Lab C: Colliding Carts

2.1)

- Directly: Position, time, inertia of carts
- Calculated: Velocity, acceleration, force, energy, … (anything that can use derivations of position, time, and inertia)
- A phone/clock can give as more precise time measurements and video the process to measure time & movement together
- Make sure the speed is very slow to allow for human time to react and to reduce motion blur in the video. Also, the slower velocities will prevent the carts from traveling past the bounds of the track.
- Top down in the center is ideal (this is to keep everything in frame and reduce perspective shifts, but something that can see the yellow position line equally in the center will work.

2.2)

- Instrumental uncertainty for position: 0.5 mm
- Negligible time uncertainty (we can measure that from the video)
- Random uncertainty for position will be calculated based on our uncertainties as a group (we take the average of our numbers and find the maximum difference between our average and each of our individual measurements)
- Inertia of both carts is 1.5 kg (0.5 kg base, + 1 kg of extra weight)
- We will be colliding the carts with their magnetized sides facing each other to simulate an elastic collision.

Data measurement strategy

- Record the collision in slow motion with a clock in view for time reference
- Do the collision
- We will take 4 measurements per cart of position and time:
    - Start or a bit after
    - Sometime after push and before collision
    - Sometime after collision
    - Stop point
- Because of the “frictionless” track, we can assume all movement is with constant velocity. Therefore, we only need to take 4 measurements per cart as each pair will tell us the “constant” velocity of the cart before and after the collision.
- Measurements positional are taken from the inner edge of the carts (the side where the carts are facing each other). Measurements were taken as absolute position on the track and recorded in cm to the hundredth place (instrumental uncertainty lets us put 0.00 or 0.05 cm)
- To find instrumental uncertainty, we looked in our video to see to what degree we could see the position line. Since we recorded in 4K, we had enough resolution to see the millimeter ticks. Therefore, our uncertainty is 0.5 mm

- To find random uncertainty, we take the average of our measurements for the first position of the left-hand cart, then find the largest difference between this average and the measurements of the other carts. We do the same to the right-hand cart and then use the larger value between the two carts as our random uncertainty. Then, as instructed, we use this random uncertainty for all our measurements. Later, when calculating, we will use the weakest link rule for the position (time is considered to have no uncertainty).

Data gathered from elastically colliding 2 1.5 kg carts

Cart 1 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
22.10	34.70	34.65	34.20	34.40	**34.49**
24.12	54.90	54.80	55.00	54.90	**54.90**
26.10	55.00	54.75	54.80	55.00	**54.89**
27.95	49.60	49.70	49.50	49.50	**49.58**

Cart 2 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
22.10	81.10	81.15	81.60	81.00	**81.21**
24.12	66.40	66.20	66.20	66.30	**66.28**
26.10	68.60	68.86	68.60	65.50	**67.89**
27.95	72.80	72.70	72.70	72.80	**72.75**

Positional random uncertainty: 0.39
Time random uncertainty: 0

3.1)

Student 1 equation:

$$\frac{m_2}{m_1} \text{ or } \frac{m_1}{m_2} = \frac{|v_{1i} - v_{2i}|}{|v_{1f} - v_{2f}|} k$$

Student 2 equation:

$$\frac{m_2}{m_1} \text{ or } \frac{m_1}{m_2} = \frac{|v_{1f} - v_{1i}|}{|v_{2f} - v_{2i}|} k$$

- The students did not specify what order their inertias were placed in the ratio, so we will work with both orientations. This produces 4 equation (2 per student)
- An extra unknown constant k is added to the end because these are ratios and can be scaled.

3.2)

A contrary outcome can disprove the hypothesis, but a consistent outcome cannot be used to completely prove the hypothesis. You will need more than experimental data (i.e., mathematical prove and other things) to concretely prove a hypothesis. But you can show a hypothesis is unable to be upheld. This assumes multiple trials.

4.1)

- We repeat our experiment as highlighted in 2.2, except change the mass of one of the carts (say cart 2, the one on the right side in our video recordings) to get a different inertia ratio. We will set the mass of cart 1 to 0.5 kg and cart 2 to +1.0 kg.
    - We repeat with different ratios: 1:2, 1:3, 1:4, 1:5 (we already have data for 1:1 ratio from step 2.2)
- The inertias of the carts can be gathered directly, and the velocities can be calculated (based on position and time)

4.2)

- We will use the same numbers we got from running our experiments for 4.1, except we will rearrange them to match student 2's experiment.

