||

Hissael Padilla Sánchez (A01169610)

90

Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

1. Sample the controller using Tustin's approximation

$$s = \frac{2}{h}\frac{z-1}{z+1}.$$

2. Show that the discrete controller is stable for all choices of sampling period h.
3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8$ rad/ sec. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?
4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer!

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

13 - 1

4 - x

x ≈ 0.30

0.6
aprox

$A_m = 1 - 0.64$

$A_m = 0.307$

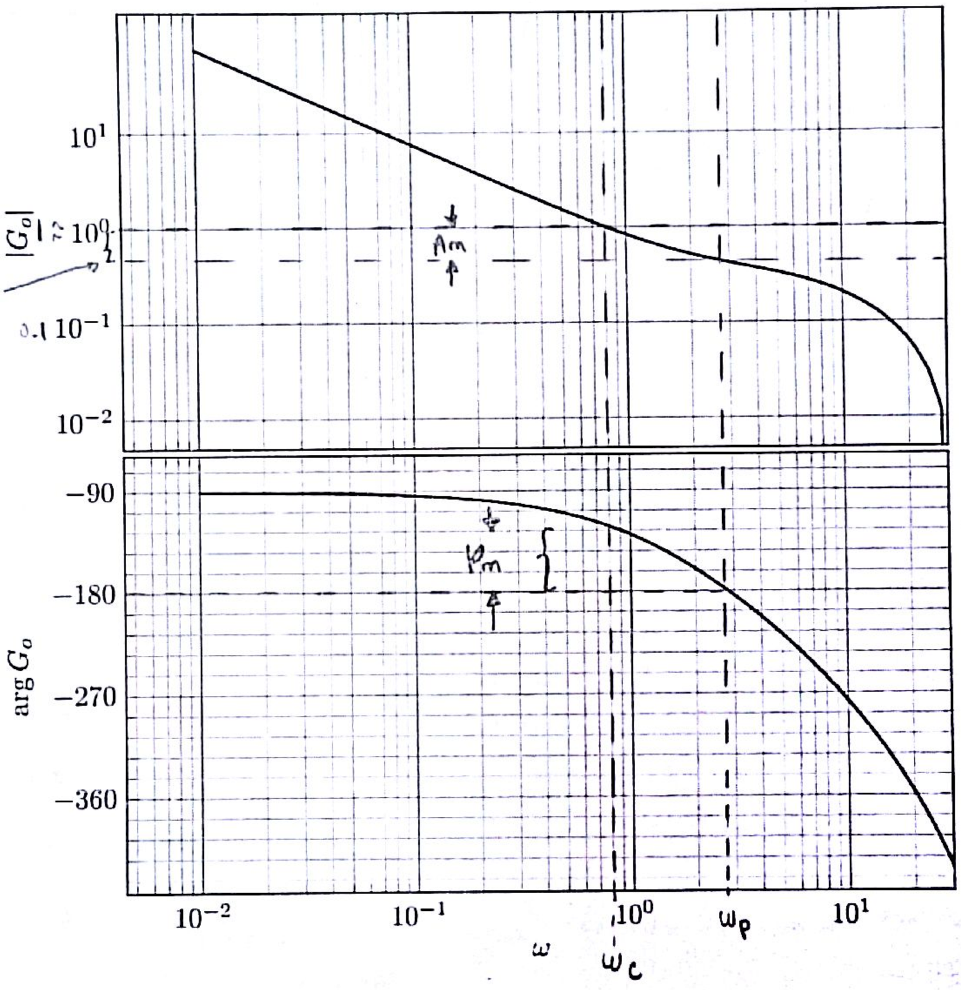


Figure 1: Bode diagram of open-loop transfer function.

From the graph $\left\{ \begin{array}{l} \omega_p \approx 2.7 \text{ rad/s} \\ \omega_c \approx 0.8 \text{ rad/s} \end{array} \right.$

From the graph $\left\{ \begin{array}{l} \varphi_m \approx 56^\circ \\ A_m \approx 0.307 \end{array} \right.$

Problem 1 Missael Padilla Sánchez (A01164610)

1)

