Computerized control - final exam (dummy)

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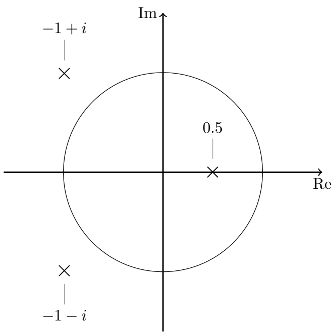
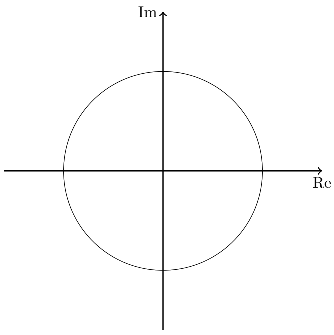
Problem 1

The figure below shows the poles of a continuous-time transfer function representing the dynamics of a system.



Choose a reasonable sampling period h, and plot the poles of the discrete-time system obtained by zero-order-hold sampling



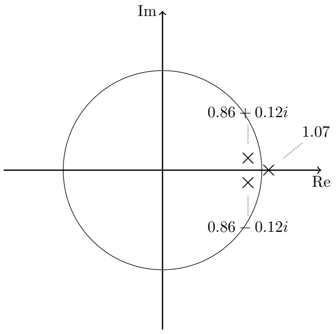
Solution

The system has two stable, complex-conjugated poles with distance $\omega_0 = \sqrt{2}$ from the origin, and one unstable pole in $s = 0.5$. The two stable poles are faster than the unstable pole, since they are farther from the origin. We can use the rule-of-thumb

$$\omega_0 h \approx 0.2 - 0.6,$$

but we should be cautious and choose a sampling period in the shorter end of the range. The reason is that zero-order-hold implies a time-delay of approximately $h/2$, and time-delays in unstable systems are problematic.

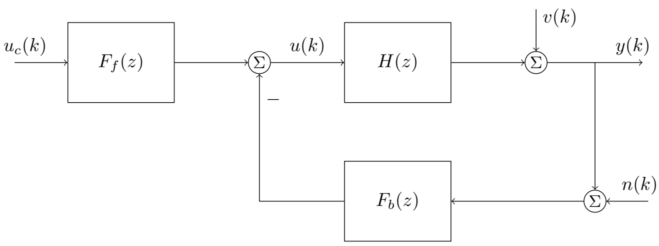
With $\omega_0 h = 0.2$ we get the discrete time poles



Problem 2

Circle the correct answer to each question. Motivate your answer briefly with 1-2 sentences.

(a)



The figure above shows a block-diagram of two-degrees-of-freedom control system. Which of the following pulse transfer operators describes the **closed-loop response**, $y(kh)$, to the measurement noise sequence, $n(kh)$.

1. $H_n(q) = \frac{1}{1+G(q)F_b(q)}$

2. $H_n(q) = -\frac{G(q)F_f(q)}{1+G(q)F_f(q)}$
3. $H_n(q) = -\frac{G(q)F_b(q)}{1+G(q)F_b(q)}$
4. $H_n(q) = \frac{G(q)F_b(q)}{1+G(q)F_b(q)}$

Solution

The correct answer is: 3. $H_n(q) = -\frac{G(q)F_b(q)}{1+G(q)F_b(q)}$. We can calculate the transfer function to find this answer, or argue as follows. There is a minus sign (negation) in the path from n to y, so the pulse transfer operator must be negative. Only alternatives 2 and 3 are negative. Of these two, alternative 2 includes the pulse transfer operator $F_f(q)$, but $F_f(q)$ is outside the signal path from n to y.

(b)

The continuous-time harmonic oscillator has transfer function

$$G(s) = \frac{\omega^2}{s^2 + \omega^2}.$$

Zero-order hold sampling of the system gives the pulse transfer function

1. $H(z) = \frac{(1-\cos\omega h)(z+1)}{(z-\cos\omega h)^2+\sin^2\omega h}$
2. $H(z) = \frac{(1-\cos\omega h)(z+1)}{(z-1)^2+\sin^2\omega h}$
3. $H(z) = \frac{(1-\cos\omega h)(z+1)}{(z+\cos\omega h)^2+\sin^2\omega h}$

