Group exercise on relative stability

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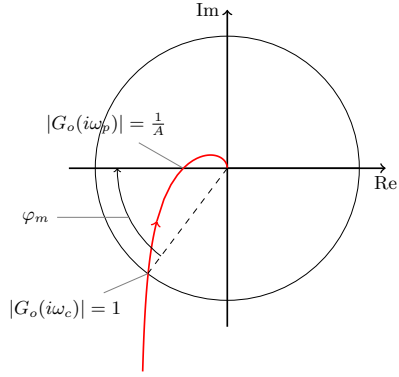
2022-07-08

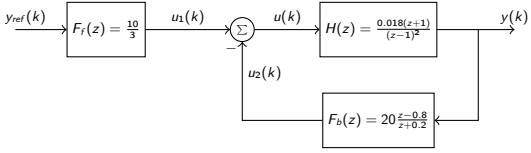
The phase margin

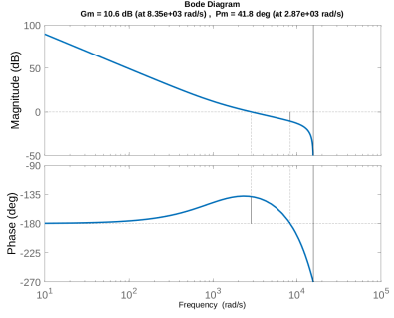


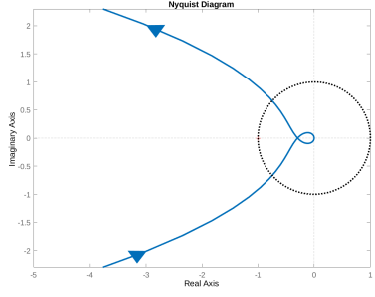
- Cross-over frequency: The frequency ω_c for which $|G_o(i\omega)| = 1$.
- Phase margin: The angle φ_m to the negative real axis for the point where the Nyquist curve intersects the unit circle.

$$\varphi_m = \arg G_o(i\omega_c) - (-180^\circ) = \arg G_o(i\omega_c) + 180^\circ$$

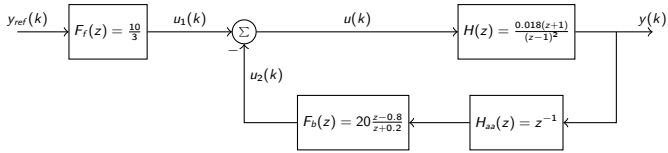
Phase margin for the hard disk drive controller





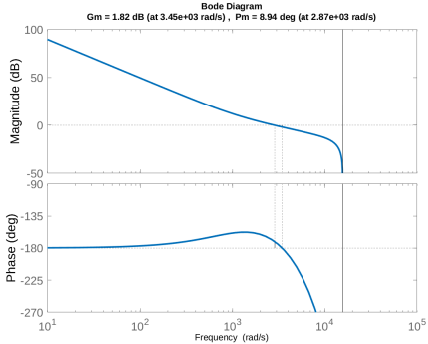


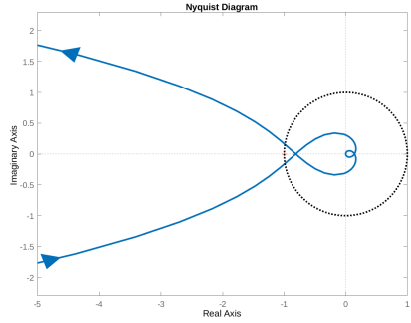
Phase margin with anti-aliasing filter



1. What is the amplitude margin (gain margin) in magnitude? (*Hint*: convert from dB)
2. What is the sampling period h? (*Hint*: The bode plot ends at the Nyquist frequency)
3. Determine the phase shift of a pure delay of h at the cutoff-frequency $\omega_c = 2.87 \times 10^3$ rad/s (*hint*: the delay of time h has transfer function e^{-sh}).
4. Determine the new phase margin with the anti-aliasing filter in the feedback path. (*Hint*: The phase of the loop gain is given by $\arg G_o = \arg H + \arg F_b + \arg H_{aa}$)

Solution


Bode Diagram
Gm = 1.82 dB (at 3.45e+03 rad/s) , Pm = 8.94 deg (at 2.87e+03 rad/s)
Magnitude (dB)
100
50
0
-50
Phase (deg)
-90
-135
-180
-225
-270
10^1
10^2
10^3
10^4
10^5
Frequency (rad/s)


Nyquist Diagram
Imaginary Axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
-4
-3
-2
-1
0
1
Real Axis