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

$$F(s) = 3\left(\frac{s+2}{s+8}\right)$$

①

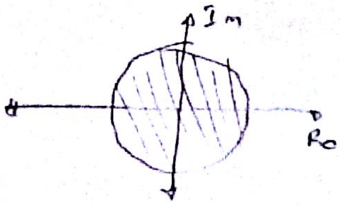
$$F(s')\Big|_{s'=\frac{2}{h}\frac{z-1}{z+1}} = 3\left[\frac{\frac{2}{h}\frac{z-1}{z+1}+2}{\frac{2}{h}\frac{z-1}{z+1}+8}\right]$$

$$= \frac{\frac{6}{h}\frac{z-1}{z+1}+6}{\frac{2}{h}\frac{z-1}{z+1}+8} = \frac{\frac{6(z-1)+6h(z+1)}{h(z+1)}}{\frac{2(z-1)+8h(z+1)}{h(z+1)}}$$

$$= \frac{6(z-1)+6h(z+1)}{2(z-1)+8h(z+1)}$$

✓

② We know that because poles in continuos time maps to locations inside the unit circle → Tustin's approximation



stable poles in cont. time ↓ Stable poles in discrete time

$$2(z-1)+8h(z+1)=0$$

$$2z-2+8zh+8h=0$$

$$z(2+8h) = 2-8h$$

$$z = \frac{2-8h}{2+8h}$$

✓

→ Controller is stable for any h

③ $w_c = 0.8$ rad/seg

$$F(j\omega) = 3\left[\frac{j\omega+2}{j\omega+8}\right]$$

✓

Same answer

$$= 3\left[\frac{0.8j+2}{0.8j+8}\right] = 0.77227 + j0.2227$$

$$= 0.8037 \angle 16.0908°$$

$$\arg[F(j\omega)] = \arg(3) + \arg(j\omega+2) - \arg(j\omega+8)$$

$$= \text{atan}\left(\frac{\omega}{2}\right) - \text{atan}\left(\frac{\omega}{8}\right) = \tan^{-1}\left(\frac{0.8}{2}\right) - \tan^{-1}\left(\frac{0.8}{8}\right) = 21.801 - 5.71$$

$$= 16.091°$$

(4) Sampled controller (Tustin's)

↳ $F(s') = \dfrac{6(z-1) + 6h(z+1)}{2(z-1) + 8h(z+1)}$

Cont. time controller

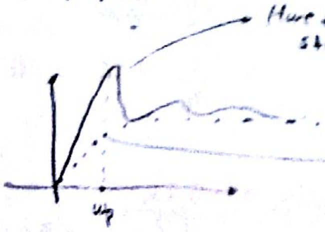
↳ $F(s) = 3\left[\dfrac{(s+2)}{s+8}\right]$

Great ω_{peak} ↓ Greater ω_c $\xrightarrow{\text{leads to}}$ Great ω_B

Analyzing the conditions we can say that...

Greater φ_m $\xrightarrow{\text{leads to}}$ Less M_{peak}

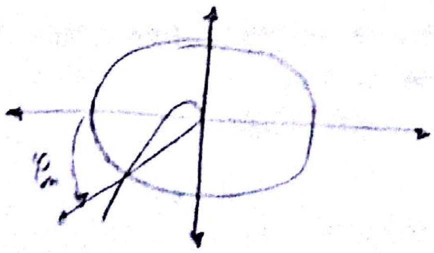
so...



More time to stabilize → Sampled cont. → Less φ_m

Less time to stabilize → Cont. time → Great φ_m

Greater φ_m



conclusion?

(-10)

Problem 2

↳

$\omega_p \approx 2.7$ rad/s

$\omega_c \approx 0.8$ rad/s

$\varphi_m \approx 56^\circ$

$A_m \approx 0.307$

César Jacob Nieto Rueda 90
A01164797

Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

1. Sample the controller using Tustin's approximation

$$s = \frac{2}{h}\frac{z-1}{z+1}.$$

2. Show that the discrete controller is stable for all choices of sampling period h.
3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8$ rad/ sec. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?
4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer!