5.1)

This is a large collection of data. Here's a few things to take note

- Δx uncertainty is $\pm\, 0.78\text{ cm} = 0.39 * 2$ (this came from the positional uncertainty being 0.39 cm as highlighted in section 2.2).
    - A percent uncertainty is computed by $\frac{0.78}{\Delta x}$. This is applied to the velocity in the end; it isn't listed for each Δx
- Velocity's uncertainty was calculated by multiplying the percent uncertainty by the computed velocity $\left(\frac{\Delta x}{\Delta t}\right)$. There is no uncertainty associated with Δt as explained in section 2.2
- All numbers here can be also used in Student 2's equation, so a rerecording is not necessary
- We are assuming a frictionless track; therefore, velocities are constant in their respective directions of movement. Therefore, we can take two positional measurements before and after the collision and determine an incoming (initial) and outgoing (final) velocity for each cart.

Data Collection for 1:2 inertia ratio cart collision

Cart 1 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
2.75	37.50	37.45	57.50	37.50	**42.49**
3.85	65.20	65.10	65.00	65.00	**65.08**
4.56	59.70	59.70	59.60	59.50	**59.63**
5.44	41.50	41.35	41.40	41.50	**41.44**

Cart 2 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{2.75}$	87.70	87.80	87.70	87.50	$\underline{\mathbf{87.68}}$
$\mathbf{3.85}$	73.50	73.50	73.40	73.50	$\underline{\mathbf{73.48}}$
$\mathbf{4.56}$	76.30	76.20	76.10	76.10	$\underline{\mathbf{76.18}}$
$\mathbf{5.44}$	84.50	84.50	84.30	84.40	$\underline{\mathbf{84.43}}$

Cart 1 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{22.59}$	1.10	20.53 ± 0.71
$\mathbf{-18.19}$	0.88	-20.67 ± 0.89

Cart 2 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{-14.20}$	1.10	-12.91 ± 0.71
$\mathbf{8.25}$	0.88	9.38 ± 0.89

Data Collection for 1:3 inertia ratio cart collision

Cart 1 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{85.93}$	43.60	43.55	43.50	43.50	$\underline{\mathbf{43.54}}$
$\mathbf{87.26}$	61.40	61.00	61.00	61.10	$\underline{\mathbf{61.13}}$
$\mathbf{88.15}$	58.60	58.20	58.10	58.40	$\underline{\mathbf{58.33}}$
$\mathbf{89.08}$	46.40	46.35	46.30	46.30	$\underline{\mathbf{46.34}}$

Cart 2 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{85.93}$	77.80	77.95	78.00	78.10	$\underline{\mathbf{77.96}}$
$\mathbf{87.26}$	68.40	68.45	68.20	68.20	$\underline{\mathbf{68.31}}$
$\mathbf{88.15}$	67.50	67.50	67.60	67.50	$\underline{\mathbf{67.53}}$
$\mathbf{89.08}$	69.30	69.20	69.20	69.30	$\underline{\mathbf{69.25}}$

Cart 1 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{17.59}$	1.33	13.22 ± 0.59
$\mathbf{-11.99}$	0.93	-12.89 ± 0.84

Cart 2 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{-9.65}$	1.33	-7.26 ± 0.59

$\mathbf{1.72}$	0.93	1.85 ± 0.84

Data Collection for 1:4 inertia ratio cart collision

Cart 1 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{132.14}$	42.00	41.70	42.00	41.20	$\underline{\mathbf{41.73}}$
$\mathbf{133.00}$	59.50	39.30	59.30	59.00	$\underline{\mathbf{54.28}}$
$\mathbf{133.57}$	46.50	46.40	46.40	46.50	$\underline{\mathbf{46.45}}$
$\mathbf{134.24}$	26.70	26.45	26.60	26.80	$\underline{\mathbf{26.64}}$

