Analytic PIDA Controller Design Technique for A Third Order System

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Abstract

In this paper a new analytical design technique for a proportional, integral, derivative and acceleration(PIDA) controller is presented to satisfy specifications for transient and steady state response of a third order control system. The PIDA controller values are analytically calculated by equating two characteristic equations: one is formed from desired root locations with specifications based on the design criteria, and another one is formed from the nominal control structure. Thus the calculation of values of the controller is so simple. The PIDA controller designs for different third order plant models are shown to illustrate the benefits of the proposed techniques. The proposed controller design techniques are also employed to show the practicality of the technique for controlling AC motor system models.

I Introduction

The PID(proportional-integral-derivative) controller is widely used in the industry because it can be implemented easily for a typical second order plant. In many cases, systems are modeled as a third order. In fact, the traditional motion control system such as an AC motor system position control is properly modeled as a third order plant without crude approximations [1, 2]. It is also known to be quite difficult to design the PID controller for a third order plant because the order of plant is greater than the number of zeros provided by the PID controller [3, 4]. Two roots considered to be dominant do not usually turn out to be completely dominant [5]. Thus, a controller for a third order plant providing robust control is desired.

Here the proportional-integral-derivative-acceleration(PIDA) controller is proposed for a third order plant. The PIDA controller consists of three zeros and three poles, but two poles may be neglected in the design process. Adding a zero to the PID controller yields the PIDA controller. The introduction of an extra zero to the PID controller is to change the root locus of the third order plant in order to make dominant roots more dominant by eliminating the effects of non-dominant roots.

In this paper a feasible analytical technique for PIDA controller design is proposed. Locating two dominant roots to provide the desired response and one root on the real axis just below dominant roots with one negligible root far from the origin on the real axis yields a desired characteristic equation. The characteristic equation of the closed loop transfer function for the controller and the plant is also found from the control structure shown in Figure 1. These two characteristic equations are equated. The system of equations is then complete and can be algebraically solved. Values for the PIDA controller are found from the solutions. The resulting PIDA controller will produce the desired response by satisfying all the requirements automatically. The proposed new design technique is robust that any effects from disturbances to the plant can also be minimized by increasing the cascade gain.

Various plant examples with different PIDA controllers are tested for the proposed methods, and their performances are compared. For each plant example, several controllers are designed and their results are compared. Furthermore, the designs of PIDA controller for AC motor drive system are carried out to evaluate the practicality of the proposed design technique.

II Design Performance Specifications

Consider the control structure shown in Figure 1 where a unity feedback is assumed. The PIDA compensator is defined as

$$G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{(s+d)} + \frac{K_A s^2}{(s+d)(s+e)}$$

$$= K\frac{(s+a)(s+b)(s+z)}{s(s+d)(s+e)} \qquad a,b,z \ll d,e \quad (1)$$

where a, b, z and d, e are zeros and poles of the PIDA controller, respectively. Since $a, b, z \ll d, e$ the poles d, e are considered to be negligible. The goal of a PIDA controller is to determine a set of controller values satisfying the transient performance of a plant.

The closed loop transfer function $T(s)$ is

$$T(s) = \frac{G(s)G_c(s)}{1+G(s)G_c(s)} \quad (2)$$

where $1 + G(s)G_c(s) = 0$ is the characteristic equation. The performance of control system shown in Figure 1 is often defined by the transient response which includes parameters such as percent overshoot($P.O$), settling time(T_s), and peak time(T_p). Here we have set the desired specification as follows:

$$P.O \leq L \quad (3)$$

$$T_s \leq M \quad (4)$$

$$max|\frac{Output\ response}{Disturbance\ signal}| = max\frac{|C(t)|}{|D(t)|} < W \quad (5)$$

The values of L, M and W are selected by the designer. We know that the steady state error $e_{ss} = 0$ with the PIDA controller for a step input. The specifications determine how fast, accurate, and robust the system is required. Defining L and M determines the desired dominant root location region in an s-plane as shown in Figure 2 where $\theta = cos^{-1}(\sqrt{\frac{(ln\frac{L}{100})^2}{\pi^2+(ln\frac{L}{100})^2}})$. Since the designed controller after the first attempt with the conventional root locus technique does usually not satisfy all the specifications, repeated attempts are needed until it meets all the specifications. This process is quite time consuming. Furthermore, the traditional PID controller design technique becomes more difficult for a third order plant. Thus the purpose of this paper is to provide a new analytical method of designing a PIDA controller for a third order plant satisfying the given specifications.

