Precalc Practice Exam 1 Solns

Monday, September 28, 2020 8:44 AM

1. $$2|x-7|-4 \geq 42$$

$$+4 \qquad +4$$

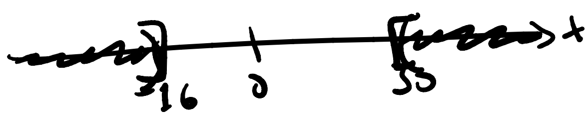
$$\frac{2|x-7|}{2} \geq \frac{46}{2}$$

$$|x-7| \geq 23$$

Two possibilities:

$$x-7 \leq -23 \qquad\qquad x-7 \geq 23$$

$$\boxed{x \leq -16} \qquad\qquad \boxed{x \geq 30}$$



2. $$A(t) = 115(1.025)^t$$

$$B(t) = 82(1.029)^t$$

After 20 years:

$$A(20) = 115(1.025)^{20}$$

$$= 188$$

$$B(20) = 82(1.029)^{20}$$

$$= 145.$$ A has more.

$= 145.$ A has more.

[Note: this is not a great exam problem since you need a calculator.]

3. $f(x) = \dfrac{x^2+2x+1}{x^2+100}$

Domain: does $x^2+100=0$?

No! $x^2+200 > 0$, for any x.

So domain is all real numbers.

y-intercept: Set $x=0$,

$f(0) = \boxed{\frac{1}{100}}$

Zeros: Set $f(x)=0$.

$$0 = \frac{x^2+2x+1}{x^2+100}$$

$$x^2+2x+1=0$$

$$(x+1)^2=0$$

$\boxed{x=-1, \text{ multiplicity } 2}$.

Vertical Asymptotes: none, because

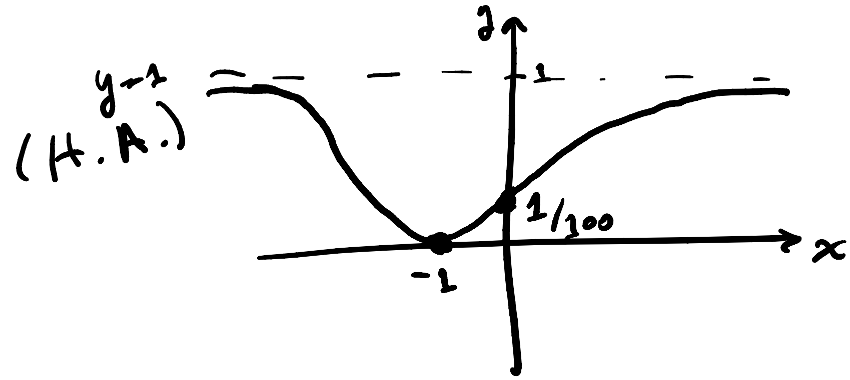
Vertical Asymptotes: none, because denominator is never zero.

Horizontal Asymptotes:
degree of numerator

= degree of denominator,

So there is a horizontal asymptote at $y=1$ (the ratio of the leading coefficients is $\frac{1}{1}$.)

At the only zero, $x=-1$, it has multiplicity 2, so that the function will bounce off:



You did not have to graph it

4. $f(x) = 4\sqrt{x} + 2$.

Domain: $x > 0$, square root can't have a negative number inside.

f is one-to-one, since the square root function is one-to-one. It only has one x for each y, & vice versa. However, if we had the $-\sqrt{\ }$ as well, it would no longer be one-to-one.