Cart 2 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{132.14}$	75.20	74.90	75.00	75.10	$\underline{\mathbf{75.05}}$
$\mathbf{133.00}$	62.70	62.60	62.40	62.50	$\underline{\mathbf{62.55}}$
$\mathbf{133.57}$	61.30	61.20	61.20	61.30	$\underline{\mathbf{61.25}}$
$\mathbf{134.24}$	61.20	61.10	61.20	61.10	$\underline{\mathbf{61.15}}$

Cart 1 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{12.55}$	0.86	14.59 ± 0.91
$\mathbf{-19.81}$	0.67	-29.57 ± 1.16

Cart 2 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{-12.50}$	0.86	-14.53 ± 0.91
$\mathbf{-0.10}$	0.67	-0.15 ± 1.16

Data Collection for 1:5 inertia ratio cart collision

Cart 1 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{149.60}$	31.80	31.75	31.80	31.60	$\underline{\mathbf{31.74}}$
$\mathbf{151.65}$	51.40	51.25	51.00	51.20	$\underline{\mathbf{51.21}}$
$\mathbf{152.60}$	51.80	51.50	51.80	51.50	$\underline{\mathbf{51.65}}$
$\mathbf{153.65}$	34.50	34.40	34.80	34.60	$\underline{\mathbf{34.58}}$

Cart 2 position at each time stamp

Time (sec)	Sample 1 (cm)	Sample 2 (cm)	Sample 3 (cm)	Sample 4 (cm)	Average (cm)
$\mathbf{149.60}$	82.40	82.20	82.20	82.30	$\underline{\mathbf{82.28}}$

$\mathbf{151.65}$	63.40	63.25	63.50	63.10	$\underline{\mathbf{63.31}}$
$\mathbf{152.60}$	57.40	57.35	57.30	57.20	$\underline{\mathbf{57.31}}$
$\mathbf{153.65}$	57.00	57.10	57.00	57.00	$\underline{\mathbf{57.03}}$

Cart 1 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{19.48}$	2.05	9.50 ± 0.38
$\mathbf{-17.08}$	1.05	-16.26 ± 0.74

Cart 2 computed velocity (listed as initial, then final)

Δx (cm)	Δt (sec)	Velocity (cm/s)
$\mathbf{-18.96}$	2.05	-9.25 ± 0.38
$\mathbf{-0.29}$	1.05	-0.27 ± 0.74

5.1 Computing Students' Hypotheses)

Student 1)

Student 1's hypothesis was a ratio of each cart's relative velocities. We will be using a ratio of the incoming (initial) relative velocities of the cart to the outgoing (final) relative velocities.

Relative velocities of carts at each inertia ratio

Inertia ratio	Incoming relative velocities (cm/s)	Outgoing relative velocities (cm/s)	Student 1 hypothesized ratio
1:1	17.50 ± 0.77	5.50 ± 0.84	$\mathbf{3.18 \pm 0.49}$
1:2 or 2:1	33.44 ± 1.42	30.04 ± 1.77	$\mathbf{1.11 \pm 0.07}$
1:3 or 3:1	20.48 ± 1.17	14.74 ± 1.68	$\mathbf{1.39 \pm 0.16}$
1:4 or 4:1	34.94 ± 1.81	29.42 ± 2.33	$\mathbf{0.99 \pm 0.08}$
1:5 or 5:1	18.75 ± 0.76	15.99 ± 1.49	$\mathbf{1.17 \pm 0.11}$

Student 2)

Student 2's hypothesis was a ratio of the change in velocities for each cart. We will be using a ratio of cart 1's change in velocity to cart 2's change in velocity.

