Computerized Control partial exam 2 (15%)

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Time April 3 17:35-18.55

Place 5105

Permitted aids The single colored page with your own notes, table of Laplace transforms, calculator

All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

Good luck!

Matricula and name:

Problem 1

The euler forward approximation of a derivative is

$$\frac{d}{dt}f(t) \approx \frac{f(t+h)-f(t)}{h},$$

which can be written using the derivative operator $\mathrm{p} = \frac{d}{dt}$ and the shift operator q as

$$\mathrm{p}\, f \approx \frac{\mathrm{q}-1}{h} f.$$

(a)

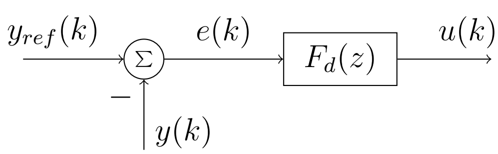
Use the forward approximation to find a discrete approximation $F_d(z)$ of the PI-controller

$$F(s) = K(1 + \frac{1}{T_i s}).$$

Calculations and answer:

(b)

The block diagram below shows the discrete-time controller.

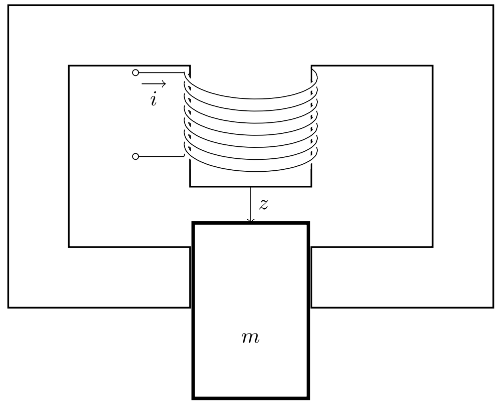


Write the controller as a *difference equation* using $F_d(z)$ determined in (a).

Calculations and answer:

Problem 2

The magnetic levitation (maglev) system below is an example of an unstable system.



Ignoring friction in the system, a linearized model is given by the state space system

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t), \end{aligned} \tag{1}$$

with poles in $s = \pm\omega$. The input signal $u(t)$ is a deviation in the current $i(t) = i_0 - u(t)$ applied to the windings, the state variable $x_1(t)$ is a deviation in the position of the suspension mass $z(t) = z_0 + x_1(t)$, and the state variable $x_2(t)$ is the velocity of the mass. The tuple (i_0, z_0, $\dot{z} = 0$) forms an operating point, for which the system is in (unstable) equilibrium.

Discretizing the system using zero-order-hold with $\omega = 1$ and $\omega h = 0.2$ gives the discrete-time state space system

$$\begin{aligned} x(k+1) &= \underbrace{\begin{bmatrix} 1.02 & 0.20 \\ 0.20 & 1.02 \end{bmatrix}}_{\Phi} x(k) + \underbrace{\begin{bmatrix} 0.02 \\ 0.20 \end{bmatrix}}_{\Gamma} u(k) \\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(k). \end{aligned} \tag{2}$$

(a)

Determine the poles of the discrete-time open-loop system.

Calculations and answer:

(b)

Assume that the state vector $x(k)$ must be estimated using an observer. Determine an observer gain vector K such that the observer has poles in the origin.

Calculations and answer:

Problem 3

A discrete-time controller with sampling period $h = 0.2$ has been designed for the maglev system, according to the closed-loop block diagram in figure 1.

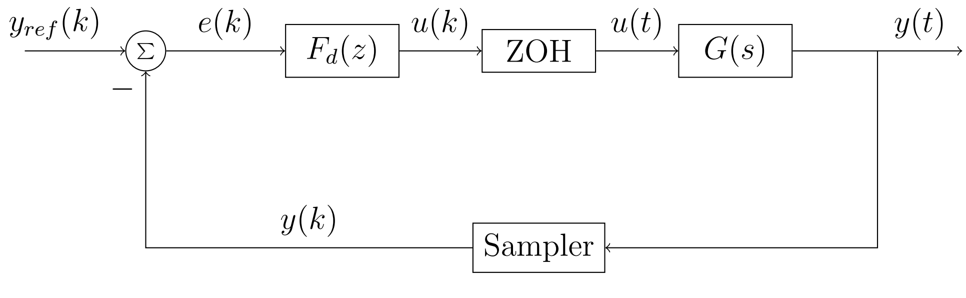


Figure 1: Discrete-time feedback control of the maglev system.

