Computerized Control partial exam 1 (15%)

Kjartan Halvorsen

Time February 13 17:35-18.55

Place 5105

Permitted aids The single colored page with your own notes, table of Laplace transforms, calculator

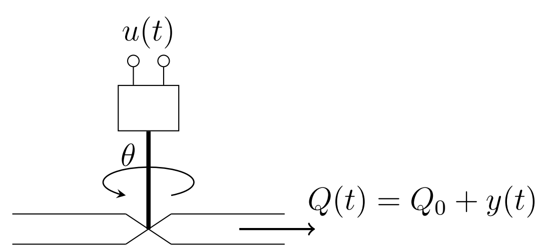
All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

Good luck!

Matricula and name:

Flow control in a valve

A butterfly valve is used to control the flow of petroleum in an oil refinery. The valve is a so-called quarter-turn valve, and goes from fully closed $\theta = 0°$ to fully open in a quarter rotation of the valve shaft $\theta = 90°$. The valve shaft is actuated by a DC motor via a transmission (gear box).



The signal $u(t)$ [V] is the voltage over the DC motor and the signal $Q(t)$ [m^3/s] is the flow through the valve.

Problem 1

In order to use methods for discrete-time control you need to determine a discrete model of the plant. In continuous time the dynamical system from the input signal $u(t)$ to the angle of the valve shaft $\theta(t)$ is well described by the transfer function

$$\Theta(s) = \frac{k_V}{s(sT_V + 1)} U(s).$$

Sample the system using zero-order-hold. You can make use of the partial fraction expansion

$$\frac{k_V}{s^2(sT_V + 1)} = k_V \left(\frac{1}{s^2} - \frac{T_v}{s} + \frac{T_V^2}{sT_V + 1} \right).$$

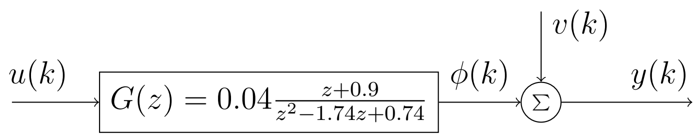
Calculations:

Problem 2

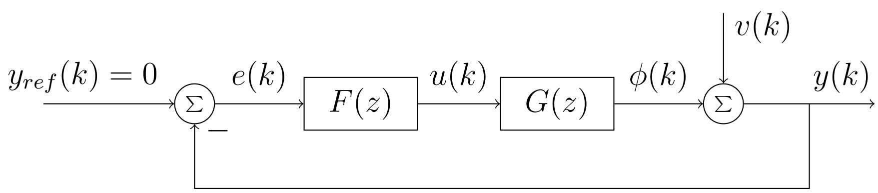
The flow through the valve depends on the product of the valve angle θ and the square root of the pressure difference $\Delta P = P_u - P_d$ across the valve

$$Q(t) = k_\theta \theta(t)\sqrt{\Delta P(t)} = f\big(\theta(t), \Delta P(t)\big),$$

where k_θ is a positive constant. Defining deviations around an operating point $\Delta P(t) = \Delta P_0 + v(t)$, $\theta(t) = \theta_0 + \phi(t)$, the nonlinear model of the flow can be linearized. In suitable units, the linear discrete model of a specific, electrically actuated butterfly valve can be represented by the block-diagram below



You are aware of the fact that the pressure in the pipe can vary, so you consider using a feedback controller in order to try to keep the flow through the valve constant even in the presence of changes in the pressure. A flow meter is installed that measures the flow $Q(t)$. By subtracting the operating point value Q_0, the sampled signal $y(k) = Q(k) - Q_0$ is available for feedback control.



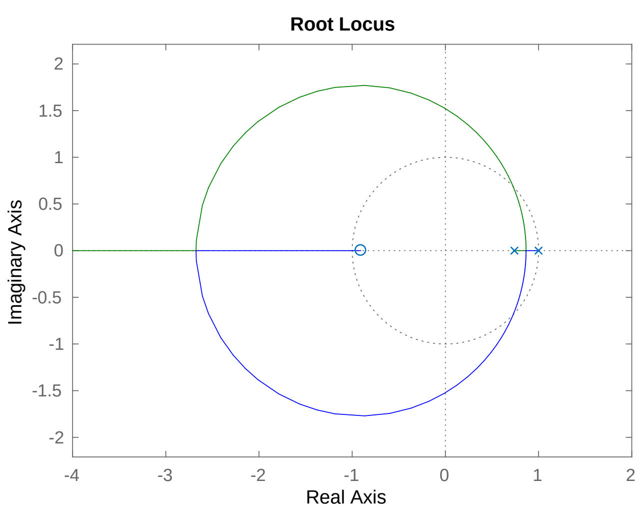
(a)

Determine the closed-loop pulse transfer function from the pressure disturbance $v(k)$ to the output signal (flow deviation) $y(k)$.

