Computerized control - partial exam 2 (20%)

Kjartan Halvorsen

2015-10-23

Problem 1

In the preparation exercise for this exam a controller was designed for the system

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right)$$

which has a zero in the right half plane. The continuous-time controller was given by the transfer function

$$F(s) = 3\frac{s+2}{s+8}.$$

1. Sample the controller using Tustin's approximation
$$s = \frac{2}{h}\frac{z-1}{z+1}.$$
2. Show that the discrete controller is stable for all choices of sampling period h.
3. The cross-over frequency of the continuous-time open loop transfer function was found to be $\omega_c = 0.8\,\text{rad/sec}$. What is the phase of the continuous-time controller at this frequency (what is its complex argument)?
4. Will the open-loop system using the sampled controller you obtained have a phase margin which is greater than or less than the phase margin using the continuous-time controller? Motivate your answer!

Problem 2

In figure 1 the open-loop transfer function for the system in Problem 1 with a discrete controller ($h = 0.2$) is given. Identify (mark in the figure):

1. The cross-over frequency ω_c.
2. The phase margin φ_m.
3. The phase-cross over frequency ω_p.
4. The amplitude margin A_m.

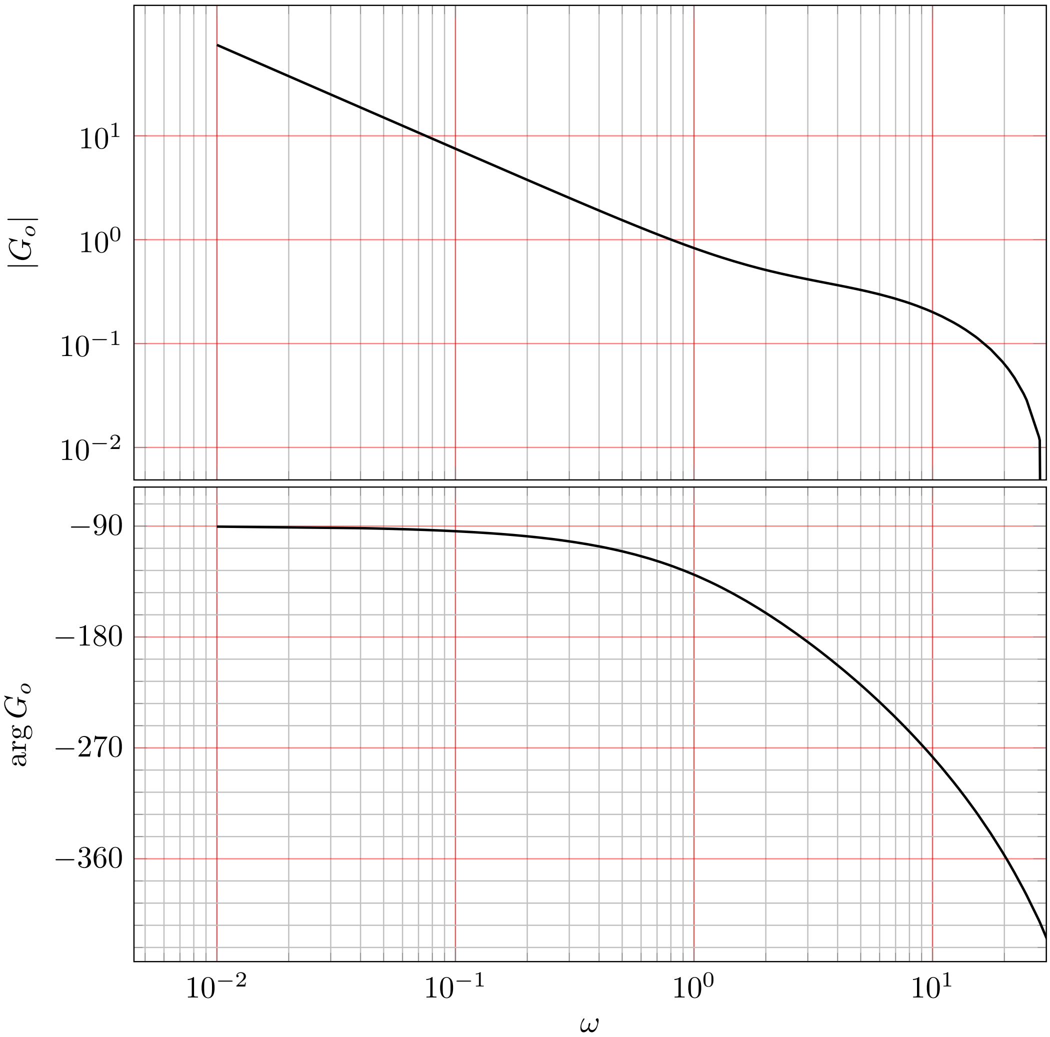


Figure 1: Bode diagram of open-loop transfer function.

Solutions

Problem 1

1. The sampled controller using Tustin's approximation becomes

$$F_d(z) = F(s')|_{s'=\frac{2}{h}\frac{z-1}{z+1}} = 3\frac{\frac{2}{h}\frac{z-1}{z+1}+2}{\frac{2}{h}\frac{z-1}{z+1}+8} = 3\frac{(1+h)z-(1-h)}{(1+4h)z-(1-4h)}.$$

2. The characteristic equation of the discrete controller is

$$(1+4h)z-(1-4h) = 0.$$

   Clearly, the controller has a single pole in $\frac{1-4h}{1+4h}$ which is inside the unit disk for all positive values of h.