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

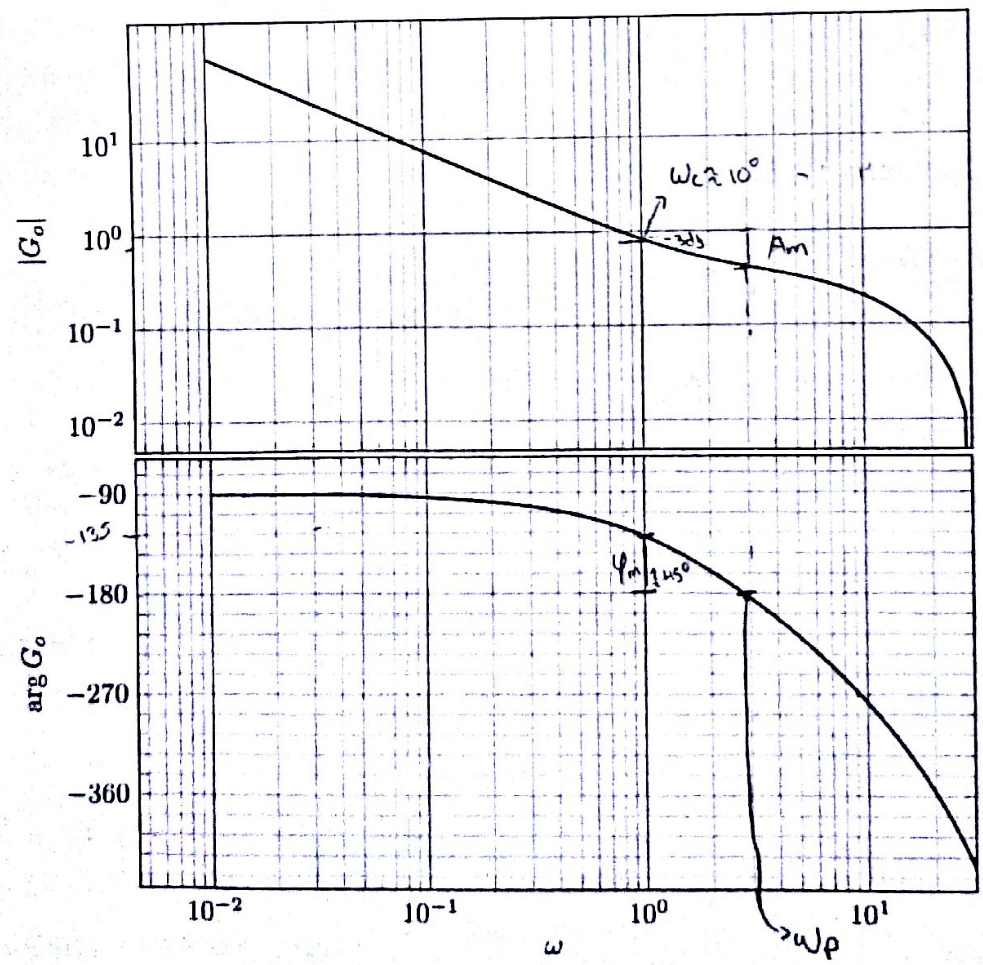


Figure 1: Bode diagram of open-loop transfer function.

César Nieto Rueda
A01164797

1.- $G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$

$F(s) = 3\frac{s+2}{s+8}$

Using Tustins

$$F(z) = F(s')\Big|_{s' = \frac{2}{h}\frac{z-1}{z+1}}$$

$$F(z) = 3\frac{\left(\frac{2}{h}\frac{z-1}{z+1} + 2\right)}{\left(\frac{2}{h}\frac{z-1}{z+1}\right) + 8} =$$

$$= 3\cdot\frac{2\cdot(z-1) + 2\cdot h\cdot(z+1)}{2(z-1) + 8\cdot h\cdot(z+1)}$$

$$= 3\cdot\frac{\cancel{z}[(z-1) + h(z+1)]}{\cancel{z}[(z-1) + 4\cdot h(z+1)]}$$

$$= 3\cdot\frac{(z-1) + h(z+1)}{(z-1) + 4h(z+1)}$$

$$= 3\cdot\frac{(z-1+hz+h)}{z-1+4hz+4h}$$

1. $$= 3\cdot\frac{z(h+1) - 1 + h}{z(1+4h) - 1 + 4h}$$ //

✓

2. $z(1+4h) - 1 + 4h = 0$

$$z = \frac{+1-4h}{1+4h}$$

✓

$$z = \frac{1}{1+4h} - \frac{4h}{1+4h}.$$

for every choice of sampling period h
the pole will be inside the unit circle
it would be close to -1 but it will never
[illegible] stable.

3. open loop
$W_c = 0.8$ rad/sec

$F(s) = 3\frac{s+2}{s+8}$

$$F(iw_c) = 3\frac{(iw_c+2)}{(iw_c+8)}$$

phase $= \arg|F(iw_c)|$

$\arg|F(iw_c)| = \cancel{\arg(3)}^{0} + \arg(iw_c+2) - \arg(iw_c+8)$ ✓

$\arg|F(iw_c)| = \arctan(w_c, 2) - \arctan(w_c, 8)$

phase $= 16.0908°$ //

4. It will be the same because we are using Tustin's approximation and one of the features of this sampling approximation is to have the same phase margin as in continous time.