Calculations:

(b)

As a good engineer, you try the simplest solution first and test if that is good enough. So, you try a proportional controller $F(z) = K$. The root-locus below shows how the closed-loop poles depend on the controller gain K. Describe in words (3-5 sentences) some interesting properties (stability, damping, speed) of the system that the root locus below tells us as the gain K varies from 0 to ∞.

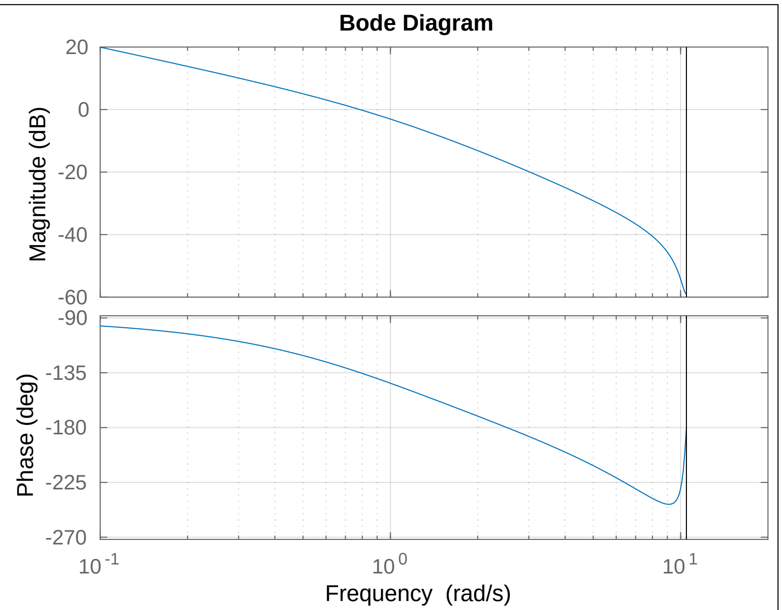


Description:

(c)

With the gain $K = 1$, the next Bode diagram is obtained for the loop gain $G_o(z) = KG(z)$. Determine and indicate in the diagram (draw) the

1. cross-over frequency
2. phase margin
3. phase cross-over frequency
4. amplitude margin



Bonus question (worth 5p):

What is the sampling period of the discrete-time system? Motivate!

(d)

You find it difficult to achieve the desired performance for the closed-loop system with a simple proportional controller. So, instead you try a lead-compensator

$$F(z) = K\frac{z-b}{z-a}, \qquad b > a.$$

Why would a lead-compensator give better performance than the proportional controller in this case?

Answer:

(e)

The lead-compensator in (d) can be written

$$U(z) = F(z)E(z) \quad \text{or} \quad u(k) = F(q)e(k).$$

Write the lead-compensator as a difference equation, so that it can be implemented on a microcontroller.

Calculations:

Solutions

Problem 1

We first need to determine the step response of the continous-time system.

$$\begin{aligned}\theta(t) &= \mathcal{L}^{-1}\left\{\frac{k_V}{s(sT_V+1)}\cdot\frac{1}{s}\right\} = k_V\left(\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\frac{T_V}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{T_V^2}{sT_V+1}\right\}\right)\\ &= k_V\left(t - T_V u_H(t) + T_V \mathrm{e}^{-\frac{t}{T_V}}\right).\end{aligned}$$

Then sample (set $t = kh$) and apply the z-transform

$$\begin{aligned}\Theta(z) &= k_V\left(\mathcal{Z}\left\{kh\right\} - T_V\mathcal{Z}\left\{u_H(kh)\right\} + T_V\mathcal{Z}\left\{\mathrm{e}^{-\frac{kh}{T_V}}\right\}\right)\\ &= k_V\left(\frac{zh}{(z-1)^2} - \frac{T_V z}{z-1} + \frac{T_V z}{z-\mathrm{e}^{-\frac{h}{T_V}}}\right).\end{aligned}$$

