Computerized control - preparation for partial exam 2

Kjartan Halvorsen

October 23, 2015

1 Controller design

A continous-time system is given by the transfer function

$$G(s) = \frac{1}{s}\left(\frac{-s+2}{s+2}\right).$$

The system is to be controlled using error feedback

$$U(s) = F(s)\big(U_c(s) - Y(s)\big).$$

(a) Determine the controller $F(s)$ so that the closed-loop system has the transfer function

$$G_c(s) = \left(\frac{-s+2}{s+2}\right)\frac{1}{1+s/3}.$$

Hint: Write $F(s)$ as a function of G and G_c.

(b) Figure 1 shows the Bode diagram of two different systems. Which of the graphs (solid or dashed lines) correspond to the closed-loop system? OBS: only the phase curves differ. What is the band-width of the closed-loop system?

(c) Figure 2 shows the Bode diagram of the open-loop transfer function using the controller from exercise (a). What cross-over frequency and phase-marginal was achieved with the controller?

(d) Obtain a sampled controller using Tustin's approximation.

2 PID tuning

Figure 3 shows the step-response from an experiment to find the ultimate gain and the ultimate period for the system in exercise 1.

(a) What is the ultimate period?

(b) The gain is $K_u = 2$. Write down the PID-controller obtained from the Ziegler-Nichols tuning method using the table 8.3 in Å&W.

(c) Figure 4 shows the step-responses of the closed-system obtained with the controller in exercise 1 (dashed) and with the controller using PID tuning (solid). How do they differ, and what does this say about the difference in phase margin and cross-over frequency? Why does the response with PID controller have a jump at $t = 0$? Why do both responses start in the wrong direction (negative)?

(d) Obtain a sampled controller using backward difference (for all parts of the PID).

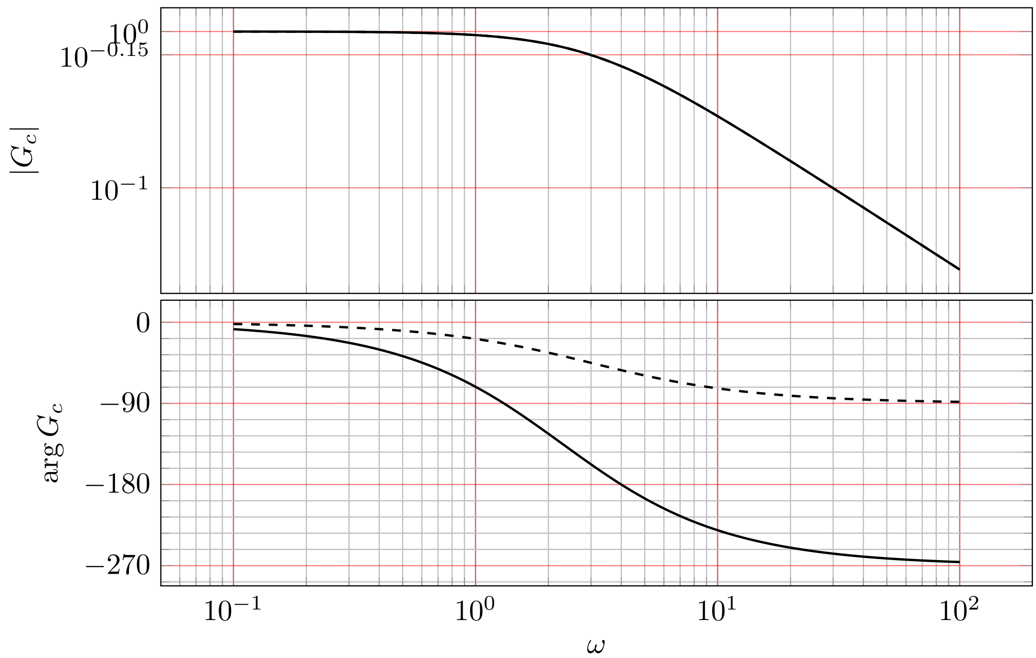


Figure 1: Bode diagram of the closed-loop system. Which is the correct phase-curve?

