FINAL EXAM: PRECALCULUS, FALL 2020 - NEWTON

Instructions. You may work on this exam on your own time, using any resource you like (open notes, open book, open Internet). However, you still need to understand the material yourself, even if you are looking up some things. Show all your work and explain your solutions fully in order to get full credit. Copying from the book without understanding is not enough, and many internet solutions are outright wrong or missing information and explanations. Please avoid using a calculator. You won't need one, since you don't have to simplify your numerical answers. **Leave your answers exact, do not convert to decimal** (as in, leave things with the square roots, fractions, e, π).

Your work on this exam should be your own. Working with other students can get you both a zero. Trust and believe in yourself over other students and the internet. **Please do not post the questions online, I will search online for my questions before grading. Solutions that are line-by-line similar to solutions online will be graded harshly - if you copy from an online resource, I expect you to be giving your own explanations of the process in addition to using the resource.**

When the question asks for an explanation of the work or your process, that means you should give an explanation in words, in addition to the mathematical work you showed to get to the answer. However, this isn't an English test, I don't care about whether it's a complete sentence, or whether you mispelled something, had bad grammar, etc. As long as you give some sort of correct explanation in words, it will be fine.

Good luck! Do your best! You'll do great.

(1) Let $f(x) = x^2 + 2x + 2$. Solve $f(x) = 0$ for x over the complex numbers. Convert your solutions to the polar form of a complex number. Show and explain all your work.

(2) Parameterize the equation $4x^2 + 9y^2 = 36$. Graph the curve, or describe the process of graphing it and what shape it would make. Explain each step of your work and your process.

(3) Moore's law says that the number of transistors on a microchip doubles every two years. In the year 2000, we could fit 1 billion transistors on a microchip. Write a function describing the number of transistors (in billions) on a microchip as a function of time (in years). In what year will we be able to fit 40 billion transistors on a microchip? (Not for credit: but if this interests you, you can also look up how many transistors are on modern microchips now.) Give an exact answer, then if you're curious you can put the number into a calculator to get a decimal answer. Justify each step of your solution.

(4) Let $f(x) = 2x^2 + 4x + 2$. Find the domain and range of f, find the zeros of f and their multiplicities, find the y-intercept, and determine the end behavior. Find the inverse function of f and its domain and range. Give a reason for each step of your work.

(5) Let $\vec{u} = 2\hat{i} + 3\hat{j}$. Find a unit vector in the direction of $\vec{u}$. What is the angle between $\vec{u}$ and the x-axis? *[Hint: it might help you to draw the vector on the xy-plane.]* Give a reason for each step of your work.

(6) A triangular sign measures 1 foot on one side and 4 feet on another side. These sides form an angle of $\frac{\pi}{3}$. How long is the third side of the sign? State what formula you are using and explain each step of your work.