Divide by the z-transform of the input signal (step) to obtain

$$\begin{aligned}G(z) = \frac{\Theta(z)}{U(z)} &= k_V\frac{z-1}{z}\left(\frac{zh}{(z-1)^2} - \frac{T_V z}{z-1} + \frac{T_V z}{z-\mathrm{e}^{-\frac{h}{T_V}}}\right)\\ &= k_V\left(\frac{h}{z-1} - T_V + \frac{T_V(z-1)}{z-\mathrm{e}^{-\frac{h}{T_V}}}\right)\\ &= k_V\frac{h(z-\mathrm{e}^{-\frac{h}{T_V}}) - T_V(z-1)(z-\mathrm{e}^{-\frac{h}{T_V}}) + T_V(z-1)^2}{(z-1)(z-\mathrm{e}^{-\frac{h}{T_V}})}.\end{aligned}$$

Problem 1

(a)

The closed-loop system is

$$Y(z) = V(z) + G(z)F(z)E(z) = V(z) - G(z)F(z)Y(z)$$

so

$$Y(z) = \frac{1}{1+G(z)F(z)}V(z)$$

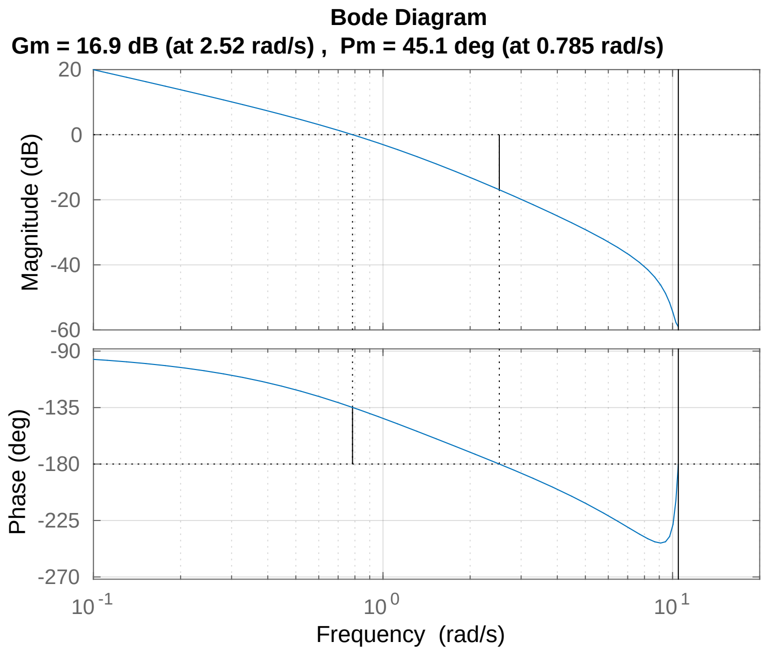
and the desired closed-loop pulse transfer function is

$$G_c(z) = \frac{1}{1+G(z)F(z)} = \frac{1}{1+F(z)0.04\frac{z+0.9}{z^2-1.74z+0.74}} = \frac{z^2-1.74z+0.74}{z^2-1.74z+0.74+F(z)0.04(z+0.9)}.$$

(b)

We see starting points in $z = 1$ and $z = 0.74$ (the roots of $z^2 - 1.74z + 0.74$). For small K the system will be stable and dominated by the slow pole near 1. As K increases, the two poles will meet on the real axis. Such a system is critically damped. Further increasing K causes the branches to go out into the complex plane and the system becomes under-damped. The branches break out of the unit circle at some point, and so the system becomes unstable.

(c)



(d)

The lead-compensator moves the branches of the root locus further to the left. This means that it is possible to achieve a closed-loop system with poles that are further from the point 1 and at the same time further inside the unit circle than what is possible with the proportional controller. This gives a faster and more damped closed-loop system.

(e)

We have

$$u(k) = F(q)e(k) = K\frac{\mathrm{q} - b}{\mathrm{q} - a}e(k)$$
$$(\mathrm{q} - a)u(k) = K(\mathrm{q} - b)e(k)$$
$$\mathrm{q}\,u(k) - au(k) = K\,\mathrm{q}\,e(k) - Kbe(k)$$
$$u(k+1) = au(k) + Ke(k+1) - Kbe(k).$$