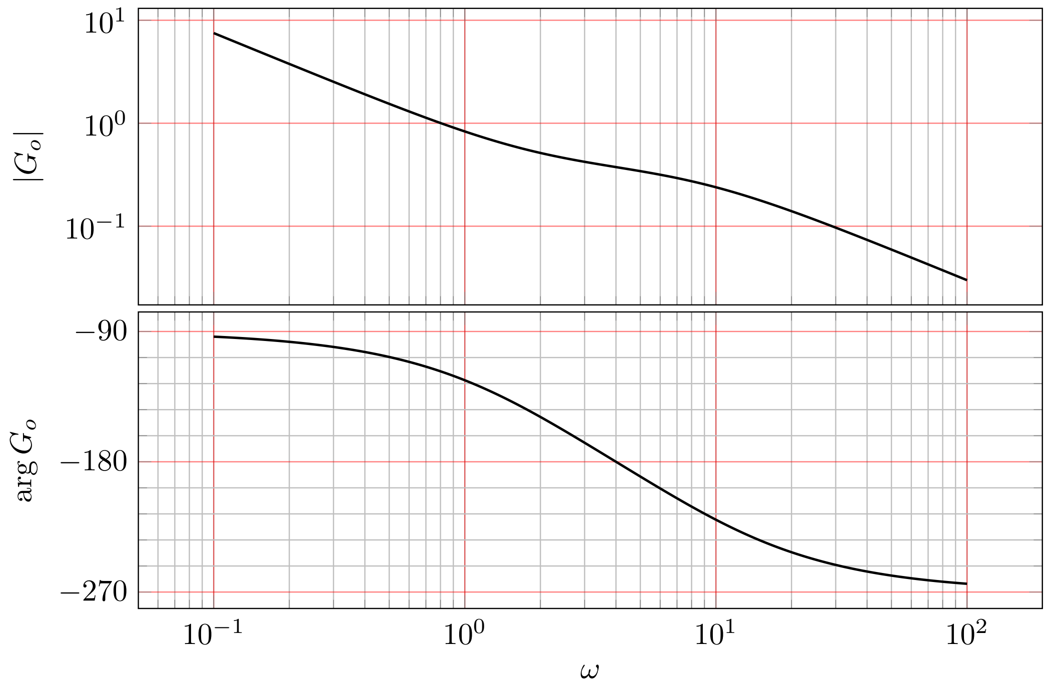


Figure 2: Bode diagram of the open-loop system. What is the cross-over frequency and phase margin?

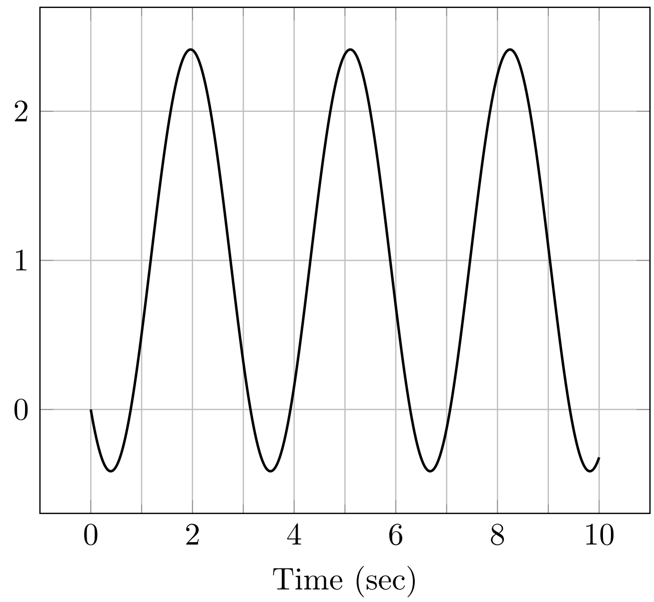


Figure 3: Step-response with ultimate gain. What is the ultimate period?

