

# Chapter 6

## Underwater granular flows

### 6.1 Introduction

Avalanches, landslides, and debris flows are geophysical hazards, which involve rapid mass movement of granular solids, water, and air as a single phase system. Globally, landslides cause billions of pounds in damage, and thousands of deaths and injuries each year. Hence, it is important to understand the triggering mechanism and the flow evolution. The momentum transfer between the discrete and continuous phases significantly affects the dynamics of the flow as a whole (Topin et al., 2012). Although certain macroscopic models are able to capture simple mechanical behaviours (Peker and Helvacı, 2007), the complex physical mechanisms occurring at the grain scale, such as hydrodynamic instabilities, formation of clusters, collapse, and transport, (Topin et al., 2011) have largely been ignored. In particular, when the solid phase reaches a high volume fraction, the strong heterogeneity arising from the contact forces between the grains, and the hydrodynamic forces, are difficult to integrate into the homogenization process involving global averages.

In order to describe the mechanism of immersed granular flows, it is important to consider both the dynamics of the solid phase and the role of the ambient fluid (Denlinger and Iverson, 2001). The dynamics of the solid phase alone is insufficient to describe the mechanism of granular flow in a fluid. It is important to consider the effect of hydrodynamic forces that reduce the weight of the solids inducing a transition from dense-compacted to dense-suspended flows, and the drag interactions which counteract the movement of the solids (Meruane et al., 2010). Transient regimes characterized by change in the solid fraction, dilation at the onset of flow and development of excess pore pressure, result in altering the balance between the stress carried by the fluid and that carried by the grains, thereby changing the overall behaviour of the flow.

The presence of a fluid phase in a granular medium has profound effects on its mechanical behaviour. In dry granular media the rheology is governed by grain inertia and static stresses sustained by the contact network depending on the shear-rate and confining pressure, respectively (Midi, 2004). As the fluid inertia and viscosity come into play, complications arise as a result of contradictory effects. On one hand, the fluid may delay the onset of granular flow or prevent the dispersion of the grains by developing negative pore pressures (Pailha et al., 2008; Topin et al., 2011). On the other hand, the fluid lubricates the contacts between grains, enhancing in this way the granular flow, and it has a retarding effect at the same time by inducing drag forces on the grains. The objective of the present study is to understand the differences in the mechanism of flow initiation and kinematics between dry and submerged granular flow. In the present study, 2D Lattice-Boltzmann and Discrete Element Method is used to model the fluid-soil interactions in underwater granular flows.

## 6.2 Granular collapse in fluid

### 6.2.1 LBM-DEM Coupling

Lattice Boltzmann approach can accommodate large grain sizes and the interaction between the fluid and the moving grains can be modelled through relatively simple fluid – grain interface treatments. Further, employing the Discrete Element Method (DEM) to account for the grain – grain interaction naturally leads to a combined LB – DEM procedure. The Eulerian nature of the LBM formulation, together with the common explicit time step scheme of both LBM and DEM makes this coupling strategy an efficient numerical procedure for the simulation of grain – fluid systems. Such a coupled methodology is used in simulating grain – fluid systems dominated by grain – fluid and grain – grain interactions. To capture the actual physical behavior of the fluid – grain system, it is essential to model the boundary condition between the fluid and the grain as a non-slip boundary condition, i.e. the fluid near the grain should have similar velocity as the grain boundary. The solid grains inside the fluid are represented by lattice nodes. The discrete nature of lattice, results in a stepwise representation of the surfaces, which are circular, hence sufficiently small lattice spacing is adopted.

In this study, a 2D poly-disperse system ( $d_{max}/d_{min} = 1.8$ ) of circular discs in fluid was used to understand the behaviour of granular flows on inclined planes (see Figure 6.1). The soil column was modelled using 1000 discs of density  $2650 \text{ kg m}^{-3}$  and a contact friction angle of  $26^\circ$ . The collapse of the column was simulated inside a fluid with a density of  $1000 \text{ kg m}^{-3}$  and a kinematic viscosity of  $1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ . The choice of a 2D geometry has the advantage of cheaper computational effort than a 3D case, making it feasible to simulate very large systems.

- 33 A granular column of aspect ratio ‘a’ of 0.8 was used. A hydrodynamic radius  $r = 0.9R$  was adopted during the LBM computations. Dry analyses were also performed to study the effect of hydrodynamic forces on the run-out distance.

### 6.2.2 Permeability

In DEM, the grain – grain interaction is described based on the contact interactions. In a 3D granular assembly, the pore spaces between grains are interconnected, whereas in 2-D assembly, the grains are in contact with each other that result in a non-interconnected pore-fluid space. This results in a no flow condition in a 2-D case. In order to overcome this difficulty, a reduction in radius is assumed only during LBM computations (fluid and fluid – solid interaction). The reduction in radius allows interconnected pore space through which the surrounding fluid can flow (reduced  $R=0.7r$  to  $0.95r$ , ‘r’ is grain radius). The reduction in radius is assumed only during LBM computations, hence this technique has no effect on the grain – grain interactions computed using DEM. Different permeability can be obtained, for any given initial packing, by varying the reduction in the radius for the grains, without changing the actual granular packing. Cumulative  $\beta$  method is adopted to generate a randomly packed granular assembly with a polydispersity of 1.8 (see Figure 2).<sup>8</sup> The size of the grains varies between 1.25mm to 2.2mm.