III Problems of PID Controller for a Third Order Plant

The difficulty of satisfying specifications with PID controller for a third order plant can easily shown by the root locus plot. For a Type 2 third order plant(two poles at the origin) with a PID controller, the root locus plot is different from those of Type 0 or 1 plant as shown in Figure 3 which shows the closed loop system of a Type 2 plant is unstable. It is quite difficult to design a PID controller for a third order plant. So the idea here is to give more flexibility to the PID controller by adding one more zero.

IV Analytic PIDA Controller Design Technique

As a solution, we propose the PIDA controller design technique which incorporates an additional zero in the PID controller. The idea behind the PIDA design technique on s-plane is to modify the root locus plot by adding an additional zero appropriately so that the dominant roots become entirely dominant. Four roots excluding two negligible roots are automatically prelocated with three of them at least lining up on one vertical line in the left half s-plane with one additional root far from the origin considered as not significant as shown in Figure 2 if the largest pole of a plant not at the origin is significantly greater than $-\zeta\omega_n$. When the largest pole of a plant not at the origin is smaller than $\zeta\omega_n$ two roots are placed at dominant locations with one at the left of the largest pole of a plant and another at far from the origin in an s-plane in order to minimize the effects of non-dominant poles. The resulting performance for the system with PIDA controllers is slightly different for the different types of the plant (zero, one, two) since they have different root locus plots. In other words, the effects of non dominant roots become larger for a Type 1 plant and the largest for a Type 2 plant due to poles at the origin.

The procedure for designing the PIDA controller in steps is as follows:

- STEP 1. Determine $\zeta\omega_n$ of the dominant roots from T_s specification where $T_s = \frac{4}{\zeta\omega_n}$. The dominant roots are $s = q$ and $\hat{q}$ where $q = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$.

- STEP 2. Determine ζ of dominant roots from P.O specification where $\zeta = \sqrt{\frac{(ln\frac{L}{100})^2}{\pi^2+(ln\frac{L}{100})^2}}$.

- STEP 3. Select the real root equal to the real part of the dominant roots: $R = Re\{dominantroots\} \leq$ -$\zeta\omega_n$. In order to minimize the effect from non-dominant closed loop roots it is suggested to select R where it is to the left of the largest open loop pole of a plant not at the origin in left half s-plane.

- STEP 4. Select the real root r so that $r \ll -\zeta\omega_n$.

- STEP 5. Write the characteristic equations for $1 + GG_c = 0$
  and set $(s + r)(s + R)(s + q)(s + \hat{q}) = 0$
- STEP 6. Equate the characteristic equations.
- STEP 7. Solve the simultaneous set of four equations:
- STEP 8. Plot the responses:
  a. For c(t) given $r(t)$ = unity step.
  b. For c(t) given $r(t) = 0$ and $d(t)$ = unity step.
  If $P.O$ is not satisfied, then increase the cascade gain K(see equation (1)).

V Controller Design Examples with PIDA Controller

The desired performance specifications are $P.O \leq 5\%, T_s \leq 2secs$,and $\frac{|C(t)|}{|D(t)|} \leq 0.05$. Consider the plant

Type 0 Plant : $G(s) = \frac{1}{(s+1)(s+3)(s+6)}$

Following the procedure steps given above we have STEPs 1,2,3 and 4. $\zeta\omega_n \leq -2$ and $\zeta \geq 0.707$ for dominant root locations region. Then select the desired dominant roots at $-2.1 \pm 2.0j$, one real root, R at -2.1 since the largest pole of $G(s)$ is at -1 which is larger than $-\zeta\omega_n = -2$, and one negligible root , r at -30.
STEP 5. $1+G(s)G_c(s) = (s+2.1)(s+30)(s+2.1\pm2.0j)$
STEP 6. $s^4+(10+K)s^3+(K(a+b+z)+27)s^2+(18+K(ab+z(a+b)))s+Kabz = s^4+36.3s^3+206.23s^2+534.561s+529.83$
STEP 7. Four equations for four unknown values can be solved as :
$10 + K = 36.3$,
$K(a + b + z) + 27 = 206.23$,
$18 + K(ab + z(a + b)) = 534.561$,
$Kabz = 529.83$.
Solving for a, b, z and k yields $a, b = 2.3931 \pm 2.0501j$, $z = 2.0288$ and $K = 26.3$.