Time-delay because of sample-and hold
(-10)

Alejandra Sandoval Casimiro A01371127

65

① $T(S) = F(s)\Big|_{s=\frac{2z-1}{hz+1}} = \dfrac{3\,\frac{2}{h}\left(\frac{z-1}{z+1}\right)+2}{\frac{2}{h}\left(\frac{z-1}{z+1}\right)+8}$

$$= \frac{\frac{6}{h}\left(\frac{z-1}{z+1}\right)+2}{\frac{2}{h}\left(\frac{z-1}{z+1}\right)+8} = \frac{\frac{6z-6+6hz+6h}{h(z+1)}}{\frac{2z-2+8hz+8h}{h(z+1)}}$$

$$= \frac{6(z-1)+6h(z+1)}{2(z-1)+8h(z+1)}$$

✓

② $2(z-1)+8h(z+1)$

$2z-2+8hz+8h$

having any h is imposible that you get 0

(-5)

③ $\omega_c = 0.8$ rad/s

$\angle G(\omega)\Big|_{\omega c}$ $\quad F(s) = 3\,\dfrac{s+2}{s+8}$

$\arg(\underset{0}{3}) + \arg(i\omega+2) - \arg(i\omega+8)$

$\arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{\omega}{8}\right) = 0.28$ rad

$0.28 \text{ rad}\left(\frac{360}{2\pi}\right) = 16.04°$

✓

(4) Comparando phase margin

$$\frac{6(z-1)+2h(z+1)}{2(z-1)+8h(z+1)} \quad \text{vs} \quad 3\,\frac{s+2}{s+8}$$

$$\varphi_m = 180 - \omega_p$$

The sampled controller has a greater. Phase margin because. is less close to -1 than the continuous-time controller.

Time-delay...

(-10)

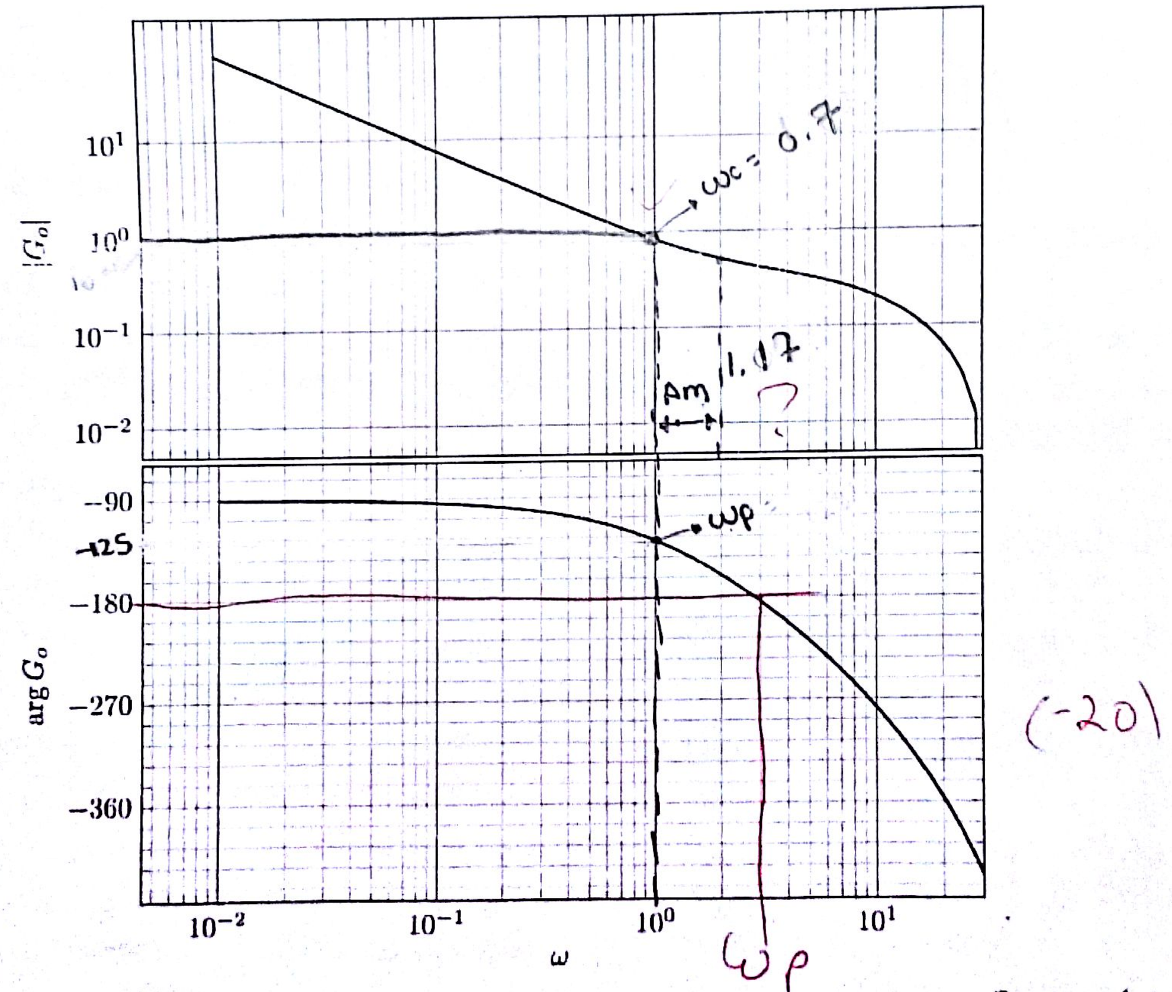


Figure 1: Bode diagram of open-loop transfer function.