A square sample of 50 mm x 50 mm is used to determine the transverse permeability. Dirichlet boundary condition,<sup>9</sup> i.e. pressure/density constrain is applied at the left and the right boundaries. A small increment ( $10^{-4}$ ) in density is applied at the left hand side boundary and a constant density is maintained at the right hand side boundary. This results in a gradient of pressure causing the fluid in the domain to flow. The mean velocity of flow ( $v$ ) is determined and the permeability of the sample ( $k$ ) is computed as:

$$k = v \cdot \mu \cdot \frac{\Delta x}{\Delta P}, \quad (6.1)$$

where  $\mu$  is the dynamic viscosity of the fluid (Pa s),  $\Delta x$  is the thickness of the bed of porous medium  $m$ , and  $\Delta P$  is the applied pressure difference Pa. In the present study, the radius is varied from 0.7 to 0.95 to obtain a wide range of permeability for the sample. Increase in the size of the grain from 0.7 to 0.95 reduces the porosity from 0.60 to 0.27. The permeability computed from LB – DEM method is verified by comparing it with an analytical solution. One of the widely used analytical solutions for permeability is the Carman – Kozeny equation (i.e. the CK Model), which is based on Poiseuille flow through pipe and is mainly used for 3D, homogenous, isotropic, granular porous media at moderate porosities. In the present study, a modified Carman – Kozeny equation that takes into account the microstructure of the fibers and

is valid in a wide range of porosities is adopted. The normalized permeability is defined as

$$\frac{k}{d^2} = \frac{\varepsilon}{\psi_{CK}(1-\varepsilon)^2}. \quad (6.2)$$

In the CK model, the hydraulic diameter  $D_h$ , is expressed as a function of measurable quantities: porosity and specific surface area

$$D_h = \frac{4\varepsilon V}{S_v} = \frac{\varepsilon d}{(1-\varepsilon)}, \quad (6.3)$$

$$a_v = \frac{\text{grain surface}}{\text{grain volume}} = \frac{S_v}{(1-\varepsilon V)} = \frac{4}{d}, \quad (6.4)$$

With the total wetted surface,  $S_v$ , and the specific surface area,  $a_v$ . The above value of  $a_v$  is for circles (cylinders) - for spheres  $a_v = 6/d$ .  $\psi_{CK}$  is the empirically measure CK factor. Which represents both the shape factor and the deviation of flow direction from that in a duct. It is approximated for randomly packed beds of spherical grains. The variation in the flow rate for different reduction in radius is presented in Figure 3a. The normalized permeability for different porosity obtained by varying the radius from 0.7 to 0.95 is presented in Figure 3b. It can be observed from the figure that the permeability decreases drastically as the radius is varied from 0.7r to 0.95r. The granular assembly is almost impermeable for a radius of 0.95r. The normalized permeability is found to match the qualitative trend of the Carman-Kozeny equations. The LB – DEM permeability curve lies between the permeability curves for spherical and cylindrical grain arrangements implying a better approximation of permeability in 2D granular assembly by reducing the radius during LBM computations.

### 6.3 Submarine granular flows down incline plane

The flow of dense granular material is a common phenomenon in engineering predictions, such as avalanches, landslides, and debris-flow modelling. Despite the huge amount of research that has gone into describing the behaviour of granular flows, a constitutive equation that describes the overall behaviour of a flowing granular material is still lacking. The initiation and propagation of submarine granular flows depend mainly on the slope, density, and quantity of the material destabilised. Although certain macroscopic models are able to capture the simple mechanical behaviours, the complex physical mechanisms that occur at the grain scale, such as hydrodynamic instabilities, the formation of clusters, collapse, and transport, have largely been ignored (Topin et al., 2011). The momentum transfer between the discrete and the continuous phases significantly affects the dynamics of the flow (Peker and Helvaci, 2007). Grain-scale

description of the granular material enriches the macro-scale variables, which poorly account for the local rheology of the materials. In order to describe the mechanism of saturated and/or immersed granular flows, it is important to consider both the dynamics of the solid phase and the role of the ambient fluid (Denlinger and Iverson, 2001). In particular, when the solid phase reaches a high volume fraction, it is important to consider the strong heterogeneity arising from the contact forces between the grains, the drag interactions which counteract the movement of the grains, and the hydrodynamic forces that reduce the weight of the solids inducing a transition from dense compacted to a dense suspended flow (Meruane et al., 2010). The case of the collapse in presence of an interstitial fluid has been less studied. In this paper, we study the submarine granular flows in the inclined configuration. We study the effect of permeability, density and slope angle on the run-out evolution.

