Computerized Control partial exam 1 (18%)

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Time September 12 19:05-20.35

Place 4203

Permitted aids The single colored page with your own notes, table of Laplace transforms, calculator

All answers should be readable and well motivated (if nothing else is written). Solutions/motivations should be written on the provided spaces in this exam. Use the last page if more space is needed.

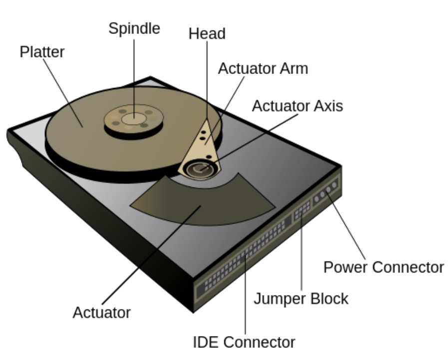
Good luck!

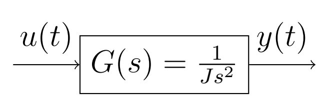
Matricula and name:

Control of a frictionless mechanical system

Friction is ubiquous in mechanical systems, but can sometimes be neglected in a model. The textbook gives one such example: Control of the position of the arm of a hard disk drive. With the input signal $u(t)$ being the torque applied to the arm, $y(t)$ the angular position of the arm and J its moment of inertia, the system is described by the differential equation

$$J\ddot{y}(t) = u(t). \tag{1}$$





Problem 1

(a) Show that zero-order-hold sampling of the model (1) gives the pulse-transfer function

$$H(z) = \frac{\frac{h^2}{2J}(z+1)}{(z-1)^2}.$$

Calculations:

(b) State as a mathematical expression how the continuous-time poles (in the s-plane) and the corresponding discrete-time poles (in the z-plane) are related, and verify that the relationship holds in this particular case.

Answer:

Problem 2 In the rest of the exam, consider the sampled system obtained with $J = 0.5$ and sampling time $h = 1$ (the time unit is 100 µs).

$$H(z) = \frac{z+1}{(z-1)^2}.$$

The system is being controlled by the discrete-time controller

$$U(z) = F(z)E(z) = 0.4\frac{z-0.8}{z-0.2}\big(Y_{ref}(z) - Y(z)\big). \tag{2}$$

(a) Draw a block-diagram of the closed-loop system

Block diagram:

(b) Show that the closed-loop pulse-transfer function from the reference signal $y_{ref}(k)$ to the control error $e(k)$ is

$$H_e(z) = \frac{(z-1)^2(z-0.2)}{(z-0.2)(z-1)^2 + 0.4(z-0.8)(z+1)}. \tag{3}$$

Calculations:

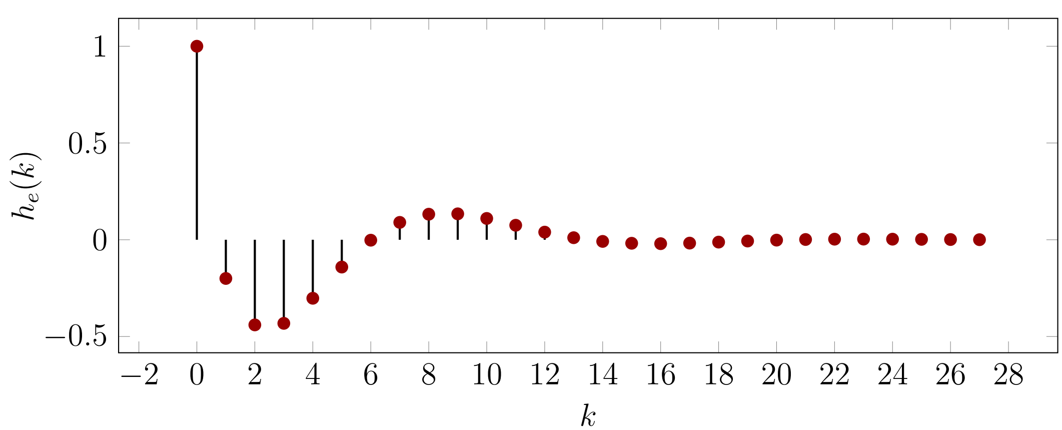
(c) What is the gain of the pulse-transfer function $H_e(z)$ for constant signals? Explain what this means for the response of the system to a step change in $y_{ref}(k)$.

