Digital PID

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Mapping of the stable region of the s-plane when discretizing the controller

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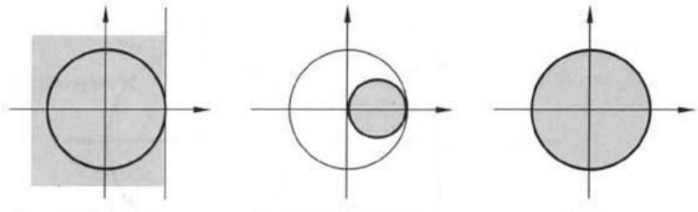
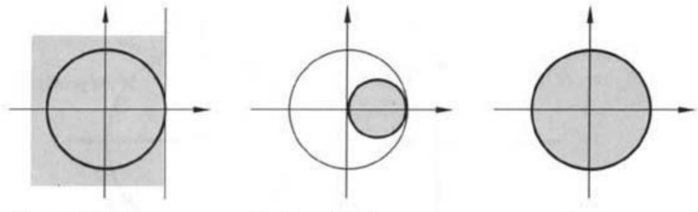


Mapping of the stable region of the s-plane when discretizing the controller



Activity Which of the above mappings of the left-half plane of the s-plane corresponds to A) the bilinear transformation (Tustin's approximation), B) the forward difference approximation, C) the backward difference approximation?

Mapping of the stable region of the s-plane

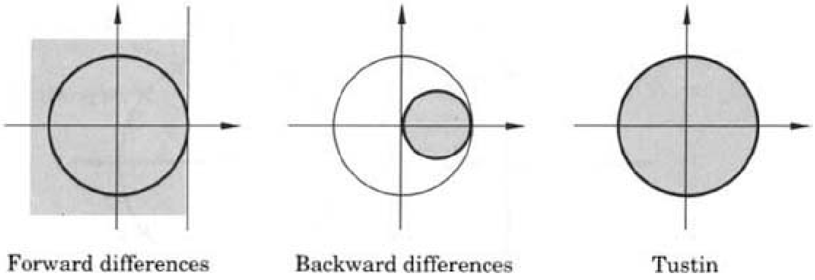


Figure 8.2 Mapping of the stability region in the s-plane on the z-plane for the transformations (8.4), (8.5), and (8.6).

Åström and Wittenmark *Computer-controlled systems*

ISA form of the PID

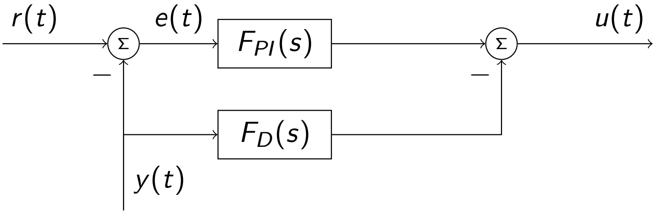
ISA - International Society of Automation

$$F(s) = K_c \left(1 + \frac{1}{T_i s} + T_d s\right)$$

With low-pass filter for the derivative part

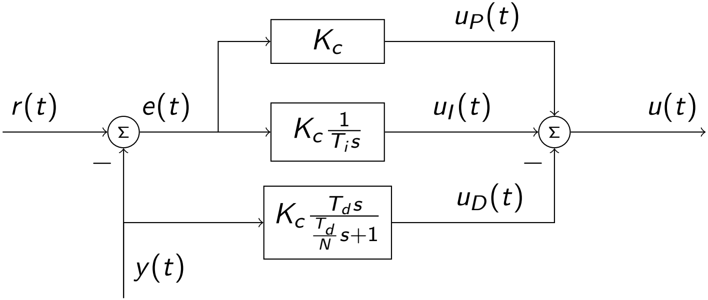
$$F(s) = K_c \left(1 + \frac{1}{T_i s} + \frac{T_d s}{\frac{T_d}{N} s + 1}\right), \quad N \approx 3 \ - \ 10$$

PID with derivative action only on the process variable



$$U(s) = \underbrace{K_c\left(1 + \frac{1}{T_i s}\right)}_{F_{PI}(s)} E(s) - \underbrace{K_c \frac{T_d s}{\frac{T_d}{N}s + 1}}_{F_D} Y(s)$$

Common discretization of the PID



$$U(s) = U_P(s) + U_I(s) - U_D(s) = K_c E(s) + K_c \frac{1}{T_i s} E(s) - K_c \frac{T_d s}{\frac{T_d}{N} s + 1} Y(s)$$

Activity 1) Use the forward difference $s \approx \frac{z-1}{h}$ for the integral part, and the backward difference $s \approx \frac{z-1}{zh}$ for the derivative part. 2) Apply the inverse z-transform to obtain the controller in the form of a difference equation.

Common discretization of the PID - Solution

Proportional part

Very simple: $u_P(kh) = K_c e(kh)$

Integral part

Substitute $s = \frac{z-1}{h}$ in the transfer function $F_I(s) = \frac{K_c}{T_i}\frac{1}{s}$

$$F_{I,d}(z) = \frac{K_c}{T_i}\frac{1}{\frac{z-1}{h}} = \frac{K_c}{T_i}\frac{h}{z-1}$$

$$U_I(z) = \frac{K_c}{T_i}\frac{h}{z-1}E(z),$$

$$U_I(z)(z-1) = \frac{K_c}{T_i}hE(z), \qquad \text{Apply inverse z-transform}$$

$$u_I(kh+h) - u_I(kh) = \frac{K_c}{T_i}he(kh) \qquad \Leftrightarrow \qquad u_I(kh+h) = u_I(kh) + \frac{K_c}{T_i}he(kh)$$

Common discretization of the PID - Solution

Derivative part

Substitute $s = \frac{z-1}{zh}$ in the transfer function $F_D(s) = K_c \frac{T_d s}{\frac{T_d}{N}s+1}$

$$F_{D,d}(z) = K_c \frac{T_d \frac{z-1}{zh}}{\frac{T_d}{N}\cdot\frac{z-1}{zh}+1} = K_c \frac{T_d(z-1)}{\frac{T_d}{N}(z-1)+zh} = K_c \frac{T_d(z-1)}{(\frac{T_d}{N}+h)z-\frac{T_d}{N}}$$

$$U_D(z) = K_c \frac{T_d(z-1)}{(\frac{T_d}{N}+h)z-\frac{T_d}{N}} Y(z)$$

$$\Big((\frac{T_d}{N}+h)z-\frac{T_d}{N}\Big)U_D(z) = K_c T_d (z-1)Y(z), \qquad \text{Apply the inverse z-transform}$$

$$(\frac{T_d}{N}+h)u_D(kh+h)-\frac{T_d}{N}u_D(kh) = K_c T_d\big(y(kh+h)-y(kh)\big)$$

The discrete PID algorithm

Calculated: $\alpha_1 = \dfrac{\frac{T_d}{N}}{\frac{T_d}{N} + h}$, $\alpha_2 = K_c \dfrac{T_d}{\frac{T_d}{N} + h}$, $\beta = K_c \dfrac{h}{T_i}$

Stored: $y(kh-h),\ u_I(kh-h),\ u_D(kh-h)$

Sample signals: $r(kh),\ y(kh)$

$e(kh) = r(kh) - y(kh)$

$u_P(kh) = K_c e(kh)$

$u_D(kh) = \alpha_1 u_D(kh-h) + \alpha_2 \big(y(kh) - y(kh-h)\big)$

$u(kh) = u_P(kh) + u_I(kh-h) - u_D(kh),$ Send to DAC

$u_I(kh) = u_I(kh-h) + \beta e(kh)$