Change in velocity for each cart at each inertia ratio

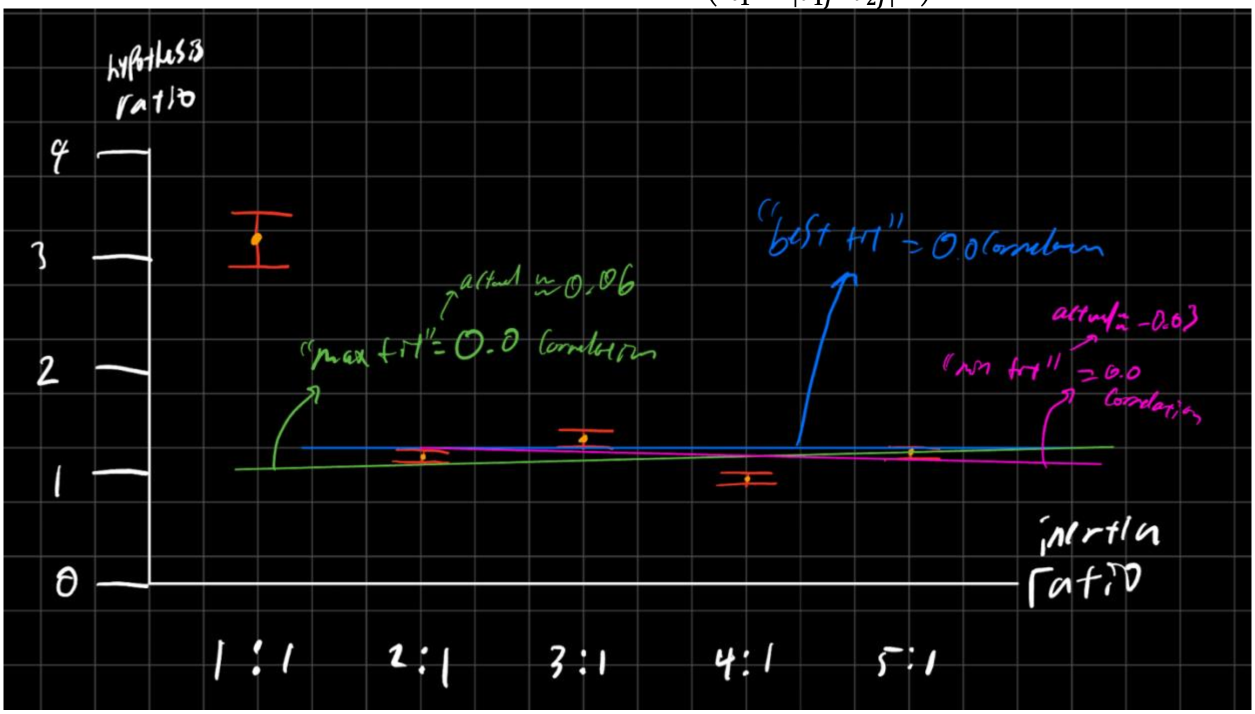
Inertia ratio	Cart 1 change in velocity (cm/s)	Cart 2 change in velocity (cm/s)	Student 2 hypothesized ratio
1:1	12.98 ± 0.81	10.02 ± 0.81	$\mathbf{1.29 \pm 0.10}$
1:2 or 2:1	41.20 ± 1.60	22.28 ± 1.60	$\mathbf{1.85 \pm 0.13}$
1:3 or 3:1	26.11 ± 1.43	9.11 ± 1.43	$\mathbf{2.87 \pm 0.45}$
1:4 or 4:1	49.98 ± 2.07	14.39 ± 2.07	$\mathbf{3.07 \pm 0.44}$
1:5 or 5:1	25.76 ± 1.12	8.98 ± 1.12	$\mathbf{2.87 \pm 0.36}$

5.1 Graphs and Conclusion)

Student 1)

Since student 1’s hypothesis was based on the relative velocities of the carts, the calculated ratio was dependent on how hard we pushed the carts. As can be seen in the following graphs, we pushed the carts much harder in the 1:1 inertia ratio experiment as compared to the others. In an ideal world, all these values should be exactly equal, but since the ratio is sensitive to how hard we pushed the carts, it is subject to human error (on the amount of force we applied).

