Computerized control partial exam 1 (15%)

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Time September 13 17:30

Place 4101

Permitted aids The single colored page with your own notes, table of Laplace transforms, calculator

All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

Good luck!

Matricula and name

The system

The dynamic model of a ship with input u being the rudder angle and the output y being the heading (see figure 1) can be described as a continuous-time second order system with a pole in the origin

$$G(s) = \frac{K}{s(s+a)}.$$

For fully loaded, large tankers this dynamics is often unstable, meaning that $a < 0$.

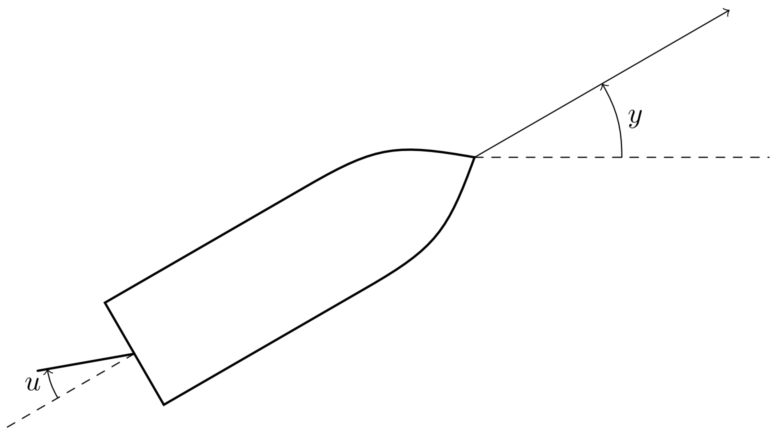


Figure 1: Heading of a ship controlled by rudder input.

Consider for this exam the normalized continuous-time model of the tanker

$$G(s) = \frac{1}{s(s-1)}.$$

Problem 1 (50p)

The system is sampled with sampling interval h using step-invariant (zero-order hold) sampling. **Circle the correct pulse-transfer function below, and show your calculations**

1. $H(z) = \frac{(1-\mathrm{e}^h-h)z-\big((1-h)\mathrm{e}^h-1\big)}{(z-1)(z-\mathrm{e}^h)}$

2. $H(z) = \frac{(-1+\mathrm{e}^h-h)z-\big((1-h)\mathrm{e}^h-1\big)}{(z-1)(z-\mathrm{e}^{2h})}$

3. $H(z) = \frac{(-1+\mathrm{e}^h-h)z-\big((1-h)\mathrm{e}^h-1\big)}{(z-1)(z-\mathrm{e}^h)}$

Derivation:

Problem 2 (20p)

Assume that the sampling period is $h = 0.2$. In figure 2 draw the poles (crosses) and zero (circle) for both the continuous-time transfer function $G(s)$ and the discretized pulse-transfer function $H(z)$ you determined in Problem 1.

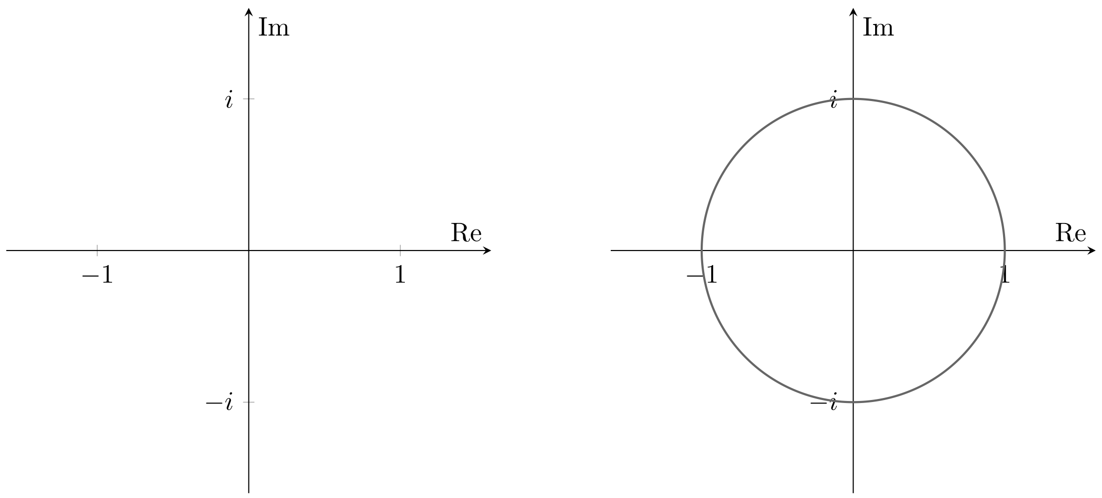
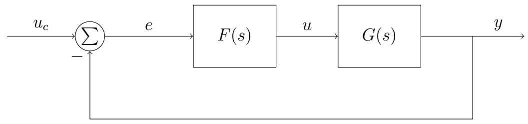


Figure 2: Problem 2: Plot the poles of the continuous-time system (on the left) and the poles and zero of the discrete-time system (on the right). Indicate (with arrows and/or colors) corresponding pairs of continuous-time and discrete-time poles.