In this study, a 2D poly-disperse system ( $d_{max}/d_{min} = 1.8$ ) of circular discs in fluid was used to understand the behaviour of granular flows on inclined planes (see Figure 6.1). The soil column was modelled using 1000 discs of density  $2650 \text{ kg m}^{-3}$  and a contact friction angle of  $26^\circ$ . The collapse of the column was simulated inside a fluid with a density of  $1000 \text{ kg m}^{-3}$  and a kinematic viscosity of  $1 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ . The choice of a 2D geometry has the advantage of cheaper computational effort than a 3D case, making it feasible to simulate very large systems. A granular column of aspect ratio ‘a’ of 0.8 was used. A hydrodynamic radius  $r = 0.9R$  was adopted during the LBM computations. Dry analyses were also performed to study the effect of hydrodynamic forces on the run-out distance.

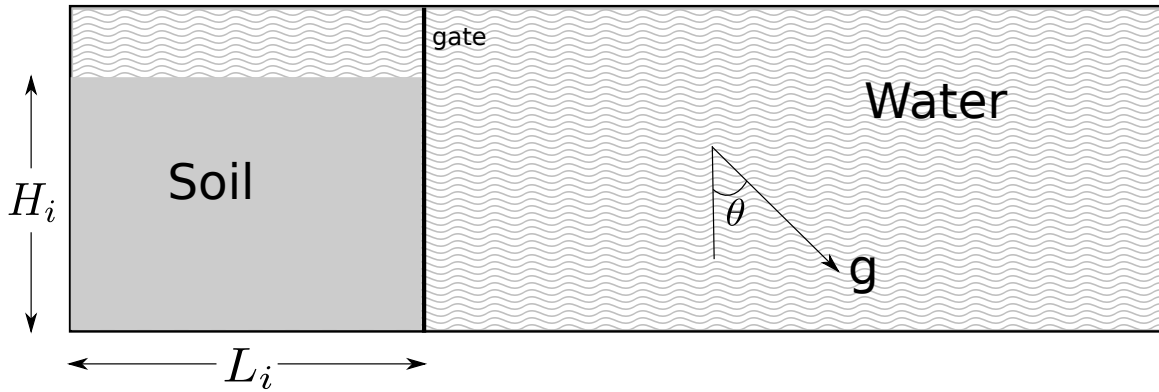


Figure 6.1 Underwater granular collapse set-up

### 6.3.1 Effect of initial density

The morphology of the granular deposits in fluid is shown to be mainly controlled by the initial volume fraction of the granular mass and not by the aspect ratio of the column (Pailha et al., 2008; Rondon et al., 2011). In order to understand the influence of the initial packing density

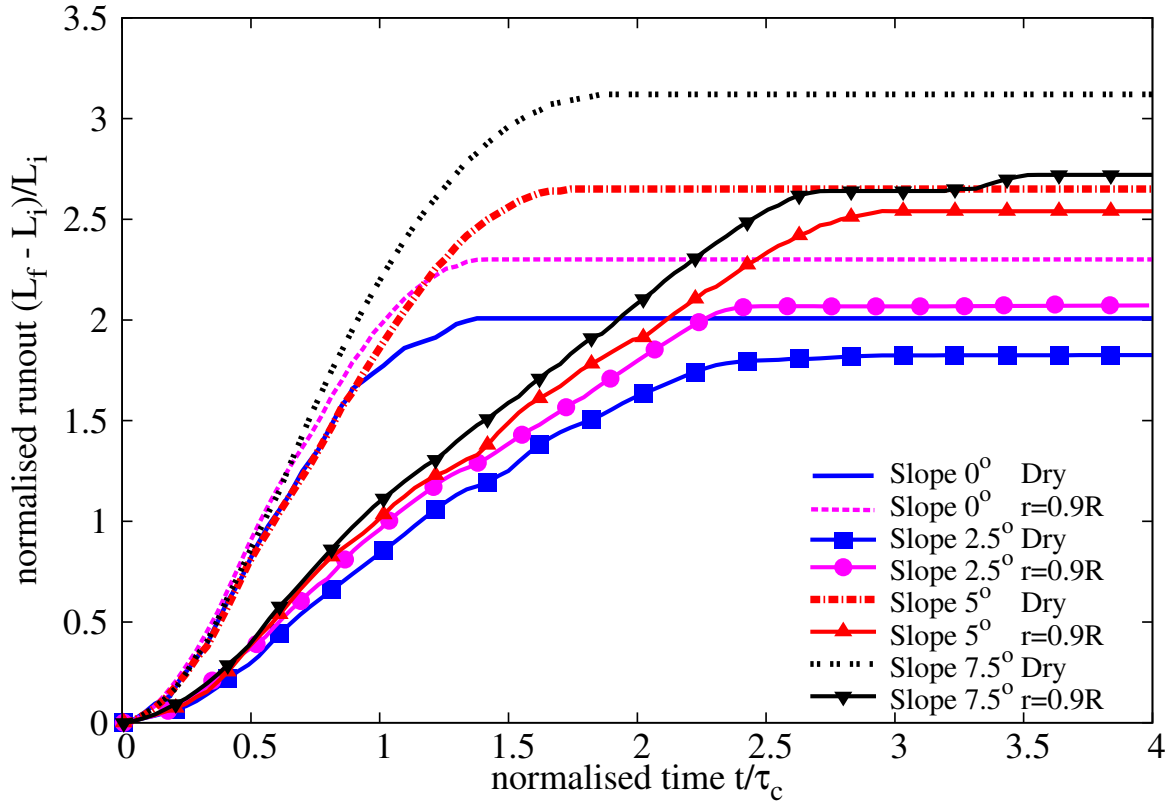


Figure 6.2 Evolution of run-out with time (dense)