Let $G_o(z)$ denote the loop gain (gain from $e(k)$ to $y(k)$) of the system, with bode diagram given in figure 2.

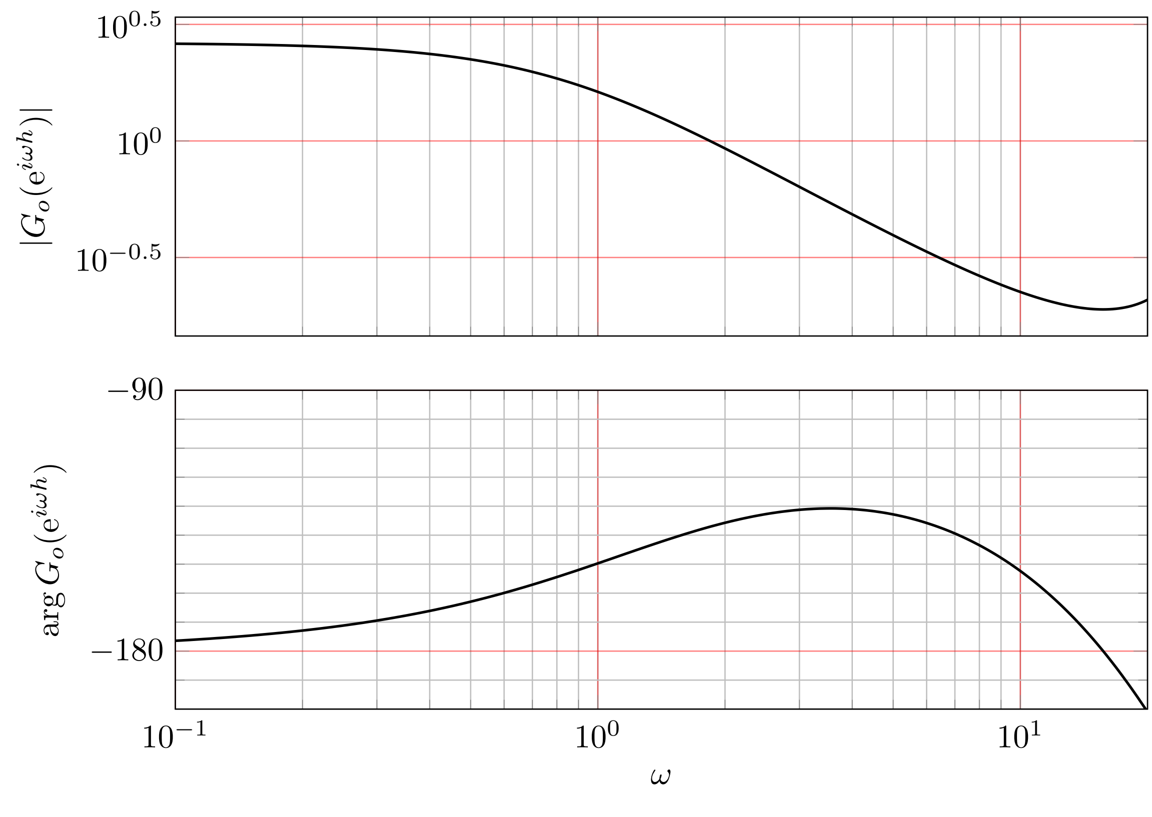


Figure 2: Bode diagram of the loop gain for the discrete-time control system in figure 1.

(a)

Determine the cross-over frequency and phase margin of the system.

Answer:

(b)

In the design of the controller in the feedback system of figure 1 it was assumed that an anti-aliasing filter was not needed. However, later it was found that it was indeed necessary to filter the noise in the measured output signal $y(t)$. The modified control system is shown in figure 3.

