Stability

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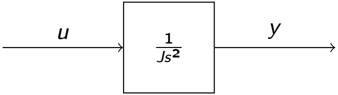
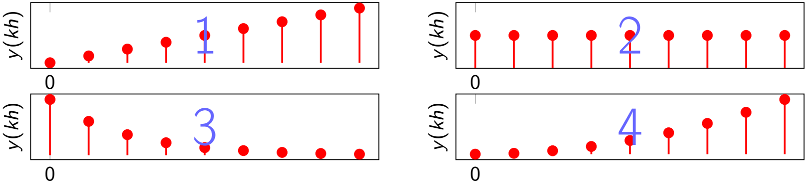
Sampling the hard disk drive arm model

$$H(z) = \frac{z-1}{z}\mathcal{Z}\left\{\mathcal{L}^{-1}\{\frac{G(s)}{s}\}\right\}$$

$$J\ddot{y} = u$$



Which of the below graphs show the sampled step-response of the system?



Sampling the hard disk drive arm model

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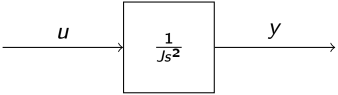


Sampled step-response: $y(kh) = \frac{1}{2J}(kh)^2$

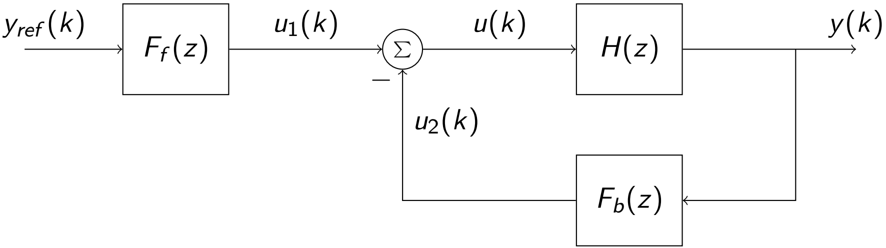
$$k^2 \quad \stackrel{\mathcal{Z}}{\longleftrightarrow} \quad \frac{z(z+1)}{(z-1)^3}$$

Which of the below pulse-transfer functions corresponds to the discretized hard disk drive model?

1	2	3
$H(z) = \frac{h^2 z}{2J(z+1)^2}$	$H(z) = \frac{h^2(z+1)}{2Jz^2}$	$H(z) = \frac{h^2(z+1)}{2J(z-1)^2}$

Block-diagram algebra

Same rules as in the continuous-time case!



With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c z} Y_{ref}(z).$$

Block-diagram algebra - steps in detail

With

$$U(z) = U_1(z) - U_2(z) = F_f(z)Y_{ref}(z) - F_b(z)Y(z), \quad \text{and}$$

$$Y(z) = H(z)U(z), \quad \text{we obtain}$$

$$Y(z) = H(z)U(z) = H(z)\left(F_f(z)Y_{ref}(z) - F_b(z)Y(z)\right)$$

Move all terms with Y to the left side:

$$Y(z) + H(z)F_b(z)Y(z) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z)\big(1 + H(z)F_b(z)\big) = H(z)F_f(z)Y_{ref}(z)$$

$$Y(z) = \frac{H(z)F_f(z)}{1 + H(z)F_b(z)}Y_{ref}(z)$$

Stability for the closed-loop system

$$Y(z) = \underbrace{\frac{F_f(z)H(z)}{1 + F_b(z)H(z)}}_{H_c z} Y_{ref}(z).$$

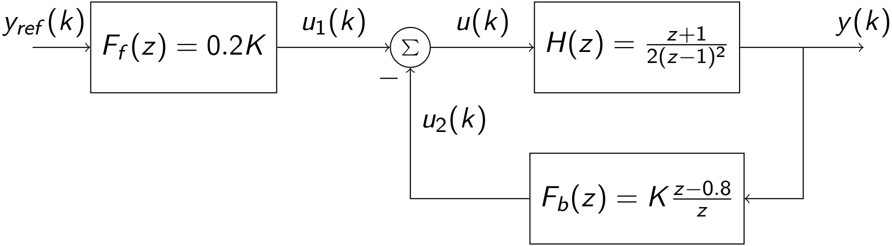
Stability requires that all poles of the system, that is all solutions to the characteristic equation

$$1 + F_b(z)H(z) = 0$$

are located inside the unit circle of the z-plane.

Stability for the disk drive arm

Case $\frac{h^2}{J} = 1$.



Characteristic equation

$$1 + H(z)F_b(z) = 0$$

$$1 + \frac{z+1}{2(z-1)^2}K\frac{z-0.8}{z} = 0$$

$$(z-1)^2 z + \frac{K}{2}(z+1)(z-0.8) = 0$$

Is the system stable for the gain $K = 2$?