$Am = \omega p - \omega c$

$1.87 - 0.7 =$

1.17

$\varphi m = 180 - 125 = 55°$

$\left| \frac{3(s+2)}{s+8} \right| = 1$

$\omega p: \quad \frac{3\sqrt{\omega^2+4}}{\sqrt{\omega^2+64}} = 1 \rightarrow (3\sqrt{\omega^2+4})^2 = (\sqrt{\omega^2+64})^2$

$9(\omega^2+4) = \omega^2+64$

$9\omega^2+36 = \omega^2+64$

$8\omega^2 = 28$

$\omega = \sqrt{\frac{28}{8}} = 1.87$

||

Hugo Rosas Sandoval A00513308 75

Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

a) 1. Sample the controller using Tustin's approximation

$$s = \frac{2}{h}\frac{z-1}{z+1}.$$

b) 2. Show that the discrete controller is stable for all choices of sampling period h.

c) 3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8$ rad/sec. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?

d) 4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer!

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

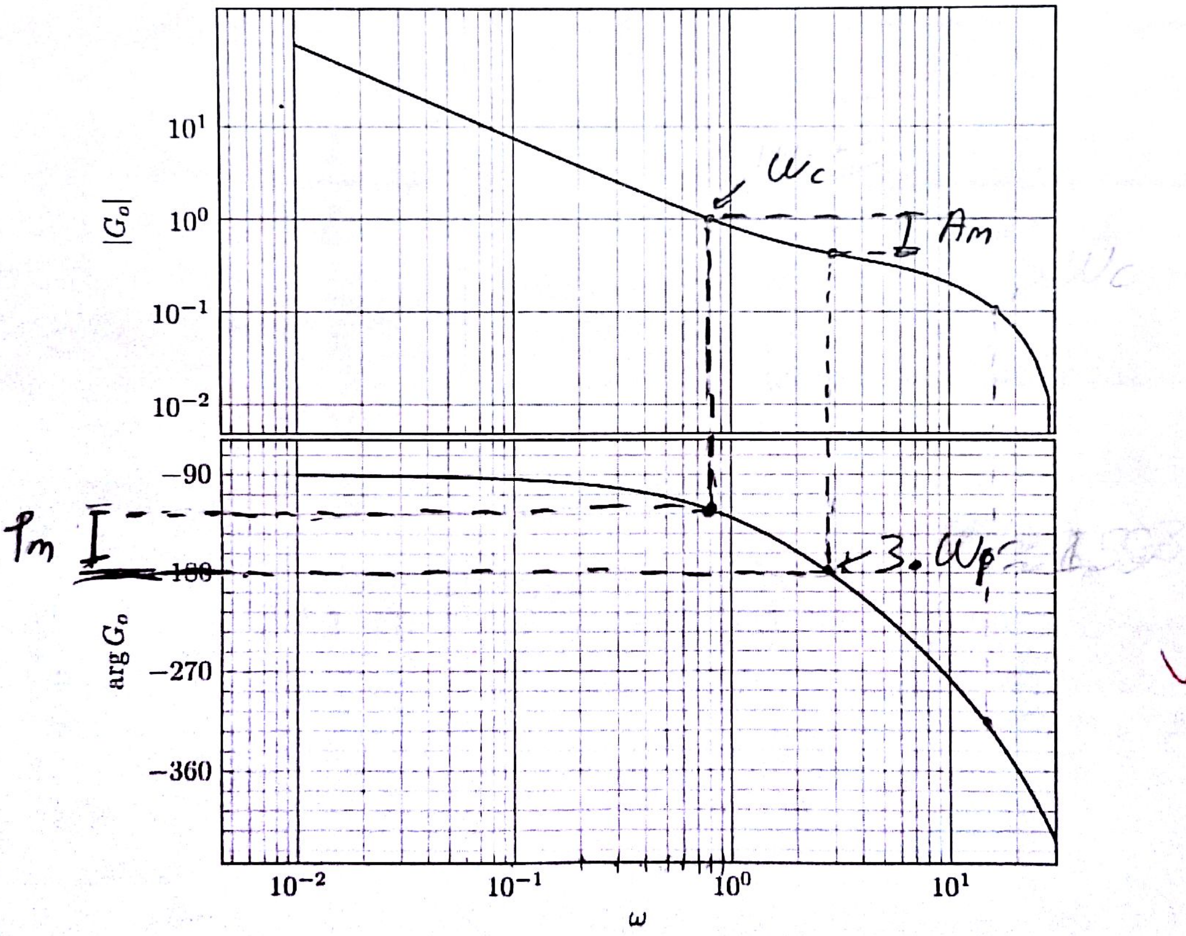


Figure 1: Bode diagram of open-loop transfer function.