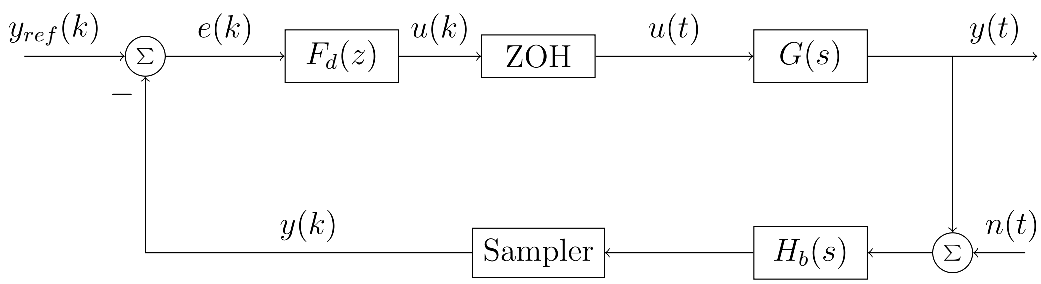
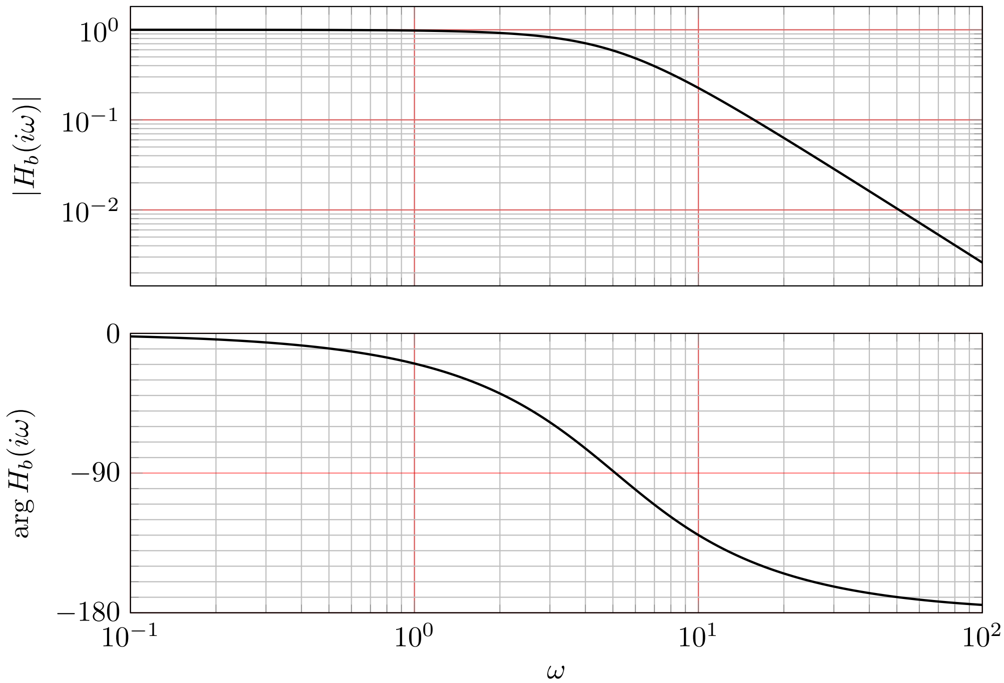


Figure 3: Discrete-time feedback control of the maglev system with anti-aliasing filter.

The anti-aliasing filter $H_b(s)$ is a second-order Bessel filter with Bode diagram given below.



What is the phase margin of the control system **with** anti-aliasing filter? How would you **describe** such a closed-loop system?