on the run-out behaviour, a dense sand column (initial packing density,  $\Phi = 83\%$ ) and a loose sand column ( $\Phi = 79\%$ ) were used. The granular columns collapse and flow down slopes of varying inclinations ( $2.5^\circ$ ,  $5^\circ$  and  $7.5^\circ$ ).

The evolution of run-out distances for a dense sand column with time in dry and submerged conditions for varying slope inclinations are presented in figure 6.2. The run-out distance is longer in submerged condition than the dry condition for a flow on a horizontal surface. However, with increase in the slope angle the run-out in the fluid decreases.

Dense granular columns in fluid take a longer time to collapse and flow, due to the development of large negative pore-pressures, as the dense granular material dilates during the initial phase of the flow. The morphology of dense granular flows down slopes of varying inclinations at the critical time ( $t = \tau_c = \sqrt{H/g}$ , when the flow is fully mobilised) are shown in figure 6.4.

It can be seen that the viscous drag on the dense column tend to predominate over the influence of hydroplaning on the run-out behaviour. This influence can be observed in the smaller peak kinetic energy for granular column in fluid compared to it's dry counterpart (see Figure 6.3). With increase in slope angle, the volume of material that dilates increases. This results in large negative pore pressures and more viscous drag on the granular material.

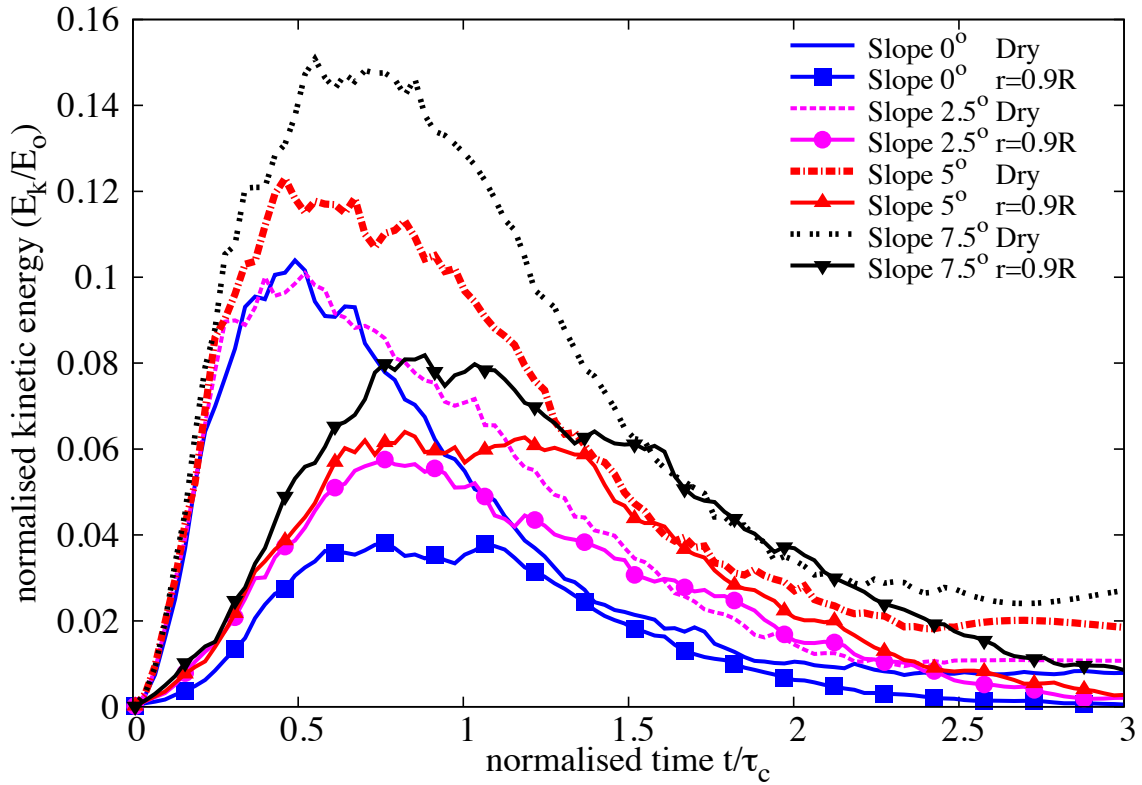
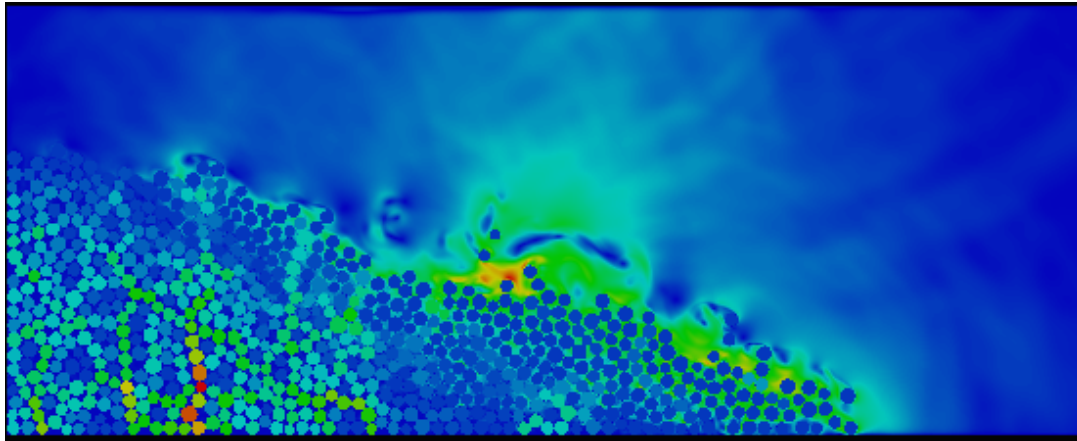


