# Stability

Kjartan Halvorsen

2022-07-04

# Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\{\frac{G(s)}{s}\}\right\}$$

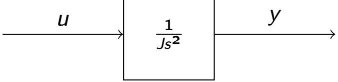
$J\ddot{y} = u$

$$u \longrightarrow \boxed{\frac{1}{Js^2}} \xrightarrow{\;y\;}$$

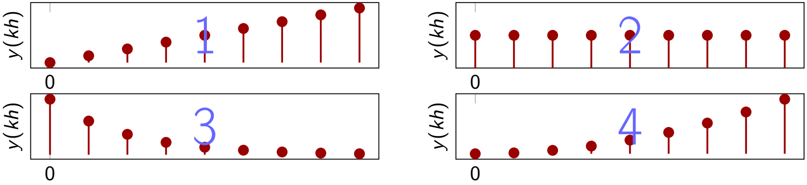
# Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z} \mathcal{Z}\left\{ \mathcal{L}^{-1}\left\{ \frac{G(s)}{s} \right\} \right\}$$

$J\ddot{y} = u$



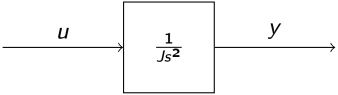

Which of the below graphs show the sampled step-response of the system?



# Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\{\frac{G(s)}{s}\}\right\}$$

$$J\ddot{y} = u$$




Sampled step-response: $y(kh) = \frac{1}{2J}(kh)^2$

$$k^2 \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad \frac{z(z+1)}{(z-1)^3}$$

# Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\{\frac{G(s)}{s}\}\right\}$$

$$J\ddot{y} = u$$

$$\xrightarrow{u} \boxed{\frac{1}{Js^2}} \xrightarrow{y}$$

Sampled step-response: $y(kh) = \frac{1}{2J}(kh)^2$

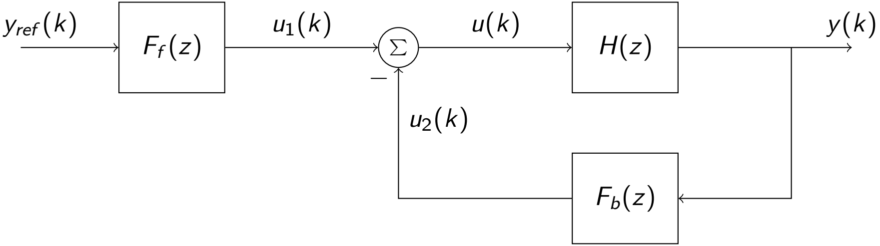
$$k^2 \quad \overset{\mathcal{Z}}{\longleftrightarrow} \quad \frac{z(z+1)}{(z-1)^3}$$

Which of the below pulse-transfer functions corresponds to the discretized hard disk drive model?

| 1 | 2 | 3 |
|---|---|---|
| $H(z) = \frac{h^2 z}{2J(z+1)^2}$ | $H(z) = \frac{h^2(z+1)}{2Jz^2}$ | $H(z) = \frac{h^2(z+1)}{2J(z-1)^2}$ |

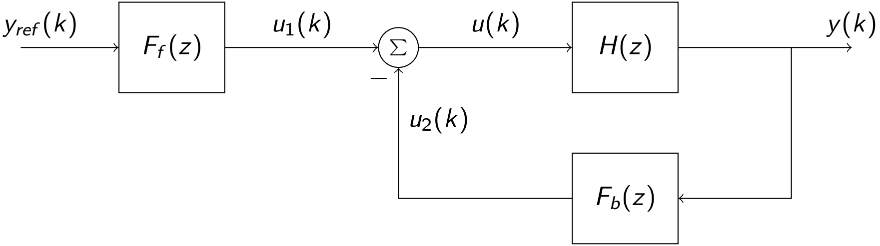
# Block-diagram algebra

Same rules as in the continuous-time case!



# Block-diagram algebra

Same rules as in the continuous-time case!




With

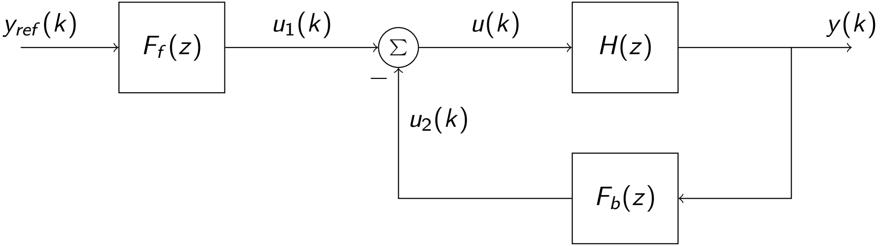
$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c(z)} Y_{ref}(z).$$

# Block-diagram algebra

Same rules as in the continuous-time case!




With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c(z)} Y_{ref}(z).$$

## Resource

Block diagram basics from LearnChemE

# Block-diagram algebra - steps in detail

With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = H(z)U(z) = H(z)\left(F_f(z)Y_{ref}(z) - F_b(z)Y(z)\right)$$

Move all terms with $Y$ to the left side:

$$Y(z) + H(z)F_b(z)Y(z) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z)\big(1 + H(z)F_b(z)\big) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z) = \frac{H(z)F_f(z)}{1 + H(z)F_b(z)} Y_{ref}(z)$$

# Stability for the closed-loop system

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c(z)} Y_{ref}(z).$$

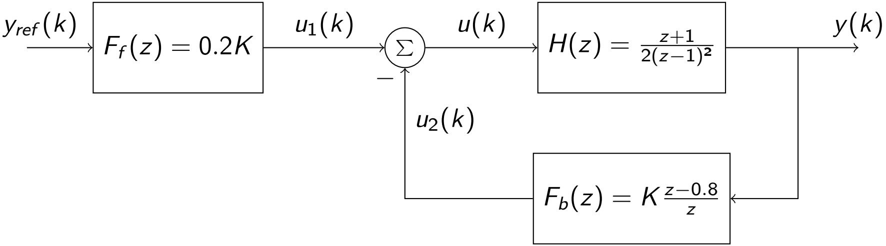
Stability requires that all poles of the system, that is all solutions to the characteristic equation

$$1 + F_b(z)H(z) = 0$$

are located inside the unit circle of the z-plane.

# Stability for the disk drive arm

Case $\frac{h^2}{J} = 1$.




Characteristic equation

$$1 + H(z)F_b(z) = 0$$

$$1 + \frac{z+1}{2(z-1)^2} K \frac{z-0.8}{z} = 0$$

$$(z-1)^2 z + \frac{K}{2}(z+1)(z-0.8) = 0$$

Is the system stable for the gain $K = 1$, and for $K = 2$?