Solution

The correct answer is: 1. $H(z) = \frac{(1-\cos\omega h)(z+1)}{(z-\cos\omega h)^2+\sin^2\omega h}$. The harmonic oscillator has poles on the imaginary axis in the continuous-time case, and on the unit circle in the discrete-time case. Both alternative 1 and 3 have poles on the unit circle. However, the discrete-time poles are obtained from the continuous-time poles $\pm i\omega$ according to the mapping

$$p = \mathrm{e}^{\pm i\omega h} = \cos\omega h \pm i\sin\omega h,$$

where the last equality is the famous Euler's formula. Alternative 1 has indeed poles in

$$\cos\omega h \pm i\sin\omega h,$$

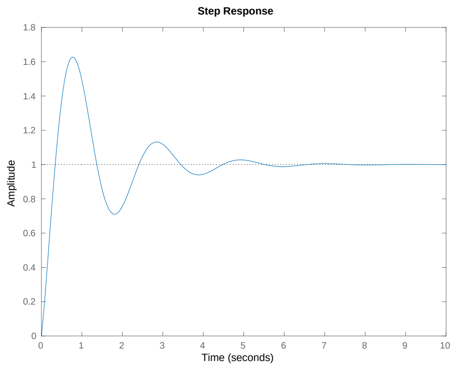
whereas Alternative 3 has poles in

$$-\cos\omega h \pm i\sin\omega h.$$

(c)

The figure below shows the step-response of a closed-loop system with a discretized PID controller for some values of the controller parameters. In continuous-time the controller has the form

$$F(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_d s/N}\right).$$



How should the controller be modified if we want the response of the closed-loop system to be **faster with less overshoot**?

1. Increase N and decrease T_i.
2. Increase K and T_d.
3. Decrease K and increase T_d.
4. Increase K and decrease T_i.

Solution

The correct answer is: 2. Increase K and T_d. To make the response faster, we must increase the gain of the controller K. Only alternative 2 and 4 suggest this. To make the overshoot smaller, we must increase the damping. This is done by increasing T_d.

Problem 3

Consider the discrete-time double integrator

$$H(z) = \frac{h^2(z+1)}{2(z-1)^2}.$$

(a)

Write the system on state-space form, using the controllable canonical form

$$x(k+1) = \underbrace{\begin{bmatrix} -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}}_{\Phi} x(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{\Gamma} u(k),$$

$$y(k) = \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} x(k)$$

where

$$H(z) = \frac{b_1 z^{n-1} + b_2 z^{n-2} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n}.$$

Solution

The harmonic oscillator can be written

$$H(z) = \frac{h^2(z+1)}{2(z-1)^2} = \frac{h^2/2(z+1)}{z^2 - 2z + 1},$$

which on controllable canonical form is

$$x(k+1) = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} h^2/2 & h^2/2 \end{bmatrix}.$$

(b)

Determine a linear state feedback

$$u(k) = -Lx(k) + u_c(k)$$

such that the closed-loop system has poles in $\pm i0.4$

Solution

Linear feedback control of a system on controllable canonical form is particularly easy, since the resulting system is also on controllable canonical form. The closed loop system with

$$u(k) = -Lx(k) + u_c(k)$$

has pulse transfer function

$$H_c(z) = \frac{h^2/2(z+1)}{z^2(-2+l_1)z+1+l_2}$$

and the desired denominator is

$$(z-i0.4)(z+i0.4) = z^2+0.4^2,$$

Equating the coefficients gives the feedback gains

$$\begin{aligned} -2+l_1 = 0 \quad &\Rightarrow \quad l_1 = 2 \\ 1+l_2 = 0.16 \quad &\Rightarrow \quad l_2 = -0.84 \end{aligned}$$

Using the general procedure for pole-placement we get the characteristic polynomial

$$\begin{aligned} \det\left(zI-(\Phi-\Gamma L)\right) &= \det\left(\begin{bmatrix} z & 0 \\ 0 & z \end{bmatrix} - \left(\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} l_1 & l_2 \\ 0 & 0 \end{bmatrix}\right)\right) \\ &= \det\begin{bmatrix} z-2+l_1 & 1+l_2 \\ -1 & z \end{bmatrix} \\ &= (z+l_1-2)z+1+l_2 = z^2+(l_1-2)z+1+l_2 \end{aligned}$$

And comparing with the desired characteristic polynomial

$$z^2+0.4^2$$

we get the system of equations

$$\begin{aligned} l_1 &= 0+2 = 2 \\ l_2 &= 0.4^2-1 = -0.84 \end{aligned}$$

A step-response with $h=1$ and a step in u_c occurring at $t=1$ is shown below.