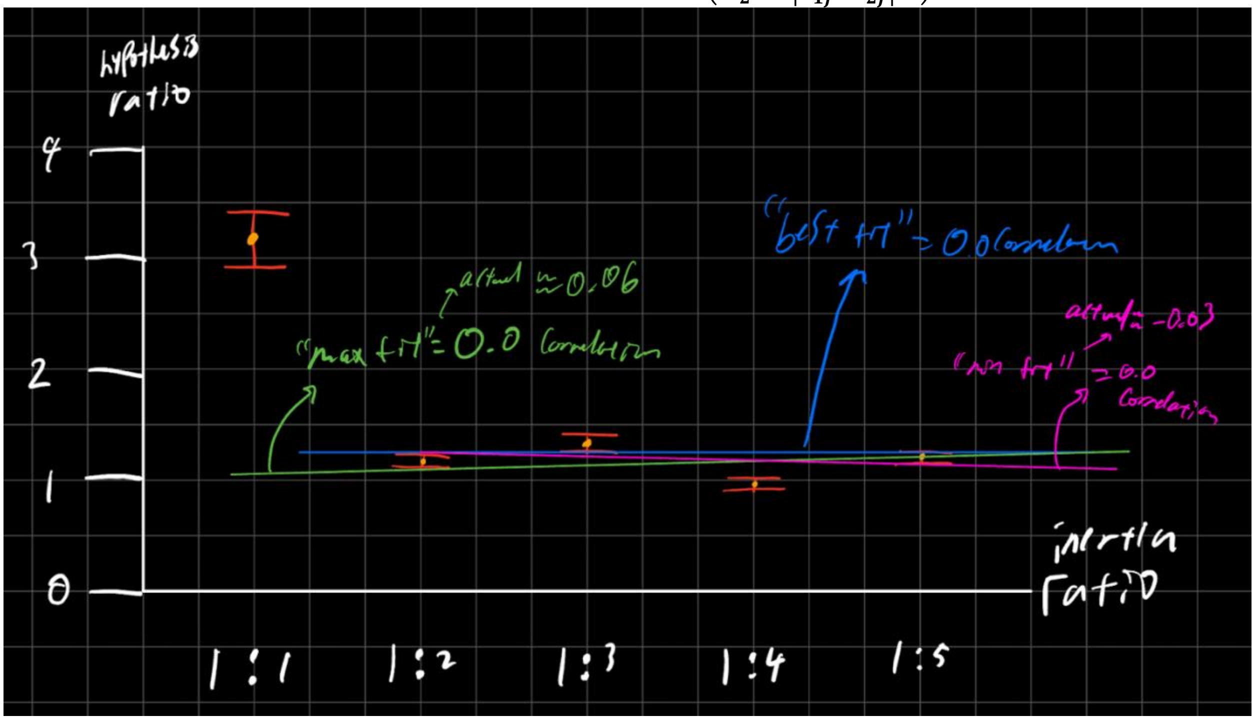
Figure 1: Inertia ratios vs hypothesized ratios $\left(\frac{m_2}{m_1}=\frac{|v_{1i}-v_{2i}|}{|v_{1f}-v_{2f}|}k\right)$



This graph is showing the inertia ratio of cart 2 to cart 1. If the hypothesis were to be correct, there should be a positive correlation (ideally one-to-one, but a scalar is allowed). However, we can see that there is no correlation between the increase in inertia ratio and the hypothesized ratio.

Again, since the hypothesis ratios (independent variable) was based on how hard we pushed the carts, most of the values are approximately the same with the variability coming from human error. In a perfect scenario all these points will be exactly aligned. Therefore our “fit” lines are all 0 to 1 significant digit (our calculations are in 2 significant digits, which is reduced to one when fitting). Because of the slower speeds and high resolution of our data capture, we have relatively small error bars meaning it was very difficult to properly “fit” a line to our data (especially since they are not supposed to be correlated).

Figure 2: Inertia ratios vs hypothesized ratios $\left(\frac{m_1}{m_2} = \frac{|v_{1i}-v_{2i}|}{|v_{1f}-v_{2f}|}k\right)$



