Computerized control - Final Exam - modified from Fall 2015

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2015-11-24

The dynamic model of a ship with input u being the rudder angle and the output y being the heading (see figure 1) can be described as a continuous-time second order system with a pole in the origin

$$G(s) = \frac{K}{s(s+a)}.$$

For fully loaded, large tankers this dynamics is often unstable, meaning that $a < 0$ [1] .

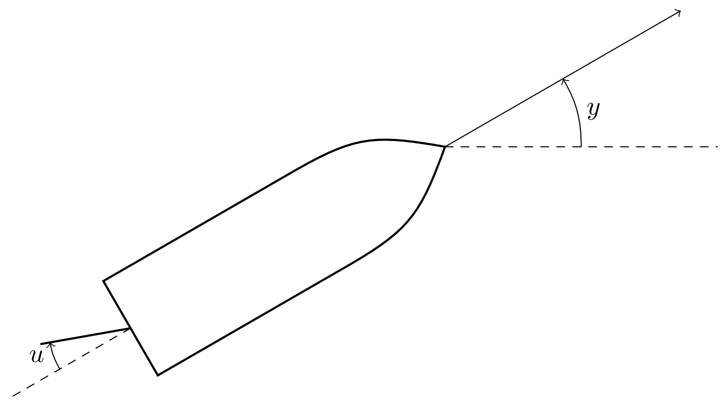


Figure 1: Heading of a ship controlled by rudder input.

Consider for this exam the normalized continuous-time model of the tanker

$$G(s) = \frac{1}{s(s-1)}.$$

with the discrete-time model obtained by zero-order hold

$$H(z) = \frac{(-1 + \mathrm{e}^h - h)z + 1 - (1-h)\mathrm{e}^h}{(z-1)(z-\mathrm{e}^h)}.$$

Specifically, use sampling time $h = 0.2$, which gives the (approximate) model

$$H(z) = \frac{0.02z + 0.02}{(z-1)(z-1.2)} = \frac{0.02z + 0.02}{z^2 - 2.2z + 1.2}. \tag{1}$$

All answers should be well motivated!

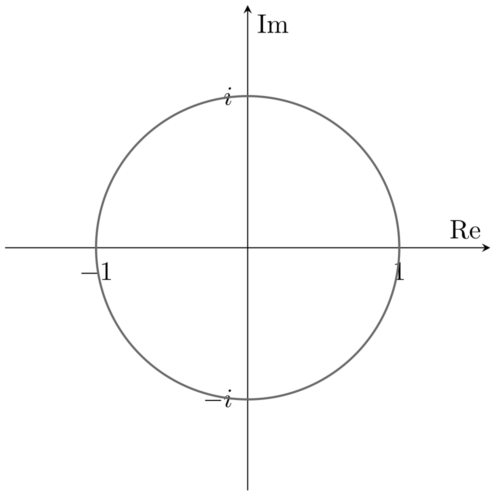


Figure 2: Problem 1: Plot the poles and zeros of the discrete-time system.

Problem 1

1. In figure 2 draw the poles (crosses) and zero (circle) for the discrete-time pulse-transfer function in (1).
2. Assume that the tanker with model (1) is stabilized using error-feedback and a PD-controller. The Bode-diagram of the resulting **closed-loop** system is given in figure 3. What is the bandwidth of the closed-loop system? At what frequency is the resonance peak?

