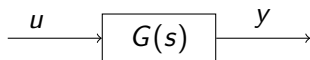


Frequency response

Kjartan Halvorsen

2021-06-29

Response of LTI systems to sinusoids



Let $u(t) = \sin \omega_1 t$. Then, after transients have died out,

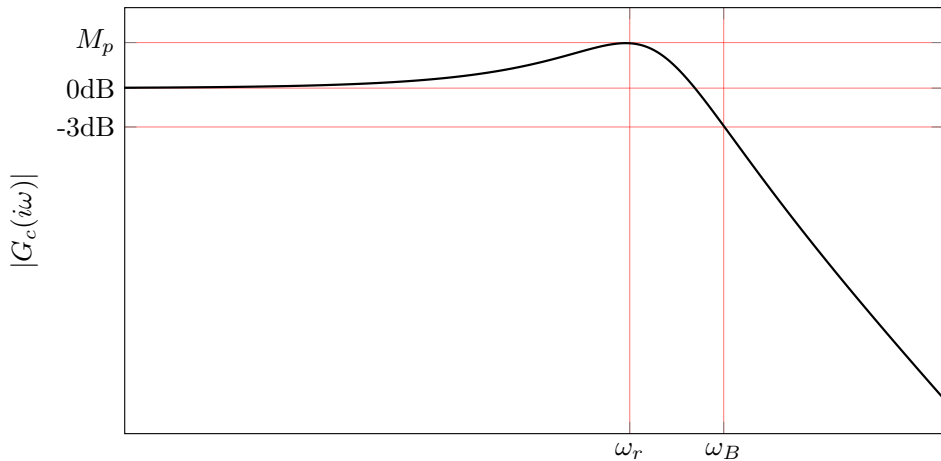
$$y(t) = |G(i\omega_1)| \sin (\omega_1 t + \arg G(i\omega_1)).$$

The Bode diagram

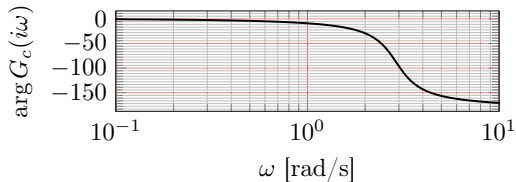
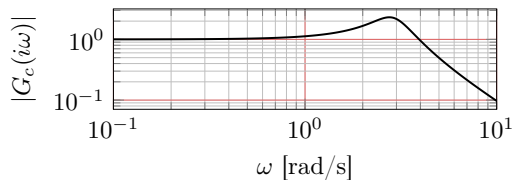
$$y(t) = \underbrace{|G(i\omega_1)|}_{\text{amplification}} \sin(\omega_1 t + \underbrace{\arg G(i\omega_1)}_{\text{phase shift}})$$

The Bode diagram shows the magnitude and phase of the transfer function evaluated on the positive imaginary axis. It thus contains all information about the steady-state response of the system to input signals of different frequency.

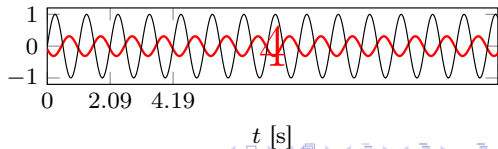
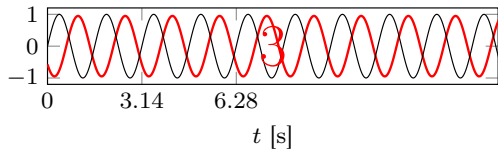
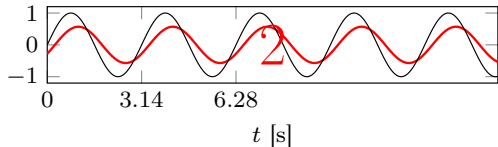
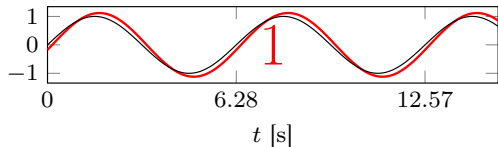
Specifications on the frequency properties of the closed-loop system



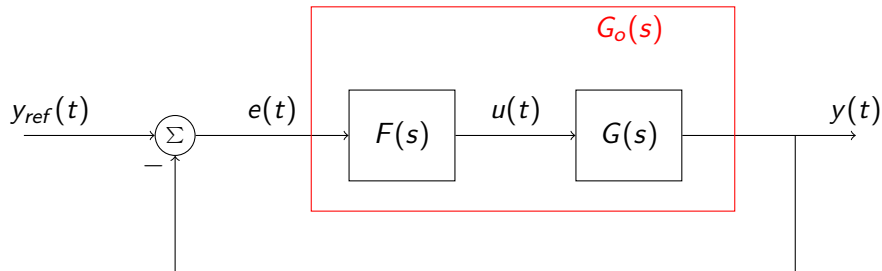
Exercise: Reading the Bode diagram



which of the below frequency responses **is not** compatible with the Bode diagram?



From loop gain to closed-loop gain



$$G_c(i\omega) = \frac{G(i\omega)F(i\omega)}{1 + G(i\omega)F(i\omega)} = \frac{G_o(i\omega)}{1 + G_o(i\omega)}$$

$$|G_c(i\omega)| = \frac{|G_o(i\omega)|}{|1 + G_o(i\omega)|} = \frac{|G_o(i\omega)|}{|G_o(i\omega) - (-1)|}$$

Keep the loop gain $G_o(i\omega)$ away from -1!