Graphing Polynomials

Tuesday, September 15, 2020 2:54 PM

$$f(x) = ax^{n} + bx^{n-1} + \cdots cx^{1} + d$$

(n: any natural number)

Quadratic: $ax^2 + bx + c$

Cubic: $ax^3 + bx^2 + cx + d$

Linear: $mx + b$

Graph:

- zero
- end behaviour
- shape: how many wiggles it has

Zeros: x-intercept

Where is $f(x) = 0$?

Linear: $f(x) = 3x + 2$

Set $f(x) = 0$

$$0 = 3x + 2$$
$$-2 \qquad -2$$

$$\frac{-2}{3} = \frac{3x}{3}$$

$$\boxed{x = \frac{-2}{3}}$$

"Zero" of $f(x) = 3x + 2$

$= 3x+2$

Quadratic: $f(x) = x^2 + 4x + 4$

Two options for finding zeros:

- factoring: Set $f(x) = 0$

$$0 = x^2 + 4x + 4$$

$$0 = (x+2)(x+2)$$

- quadratic formula:

$$0 = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Find the zeros of $f(x) = x^2 + 4x + 4$.

$$0 = x^2 + 4x + 4$$

$$0 = (x+2)(x+2)$$

either $x+2 = 0$ or $x+2 = 0$.

both give $x = -2$.

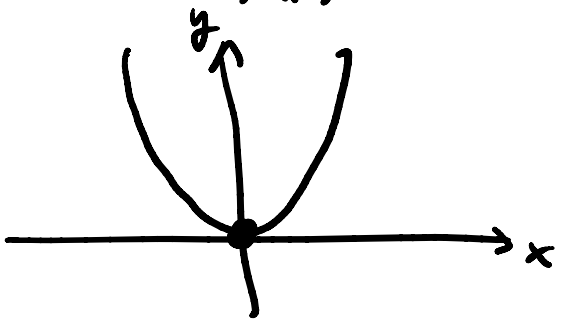
$\Rightarrow$ "Zeros of f are $x = -2$, with multiplicity 2."

Multiplicity: a zero of f has multiplicity n if that solution

multiplicity n if that solution appears n times.

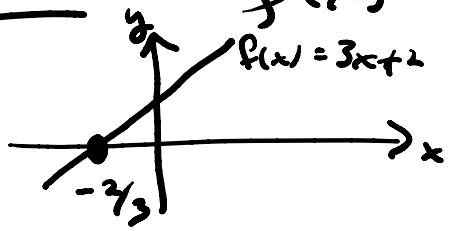
Multiplicity & Graphs.

$f(x) = x^2$.



Zeros: $x^2 = 0$
$\Rightarrow x = 0$,
mult. 2.

vs. $f(x) = 3x + 2$ Zero: $x = -2/3$,
mult. 1.

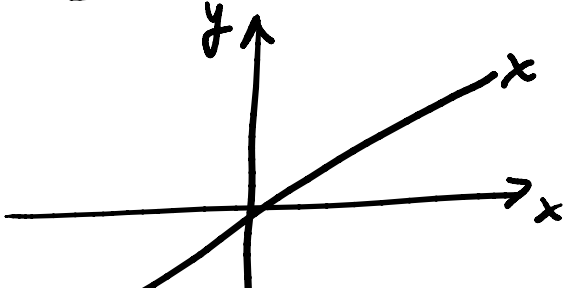


even multiplicity: doesn't cross x-axis

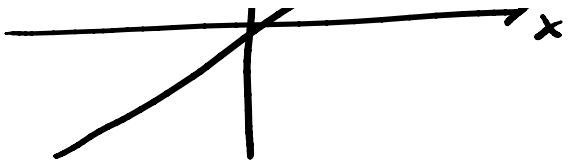
odd multiplicity: does cross x-axis.

End Behaviour.

Ex. $f(x) = x$.



as x gets really big, 100, 1000, to ∞,



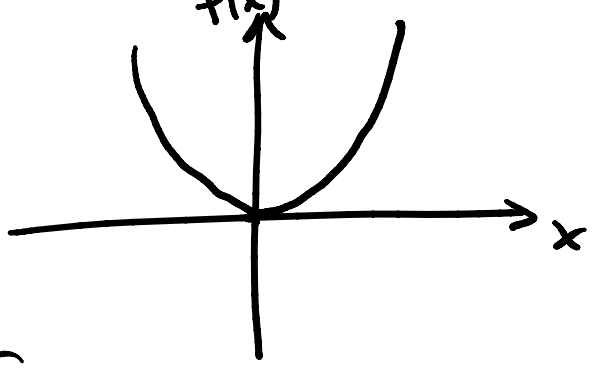
as x gets very small -100, -1000, -10000, what happens to f?

100, to ∞, does f get bigger or smaller?

Ex. $f(x) = x^2$

- As x goes to ∞, so does $f(x)$
- As x goes to $-\infty$

x	$f(x)$
10	100
100	$100 \cdot 100$
-10	100
-100	$(-100)\cdot(-100)$

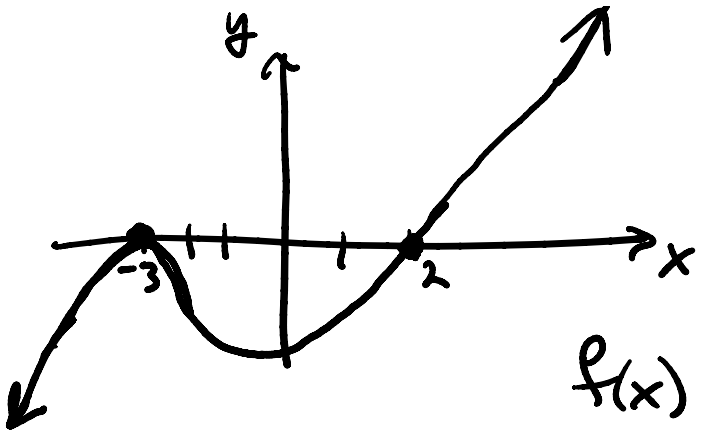


42. Section 3.4

$$f(x) = (x+3)^2(x-2)$$

- zeros $\longrightarrow$ $0 = \underbrace{(x+3)^2}_{=0}\underbrace{(x-2)}_{=0}$
- end behavior.

$y \uparrow$ $\nearrow$ $x = -3$ mult. 2



$x = -3$ mult. 2

$x = 2$ mult. 1.

$$f(x) = (x+3)(x+3)(x-2)$$

$$= (x^2 + 3x + 3x + 9)(x-2)$$

$$= (x^2 + 6x + 9)(x-2)$$

$$= x^3 + \dots$$

$\downarrow$

as $x \to \infty$, $x^3 \to \infty$

$x \to -\infty$, $x^3 \to -\infty$