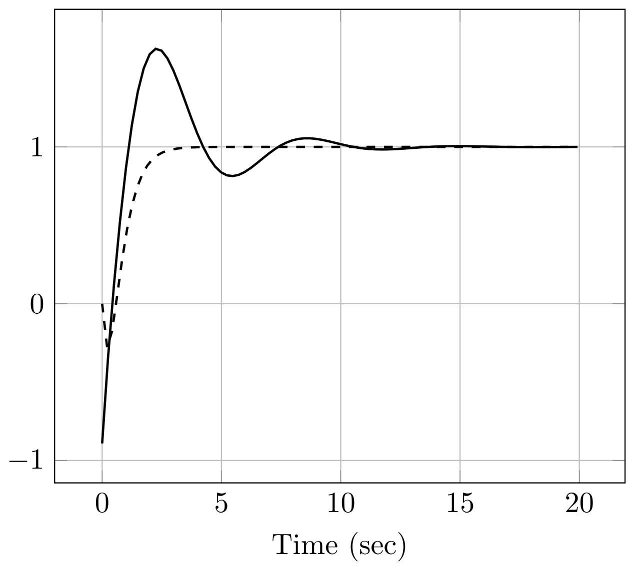


Figure 4: Step-responses using the two controllers from exercise 1 and 2. What are the interesting differences?

3 Solutions

3.1 Controller design

(a) The close-loop system is given by

$$G_c = \frac{GF}{1+GF}.$$

Solving this for F gives

$$\begin{aligned} F = \frac{G_c}{G(1-G_c)} &= \frac{\left(\frac{-s+2}{s+2}\right)\frac{1}{1+s/3}}{\frac{-s+2}{s(s+2)}\left(1-\left(\frac{-s+2}{s+2}\right)\frac{1}{1+s/3}\right)} \\ &= \frac{s}{(1+s/3)\left(\frac{(s+2)(1-s/3)-(-s+2)}{(s+2)(1+s/3)}\right)} \\ &= \frac{s(s+2)}{2s-s^2/3} = \frac{s+2}{s/3+8/3}. \end{aligned}$$

(b) The argument of the closed loop system is given by

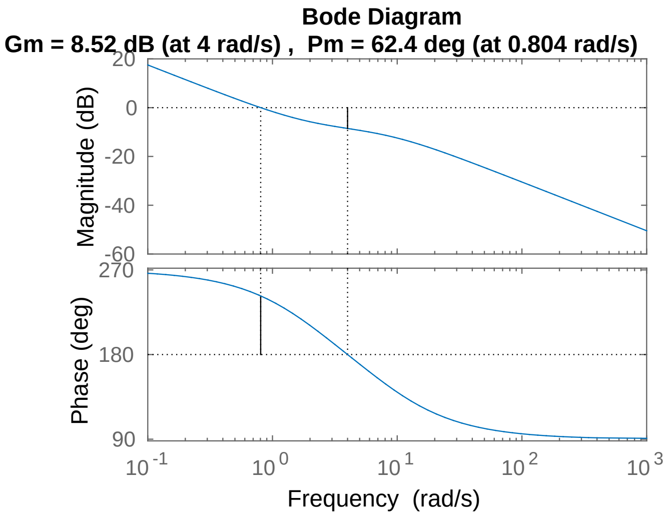
$$\arg G_c(i\omega) = \arg(-i\omega+2) - \arg(i\omega+2) - \arg(1+i\omega/3)$$

For large frequencies, this is approximately

$$\arg G_c(i\omega) \approx \arg -i\omega - \arg i\omega - \arg i\omega/3 = -3\pi/2,$$

or -270 degrees. **It must be the solid line in the bode diagram of figure 1.**

(c)