Answer:

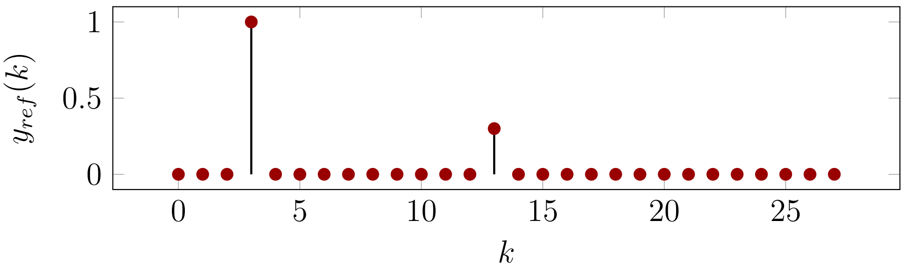
(d) Write the control law (2) as a difference equation.

Calculations:

Problem 3 The closed-loop system with pulse-transfer function (3) has pulse-response as shown below



Which of the responses in figure 1 is the reponse of the system when the reference signal $y_{ref}(k)$ is as shown below? **Motivate!**



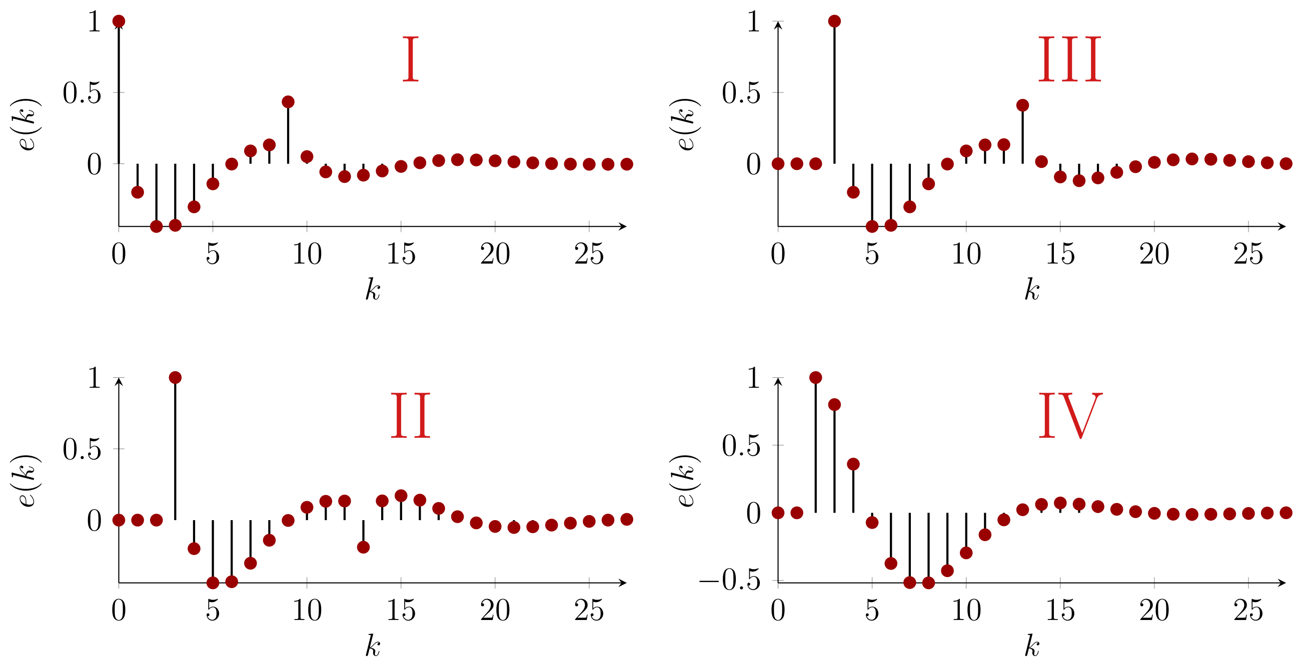


Figure 1: Responses to the closed-loop system.