$$G_{c_1}(s) = 26.3\frac{(s^2 + 4.7390s + 9.3611)(s + 2.0288)}{s} \tag{6}$$

We also solve for two other designed controllers, $G_{c_2}(s)$ where $R = 2.9$

$$G_{c_2}(s) = 27.1\frac{(s^2 + 4.7309s + 9.3606)(s + 2.8843)}{s} \tag{7}$$

and $G_{c_3}(s)$ where $R = 5.9$

$$G_{c_3}(s) = 30.1\frac{(s^2 + 4.3556s + 8.3585)(s + 5.9168)}{s} \tag{8}$$

The step response for these 3 controllers is shown in Figure 4 and recorded in Table 1. Also listed in Table 1 are the resulting performances of other controllers where the different values for R are used. All step responses that satisfy the specifications are shown in Figure 4. The resulting responses meet all specifications through the first design process. As R moves further from the origin, the percent overshoot is increased, but the settling time and the peak time become fast. The best compromise controller was $G_{c_2}(s)$.

Table 1. Performances with different controllers by locating R differently

Controller	G_{c_1}	G_{c_2}	G_{c_3}	Specifications
R	2.1	2.9	5.9	$--$
K	26.3	27.1	30.1	$--$
$P.O\%$	2.47	2.74	8.95	$\leq 5\%$
$T_p(sec)$	0.73	0.4	0.08	≤ 1.0
$T_s(sec)$	1.06	0.71	0.19	≤ 2.0
$max\frac{\|C(t)\|}{\|D(t)\|}$	0.0016	0.0011	0.0006	0 or < 0.05

Now consider the Type 1 Plant : $G(s) = \frac{1}{s(s+1)(s+7)}$

STEP 6. $s^4 + (8 + K)s^3 + (Kz + K(a + b) + 7)s^2 + K(ab + z(a + b))s + Kabz = s^4 + 36.3s^3 + 206.23s^2 + 534.561s + 529.83$
STEP 7. $a, b = 2.2965 \pm 1.5418j$, $z = 2.4471$ and $K = 28.3$ where $R = 2.1$.

$$G_{c_1}(s) = 28.3\frac{(s^2 + 4.5930s + 7.6511)(s + 2.4471)}{s} \tag{9}$$

We also consider two additional controllers, $G_{c_2}(s)$ where $R = 6$

$$G_{c_2}(s) = 32.2\frac{(s^2 + 4.0396s + 7.4748)(s + 6.2896)}{s} \tag{10}$$

and $G_{c_3}(s)$ where $R = 10$

$$G_{c_3}(s) = 36.2\frac{(s^2 + 4.0796s + 7.8422)(s + 8.8872)}{s} \tag{11}$$

The $P.O = 6.0, 7.16, 8.8\%$ for $G_{c_1}(s), G_{c_2}(s), G_{c_3}(s)$ respectively which is greater than specified. We then increase K for each controller to $35, 50, 75$ for $G_{c_1}(s), G_{c_2}(s), G_{c_3}(s)$ to satisfy the desired percent overshoot. The step responses are shown in Figure 5 and recorded in Table 2 and meets the specifications. In this case $G_{c_3}(s)$ provides the best response.

Table 2. Performance with different controllers by locating R differently

Controller	G_{c_1}	G_{c_2}	G_{c_3}	Specifications
R	2.1	6	10	$--$
K	35	50	75	$--$
$P.O\%$	4.87	4.72	4.95	$\leq 5\%$
$T_p(sec)$	0.45	0.15	0.08	≤ 1.0
$T_s(sec)$	0.95	0.53	0.3	≤ 2.0
$max\frac{\lvert C(t)\rvert}{\lvert D(t)\rvert}$	0.0013	0.0004	0.0002	0 or < 0.05

Now consider the Type 2 Plant : $G(s) = \frac{1}{s^2(s+1)}$

STEP 6. $s^4+(1+K)s^3+K(a+b+z)s^2+K(ab+z(a+b))s+Kabz = s^4+36.3s^3+206.23s^2+534.561+529.83$
STEP 7. $a, b = 1.9090 \pm 1.9418j$, $z = 2.0242$ and $K = 35.3$ where $R = 2.1$.