(d) Substitute

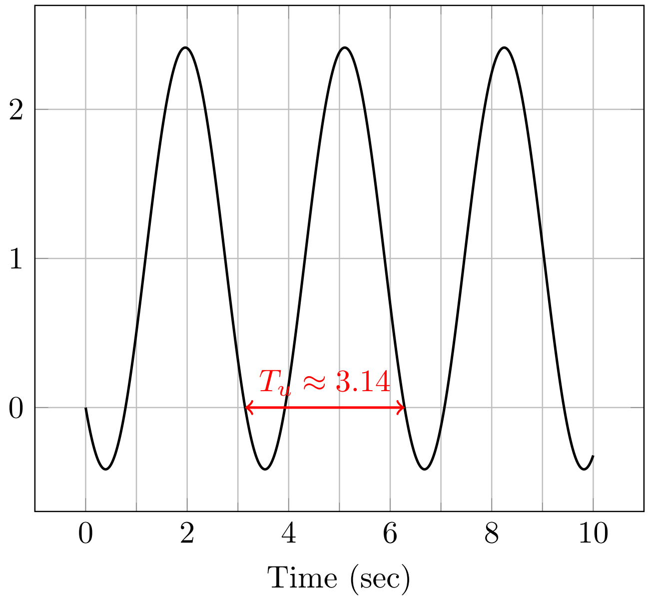
$$s' = \frac{2}{h}\frac{z-1}{z+1}$$

in the transfer function for the controller this gives

$$F_d(z) = F(s')|_{s'=\frac{2}{h}\frac{z-1}{z+1}} = 3\frac{\frac{2}{h}\frac{z-1}{z+1}+2}{\frac{2}{h}\frac{z-1}{z+1}+8} = 3\frac{z-1+h(z+1)}{z-1+4h(z+1)}$$

3.2 PID Tuning

(a) The ultimate period is actually $T_u = \pi$:



(b) With $K_u = 2$ and $T_u = \pi$, the Ziegler-Nicholls tuning is

$$F(s) = 1.2\left(1 + \frac{2}{\pi s} + \frac{\pi}{8}s\right)$$

(c) Interesting differences between the two step responses:

- The tuned PID-controller has a rather high overshoot (more than 50%). The controller from exercise 1 has no overshoot at all.
- The tuned PID-controller has a short rise-time, but much longer settling time.
- The tuned PID-controller gives a jump in the wrong direction at $t = 0$. The other controller starts from zero, but starts off in the wrong direction.

The open-loop transfer function with the tuned PID must have a much smaller phase margin (in fact the phase margin is 24.7° compared to 62.4°).

The PID controlled system has an initial jump because there is a direct term from the input to the output. The open-loop system is given by

$$G_{o_{ZN}} = \frac{-0.47124(s-2)(s+1.273)^2}{s^2(s+2)} = -0.47124 + \frac{0.685s^2 + 1.636s + 1.527}{s^2(s+2)}$$

So at time $t = 0$ the step in the command signal shows directly in y multiplied with a negative factor.

The controller from exercise 1 gives a closed-loop system that starts in the wrong direction due to the right-hand side zero of $G(s)$. This can be seen by taking the derivative of the step-response and use the inital-value theorem. The derivative will be negative at time $t = 0$. The derivative of the step response is simply the impulse response. Since the close loop system is given by

$$G_{c_{exc1}} = \left(\frac{-s+2}{s+2}\right)\frac{1}{1+s/3}$$

the initial-value theorem gives

$$\lim_{t\to 0}\left(\frac{d}{dt}y_{step}(t)\right) = \lim_{t\to 0} y_{impulse}(t) = \lim_{s\to\infty} sG_{c_{exc1}}(s) = -\frac{1}{3}.$$

(d) Backward difference gives the sampled controller

$$
\begin{aligned}
F_d(z) &= F_{ZN}(s')|_{s'=\frac{z-1}{zh}} = 1.2\left(1+\frac{2}{\pi s}+\frac{pi}{8}s\right)|_{s'=\frac{z-1}{zh}} \\
&= 1.2\left(1+\frac{2}{\pi\frac{z-1}{zh}}+\frac{\pi}{8}\frac{z-1}{zh}\right)
\end{aligned}
$$