Calculations and answer:

Solutions

Problem 1

(a)

The forward approximation becomes

$$F_d(z) = F(s)|_{s=\frac{z-1}{h}} = K(1 + \frac{1}{T_i\frac{z-1}{h}}) = K(1 + \frac{h}{T_i(z-1)}).$$

(b)

The controller is

$$U(z) = F_d(z)E(z),$$

which using the pulse transfer operator can be written

$$u(kh) = K(1 + \frac{h}{T_i(\mathrm{q}-1)})e(kh) = K\frac{\mathrm{q}-1+\frac{h}{T_i}}{\mathrm{q}-1}e(kh)$$

$$(\mathrm{q}-1)u(kh) = K(\mathrm{q}-1+\frac{h}{T_i})e(kh)$$

$$u(kh+h) - u(kh) = K\big(e(kh+h) - (1-\frac{h}{T_i})e(kh)\big)$$

$$u(kh+h) = u(kh) + K\big(e(kh+h) - (1-\frac{h}{T_i})e(kh)\big)$$

Problem 2

(a)

The poles are given by the eigenvalues of the Φ matrix, which in turns are the solutions to the characteristic equation $\det(zI - \Phi) = 0$. With

$$\Phi = \begin{bmatrix} 1.02 & 0.20 \\ 0.20 & 1.02 \end{bmatrix},$$

we get

$$\det \begin{bmatrix} z-1.02 & -0.20 \\ -0.20 & z-1.02 \end{bmatrix} = (z-1.02)^2 - 0.2^2 = z^2 - 2.04z + 1.02^2 - 0.2^2.$$

with solutions

$$z = 1.02 \pm \frac{1}{2}\sqrt{(-2.04)^2 - 4(1.02^2 - 0.2^2)} = \begin{cases} 1.22 \\ 0.82 \end{cases}.$$

Evidently one of the poles are outside the unit circle.

(b)

The dynamics of the observer is given by

$$\hat{x}(k+1) = \big(\Phi - KC\big)\hat{x}(k) + \Gamma u(k) + Ky(k),$$

where

$$KC = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ k_2 & 0 \end{bmatrix}.$$

The characteristic polynomial becomes

$$\begin{aligned}
\det\big(zI - (\Phi - KC)\big) &= \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \begin{bmatrix} 1.02 - k_1 & -0.2 \\ 0.2 - k_2 & 1.02 \end{bmatrix}\right) \\
&= \det\begin{bmatrix} z - 1.02 + k_1 & -0.2 \\ -0.2 + k_2 & z - 1.02 \end{bmatrix} = (z - 1.02 + k_1)(z - 1.02) + 0.2(-0.2 + k_2) \\
&= z^2 + (-2.04 + k_1)z - 1.02(k_1 - 1.02) - 0.2^2 + 0.2k_2 = z^2 + (k_1 - 2.04)z - 1.02k_1 +
\end{aligned}$$

Comparing with the desired characteristic polynomial (poles in the origin) z^2, we get the equations

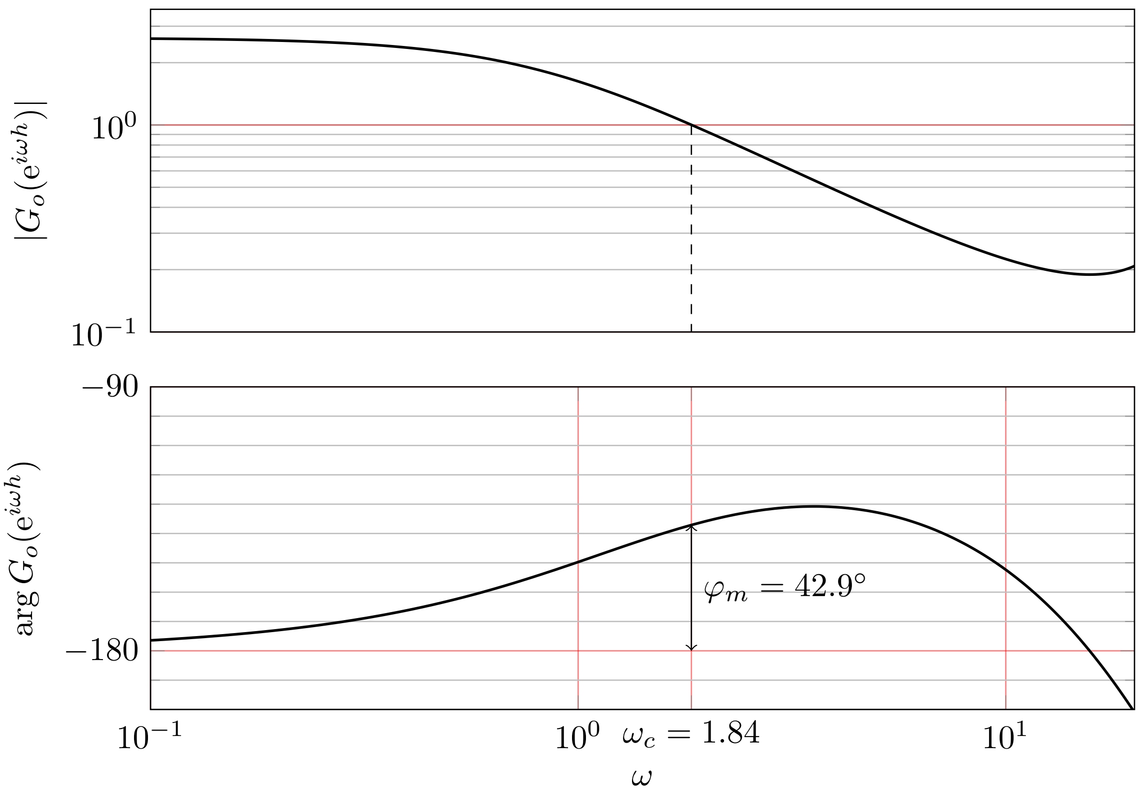
$$\begin{aligned}
k_1 - 2.04 &= 0 \\
-1.02k_1 + 0.2k_2 + 1.02^2 - 0.2^2 &= 0
\end{aligned}$$