$$G_c(s) = 35.3\frac{(s^2+3.8180s+7.4149)(s+2.0242)}{s} \tag{12}$$

STEP 8. Plot a step response. We also determine two other controllers, $G_{c_2}(s)$ where $R = 0.1$

$$G_{c_2}(s) = 33.3\frac{(s^2+3.91s+7.3568)(s+0.1)}{s} \tag{13}$$

and $G_{c_3}(s)$ where $R = 30$

$$G_{c_3}(s) = 63.2\frac{(s^2+4.1870s+8.4498)(s+14.1739)}{s} \tag{14}$$

Since the overshoot is not satisfied with $P.O = 7.59, 10.74, 16.09\%$ for $G_{c_1}(s), G_{c_2}(s), G_{c_3}(s)$, increasing the cascade gain K for each controller to 60, 85, 280 satisfies the 5% overshoot. Note that $max\frac{|C(t)|}{|D(t)|}$ for $G_{c_2}(s)$ is 10 times larger than that of other controllers. The step responses are shown in Figure 6 and recorded in Table 3.

Table 3. Performances with different controllers by locating R differently

Controller	G_{c_1}	G_{c_2}	G_{c_3}	Specifications
R	2.1	0.1	30	$--$
K	60	85	280	$--$
$P.O\%$	4.46	4.95	4.9	$\leq 5\%$
$T_p(sec)$	0.12	0.08	0.02	≤ 1.0
$T_s(sec)$	0.45	0.34	0.09	≤ 2.0
$max\frac{\lvert C(t)\rvert}{\lvert D(t)\rvert}$	0.0011	0.0145	0.00003	0 or < 0.05

Based on controller performances it is found that forcing R to be right below the dominant roots is reasonable to satisfy desired specifications, specially for percent overshoot. Other choices for R provide faster settling time and peak time, but larger percent overshoot. For a Type 2 system, larger controller cascade gains are required when compared with other plant types.

VI Applications to Induction Motor Control

In this section, we like to apply the proposed design technique to a real world problem such as industrial mechanical motor controls. In industry the induction motor has been increasingly used because of its power and efficiency despite of its size and price and work environment even though the induction motor is highly nonlinear and its control is quite complicated [6]. Here we are designing the PIDA controller for the simplified induction motor position control model that has been implemented in the paper [7]. The induction motor model used here is the three-phase Y-connected two-pole 800w 60Hz 120V/5.4 A. By linearization method the control structure with a simplified motor model is shown in Figure 7. The rotor speed is controlled by a PI controller. The transfer function of induction motor is given by

$$G(s) = \frac{\theta_d(s)}{\theta(s)} = \frac{K_I K_t}{s(Js^2+(f+K_P K_t)s+K_I K_t)} \tag{15}$$

where K_P, K_I are PI controller gains, K_t is motor constant. The estimated motor parameters used here are as follows: $J = 0.305, f = 0.2725, K_P = 14.0242, K_I = 94.1637$, and $K_t = 0.5443$ [7]. Then $G(s)$ becomes a third order transfer function

$$G(s) = \frac{168.0436}{s(s^2+25.921s+168.0436)} \tag{16}$$

where poles are located at $0, -12.96 \pm 0.2864j$. We desire to design a PIDA controller. In this case, the careful selection of R is required because the poles with the largest negative real part not at the origin are $-12.93 \pm 0.2864j$ which are the complex poles. If we select R such that $R > -12.93$, then the effect of non-dominant poles will be increased. Therefore, the desired closed loop pole locations $R = -13$ where it is located at the left of the largest pole of a plant and $r = -30$ are selected. The desired characteristic equation is

$$1+G(s)G_c(s) = (s+13)(s+30)(s+2.1\pm 2.0j) \tag{17}$$

Equating with the nominal characteristic equation yields

$$\begin{aligned} s^4 + (25.9210+K')s^3 + (168.0436+K'(a+b+z))s^2 \\ +(K'(a+b)z+K'ab)s+K'abz \\ = s^4+47.2s^3+579s^2+1999.6s^2+3279.9 \end{aligned} \tag{18}$$

