Precalc Practice Final Solns

Friday, December 4, 2020 3:00 PM

1.

$$\frac{i^3+1}{2-i} = \frac{i^2\cdot i+1}{2-i} \quad \text{(Simplify } i^3\text{)}$$

$$= \frac{-i+1}{2-i}\cdot\frac{(2+i)}{(2+i)} \quad \text{multiply by conjugate}$$

$$= \frac{(1-i)(2+i)}{4+2i-2i-i^2}$$

$$= \frac{2+i-2i+i^2}{4-(-1)}$$

$$= \frac{2-i-1}{5}$$

$$= \boxed{\frac{1-i}{5}}$$

in polar form,

$$r = \sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{-1}{5}\right)^2}$$

$$= \sqrt{\frac{2}{25}}$$

$$\boxed{r = \frac{\sqrt{2}}{5}}$$

and

$$\tan\theta = \frac{-1/5}{1/5}$$

$$\theta = \tan^{-1}(-1)$$

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$\boxed{\theta = \frac{3\pi}{4}}$ (just pick the first angle you find.)

Then the polar form is

$$z = r(\cos\theta + i\sin\theta)$$
$$= \boxed{\frac{\sqrt{2}}{5}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)}$$

2.

$x = t + 1$

$y = 3 - t^2$

Then $t = x - 1$, so $y = 3 - (x-1)^2$.

The domain and range are all real numbers, since this is a polynomial.

At $f(x) = 0$,

$0 = 3 - (x-1)^2$

$(x-1)^2 = 3$

$x - 1 = \pm\sqrt{3}$

$\boxed{x = 1 \pm \sqrt{3}}$

3.

$$T(t) = 20 + 10e^{2t}$$

At $t = 0$:

$$T(0) = 20 + 10e^{0}$$

$$\boxed{T(0) = 30^{\circ} F.}$$

When does $T(t) = 40$?

$$40 = 20 + 10e^{2t}$$

$$20 = 10e^{2t}$$

$$2 = e^{2t}$$

$$\ln(2) = 2t$$

$$\boxed{t = \frac{\ln(2)}{2}}$$

4. Vertical asymptotes at $x = 1$ & -1:

$$\frac{1}{(x-1)(x+1)}$$

x-intercepts at $(2,0)$ & $(3,0)$:

$$(x-2)(x-3)$$

$$\frac{(x-2)(x-3)}{(x-1)(x+1)}$$

$$= \frac{x^2-5x+6}{x^2-1}$$

This currently has a horizontal asymptote at $y = 1$, so to get it to be at $y=2$, multiply by 2.

$$\boxed{f(x) = 2\left(\frac{x^2-5x+6}{x^2-1}\right)}$$

5. $\vec{u} = 1\hat{\imath} + 3\hat{\jmath}$

$\vec{v} = -2\hat{\imath} - 4\hat{\jmath}$

$$2\vec{u} + 3\vec{v} = 2(1\hat{\imath} + 3\hat{\jmath}) + 3(-2\hat{\imath} - 4\hat{\jmath})$$

$$= 2\hat{\imath} + 6\hat{\jmath} - 6\hat{\imath} - 12\hat{\jmath}$$

$$= \boxed{-4\hat{\imath} - 6\hat{\jmath}.}$$

The magnitude of this vector is

$$\sqrt{4^2+6^2} = \sqrt{16+36}$$

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$$= \sqrt{52}$$
$$= \boxed{2\sqrt{13}}$$

6.

$$\cot(x) - \tan(x) = 2\cot(2x)$$

right side $= \dfrac{2\cos(2x)}{\sin(2x)}$ Double angle identities

$$= \cancel{2}\left(\frac{\cos^2(x) - \sin^2(x)}{\cancel{2}\sin(x)\cos(x)}\right)$$

$$= \frac{\cos^{\cancel{2}}(x)}{\sin(x)\cancel{\cos(x)}} - \frac{\sin^{\cancel{2}}(x)}{\cancel{\sin(x)}\cos(x)}$$

$$= \frac{\cos(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)}$$

$$= \cot(x) - \cot(x).$$

yep.