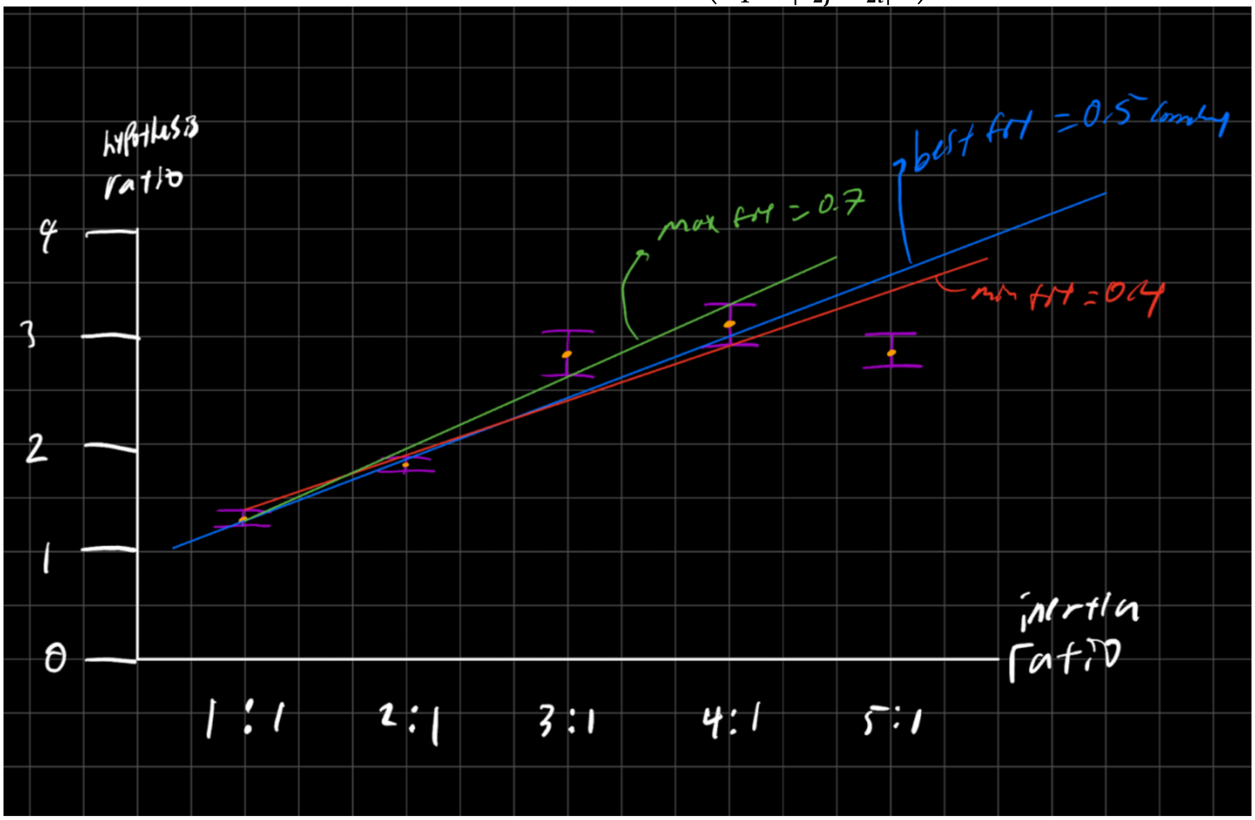
To account for the alternative equation for Student 1 (where we are comparing the ratio of cart 1 to cart 2 to the hypothesized ratio), we flip the inertia ratio. We keep the data in the same order for simplicity. In this graph there should be a negative correlation (ideally one-to-one again) but like before the correlation is 0.0.

Student 2 is on next page

Student 2)

Student 2's data is based on differences in velocity.

