EXAM 1: PRECALCULUS, FALL 2020 - NEWTON

Instructions. You may work on this exam on your own time, using any resource you like (open notes, open book, open Internet). However, you still need to understand the material yourself, even if you are looking up some things. Show all your work and explain your solutions fully in order to get full credit. Copying from the book without understanding is not enough, and many internet solutions are outright wrong or missing information and explanations. Please avoid using a calculator. You won't need one, since you don't have to simplify your numerical answers. **Leave your answers exact, do not convert to decimal** (as in, leave things with the square roots, fractions, e, π).

Your work on this exam should be your own. Working with other students can get you both a zero. Trust and believe in yourself over other students and the internet.

When the question asks for an explanation of the work or your process, that means you should give an explanation in words, in addition to the mathematical work you showed to get to the answer. However, this isn't an English test, I don't care about whether it's a complete sentence, or whether you misspelled something, had bad grammar, etc. As long as you give some sort of correct explanation in words, it will be fine.

Good luck! Do your best! You'll do great.

(1) Let $f(x) = 2x + 1$ and $g(x) = e^x$. Find $f(g(x))$. What is the domain and range of $f(g(x))$? What are the x and y-intercepts of $f(g(x))$, if they exist? Find the asymptotes of $f(g(x))$, if there are any. (You can show this information on a graph, if you would like to.) Find the inverse function of $f(g(x))$, and find the domain and range of the inverse function.

(2) Solve the equation: $\ln(x + 3) - \ln(2x + 1) = 0$ for x. Give a justification for each step of your work.

(3) Verify the identity: $\dfrac{1 + \cos(2t)}{\sin(2t) - \cos(t)} = \dfrac{2\cos(t)}{2\sin(t) - 1}$. Justify each step.

(4) You are building a ramp that goes up to the door of your house. The door is 2 feet off the ground. You want the ramp to meet the ADA requirements, which is that it can only go 1 inch up for every horizontal foot. How far away from the house does the ramp have to start? How long will the ramp be? Show your work using trig functions and/or a triangle. What angle, with respect to the ground, does the ramp make? (Leave your answer for the angle as an inverse trig function.) Give a reason for each step of your work.

(5) Simplify the expression: $\sin\left(\cos^{-1}\left(\frac{x}{x+1}\right)\right)$. Give a reason for each step of your work.

(6) Let $f(x) = 2\cos(2\pi x - 8\pi) - 1$. Find the period, amplitude, midline, and horizontal shift (both how far it is shifted, and which direction) of $f(x)$.

(7) Let $f(x) = \sin^2(x) - 2\cos^2(x)$. Solve the equation $f(x) = 1$ for x, for $x \in [0, 2\pi)$. Find the inverse function of f, $f^{-1}(x)$. What is the domain of $f^{-1}(x)$? Solve the equation $f^{-1}(x) = 0$ for x.

(8) Convert the equation to polar coordinates: $x^2 + y^2 = 2xy$. Once it is in polar coordinates, solve the equation for $\theta \in [0, 2\pi)$.