Inverse:

$$y = 4\sqrt{x} + 2 \qquad \text{range: } y \geq 2.$$

$$y - 2 = 4\sqrt{x}$$

$$\frac{y-2}{4} = \sqrt{x}$$

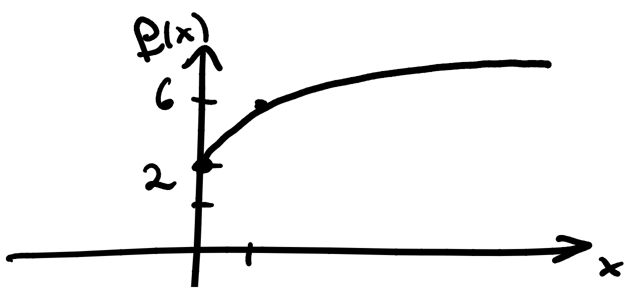
$$x = \left(\frac{y-2}{4}\right)^2, \quad \left[\text{Note } x > 0 \text{ Still!}\right]$$

$$y = \left(\frac{x-2}{4}\right)^2$$

$$f^{-1}(x) = \left(\frac{x-2}{4}\right)^2.$$

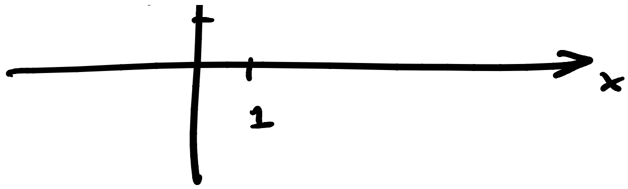
Transformations:

f is related to $\sqrt{x}$ by a vertical shift up of 2, and a vertical stretch of 4.



$f(0) = 2$

$f(1) = 4\sqrt{1} + 2 = 6.$



__Average Rate of Change:__ $\dfrac{f(1) - f(0)}{1 - 0}$

$= \dfrac{6-2}{1}$

$= 5$, which is positive.

Yes, this makes sense, since the function is increasing.

__5.__ $f(x) = \sqrt{x+1}$, $g(x) = x^2$

$f(g(x)) = \sqrt{x^2+1}$.

__Find f^{-1}:__

$y = \sqrt{x+1}$

$x = \sqrt{y+1}$

$x^2 = y + 1$

$x^2 - 1 = y$

$\boxed{f^{-1}(x) = x^2 - 1}$

$$\boxed{f^{-1}(x) = x^2 - 1.}$$

Find g^{-1}: (Note: we should restrict domain to $x > 0$, so g is one-to-one.)

$$y = x^2$$

$$x = y^2$$

$$+\sqrt{x} = y \quad (+\text{, since } y > 0\text{, since } y = x^2.)$$

$$g^{-1}(x) = \sqrt{x}.$$

Find $(f(g(x)))^{-1}$:

$$f(g(x)) = \sqrt{x^2+1} \qquad \text{Notice } y > 1.$$

$$y = \sqrt{x^2+1}$$

$$x = \sqrt{y^2+1}$$

$$x^2 - 1 = y^2$$

$$+\sqrt{x^2-1} = y \quad (+\text{since } y > 1.)$$

$$(f(g(x)))^{-1} = \sqrt{x^2-1}.$$

Since $g^{-1}(x) = \sqrt{x}$,

and $f^{-1}(x) = x^2 - 1$,

it seems like

$$g^{-1}\left(f^{-1}(x)\right) = \left(f(g(x))\right)^{-1}.$$

6. $f(x) = x^2 + 4x + 4$

<u>Domain</u>: all real numbers, since it is a polynomial.

<u>y-int</u>: Set $x = 0$,

$$f(0) = 0^2 + 4 \cdot 0 + 4 = \boxed{4.}$$

<u>zeros</u>: where is $f(x) = 0$?

$$0 = x^2 + 4x + 4$$

$$0 = (x+2)^2$$

So $\boxed{x = -2, \text{ with multiplicity } 2.}$

<u>Min/max</u>: $f(x)$ has a minimum since the leading coefficient on x^2 is 1, which is positive. It is located at the vertex, since this is a quadratic function.

The minimum at

vertex, since this is a quadratic function.

The minimum is at $\boxed{x=-2,\ y=0.}$

<u>End Behavior:</u> f is a quadratic function, so since the leading coefficient is positive, both end behavior as $x \to \infty$ and $x \to -\infty$ is to positive ∞.