Precalc Exam 2 Solutions

Monday, November 2, 2020 4:12 PM

1. $f(x) = 2x+1$ $g(x) = e^x$

$$f(g(x)) = e^{2x+1}.$$

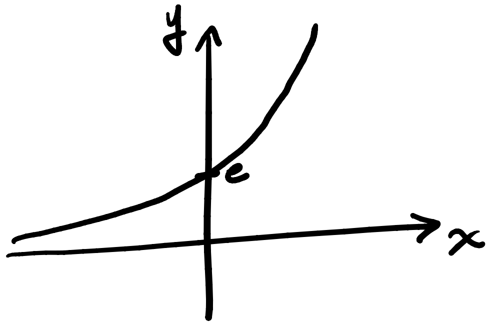
Domain: all real numbers (exponential)

Range: $y > 0$ because exponentials are always positive.

y-int: $f(g(0)) = e^{2\cdot 0+1} = e.$

There is no x-intercept because the function can't touch zero.

Asymptotes: there is no vertical asymptote
there is a horizontal asymptote at $y = 0$.



Inverse:

$$y = e^{2x+1}$$

$$2y+1$$

$$y = e$$

$$x = e^{2y+1}$$

$$\ln(x) = 2y + 1$$

$$\ln(x) - 1 = 2y$$

$$y = \frac{\ln(x) - 1}{2}$$

$$(f(g(x)))^{-1} = \frac{\ln(x) - 1}{2}.$$

<u>Domain</u>: $x > 0$ (cannot take log of a negative number.)

<u>Range</u>: all real numbers. (log function.)

<u>2.</u>

$$\ln(x+3) - \ln(2x+1) = 0.$$

log rule

$$\ln\left(\frac{x+3}{2x+1}\right) = 0$$

$\ln(1) = 0$

$$\frac{x+3}{2x+1} = 1$$

multiply both sides by x

$$x + 3 = 2x + 1$$

subtract 1, subtract x

$$2 = 2x$$

divide by 2.

$$\boxed{x = 1.}$$

<u>3.</u> $1 + \cos(2t)$ $2\cos(t)$

3. $$\frac{1+\cos(2t)}{\sin(2t)-\cos(t)} = \frac{2\cos(t)}{2\sin(t)-1}.$$

$$\text{left side} = \frac{1+\cos(2t)}{\sin(2t)-\cos(z)}$$

[Double-angle identity
$\cos(2t) = \cos^2(t) - \sin^2(t)$
$\sin(2t) = 2\sin(t)\cos(t)$]

$$= \frac{1+\cos^2(t)-\sin^2(t)}{2\sin(t)\cos(t)-\cos(t)}$$

[Pythagorean identity
$\sin^2(t)+\cos^2(t)=1$
$\cos^2(t) = 1-\sin^2(t)$
$\Rightarrow$ replace $1-\sin^2(t)$ with $\cos^2(t)$]

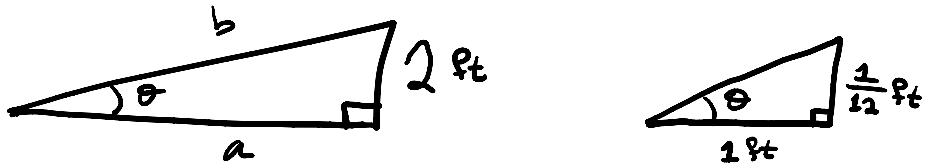
$$= \frac{2\cos^2(t)}{2\sin(t)\cos(t)-\cos(t)}$$

cancel $\cos(t)$

$$= \frac{2\cos(t)}{2\sin(t)-1}$$

= right side!

4.



$$\boxed{\theta = \tan^{-1}\left(\frac{1}{12}\right)}$$

Note: θ will be the same for both triangles.

$$\tan(\theta) = \frac{2}{a} \quad \text{(SOH CAH TOA)}$$

cross multiply

multiply ↳

$$a = \frac{2}{\tan(\theta)}$$

use that $\tan\theta = 1/12$

$$a = \frac{2}{1/12}$$

$$\boxed{a = 24 \text{ ft.}}$$

a long ramp. This is why most ADA ramps are not straight lines.

