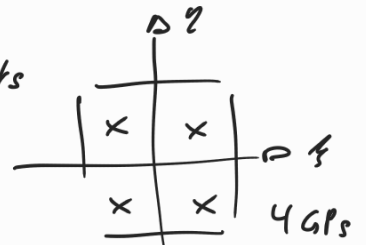


Shape functions Lagrange

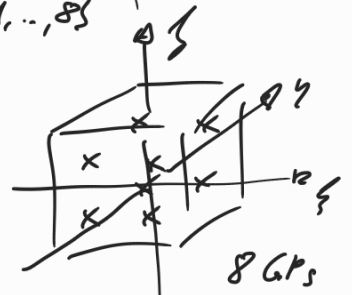
* bilinear: $N_a(\underline{\xi}) = \frac{1}{4} (1 + \xi_a \xi) (1 + \eta_a \eta)$, $a = \{1, 2, 3, 4\}$

derivatives: $N_{a,\xi}(\underline{\xi}) = \frac{1}{4} \xi_a (1 + \eta_a \eta)$
 $N_{a,\eta}(\underline{\xi}) = \frac{1}{4} \eta_a (1 + \xi_a \xi)$ } $2 \times 4 = 8$ shapefcts



* trilinear: $N_a(\underline{\xi}) = \frac{1}{8} (1 + \xi_a \xi) (1 + \eta_a \eta) (1 + \zeta_a \zeta)$, $a = \{1, \dots, 8\}$

obj. shapefunctions. $N = \underset{\text{GPs}(8)}{\overset{\text{nodes}(8)}{\begin{bmatrix} \vdots \\ \vdots \end{bmatrix}}} \in 8 \times 8$



$GP(\xi, \eta, \zeta) = ((-1)^1 (-1)^2 (-1)^3 (-1)^4 (-1)^5 (-1)^6 (-1)^7 (-1)^8)$

derivatives:

obj. shapefunctions. $dNr = \underset{\text{GP}}{\overset{\text{nodes}(8)}{\begin{bmatrix} 1 \left\{ \begin{matrix} d\xi \\ d\eta \\ d\zeta \end{matrix} \right\} \\ 2 \left\{ \begin{matrix} d\xi \\ d\eta \\ d\zeta \end{matrix} \right\} \\ \vdots \end{bmatrix}}} \in 24 \times 8$

Element routine

$dNrAll = (dNr)^T \rightsquigarrow 8 \times 24$

* Jacobimatrix:

$\underline{J}^{u,e}(\underline{\xi}) = \frac{\partial \underline{X}^{u,e}(\underline{\xi})}{\partial \underline{\xi}}$, $\underline{X}^{u,e} = \sum_a N_a^e \underline{X}_a^e$

$\rightsquigarrow \underline{J}^{u,e} = \underline{X}^{u,e} \otimes \frac{\partial}{\partial \underline{\xi}} (N_a(\underline{\xi})^e)$

matlab: $\underline{J}^e = \text{qr}(\text{edge}(e, :), 1 : \text{dim})' \cdot dNrAll$

* Shapefunctions

$\nabla_X N^a(\underline{\xi}) = \underline{J}^{-T} \nabla_{\underline{\xi}} N^a(\underline{\xi}) \rightsquigarrow dNX = (\underline{J}(:, \text{indx})') \setminus (dNrAll(:, \text{indx}))'$