Computerized control - Introduction

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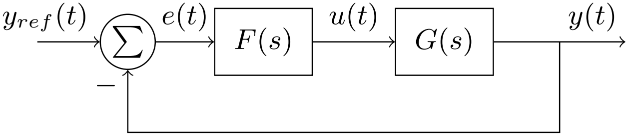
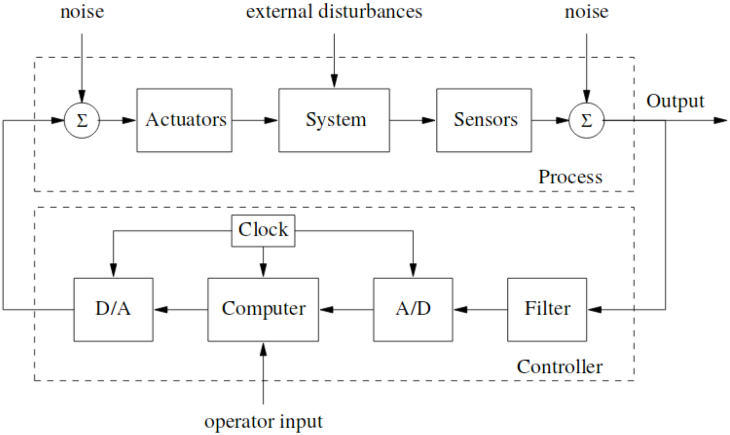
Goal of the course

To be able to analyze, design, implement and evaluate computerized control systems with a focus on practical application.

Feedback control

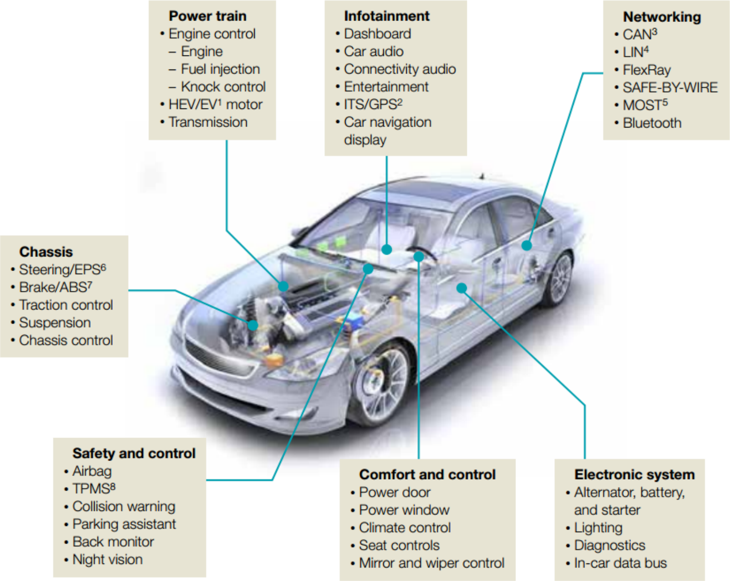


Feedback control



Why computerized control?

Computers everywhere



Two approaches to designing a discrete-time controller

1. Do design the controller in the continuous-time domain (methods from control engineering class). Then discretize the continuous-time controller.

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1. Determine discrete-time model of the plant. Do design in discrete-time domain.

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Example:

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Introducing the shift operator q: $\mathrm{q}\, x(k) = x(k+1),\ \mathrm{q}^{-1}\, x(k) = x(k-1)$

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$$\mathrm{q}\, x_k - a x_k = b u_k \quad \Leftrightarrow \quad (\mathrm{q} - a) x_k = b u_k \quad \Leftrightarrow \quad x_k = \frac{b}{\mathrm{q} - a} u_k .$$

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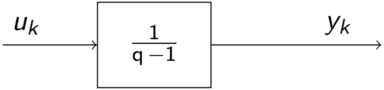
$$zX(z) - x(0) - aX(z) = bU(z) \quad \Leftrightarrow \quad (z-a)X(z) = x(0) + bU(z)$$

$$\Leftrightarrow \quad X(z) = \frac{x(0)}{z-a} + \frac{b}{z-a} U(z).$$

Exercise

Consider the following discrete-time system



Recall the definition of the shift operator $q\,x(k) = x(k+1)$, $q^{-1}\,x(k) = x(k-1)$.

1. Write the system as a difference equation $y_{k+1} = f(y_k, u_k)$.
2. What is this type of system called?

Homogenous solution

$$x(k+1) = ax(k), \quad x(0) = x_0$$

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$$\vdots$$

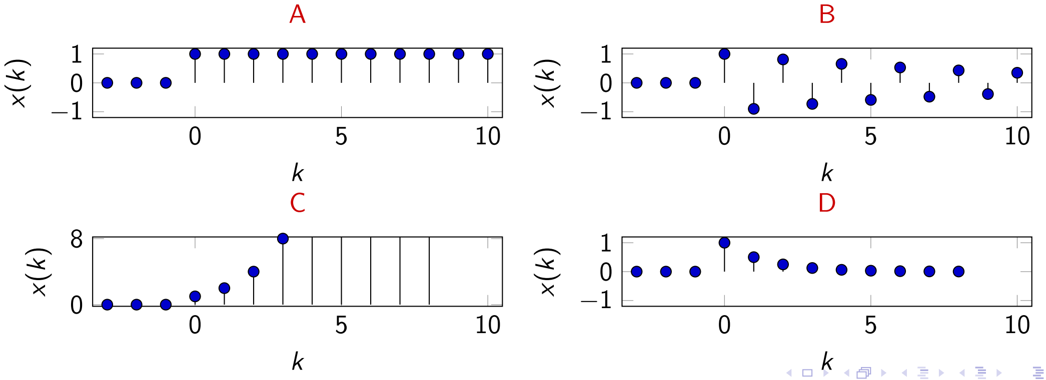
$$x(k) = a^k x_0$$

Homogenous solution to a first order system

$$x(k+1) = ax(k),\; x(0) = x_0 \quad \Rightarrow \quad x(k) = a^k x_0$$

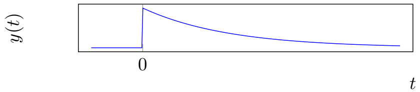
Pair each solution below to the corresponding value of a ($x_0 = 1$).