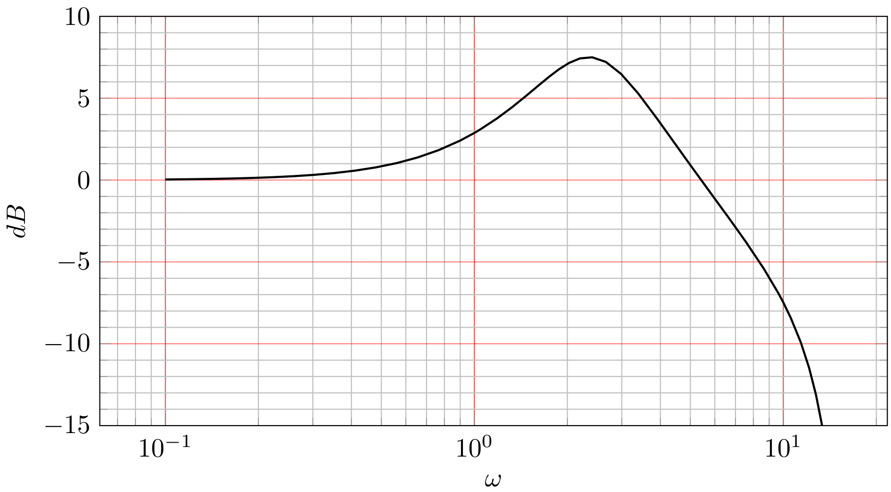


Figure 3: Problem 1: Bode diagram of closed-loop system with PD-control

Problem 2

Figure 4 shows a system controlled with an RST controller. Note that the system includes an anti-aliasing filter modelled as a pure time-delay of two sampling periods. What is the closed-loop pulse-transfer function from the disturbance d to the output y? You do not need to multiply the polynomials. It is sufficient to state your answer in terms of $A(z)$, $B(z)$, $R(z)$, $S(z)$ and z^2.

[1] Fossen, Thor I. Handbook of marine craft hydrodynamics and motion control. John Wiley & Sons, 2011.

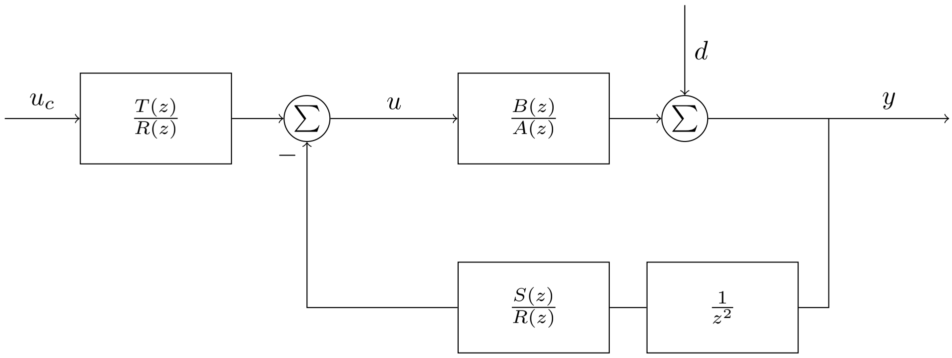


Figure 4: Problem 2: Two-degree-of-freedom controller with anti-aliasing filter.

Problem 3

When designing an RST-controller for the system in Problem 2, $R(z)$ and $S(z)$ are determined from a diophantine equation, based on the required placement of the closed-loop poles. Assume the following desired closed-loop denominator:

$$A_{cl} = \underbrace{(z-p_1)(z-p_2)z^2}_{A_c}\underbrace{(z-p_3)^3}_{A_o} \tag{2}$$

1. Write the diophantine equation in terms of $A_c(z)$, $A_o(z)$, $A(z)$, $B(z)$, $R(z)$, $S(z)$ and z^2.
2. Let the controller polynomials $R(z)$ and $S(z)$ have the same order. Determine this order, so that all the controller parameters can be determined from the diophantine equation. Note that you only need to determine the **order** of the controller. You do not need to write the equation for the controller parameters.

Problem 4

The controllable canonical state-space representation of (1) is given by

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 2.2 & -1.2 \\ 1 & 0 \end{bmatrix}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 0.02 & 0.02 \end{bmatrix} x(k), \end{aligned} \tag{3}$$

with

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}.$$

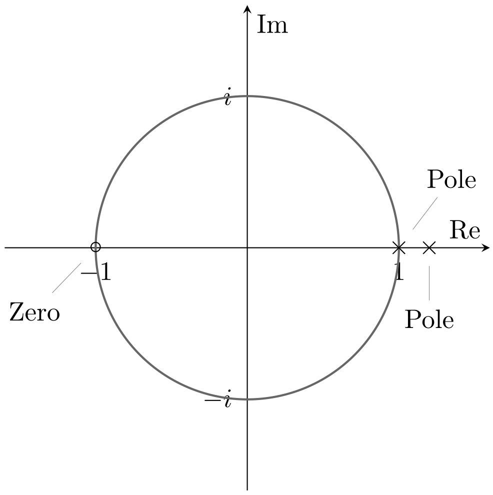
Introduce the state-feedback law $u(k) = -l_1 x_1(k) - l_2 x_2(k)$ and determine l_1 and l_2 so that the closed-loop system has the characteristic polynomial