with solution

$$K = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 2.04 \\ 5.40 \end{bmatrix}.$$

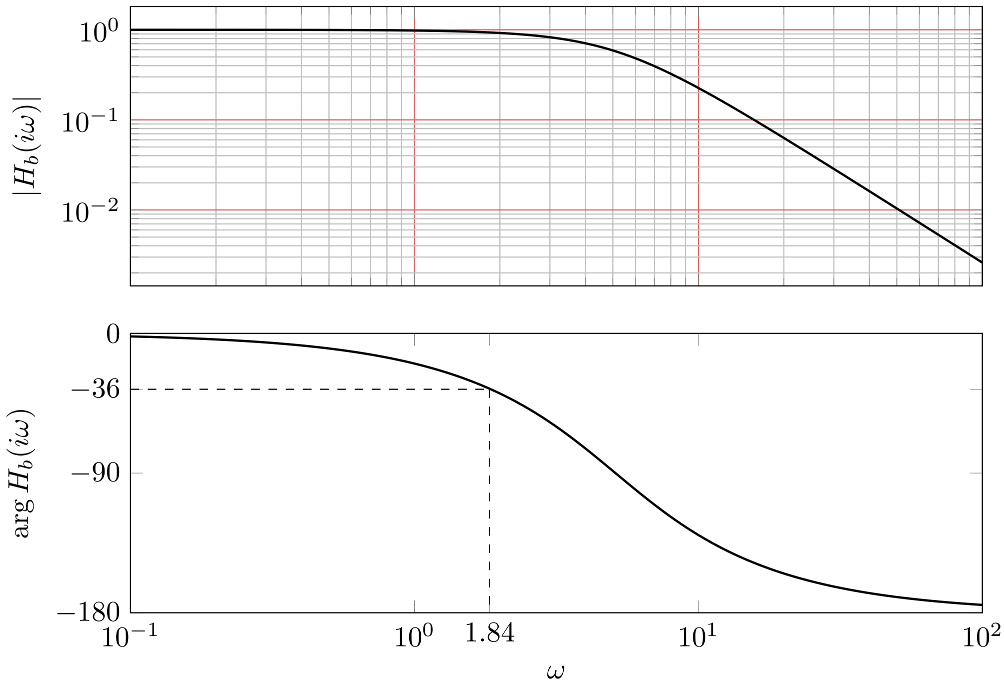
Problem 3

(a)



(b)

At the cross-over frequency the bessel filter has a phase of about $-36°$.



This means that the phase margin for the system **with** anti-aliasing filter becomes

$$\varphi_m = 42.9 - 36 = 6.9^\circ.$$

A closed-loop system with this little phase margin will have a large resonance peak and an oscillatory response.