\frac{\partial}{\partial \phi}\right)\vec{a_\phi}\right)\left(A_r\vec{a_r} + \vec{a_\theta}A_\theta + \vec{a_\phi}A_\phi\right)$$

After solving this we get.

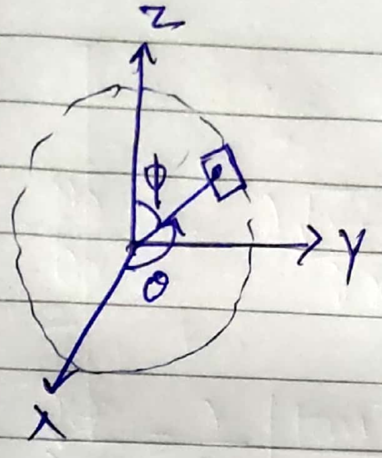
$$\vec{\nabla}\times\vec{A} = \frac{1}{r^2\sin\theta}\left(\frac{\partial}{\partial \theta}(r\sin\theta A_\phi) - \frac{\partial}{\partial \phi}(rA_\theta)\right)\vec{a_r}$$

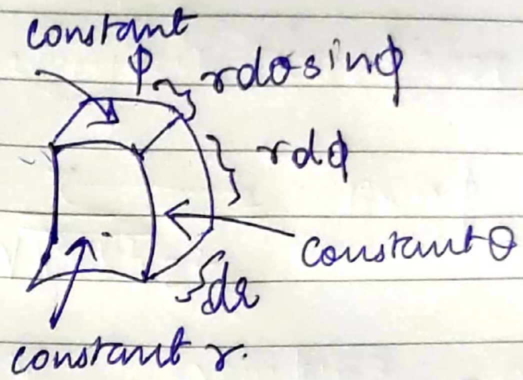
$$-\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial r}(r\sin\theta)A_\phi - \frac{\partial A_r}{\partial \phi}\right)\vec{a_\theta}$$

$$+\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\vec{a_\phi}$$

$$\left.\begin{aligned}\vec{\nabla}\times\vec{A} = &\frac{1}{r\sin\theta}\left(\frac{\partial A_\phi \sin\theta}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\vec{a_r} \\ &-\frac{1}{r\sin\theta}\left(\frac{\partial\, r\sin\theta}{\partial r}A_\phi - \frac{\partial A_r}{\partial \phi}\right)\vec{a_\theta} \\ &+\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\vec{a_\phi}\end{aligned}\right\}\text{curl formula in spherical coordinate.}$$

Ans4





For surface integral $\iint f \, dA$

$$= \iint f(r,\theta,\phi)\,(r\,d\phi\; r\,d\theta \sin\phi)$$

$$= \iint f(r,\theta,\phi)\; r^2 \sin\phi \, d\theta \, d\phi$$

This is the surface integral of a body in spherical co-ordinate system

For vol integral $\Rightarrow \iiint f \, dV$

$$= \iiint f(r,\theta,\phi)\,[(dr)(r\,d\phi)(r\sin\phi\, d\theta)]$$

$$= \iiint f(r,\theta,\phi)\, r^2 dr\, d\theta\, d\phi$$

This is the vol integral of a body in spherical co-ordinate system.