Precalc Final Exam Solutions

Friday, December 4, 2020 3:47 PM

1. $f(x) = x^2 + x + 4.$

$f(x) = 0$: $0 = x^2 + 2x + 2$

Quadratic Formula:

$$x = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= -1 \pm \frac{1}{2} \cdot 2i$$

$$\boxed{x = -1 \pm i}$$

Polar form: $r = \sqrt{1^2 + 1^2}$

$$\boxed{r = \sqrt{2}.}$$

$$\tan(\theta) = \frac{-1}{\pm 1}$$

$$\boxed{\theta = \pi/4,\ 3\pi/4.}$$

$$z = r(\cos\theta + i \sin\theta)$$

$$\boxed{= \sqrt{2}\left(\cos(\pi/4) + i \sin(\pi/4)\right)}$$

or

$$= \sqrt{2}\left(\cos(3\pi/4) + i \sin(3\pi/4)\right)$$

$$= \boxed{\sqrt{2}\left(\cos\left(3\pi/4\right) + i\sin\left(3\pi/4\right)\right)}$$

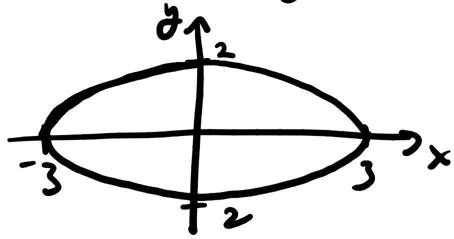
2. $4x^2 + 9y^2 = 36.$

first divide everything by 36:

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\boxed{\begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}}$$ works by the Pythag. theorem.

This is an ellipse. We could graph it by making a table of values & plotting points.



3. doubles every two years: $k = \frac{200\%}{2\text{years}}$

$k = 1$

(Huge!)

$$y = A_0 e^{kt}$$

$+$

$$= A_0 e^{t}$$

At $t = 0$ (year 2000), $y = 1$ billion.

$$1 \times 10^9 = A_0 e^0,$$

$$A_0 = 1 \times 10^9$$

So $y = 1 \times 10^9 \cdot e^{t}$

When does $y(t) = 40 \times 10^9$?

$$40 \times \cancel{10^9} = 1 \times \cancel{10^9} e^{t}$$

$$\boxed{\ln(40) = t}$$

$t = 3.69$ years,

So in 2003, maybe September.

In truth - Moore's law is wrong,

and in 2019, the state-of-the-art was 39.5 billion.

4. $f(x) = 2x^2 + 4x + 2 = 2(x+1)^2$

1. $f(x) = 2x^2 + 4x + 2 = 2(x+1)^2$

Domain: all real numbers (polynomial)

Range: $y > 0$.

Zeros: $2(x+1)^2 = 0$

$\boxed{x = -1, \text{ multiplicity } 2.}$

y-intercept: $\boxed{f(0) = 2}$

End behavior: quadratic with positive leading coefficient, so

as $x \to \infty$, $f \to \infty$

$x \to -\infty$, $f \to \infty$.

Inverse function:

$$y = 2(x+1)^2$$

Switch:

$$x = 2(y+1)^2$$

$$\frac{x}{2} = (y+1)^2$$

$$\sqrt{\frac{x}{2}} = y + 1$$

$$\boxed{y = \sqrt{\frac{x}{2}} - 1}$$

Domain: $x \geq 0$

Domain: $x > 0$

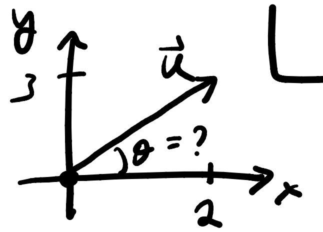
Range: $y > -1$.

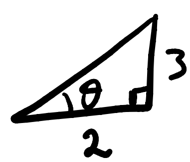
5. $\vec{u} = 2\hat{i} + 3\hat{j}$

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{2\hat{i} + 3\hat{j}}{\sqrt{2^2 + 3^2}}$$

$$= \frac{2\hat{i} + 3\hat{j}}{\sqrt{13}}$$

$$= \boxed{\frac{2}{\sqrt{13}}\hat{i} + \frac{3}{\sqrt{13}}\hat{j}}$$

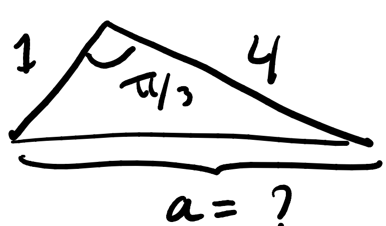




$$\tan\theta = \frac{3}{2}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{2}\right)}$$

6.



Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$a^2 = 1^2 + 4^2 - 2\cdot 1\cdot 4\cos(\pi/3)$$

$$= 17 - 8\cdot\frac{1}{\ }$$

$$= 17 - 8 \cdot \frac{1}{2}$$

$$a^2 = 13$$

$$\boxed{a = \sqrt{13}}$$