

Control computarizado - Asignación de polos (RST posicional)

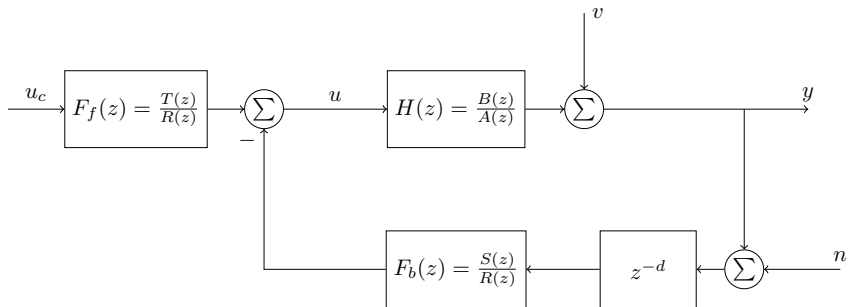
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Objetivo

- ▶ Entender diseño de un controlador por asignación de polos

Controlador de dos grados de libertad



Procedimiento

Dado modelo del proceso $H(z) = \frac{B(z)}{A(z)}$, y especificaciones de polos deseados del sistema en lazo cerrado $A_{cl}(z) = (z - \alpha_1)(z - \alpha_2) \cdots (z - \alpha_{n_c})$

1. Determina la ecuación diofántica

$$A(z)R(z)z^d + B(z)S(z) = A_{cl}(z)$$

2. Determina polinomios $R(z)$ y $S(z)$ donde $n_R \geq n_S$ que satisficán

$$A(z)R(z)z^d + B(z)S(z) = A_{cl}(z)$$

3. Factoriza el polinomio característico del lazo cerrado $A_{cl}(z) = A_c(z)A_o(z)$, donde $n_{A_o} \leq n_R$. Eliga

$$T(z) = t_0 A_o(z),$$

$$\text{donde } t_0 = \frac{A_c(1)}{B(1)}.$$

Nos da el ley de control

$$R(q)u(k) = T(q)u_c(k) - S(q)y(k).$$

Determining the order of the controller

With Diophantine equation

$$A(z)R(z)z^d + B(z)S(z) = A_{cl}(z) \quad (*)$$

and feedback controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \dots + s_n}{z^n + r_1 z^{n-1} + \dots + r_n}$$

How should we choose the order of the controller? Note:

- ▶ the controller has $n + n + 1 = 2 \deg R + 1$ unknown parameters
- ▶ the LHS of $(*)$ has degree $\deg(A(z)R(z)z^d + B(z)S(z)) = \deg A + \deg R + d$
- ▶ The diophantine gives as many (nontrivial) equations as the degree of the polynomials on each side when we set the coefficients equal.

\Rightarrow Choose $\deg R$ so that $2 \deg R + 1 = \deg A + \deg R + d$

Determining the order of the controller - Exercise 1

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b}{z + a}$$

and $d = 0$ (no delay), what is the appropriate degree of the controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0 z^n + s_1 z^{n-1} + \dots + s_n}{z^n + r_1 z^{n-1} + \dots + r_n}$$

so that all parameters can be determined from the diophantine equation

$$A(z)R(z) + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 0$
2. $n = 1$
3. $n = 2$
4. $n = 3$

Determining the order of the controller - Exercise 1

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so that all parameters can be determined from the diophantine equation

$$A(z)R(z) + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 0$
- 2.
- 3.
- 4.

Determining the order of the controller - Exercise 2

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0z + b_1}{z^2 + a_1z + a_2}$$

and $d = 2$, what is the appropriate degree of the controller

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0z^n + s_1z^{n-1} + \dots + s_n}{z^n + r_1z^{n-1} + \dots + r_n}$$

so that all parameters can be determined from the diophantine equation

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1. $n = 1$
2. $n = 2$
3. $n = 3$
4. $n = 4$

Determining the order of the controller - Exercise 2

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0z + b_1}{z^2 + a_1z + a_2}$$

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so that all parameters can be determined from the diophantine equation

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1. 2.
3. $n = 3$ 4.

Determining the order of the controller - Exercise 3

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0z + b_1}{z^2 + a_1z + a_2}$$

and $d = 2$ the appropriate degree of the controller is 3

$$F_b(z) = \frac{S(z)}{R(z)} = \frac{s_0z^3 + s_1z^2 + s_2z + s_3}{z^3 + r_1z^2 + r_2z + r_3}.$$

What are the possible choices of the degree of the observer polynomial $A_o(z)$ in

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

1. less than 2
2. less than 3
3. higher than 2
4. less than or equal to 3

Determining the order of the controller - Exercise 3

With the plant model

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0z + b_1}{z^2 + a_1z + a_2}$$

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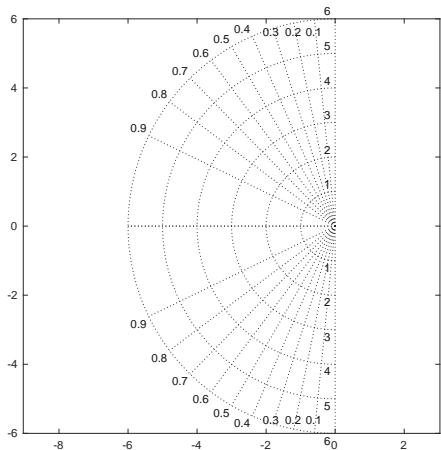
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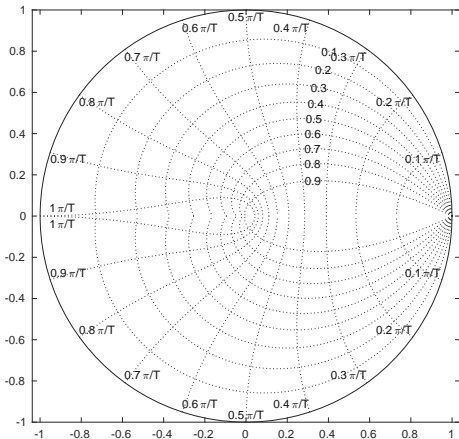
$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)?$$

- 1.
- 2.
- 3.
4. less than or equal to 3

Where to place the closed-loop poles?



s-plane



z-plane