I) $a = 1$ II) $a = 2$ III) $a = 0.5$ IV) $a = -0.9$



Discrete time vs continuous time

Continuous time



$y(t)$

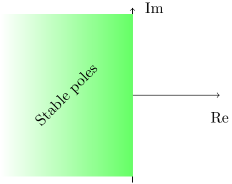
$\mathrm{p}\, y \triangleq \frac{d}{dt} y$

$(\mathrm{p} + a)y = bu \;\Leftrightarrow\; \frac{d}{dt} y + ay = bu$

$Y(s) \triangleq \mathcal{L}\{y(t)\}$

$Y(s) = G(s)U(s) = \frac{b}{s+a} U(s)$

Pole of the system: $s + a = 0 \;\Rightarrow\; s = -a$



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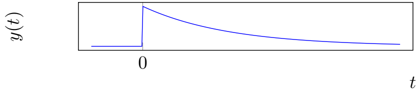

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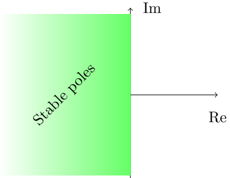
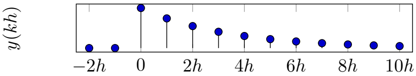


Discrete time



$y(kh)$ or $y(k)$

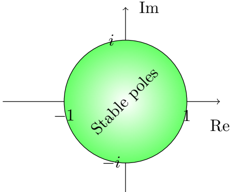
$q\,y \triangleq y(kh + h)$

$(q + \alpha)y = \beta u \;\Leftrightarrow\; y(k+1) + \alpha y(k) = \beta u(k)$

$Y(z) \triangleq \mathcal{Z}\{y(kh)\}$

$Y(z) = H(z)U(z) = \frac{\beta}{z+\alpha}U(z)$

Pole of the system: $z + \alpha = 0 \;\Rightarrow\; z = -\alpha$



Discrete design can give better performance

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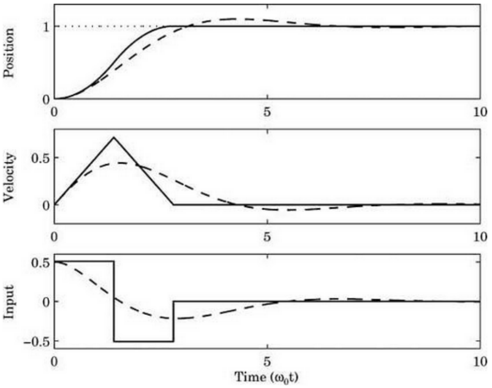
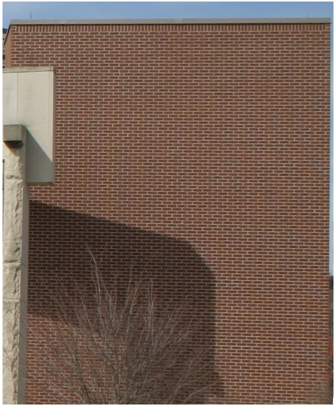
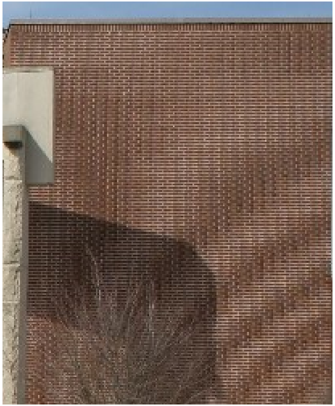


Figure 1.9 Simulation of the disk arm servo with deadbeat control (solid). The sampling period is $h = 1.4/\omega_0$. The analog controller from Example 1.2 is also shown (dashed).

Challenges with computerized control

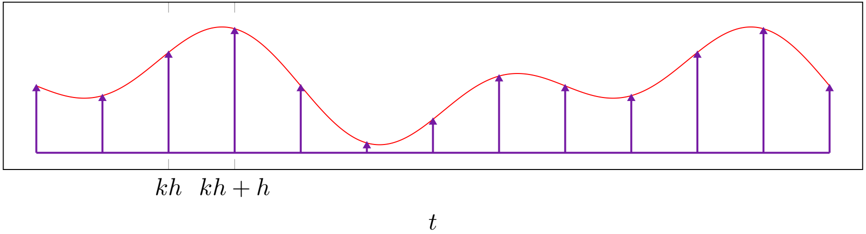
Aliasing





Challenges with computerized control

Sampling causes delay



Why learning computerized control?

- Almost all control systems are implemented on computers/microcontrollers
- Controllers designed in continuous-time must be discretized to be implemented on a computer - Performance can never be better than for continuous time.
- Design that takes into account the discrete nature of the computer can give better performance