

FINAL EXAM - DIFFERENTIAL EQUATIONS - SPRING 2021 - NEWTON

Instructions. You may work on this exam on your own time, using any resource you like (open notes, open book, open Internet). Please do not post the questions online. I will check if the questions have been posted, and solutions matching those online solutions will receive a zero. You still need to understand the material yourself, even if you are looking up some things. Show all your work and explain your solutions fully in order to get full credit. Copying from the book without understanding is not enough, and many internet solutions are outright wrong or missing information and explanations. Avoid using a calculator. You won't need one, since you don't have to simplify your numerical answers. **Leave your answers exact, do not convert to decimal** (as in, leave things with the square roots, fractions, e , π).

Your work on this exam should be your own. Working with other students can get you both a zero. Trust and believe in yourself over other students and the internet.

When the question asks for an explanation of the work or your process, that means you should give an explanation in words, in addition to the mathematical work you showed to get to the answer. However, this isn't an English test, I don't care about whether it's a complete sentence, or whether you misspelled something, had bad grammar, etc. As long as you give some sort of correct explanation in words, it will be fine.

Good luck! Do your best! You'll do great.

- (1) Find two power series solutions of the given differential equation: $y'' - x^2y' + xy = 0$. Justify each step of your solution.

- (2) Solve the differential equation: $x^2y'' + 8xy' + 6y = 0$ using any method. State what method you used and justify each step of your solution. In your own words, give a brief explanation of how the method works.

- (3) A lake has 400 fish, and it is estimated that the maximum population of fish in the lake is 10,000. The number of fish tripled in the first year. Write a differential equation that describes the population of fish in the lake. Solve your differential equation to find the number of fish in the lake as a function of time. What method did you use? Show and explain all your work. When will the number of fish reach 5,000? When is the number of fish growing the fastest?

(4) Consider the function:

$$f(x) = \begin{cases} 0, & |x| > 1 \\ e^x \cos(x), & |x| < 1 \end{cases} . \quad (1)$$

Does this function have a Fourier transform (i.e., does its Fourier transform converge)? Explain why or why not. If it does, compute its Fourier transform.