Figure 6.3 Evolution of Kinetic Energy with time (dense case)

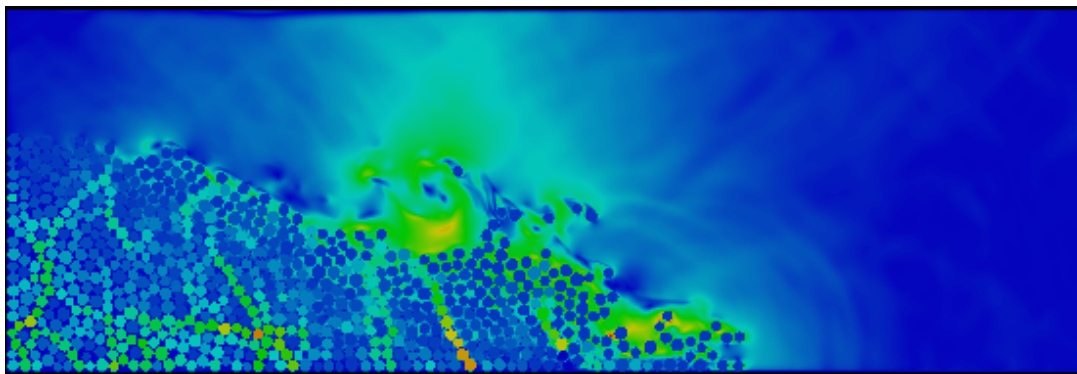
Hence, the difference in the run-out between the dry and the submerged condition, for a dense granular assembly, increases with increase in the slope angle.

In contrast to the dense granular columns, the loose granular columns (relative density  $I_D = 30\%$ ) show longer run-out distance in immersed conditions (see Figure 6.5). The run-out distance in fluid increases with increase in the slope angle compared to the dry cases. Loose granular material tends to entrain more water at the base of the flow front, creating a lubricating surface, which causes longer run-out distance (see Figure 6.6). The hydroplaning effect causes an increase in the velocity the loose condition in comparison with the dense condition (see Figure 6.7).

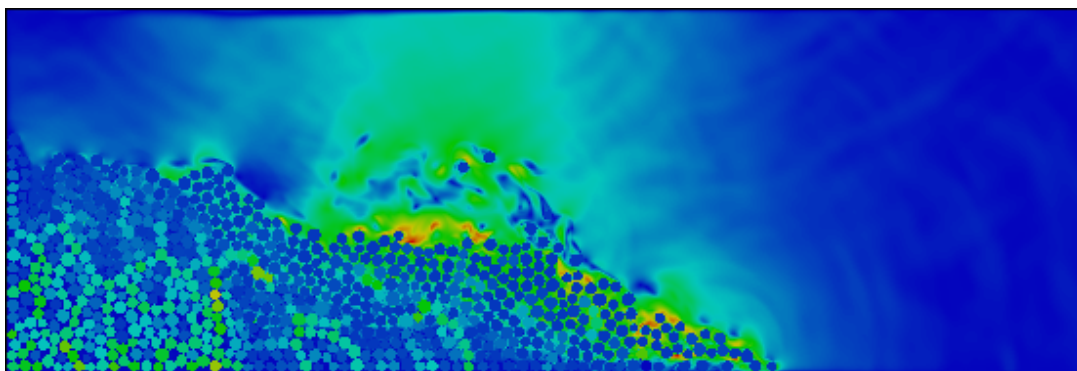
The evolution of packing density (see Figure 6.8) shows that dense and the loose conditions reach similar packing density. This indicates that the dense granular column dilates more and is susceptible to higher viscous drag forces. Whereas in the loose condition, a positive pore-pressure is observed at the base of the flow, indicating entrainment of water at the base, i.e. hydroplaning resulting in longer run-out distance.