3. The phase of the controller at $\omega_c = 0.8$ is

$$\arg F(i\omega_c) = \arg(i\omega_c+2) - \arg(i\omega_c+8) = \arctan\frac{0.8}{2} - \arctan\frac{0.8}{8} \approx 0.28 \text{ rad} = 16.1^\circ.$$

4. The sampled controller will have a phase margin which is **smaller** than that of continous-time system, since the sample-and-hold of the discrete controller introduces a time-delay of about half the sampling period. More argumentation than this is not necessary for full points. But for the interested: In the case with $h = 0.2$ sec, the (approximate) time-delay is 0.1 sec and the corresponding phase contribution of the sample-and-hold at the cross-over frequency is

$$\arg \mathrm{e}^{-i0.1\omega_c} = -0.08 \text{ rad} \approx -4.58^\circ.$$

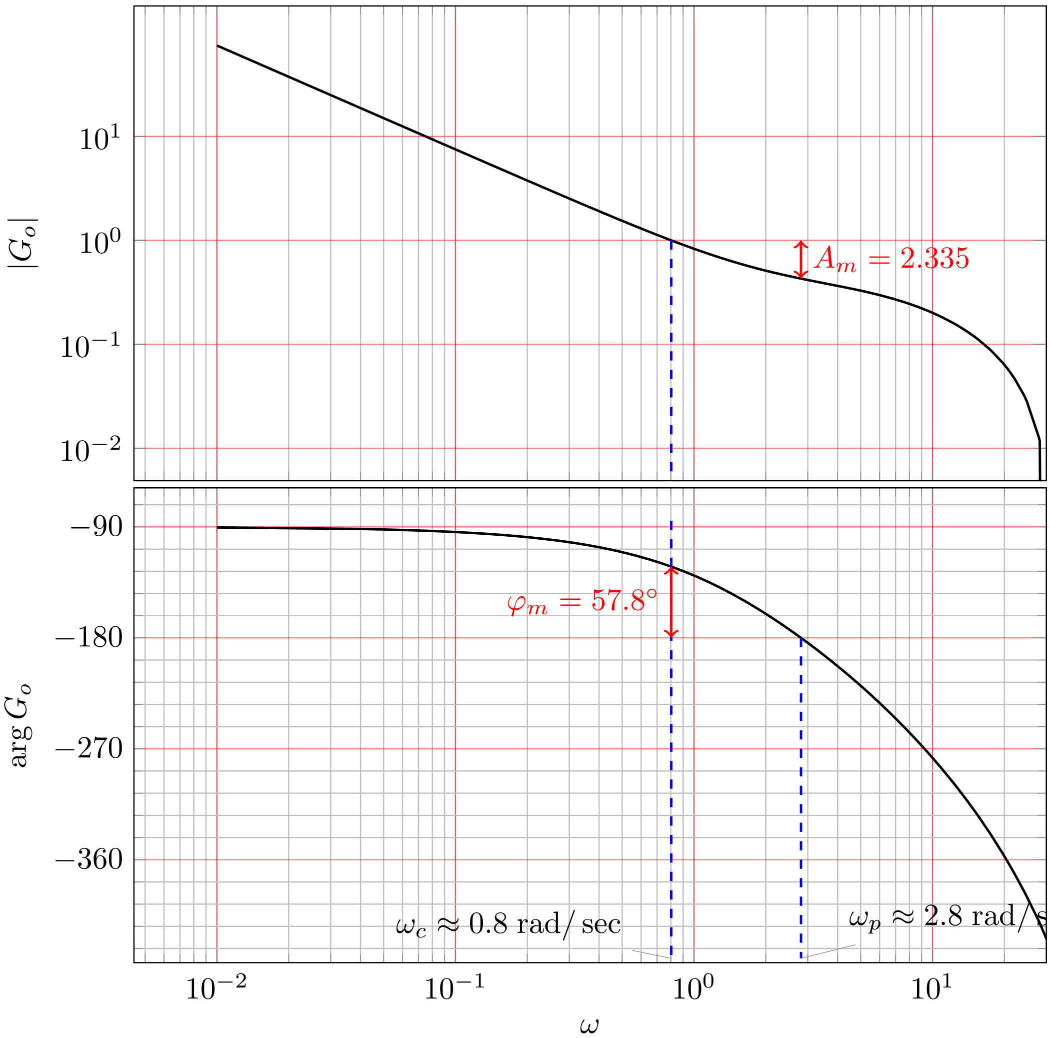
   We can also find this by considering the transfer function of the zero-order-hold block, which is given by

$$\frac{1}{s}\big(1-\mathrm{e}^{-sh}\big).$$

   The phase contribution of this block at ω_c is given by

$$\begin{aligned}
\arg\big(1-\mathrm{e}^{-i\omega_c h}\big)\frac{1}{i\omega_c} &= \arg\big(1-\mathrm{e}^{-i0.16}\big) - \arg i0.8 \\
&= \arg\big(1-\cos(-0.16) - i\sin(-0.16)\big) - \pi/2 \\
&= \arctan\frac{\sin(0.16)}{1-\cos(0.16)} - \pi/2 = -0.08 \text{ rad} \approx -4.58^\circ.
\end{aligned}$$

Problem 2



$|G_o|$
10^{1}
10^{0}
10^{-1}
10^{-2}
$A_m = 2.335$
$\arg G_o$
-90
-180
-270
-360
$\varphi_m = 57.8°$
$\omega_c \approx 0.8$ rad/ sec
$\omega_p \approx 2.8$ rad/s
10^{-2}
10^{-1}
10^{0}
10^{1}
ω