Calculations:

Problem 3 (30p)

The tanker is stabilized using error-feedback and a lead-compensator as in the figure below.



The design is done in continuous-time giving the controller

$$F(s) = 20\frac{s+1}{s+6}.$$

Suggest a discretization method (give short motivation) and discretize the controller. Write the controller as a first order difference equation

Solutions:

If necessary, you can continue your solutions on this page. Mark clearly which problem the solution corresponds to.

Solutions

Problem 1

First calculate the step-response of the continous-time system

$$Y(s) = G(s)\frac{1}{s} = \frac{1}{s^2(s-1)} = \frac{1}{s-1} - \frac{1}{s} - \frac{1}{s^2}.$$

The inverse Laplace-transform gives

$$y(t) = \mathrm{e}^t - 1 - t$$

Sampling this function gives

$$y(kh) = \mathrm{e}^{kh} - 1 - kh$$

which has the Z-transform

$$Y(z) = \frac{z}{z-\mathrm{e}^h} - \frac{z}{z-1} - \frac{hz}{(z-1)^2}$$

Dividing the z-transform of the system response to that of the input (the step) gives

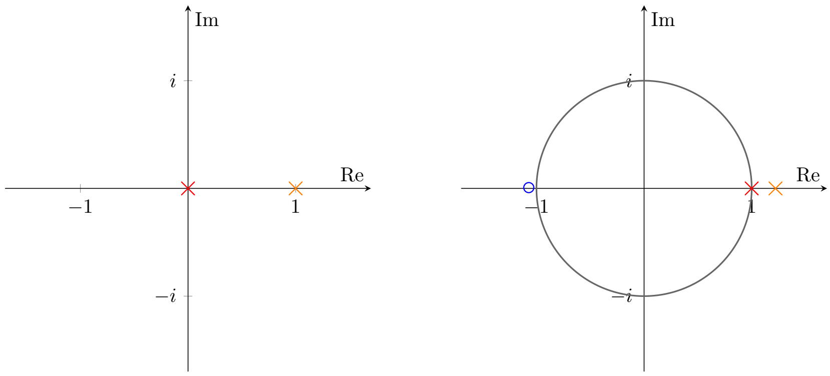
$$\begin{aligned}
H(z) = \frac{Y(z)}{U(z)} &= \frac{z-1}{z}Y(z) = \frac{z-1}{z-\mathrm{e}^h} - 1 - \frac{h}{z-1} \\
&= \frac{(z-1)^2 - (z-1)(z-\mathrm{e}^h) - h(z-\mathrm{e}^h)}{(z-1)(z-\mathrm{e}^h)} \\
&= \frac{(z-1)(z-1-z+\mathrm{e}^h) - hz + h\mathrm{e}^h}{(z-1)(z-\mathrm{e}^h)} \\
&= \frac{(\mathrm{e}^h - 1 - h)z - (\mathrm{e}^h - 1 - h\mathrm{e}^h)}{(z-1)(z-\mathrm{e}^h)} \\
&= \frac{(\mathrm{e}^h - 1 - h)z - \big((1-h)\mathrm{e}^h - 1\big)}{(z-1)(z-\mathrm{e}^h)}
\end{aligned}$$

The correct pulse transfer function is the third.

Problem 2

The discrete-time poles are in $z = 1$ and $z = \mathrm{e}^{0.2} \approx 1.22$. The zero is in

$$z = -\frac{(1-h)\mathrm{e}^{0.2} - 1}{\mathrm{e}^{0.2} - 1 - h} \approx -1.07.$$



Problem 3

Tustin's approximation should give reasonable performance, since it is a better approximation than backward or forward difference. It is also easy to apply. The approximation gives the pulse-transfer function

$$F_d(z) = F(s)\big|_{s=\frac{2(z-1)}{h(z+1)}} = 20\frac{\frac{2(z-1)}{h(z+1)}+1}{\frac{2(z-1)}{h(z+1)}+6} = 20\frac{2(z-1)+h(z+1)}{2(z-1)+6h(z+1)} = 20\frac{(2+h)z-(2-h)}{(2+6h)z-(2-6h)}$$

We get

$$U(z) = F_d(z)E(z) = 20\frac{(2+h)z-(2-h)}{(2+6h)z-(2-6h)}E(z)$$

or

$$\big((2+6h)z-(2-6h)\big)U(z) = 20\big((2+h)z-(2-h)\big)E(z).$$

Written as a difference equation

$$(2+6h)u(kh+h)-(2-6h)u(kh) = 20(2+h)e(kh+h)-20(2-h)e(kh)$$

or

$$u(kh+h) = \frac{2-6h}{2+6h}u(kh)+\frac{20(2+h)}{2+6h}e(kh+h)-\frac{20(2-h)}{2+6h}e(kh).$$