(a) Slope 2.5



(b) Slope 5.0



(c) Slope 7.5

Figure 6.4 Flow morphology at critical time for different slope angles (dense)



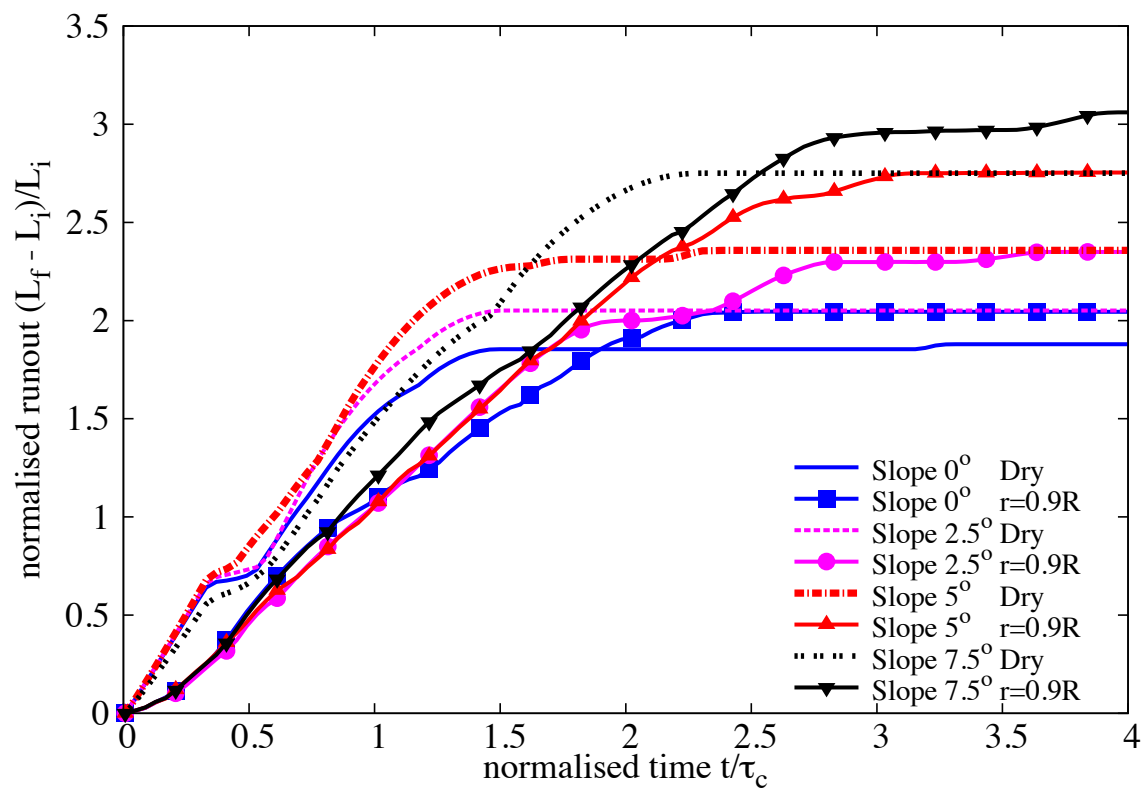
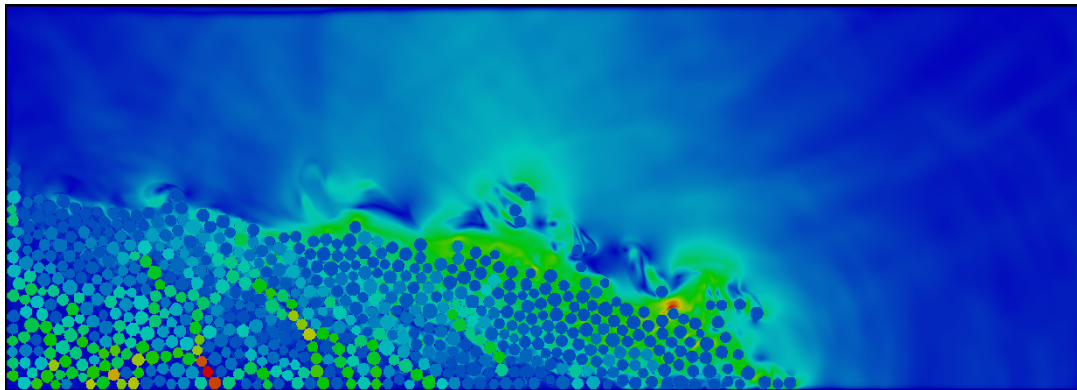
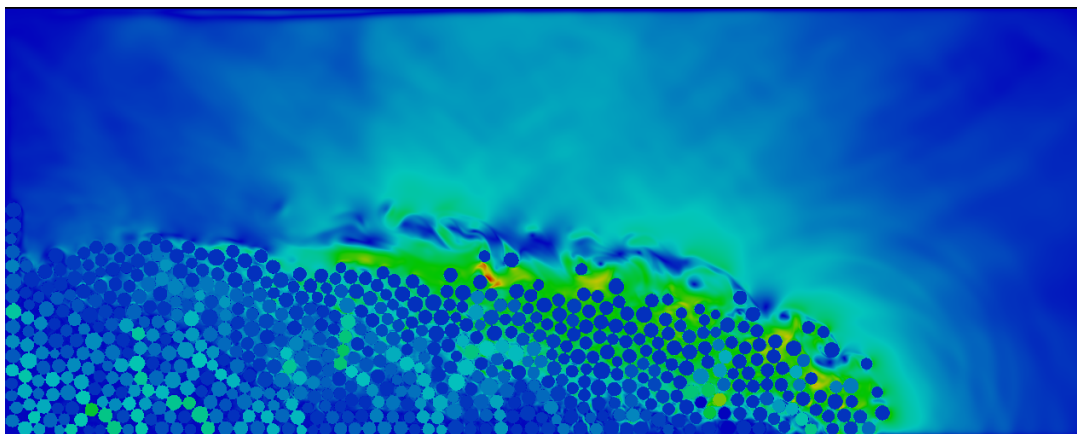


