PRACTICE FINAL EXAM - DIFFERENTIAL EQUATIONS - SPRING 2021 - NEWTON

Instructions. You may work on this exam on your own time, using any resource you like (open notes, open book, open Internet). Please do not post the questions online. I will check if the questions have been posted, and solutions matching those online solutions will receive a zero. You still need to understand the material yourself, even if you are looking up some things. Show all your work and explain your solutions fully in order to get full credit. Copying from the book without understanding is not enough, and many internet solutions are outright wrong or missing information and explanations. Avoid using a calculator. You won't need one, since you don't have to simplify your numerical answers. **Leave your answers exact, do not convert to decimal** (as in, leave things with the square roots, fractions, e, π).

Your work on this exam should be your own. Working with other students can get you both a zero. Trust and believe in yourself over other students and the internet.

When the question asks for an explanation of the work or your process, that means you should give an explanation in words, in addition to the mathematical work you showed to get to the answer. However, this isn't an English test, I don't care about whether it's a complete sentence, or whether you mispelled something, had bad grammar, etc. As long as you give some sort of correct explanation in words, it will be fine.

Good luck! Do your best! You'll do great.

(1) Solve the given initial value problem using power series: $y'' + xy' + 2y = 0, y(0) = 3, y'(0) = -2$. Explain your process.

(2) A 1kg mass is suspended on a spring with spring constant $k = 1N/m$. The mass is driven with a force $F(t) = e^{-t}$ upwards. Find a differential equation that describes the position of the mass as a function of time. Explain your process. Solve the equation to find the position of the mass as a function of time. Explain what method you used. (This is not an initial value problem, just find the general solution.)

(3) Solve the given initial value problem: $y'' + 16y = \delta(t - 2\pi), y(0) = 0, y'(0) = 0$. Justify each step of your work.

(4) Solve the given initial value problem:

$$\vec{X}' = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \vec{X}, \text{ with } \vec{X}(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{1}$$

Explain each step of your solution.