go Rosas Sandoval A00513308

) a) $F(S) = 3\frac{S+2}{S+8}$ $S' = \frac{h}{2}\frac{z-1}{z+1}$

$F(S)\Big|_{S'} = 3\frac{S+2}{S+8}\Big|_{S'}$

$F(S)\Big|_{S'} = \frac{3\left(\frac{2}{h}\cdot\frac{z-1}{z+1}\right)+2\cdot 3}{\frac{2}{h}\cdot\frac{z-1}{z+1}+8} = \frac{\frac{6z-6+2hz+2h}{\cancel{hz+h}}}{\frac{2z-2+8hz+8h}{\cancel{hz+h}}}$

$F(S)\Big|_{S'} = \frac{(6+2h)z+2h-6}{(2+8h)z+8h-2}$ $h=0.2$

OK

b) $F(S)\Big|_{S'} = \frac{[6+(0.2)(2)]z+(2)(0.2)-6}{[2+(8)(.2)]z+(8)(.2)-2} = \frac{6.4z-5.6}{3.6z-0.4}$

$F(S)\Big|_{S'} < 1$

$6.4z-5.6 < 3.6z-0.4$ (-10)

$6.4z-3.6z < +5.6-0.4$

$2.8z < 5.2$

$z < 1.857$

c) $F(s) = \dfrac{3s+6}{s+8} = \dfrac{3i\omega+6}{i\omega+8}$

$\omega_c = 0.8 \frac{rad}{s}$

$\arg(F(i\omega_c)) = -\operatorname{atan}\left(\dfrac{3\omega_c}{6}\right) - \operatorname{atan}\left(\dfrac{\omega_c}{8}\right)$

$\arg(F(i\omega_c)) = 16.1$ ✓

d) (−15)

Maria Inés Reballar A01168892

35

Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

When you use tustin aproximation you ensure that the system will be in thi area for that reuson the sistem will be stuble.

1. Sample the controller using Tustin's approximation

$$s = \frac{2}{h}\frac{z-1}{z+1}.$$

2. Show that the discrete controller is stable for all choices of sampling period h.
3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8$ rad/sec. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?
4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer! The phase margin of the sampled controller is grater than the phase margic obtained from the continuos time controller because the tustin aproximalio ensures thu de em is onle unit circle.

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

$$G_c = \frac{G(S)F(S)}{1 + G(S)(F)(S)} \qquad F(S) = 3\left(\frac{S+2}{S+8}\right) \qquad G(S) = \frac{1}{S}\left(\frac{-S+2}{S+2}\right)$$

$$G_c = \frac{\frac{1}{S}\left(\frac{-S+2}{S+2}\right)3\left(\frac{S+2}{S+8}\right)}{1 + \frac{1}{S}\left(\frac{-S+2}{S+2}\right)3\left(\frac{S+2}{S+8}\right)} \rightarrow \frac{\frac{1}{S}\left(\frac{-S+2}{S+2}\right)3\left(\frac{S+2}{S+8}\right)}{\frac{(S+2)(S+8)+3(-S+2)(S+2)}{S(S+2)(S+8)}}$$

$$G_c = \frac{\frac{3(-S+2)(S+2)}{S(S+2)(S+8)}}{\frac{(S+2)(S+8)+3(-S+2)(S+2)}{S(S+2)(S+8)}} = \frac{3(-S+2)(S+2)\cancel{[S(S+2)(S+8)]}}{(S+2)(S+8)+3(-S+2)(S+2)\cancel{[S(S+2)(S+8)]}}$$

$$G_c = \frac{3(-S+2)(S+2)}{(S+2)(S+8)+3(-S+2)(S+2)} = \frac{3(\cancel{S+2})(-S+2)}{\cancel{(S+2)}[(S+8)+3(-S+2)]}$$

$$G_c = \frac{3(-S+2)}{(S+8)+3(-S+2)}$$

2)

Using tustin aproximation.

$$G_z = \frac{3\left[-\frac{2}{h}\left(\frac{z-1}{z+1}\right)+2\right]}{\frac{2}{h}\left(\frac{z-1}{z+1}\right)+8+3\left[-\frac{2}{h}\left(\frac{z-1}{z+1}\right)+2\right]}$$

OK.

1) $$G_z = \frac{-\frac{6}{h}\left(\frac{z-1}{z+1}\right)+6}{\frac{2}{h}\left(\frac{z-1}{z+1}\right)+-\frac{6}{h}\left(\frac{z-1}{z+1}\right)+14}$$

stability?