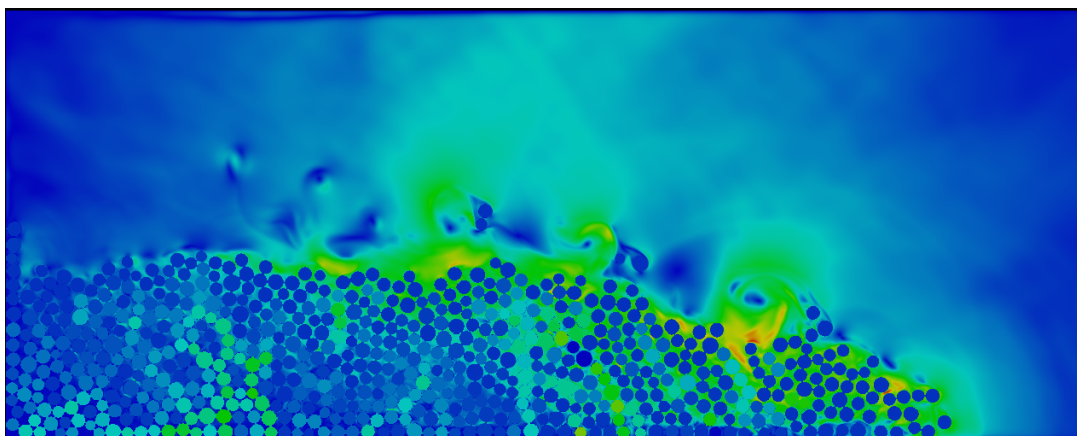
Figure 6.5 Evolution of run-out with time (loose)



(a) Slope 2.5



(b) Slope 5.0



(c) Slope 7.5

Figure 6.6 Flow morphology at critical time for different slope angles (loose)