$$(z - 0.9 + 0.1i)(z - 0.9 - 0.1i) = z^2 - 1.8z + 0.82.$$

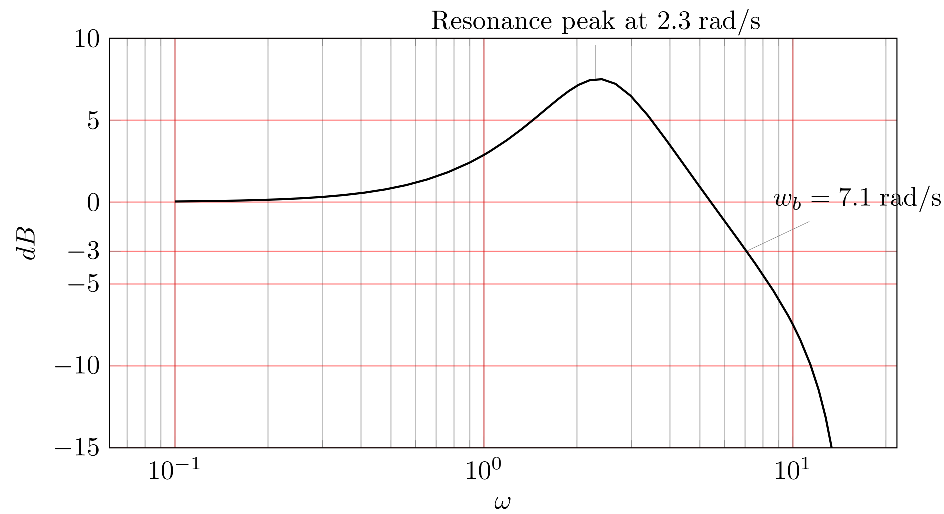
Solution

Problem 1

1. Poles and zeros



2. Bandwidth and resonance



Problem 2

To compute the pulse-transfer function from d to y, assume $u_c = 0$. We get

$$\begin{aligned} Y &= D + \frac{B}{A}U = D - \frac{B}{A}\frac{S}{R}\frac{1}{z^2}Y \\ Y + \frac{BS}{ARz^2}Y = D& \\ Y &= \frac{1}{1 + \frac{BS}{ARz^2}}D \\ &= \frac{A(z)R(z)z^2}{A(z)R(z)z^2 + B(z)S(z)}D \end{aligned}$$

Problem 3

1. The diophantine equation becomes

$$A(z)R(z)z^2 + B(z)S(z) = A_c(z)A_o(z)$$

2. The right hand side of the diophantine equation has order 7, hence the left hand side must have the same order. Since $A(z)z^2$ has order 4, then $R(z)$ must have order 3. We choose $R(z)$ and $S(z)$

to have the same order (which is a smart choice because then the pulse-transfer function of the controller does not introduce a time-delay), we get

$$\frac{S(z)}{R(z)} = \frac{s_0 z^3 + s_1 z^2 + s_2 z + s_3}{z^3 + r_1 z^2 + r_2 z + r_3}$$

which has 7 parameters. The diophantine equation gives 7 equations to determine uniquely the 7 control parameters. Note that the terms $A(z)R(z)z^2$ and $B(z)S(z)$ do **not** have to have the same order.

Problem 4

With the control law we get the closed-loop system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 2.2 & -1.2 \\ 1 & 0 \end{bmatrix} x(k) - \begin{bmatrix} l_1 & l_2 \\ 0 & 0 \end{bmatrix} x(k) \\ &= \begin{bmatrix} 2.2 - l_1 & -1.2 - l_2 \\ 1 & 0 \end{bmatrix} x(k) \end{aligned}$$

which is also on controllable canonical form. Thus we can immediately write the characteristic polynomial of the closed-loop system as

$$z^2 + (-2.2 + l_1)z + (1.2 + l_2).$$

It is also straight-forward to write the characteristic polynomial using the formula

$$\begin{aligned} \det\left(zI - \begin{bmatrix} 2.2 - l_1 & -1.2 - l_2 \\ 1 & 0 \end{bmatrix} \right) &= \det \begin{bmatrix} z - 2.2 + l_1 & 1.2 + l_2 \\ -1 & z \end{bmatrix} \\ &= (z - 2.2 + l_1)z + (1.2 + l_2) = z^2 + (-2.2 + l_1)z + (1.2 + l_2). \end{aligned}$$

Comparing coefficients with the desired characteristic polynomial

$$z^2 - 1.8 + 0.82$$

gives the solution

$$\begin{aligned} l_1 &= -1.8 + 2.2 = 0.4 \\ l_2 &= 0.82 - 1.2 = -0.38 \end{aligned}$$

With

$$m_0 = \frac{A_c(1)}{B(1)} = \frac{1 - 1.8 + 0.82}{0.02 + 0.02} = 0.5$$

and

$$u = -Lx + m_0 u_c,$$

the Bode-diagram of the closed-loop system becomes