(-10)

2) $$\frac{3(-i\omega+2)}{(i\omega+8)+3(-i\omega+2)}$$

(-10)

$\arg(-i\omega+2) - \arg(i\omega+3) + \arg(-i\omega+2) = -180^\circ$

$i\omega$ high $\quad -90 - 90^\circ - 90 = -180^\circ \qquad w_c = -\pi/2 \qquad w_c = -\pi$

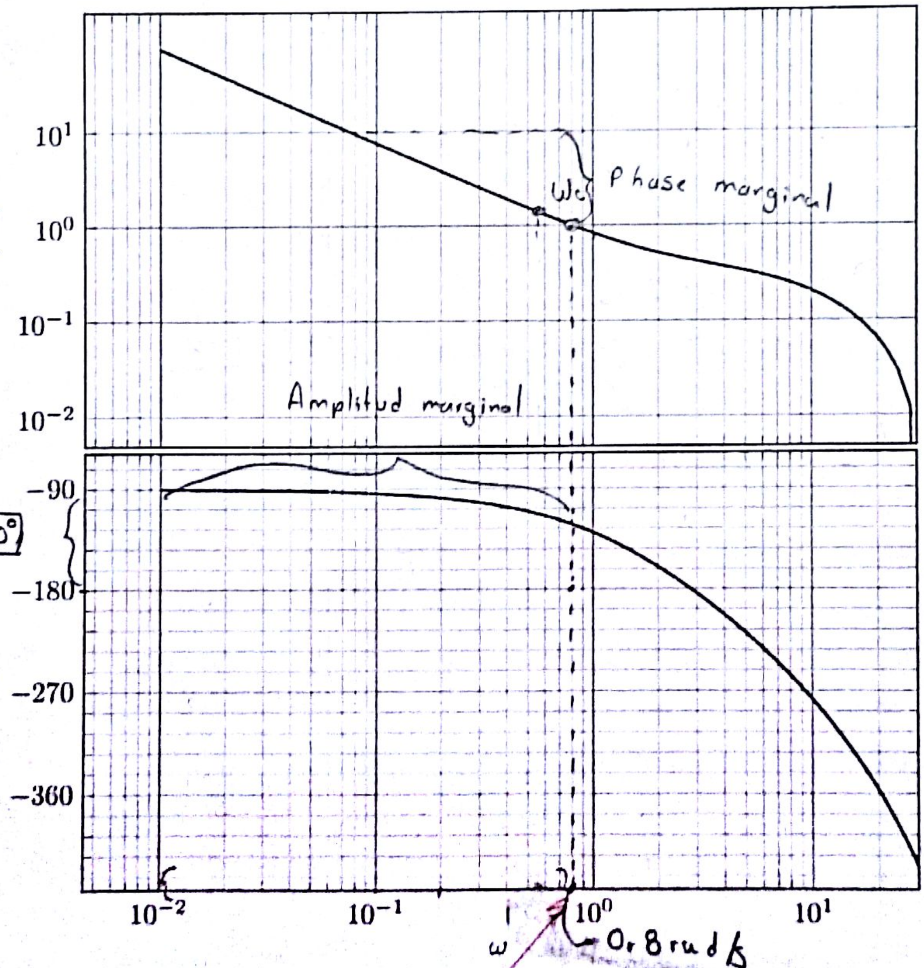
$i\omega$ low

2)

3) $W_c = 0.8 rad/s$

$arg(iW_p) = -180$

$\boxed{W_p = -90°}$



OK

Figure 1: Bode diagram of open-loop transfer function.

Confusion with Bandwidth of closed-loop system

$3 = 20 \log_{10} |a|$

$10^{\frac{3}{20}} = a$

1) The cross-over frecuency occurs when the system has a loss of $3 dB = 1.412$

$W_c = 0.8 rad/s$

(-30)

$$G_c = \frac{3(-s+2)}{(s+8)+3(-s+2)} = 0$$

$|G_c(i\omega_c)| = 1$

3(-

4) For the open loop system.

~~Gopen = F(s)(G(s)~~

$$\cancel{3\left(\frac{-s+2}{s+8}\right)\left(\frac{1}{s}\right)\left(\frac{-s+2}{s+2}\right)}$$

(-15)

$$G_{open} = \frac{3(\cancel{s+2})(-s+2)}{s(s+8)(\cancel{s+2})} = \frac{3(-s+2)}{s(s+8)}$$

~~arg (-i\omega+2) - arg(i\omega) - arg (i\omega+8)~~

High ω_p = $-90 - 90 - 90 = -180°$

Low ω_p = $-180°$

~~$\omega_p = -\pi$~~

~~$\omega_p = -2\pi$~~

Jose Gabriel Da Silva
A00808700

35

Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

1. Sample the controller using Tustin's approximation

$$s = \frac{2}{h}\frac{z-1}{z+1}.$$

2. Show that the discrete controller is stable for all choices of sampling period h.
3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8$ rad/ sec. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?
4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer!

