Computerized control final exam (25%)

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Time ?

Place ?

Permitted aids The single colored page with your own notes, table of transforms, calculator

All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

Good luck!

Matricula and name:

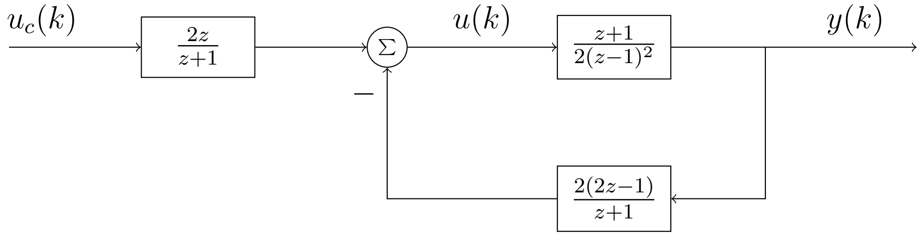
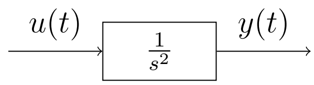


Figure 1: Closed-loop system, problem 1 (b).

Problem 1

The double-integrator



is a common model in physical systems where force (or torque) is the input and the position (or angle) is the output, and where the load is purely inertial (no friction, damping or elastic elements).

Zero-order-hold sampling of the double integrator with sampling period $h = 1$ gives the discrete-time model

$$y(k) = \frac{(q+1)}{2(q-1)^2} u(k) \tag{1}$$

(a)

Write the system in (1) as a *difference equation.*

Solution:

(b)

The plant is to be controlled by the two-degrees-of-freedom controller illustrated in figure 1. Show that the closed-loop system from the command signal $u_c(k)$ to the output $y(k)$ can be written

$$Y(z) = \frac{z(z+1)}{z^2(z+1)} U_c(z) = z^{-1} U_c(z).$$

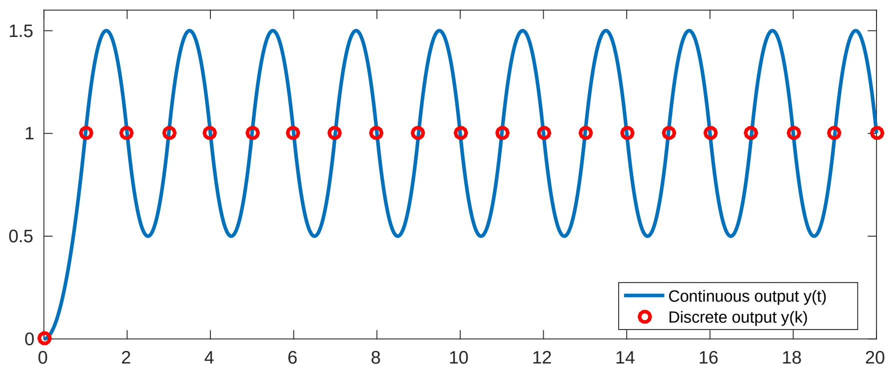
Solution:

(c)

The controller from (b) gives the closed-loop discrete-time response

$$y(k) = u_c(k-1)$$

which appears to be a system with very good performance, since the output follows the input perfectly with just a small delay of a single sampling period. Too good to be true? Yes, there is an important flaw with this controller, as shown in the simulation below. This is an example of hidden oscillations (oscillations in the continuous-time system that are invisible at the sampling instants).



Consider the feedback part of the controller

$$F_{fb}(z) = \frac{2(2z-1)}{z+1}. \tag{2}$$

Figure 2 shows four step-responses. Which of these responses corresponds to the step-response from the system (2)? Motivate!

Answer and motivation:

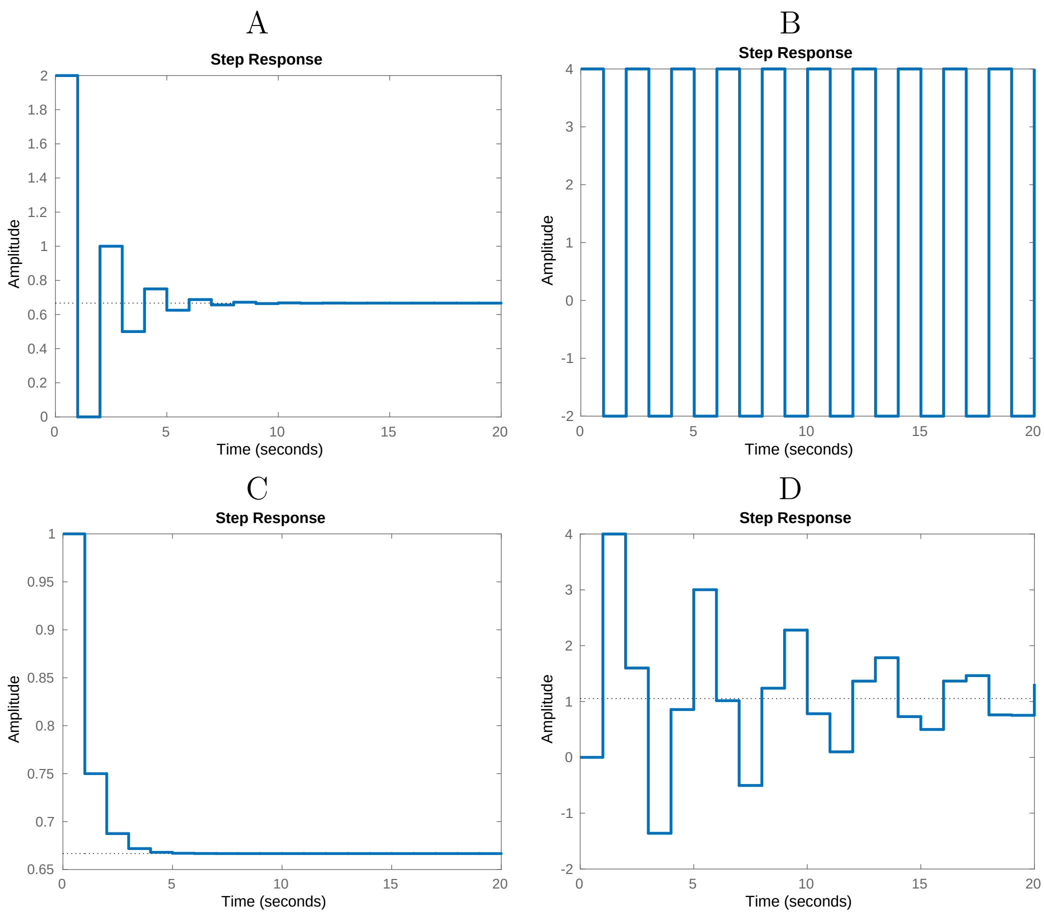


Figure 2: Step responses, problem 1 (c).

Problem 2

The double-integrator can be represented on state-space form as

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 1 & 0 \end{bmatrix} u(k) \end{aligned} \tag{3}$$

(a)

Determine *Reachability* and *Observability* for the state-space system.

Solution:

(b)

Determine a state feedback

$$u(k) = u_c(k) - k_1 x_1(k) - k_2 x_2(k)$$

which gives a closed-loop system with poles in the origin, i.e. deadbeat control.

Controller design:

If necessary, you can continue your solutions on this sheet. Mark clearly which problem the solution corresponds to.