Then using Pythagorean Theorem,

$$a^2 + b^2 = c^2$$

$$24^2 + 10^2 = c^2$$

$$676 = c^2$$

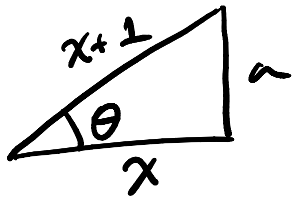
$$\boxed{c = 26 \text{ ft.}}$$

5. $\sin\left(\cos^{-1}\left(\frac{x}{x+1}\right)\right)$

let $\theta = \cos^{-1}\left(\frac{x}{x+1}\right)$

then $\cos\theta = \frac{x}{x+1} = \frac{adj}{hyp}$

$$\cos\theta = \frac{\quad}{x+1} = \frac{\quad}{hyp}$$



Using Pythagorean Theorem,

$$a^2 + x^2 = (x+1)^2$$

$$a^2 = (x+1)^2 - x^2$$

$$a = \sqrt{(x+1)^2 - x^2}$$

then

$$\sin(\theta) = \frac{opp}{hyp} = \boxed{\frac{\sqrt{(x+1)^2 - x^2}}{x+1}}.$$

6: $f(x) = 2\cos(2\pi x - 8\pi) - 1.$

Period: $T = \dfrac{2\pi}{|B|}$

$$= \frac{2\pi}{2\pi}$$

$$\boxed{T = 1.}$$

Amplitude: $\boxed{A = 2}$

Midline: $y = -1$ (due to vertical shift)

Horizontal Shift: $f(x) = 2\cos(2\pi(x-4)) - 1.$

So the horizontal shift

is 4 to the right

$\Rightarrow$ the horizontal shift is $\underline{4}$ to the right.

7. $f(x) = \sin^2(x) - 2\cos^2(x)$

$f(x) = 1$

$\Rightarrow \sin^2(x) - 2\cos^2(x) = 1.$

Use Pythag. identity:

$$\cos^2(x) = 1 - \sin^2(x)$$

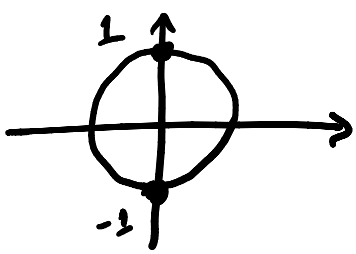
$$\sin^2(x) - 2\left(1 - \sin^2(x)\right) = 1$$

$$3\sin^2(x) - 2 = 1$$

$$3\sin^2(x) = 3$$

$$\sin^2(x) = 1$$

$$\sin(x) = \pm 1$$



So $\boxed{x = \pi/2 \text{ or } 3\pi/2.}$

8. $x^2 + y^2 = 2xy.$

$x = r\cos\theta$

$y = r\sin\theta$

$y = r\sin\theta$

Plugging this in:

$$r^2\cos^2\theta + r^2\sin^2\theta = 2r^2\cos\theta\sin\theta$$

either $r = 0$, or

$$\cos^2\theta + \sin^2\theta = 2\cos\theta\sin\theta$$

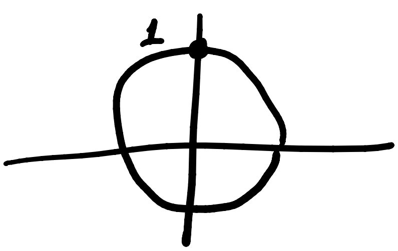
Pythag

$$1 = 2\cos\theta\sin\theta$$

double angle identity for sin

$$1 = \sin(2\theta)$$

Using the unit circle,



$$(2\theta) = \pi/2,\ \frac{\pi}{2}+2\pi$$

$$\boxed{\theta = \pi/4,\ \frac{5\pi}{4}.}$$