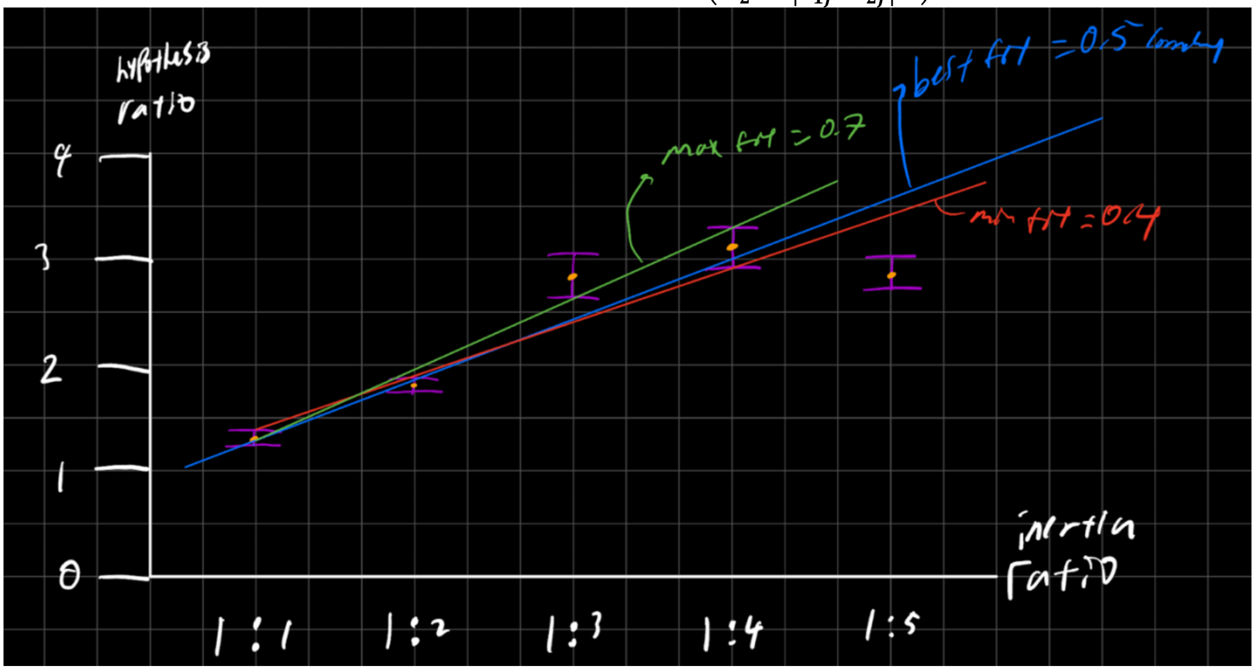
Figure 3: Inertia ratios vs hypothesized ratios $\left(\frac{m_2}{m_1} = \frac{|v_{1f}-v_{1i}|}{|v_{2f}-v_{2i}|}\boldsymbol{k}\right)$



This graph is showing the inertia ratio of cart 2 to cart 1. If the hypothesis were to be correct, there should be a positive correlation (ideally one-to-one, but a scalar is allowed). As can be seen here, there is in fact a positive correlation between the inertia ratios and the hypothesized ratio. We can also draw maximum and minimum best fit lines and come up with a final correlation of 0.5 ± 0.2, however this is not entirely necessary since the point of this graph is to compare how well correlated student 2's hypothesis is to student 1's and it is very clearly much more correlated (0.0/1.0 from student 1 to 0.5/1.0 from student 2). We acknowledge that these best fit lines only pass through 3 out of 5 data points. Ideally they pass through 4 or more but given the error bars this was the best that could be made. Again, the point is to show that there is some sort of linear correlation as opposed to having none.

To ensure that student 2's inertia ratio was not supposed to be flipped (ratio of cart 1 to cart 2), we re-plot with the inertia ratios flipped:

Figure 4: Inertia ratios vs hypothesized ratios $\left(\frac{m_1}{m_2} = \frac{|v_{1i}-v_{2i}|}{|v_{1f}-v_{2f}|}\boldsymbol{k}\right)$



Because of the ordering of the inertia ratios (like in figure 2) are in descending magnitude, there should be a negative correlation between the inertia ratios and the hypothesis ratio. Since there is a positive correlation, we know this is not the correct inertia ratio, leaving the $\frac{m_2}{m_1}$ ratio graphed in figure 3 as the correct equation.

Conclusion

Based on these graphs comparing the hypothesized ratios by student 1 and student 2 to the ratios of the inertias of the carts, we can conclude that our evidence supports student 2's hypothesis which is $\frac{m_2}{m_1} = \frac{|v_{1i}-v_{2i}|}{|v_{1f}-v_{2f}|}\boldsymbol{k}$. There appears to be a positive linear correlation between the hypothesized ratio as by student 2 and the inertia ratio of the carts. Student 1's hypothesis was a ratio of relative velocities which does not correlate with inertia. However, it should be noted that if one where to not use a consistent pushing force on the carts they could end up simulating a positive correlation for student 1's hypothesis by simply pushing harder and harder for each inertia ratio (to fake a positive correlation).