Computerized control - partial exam 2 (dummy)

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Problem 1

Figure 1 shows the step response of the system

$$G(s) = \frac{1}{(s+1)^2(s+3)}$$

for an experiment to determine the ultimate gain and ultimate period.

(a) What is the ultimate period and the phase-crossover frequency?

(b) Use the transfer function and the phase-crossover frequency to determine the ultimate gain.

(c) Determine a suitable continuous-time PID-controller using table 8.3 from Å&W.

(d) Obtain a sampled controller by using the backward difference for the D-part and Tustin's approximation for the I-part of the PID-controller.

Solutions

(a)

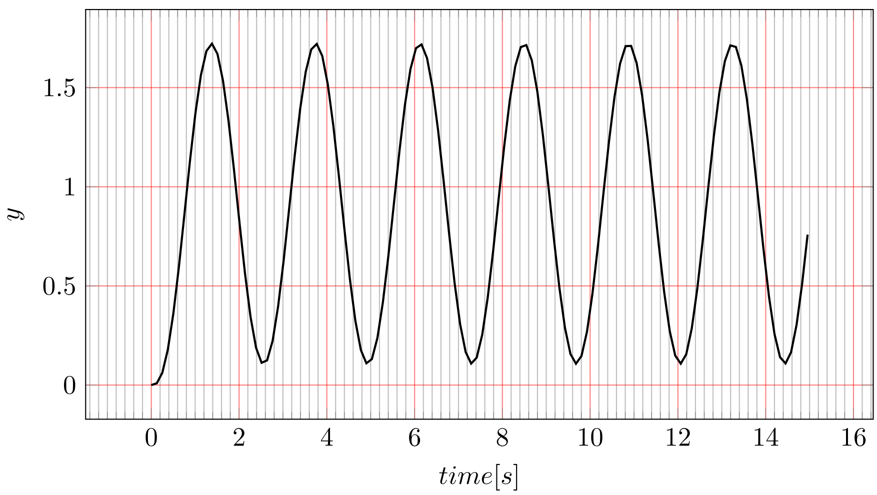
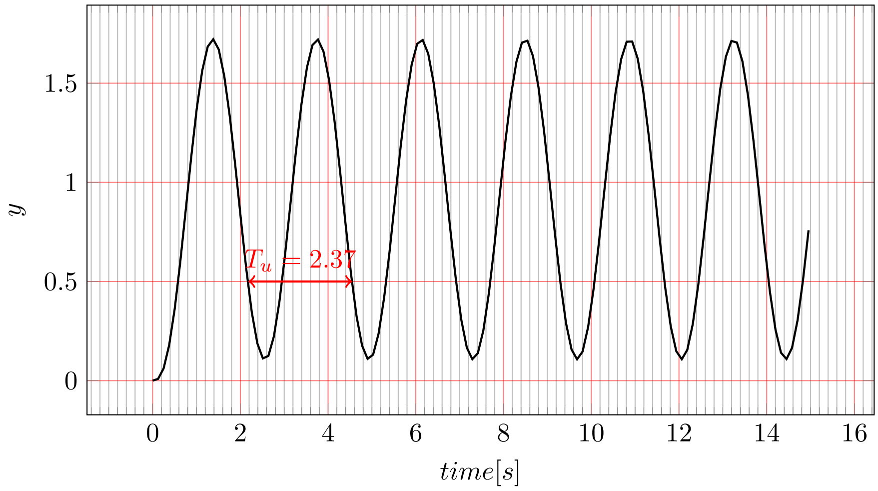


Figure 1: Response of closed-loop system using proportional control with gain equal to the ultimate gain.



The phase-crossover frequency is $\omega_p = \frac{2\pi}{T_u} = 2.65$

(b) We know that the phase-crossover frequency is 2.65 and that at this frequency the Nyquist curve of the open-loop transfer function crosses the negative real axis. Multiplying the transfer function $G(s)$ by the ultimate gain K_u causes the Nyquist curve to cross the negative real axis at -1 (that is why there are self-sustained oscillations). We get

$$|K_u G(i\omega_p)| = 1$$

or

$$\begin{aligned} K_u &= \frac{1}{|G(i\omega_p)|} = |i\omega_p + 1|^2 |i\omega_p + 3| \\ &= (\omega_p^2 + 1)\sqrt{\omega_p^2 + 9} \approx 32.11 \end{aligned}$$

(c) The controller parameters become

$$K_p = 0.6K_u = 19.27, \quad T_i = 0.5T_u = 1.185, \quad T_d = 0.125T_u = 0.296$$

(d) The controller is

$$\begin{aligned} F_d(z) &= K_p + \frac{K_p}{T_i s'}|_{s'=\frac{2}{h}\frac{z-1}{z+1}} + K_p T_d s'|_{s'=\frac{z-1}{zh}} \\ &= K_p + \frac{K_p}{T_i \frac{2}{h}\frac{z-1}{z+1}} + \frac{K_p T_d (z-1)}{zh} \\ &= K_p + \frac{K_p h(z+1)}{T_i(z-1)} + \frac{K_p T_d(z-1)}{zh} \end{aligned}$$