where $K' = \frac{K_I K_t K}{J}$ so that four unknown variables can be solved as $K' = 21.279, a, b = 3.1595 \pm 1.3711j$, and $z = 12.9938$. The designed PIDA controller is

$$G_c(s) = \frac{K(s^2 + 6.3190s + 11.8624)(s + 12.9938)}{s} \tag{19}$$

where $K = \frac{K'}{168.0436} = 21.279/168.0436 = 0.1266$. The closed loop roots are located at $-29.9962, -12.9995, -2.0998 \pm 2j$. The roots at $-2.0998 \pm 2j$ considered to be dominant should satisfy the initial purpose of our design. But the step response does not satisfy the specification since we have a higher $P.O = 8.95\%$ due to the effect from the zeros of the designed PIDA controller. In order to make the dominant roots more dominant, we increase the gain $K' = 40$ and thus $K = 0.2380$ and plot the step response in Figure 8. The performance requirement is now satisfied with $P.O = 4.9\%, T_s = 1.18sec, T_p = 0.61sec$, and $\frac{|C(t)|}{|D(t)|} = 0.035$. The closed loop roots are $-47.8446, -12.9968, -2.5370 \pm 1.8648j$. Note that the locations of dominant roots with $K = 0.2380$ are moved to further inside the desired region that provides smaller $P.O$ and T_s. Therefore, one practical tip for designing a proposed controller without iterative procedures of increasing gains is to select the dominant root locations (*e.g.*, $q, \hat{q} = -2.1 \pm 2.0j$) with some margins (*e.g.*, $q, \hat{q} = -2.3 \pm 1.9j$) so that one design iteration can meet all the specifications.

VII Conclusion

We address the difficulty of designing a PID controller for a third order plant and provide a solution by utilizing a PIDA controller with analytical merit and prove their performances by determining the response with different types of third order plants. We also proved the practicality of the proposed design technique by designing a PIDA controller for AC induction motor. The benefit of the PIDA controller design technique is so simple and analytical that one can just increase the controller cascade gain to push the closed loop dominant roots to be located at dominant positions when the first design attempt does not meet the specifications. Therefore our proposal here for a PIDA controller design is so straight forward and analytical that a novice designer can easily develop a required controller to satisfy any given specifications for a third order plant without any difficulties.

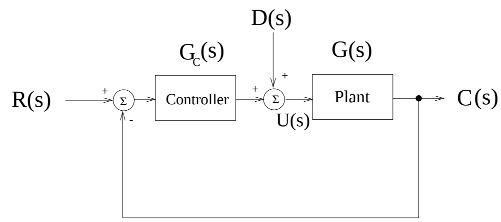

Figure 1: The Control System Structure

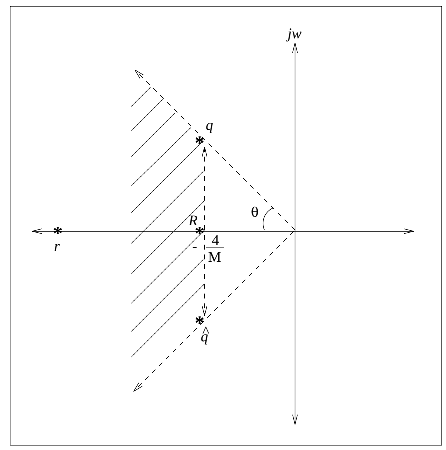

Figure 2: A desired dominant root region and desired closed loop root locations (*) in s-plane

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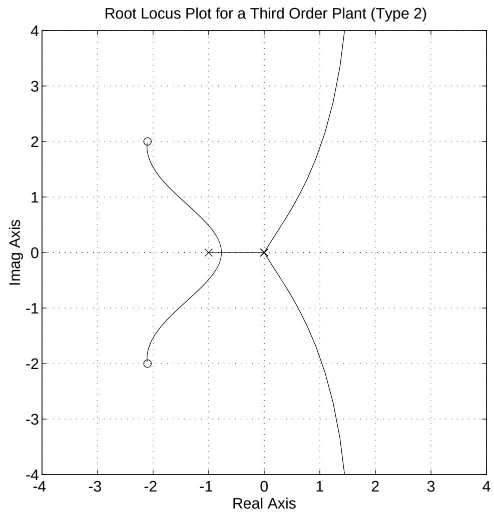