Motivation:

Solutions

Problem 1

(a) The idea with zero-order-hold sampling (a.k.a step-invariant sampling) is to solve the continuous-time system for a step input to obtain $y(t)$, then sample this signal and apply the z-transform to obtain $Y(z)$. Since the input signal (the step) has z-transform $U(z) = \frac{z}{z-1}$, we obtain the pulse-transfer function for the sampled system as $H(z) = \frac{Y(z)}{U(z)} = \frac{z-1}{z}Y(z)$.

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{Js^3}\right\} = \frac{1}{2J}t^2,$$

$$Y(z) = \mathcal{Z}\left\{y(kh)\right\} = \mathcal{Z}\left\{\frac{h^2}{2J}k^2\right\} = \frac{h^2}{2J} \cdot \frac{z(z+1)}{(z-1)^3},$$

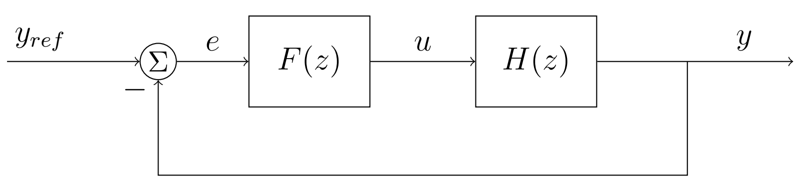
hence

$$H(z) = \frac{Y(z)}{U(z)} = \frac{z-1}{z}\frac{h^2}{2J}\frac{z(z+1)}{(z-1)^3} = \frac{\frac{h^2}{2J}(z+1)}{(z-1)^2}.$$

(b) A pole λ in the s-plane will be mapped to the pole $p = \mathrm{e}^{\lambda h}$ in the z-plane. Here we have two poles in the origin in the s-plane, $\lambda = 0$, and so both the poles are mapped to the point $p = \mathrm{e}^{0h} = 1$ in the z-plane.

Problem 2

(a)



(b) Using Mason's rule we get

$$H_e(z) = \frac{1}{1 + H(z)F(z)} = \frac{1}{1 + 0.4\frac{z-0.8}{z-0.2}\frac{z+1}{(z-1)^2}} = \frac{(z-0.2)(z-1)^2}{(z-0.2)(z-1)^2 + 0.4(z-0.8)(z+1)}.$$

(c) The static gain is $H_e(1) = 0$. So a step-change in y_{ref} will give no steady-state error.

(d) Using the shift operator q, we can write the control law as

$$u(k) = 0.4\frac{\mathrm{q} - 0.8}{\mathrm{q} - 0.2}e(k)$$

$$(\mathrm{q} - 0.2)u(k) = 0.4(\mathrm{q} - 0.8)e(k)$$

$$u(k+1) - 0.2u(k) = 0.4e(k+1) - 0.32e(k)$$
$$u(k+1) = 0.2u(k) + 0.4e(k+1) - 0.32e(k),$$

with $e(k) = y_{ref}(k) - y(k)$.

Problem 3 The correct response is **III**. The reference signal is a sum of two delayed and scaled pulses, and can be written $y_{ref}(k) = \delta(k-4)+0.3\delta(k-14)$. Correspondingly, the output should look like the superposition of two delayed and scaled pulse-responses. Responses I and IV starts at the wrong time, and can be excluded. Response II starts at $k = 4$, and starts out looking correct. However, the superposed pulse response starting at $k = 14$ is negative. This leaves response III as the correct response.