Precalc Practice Exam 2 Solns

1. $f(x) = 2e^{x+1} + 1.$

This is an increasing function, so the exponential is growing.

Domain: all real numbers (exponential)

Range: all real numbers > 1, since e^x can never be less than or equal to zero.

x-intercept: none, since $f > 1$ everywhere.

y-intercept: $f(0) = 2e^{0+1} + 1$

$= \boxed{2e + 1.}$

Note: do not convert this to decimal.

Asymptotes: there is a horizontal asymptote at $y = 1$, since that is the bottom of the range of this exponential.

There is no vertical asymptote, because it is an exponential because it is an exponential function.

Inverse function:
$$y = 2e^{x+1} + 1$$
$$x = 2e^{y+1} + 1$$
$$x - 1 = 2e^{y+1}$$
$$\frac{x-1}{2} = e^{y+1}$$
$$\ln\left(\frac{x-1}{2}\right) = y + 1$$
$$y = \ln\left(\frac{x-1}{2}\right) - 1$$
$$\boxed{f^{-1}(x) = \ln\left(\frac{x-1}{2}\right) - 1.}$$

Domain: $x > 1$, since we can't take the log of a negative number.

Range: all real numbers, because it's a logarithm.

2. $\log_{12}(2x+6) + \log_{12}(x+2) = 2.$

Using sum rule:

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$$\log_{12}((2x+6)(x+2)) = 2$$

raise 12 to both sides

$$(2x+6)(x+2) = 12^2$$

FOIL

$$2x^2+4x+6x+12 = 144$$

divide by 2

$$x^2+5x+6 = 72$$

subtract 72

$$x^2+5x-66 = 0.$$

Quadratic formula:

$$x = \frac{-5 \pm \sqrt{25+4\cdot 66}}{2}$$

$$x = \frac{-5 \pm \sqrt{289}}{2}$$

only the positive root counts, since we can't take the log of a negative number.

$$\boxed{x = \frac{-5+\sqrt{289}}{2}}$$

3. $\cos(x+y) + \cos(x-y) = 2\cos(x)\cos(y)$

Use sum & difference identities:

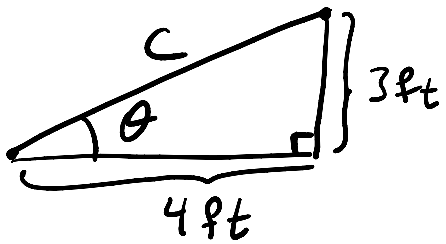
left side = $\cos(x)\cos(y) - \sin(x)\sin(y) + \cos(x)\cos(y) + \sin(x)\sin(y)$

left side $= \cos(x)\cos(y) - \cancel{\sin(x)\sin(y)} + \cos(x)\cos(y) + \cancel{\sin(x)\sin(y)}$

$= 2\cos(x)\cos(y)$

$=$ right side! This verifies the identity.

4.



To find c, Pythag:

$$c^2 = 4^2 + 3^2$$

$$c = \sqrt{16+9}$$

$$= \sqrt{25}$$

$$\boxed{c = 5 \text{ ft.}}$$

To find θ, use tan:

$$\tan\theta = \frac{3}{4}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{4}\right)},$$

Sort of big angle to change your phone at!

4. $\tan\left(\sin^{-1}\left(\frac{2}{x^2}\right)\right)$

Let $\theta = \sin^{-1}\left(\frac{2}{x^2}\right)$

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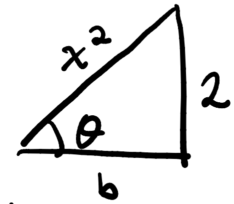
$$\sin(\theta) = \frac{2}{x^2} \Rightarrow$$



Using Pythag. Thm: $b^2 + 4 = x^4$

$$b = \sqrt{x^4 - 4}$$

Then $\boxed{\tan(\theta) = \frac{2}{\sqrt{x^4-4}}}$

5. $f(x) = 5\cos\left(\frac{\pi}{4}x - \pi\right) + 2.$

Period: $T = \dfrac{2\pi}{|\pi/4|} = \boxed{8}$

Amplitude: $A = \boxed{5.}$ (vertical stretch)

Midline: vertical shift: $\boxed{+2}$

Horizontal shift: $f(x) = 5\cos\left(\frac{\pi}{4}(x-4)\right) + 2$

So the horizontal shift is $\boxed{4}$ to the right.

6. $\sin^2(x) - \cos^2(x) - \sin(x) = 0.$

Use Pythag: $\cos^2(x) = 1 - \sin^2 x$

$\sin^2(x) - 1 + \sin^2(x) - \sin(x) = 0$

$$\sin^2(x) - 1 + \sin^2(x) - \sin(x) = 0$$

$$2\sin^2(x) - \sin(x) - 1 = 0$$

Quadratic formula:

$$\sin(x) = \frac{+1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{2}$$

$$= \frac{1 \pm \sqrt{1+8}}{2}$$

$$\sin(x) = \frac{1 \pm 3}{2}$$

$\sin(x) = -1$, or $\underbrace{\sin(x) = 2}_{\text{Not possible - not in range of sin.}}$

$$\boxed{x = \frac{3\pi}{2}}$$

7. $9xy = 1$.

$x = r\cos\theta$

$y = r\sin\theta$

The equation is: $9r^2\cos\theta\sin\theta = 1$

If $\theta = \pi/4$, then

$$9r^2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = 1$$

$$r = \sqrt{\frac{2}{}}$$

$$r = \sqrt{\frac{2}{9}}$$

At $\theta = 0, \pi/2, \pi, 3\pi/2$, etc,

r can take any value.