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

$h = 0,2$

ω_c, φ_m, ω_p, A_m

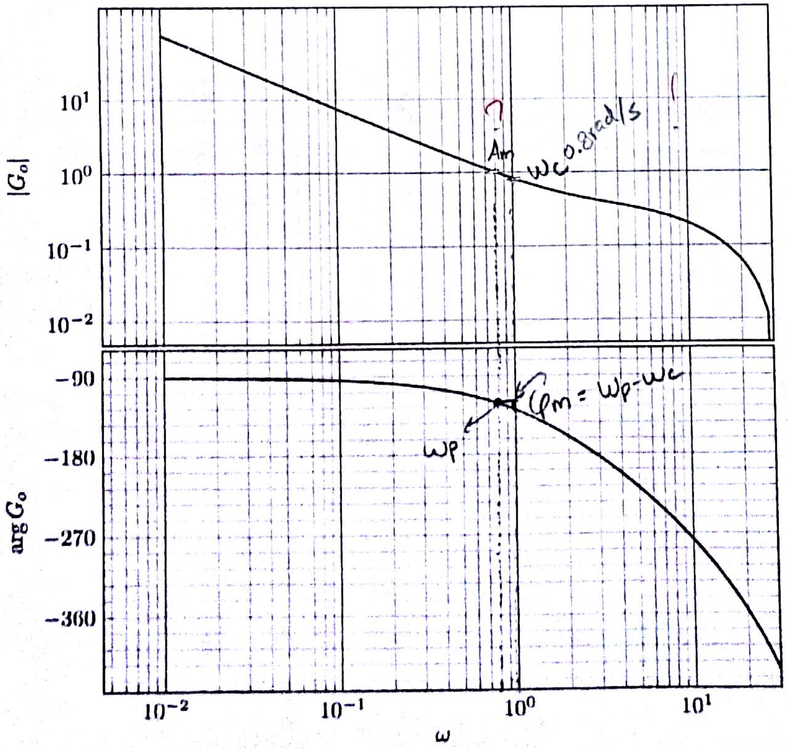


Figure 1: Bode diagram of open-loop transfer function.

$\varphi_m = \pi + \arg G(\omega_c) =$

-40

A00808700

Problem 1

① $F(s) = 3\left(\frac{s+2}{s+8}\right)$

Using Tustin ...

$$3\left(\frac{\frac{2(z-1)}{h(z+1)}+2}{\frac{2(z-1)}{h(z+1)}+8}\right) = 3\left(\frac{2(z-1)+2h(z+1)}{2(z-1)+8h(z+1)}\right)$$

$$F(s) = 3\left(\frac{z-1+hz+h}{z-1+4hz+4h}\right) = 3\left(\frac{z(1+h)+h-1}{z(1+4h)+4h-1}\right)$$ ✓

initial value

$$\lim_{h\to 0} F(s) = 3\left(\frac{z-1}{z-1}\right) = 3>0$$

?

final value

$$\lim_{h\to\infty} F(s) = 3\left(\frac{\infty}{\infty}\right) \rightarrow$$ with the derivative on both parts... $\longrightarrow 3\left(\frac{z+1}{4z+4}\right) = \frac{3}{4}\left(\frac{\cancel{z+1}}{\cancel{z+1}}\right)$

$= \frac{3}{4} > 0$

②

(-10)

The discrete controller is stable for all choices of sampling period h

$w_c = 0.8\text{rad/sec}$

③ $\varphi = \arg G_c(jw) \longrightarrow \arg G_c(jw): 3\left(\frac{jw+2}{jw+8}\right)$

(-10) Magnitude $\longrightarrow$ not argument

$$= 3\left(\frac{\sqrt{2^2+(j(0.8))^2}}{\sqrt{8^2+(j0.8)^2}}\right)$$

4 ? (-15)

$= 0.6908\ \text{rad/s}$

Problem 2

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right) \Longrightarrow G(j\omega) = \frac{-j\omega+2}{j\omega(j\omega+2)}$$

$$G(j\omega) = \frac{-j\omega}{j\omega(j\omega+2)} + \frac{2}{j\omega(j\omega+2)}$$

$$G(j\omega) = -\frac{1}{j\omega+2} + \frac{2}{j\omega(j\omega+2)}.$$

---

phase cross

$$0 = -20\log\left|-\frac{1}{j\omega+2}\right| - 20\log\left|\frac{2}{j\omega(j\omega+2)}\right|$$

$\omega_p =$