Figure 3: Root locus for $G(s)G_c(s) = K\frac{(s+2.1\pm 2.0j)}{s^3(s+1)}$

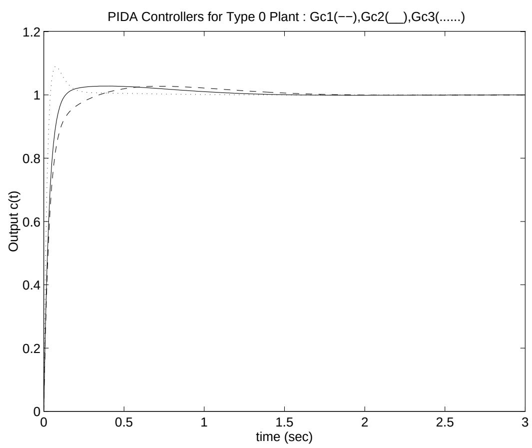

Figure 4: Type 0 Plant with three PIDA controllers

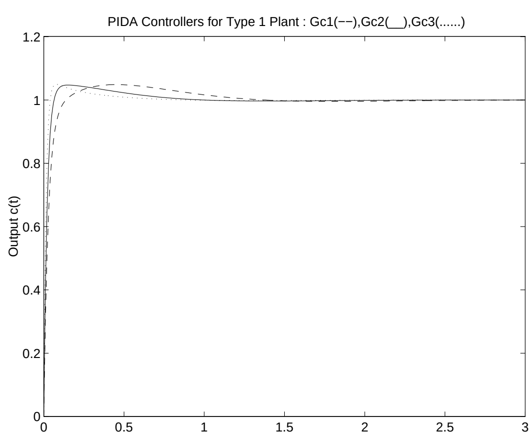

Figure 5: Type 1 Plant with PIDA controllers after increasing the cascade gain K

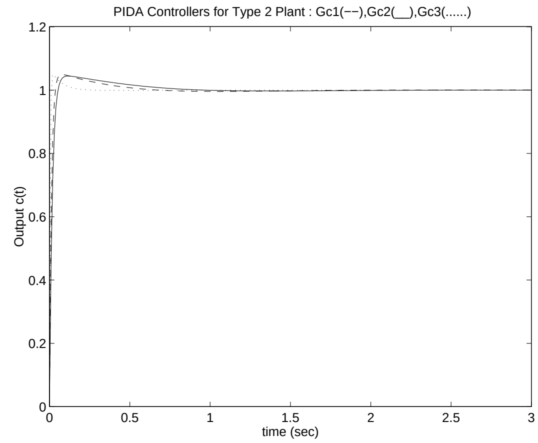

Figure 6: Type 2 Plant with PIDA controllers after increasing the cascade gain K

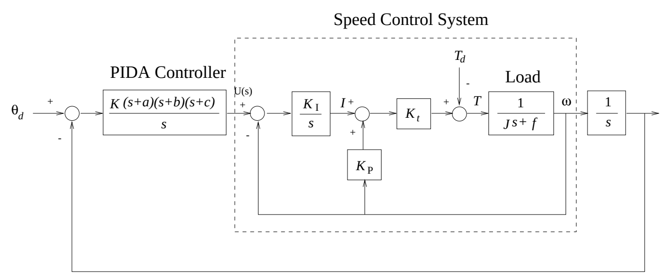

Figure 7: AC induction motor with a PIDA controller

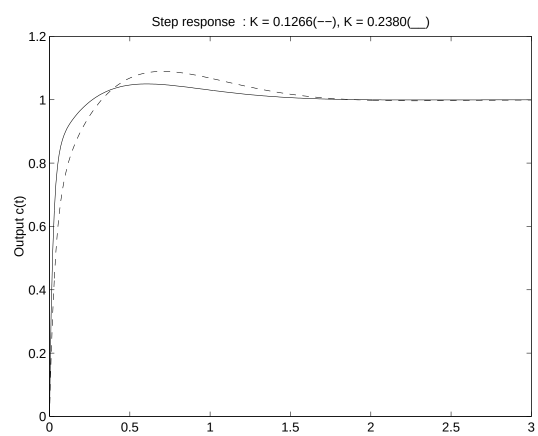

Figure 8: Transient response of AC motor with a PIDA controller