EXAM 2 - DIFFERENTIAL EQUATIONS - SPRING 2021 - NEWTON

Instructions. You may work on this exam on your own time, using any resource you like (open notes, open book, open Internet). Please do not post the questions online. I will check if the questions have been posted, and solutions matching those online solutions will receive a zero. You still need to understand the material yourself, even if you are looking up some things. Show all your work and explain your solutions fully in order to get full credit. Copying from the book without understanding is not enough, and many internet solutions are outright wrong or missing information and explanations. Avoid using a calculator. You won't need one, since you don't have to simplify your numerical answers. **Leave your answers exact, do not convert to decimal** (as in, leave things with the square roots, fractions, e, π).

Your work on this exam should be your own. Working with other students can get you both a zero. Trust and believe in yourself over other students and the internet.

When the question asks for an explanation of the work or your process, that means you should give an explanation in words, in addition to the mathematical work you showed to get to the answer. However, this isn't an English test, I don't care about whether it's a complete sentence, or whether you mispelled something, had bad grammar, etc. As long as you give some sort of correct explanation in words, it will be fine.

Good luck! Do your best! You'll do great.

(1) Find the general solution to the differential equation: $y'' + 3y' + 2y = 2x + e^{-x}$. State what method you used. Justify each step of your solution.

(2) A 1kg mass is on the ground and attached to a spring. Initially, the spring is stretched to 1 meter away from its equilibrium position, and the mass has a velocity $1m/s$ to the right (or to extend the spring). There is no damping force or gravity effects. Let the spring constant be $k = 1N/m$. At $t = \pi/4$ seconds after the mass is released, a constant external force $F = 1N$ is applied to extend the spring. (Before that, the external force is zero.) Find an initial value problem (find the differential equation and initial values) that describes the position of the mass. Solve the initial value problem to find the position of the mass as a function of t. Explain each step of your work. What happens to the mass as $t \to \infty$?

(3) Solve the initial value problem: $y'' + 2y(y')^2 = 0, y(1) = 1, y'(1) = -1/2$. Show all your work, explain each step.

(4) In San Francisco Bay, California sea lions and harbor porpoises exist in competition, meaning that they both exist in the same space and both eat mackerel, but they don't eat each other. Call the population of sea lions $s(t)$ and the population of harbor porpoises $p(t)$. Suppose in 2010, there were 1000 sea lions in SF Bay, and 5000 porpoises. Suppose each pair of sea lions has four babies per year (meaning each sea lion produces two sea lions per year) and each pair of porpoises has six babies per year (so each porpoise produces three porpoises per year). Additionally, each year, the number of sea lions decreases by 1/4 times the number of porpoises, and the number of porpoises decreases by 1/3 times the number of sea lions. Write equations describing the rate of change of $s(t)$ and $p(t)$. Solve your initial value problem to find the general solutions $s(t)$ and $p(t)$. Justify each step. What happens to the ratio of sea lions to porpoise, as $t \to \infty$? *[Note: the numbers for this problem are not particularly nice. You don't need to simplify any of your numbers, you can leave it messy, for example* $\sqrt{2/3 + 24}$*, leave it as is. Do not convert to decimal.]*