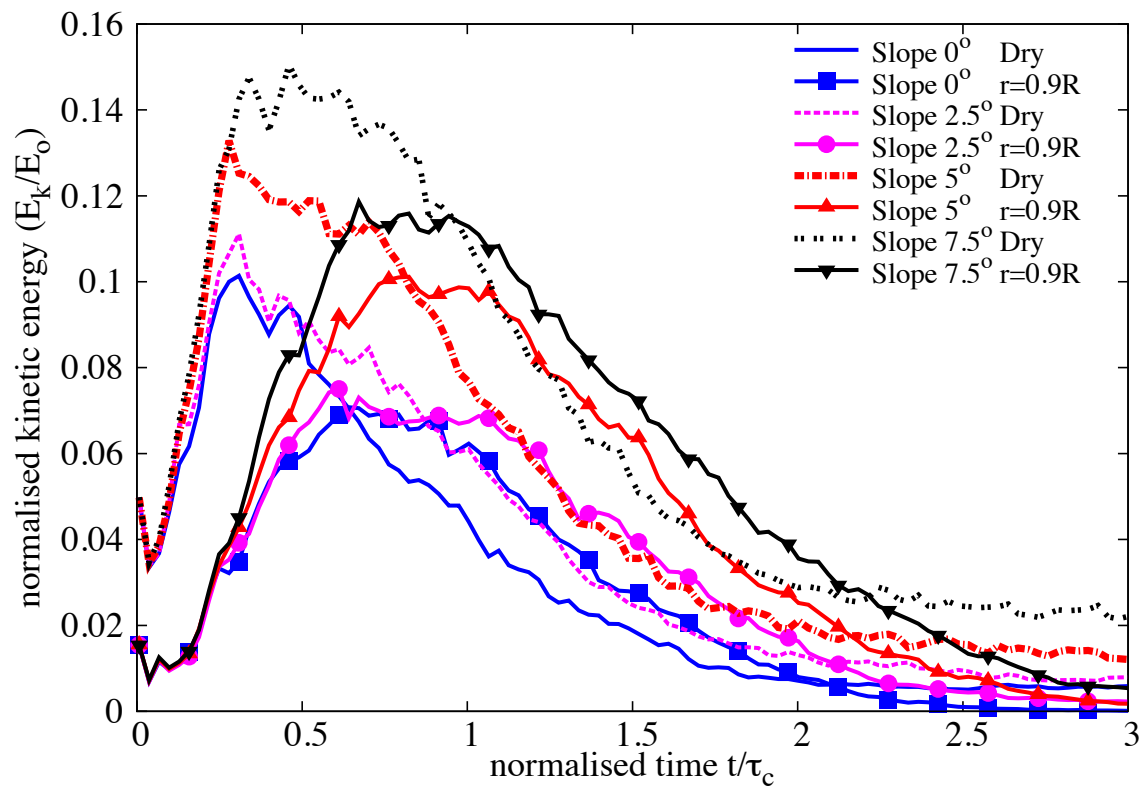


Figure 6.7 Evolution of Kinetic Energy with time (loose)

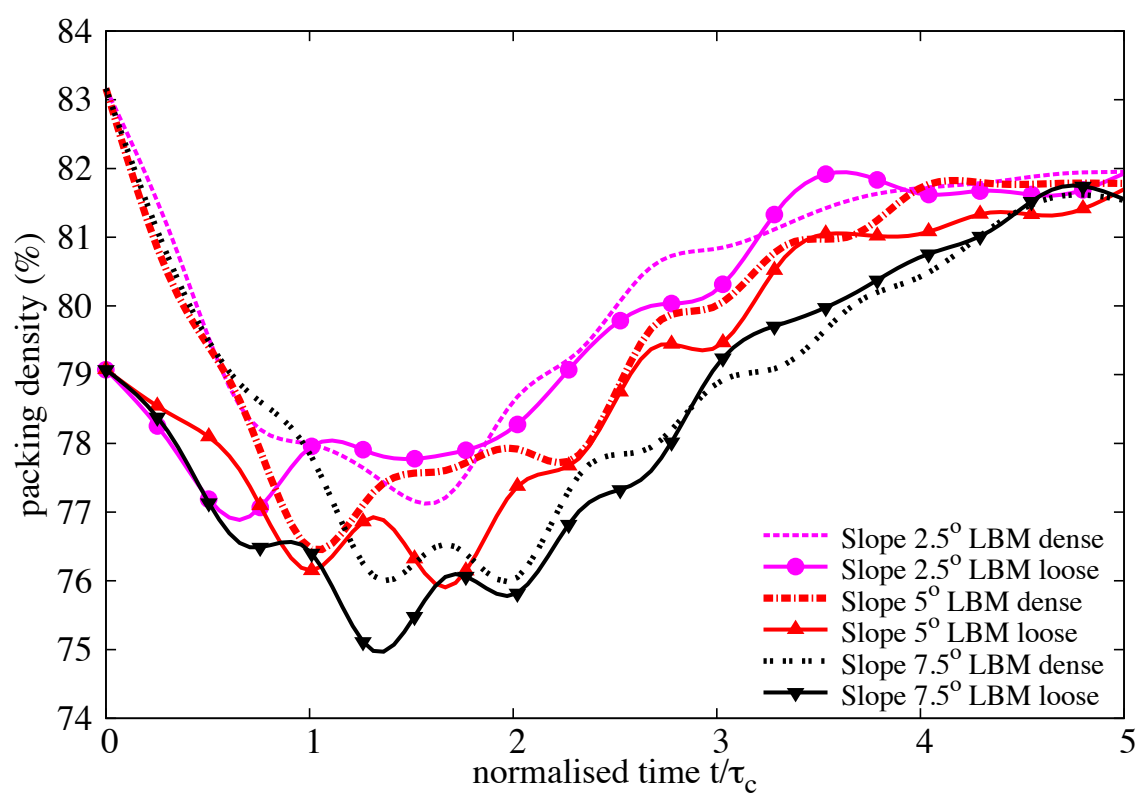


Figure 6.8 Evolution of packing density with time

### 6.3.2 Effect of permeability

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In DEM, the grain – grain interaction is described based on the overlap between the grains at the contact surface. In a 3D granular assembly, the pore spaces between grains are interconnected. Whereas in a 2-D assembly, the grains are in contact with each other that result in a non-interconnected pore-fluid space. This causes a no flow condition in a 2-D case. In order to overcome this difficulty, a reduction in radius is assumed only during the LBM computation phase (fluid and fluid – solid interaction). The reduction in radius allows interconnected pore space through which the surrounding fluid can flow. This technique has no effect on the grain – grain interactions computed using DEM. See [Kumar et al. \(2012\)](#) for more details about the relationship between reduction in radius and permeability of the granular assembly.

For a slope angle of  $5^\circ$ , the hydrodynamic radius of the loosely packed grains was varied from  $r = 0.7R$  (high permeability),  $0.75R$ ,  $0.8R$ ,  $0.85R$  to  $0.9R$  (low permeability). The run-out distance is found to increase with decrease in the permeability of the granular assembly (see Figure 6.9). The run-out distance for high permeable conditions ( $r = 0.7R - 0.8R$ ) were lower than their dry counterparts. Although, decrease in permeability resulted in an increase in the run-out distance, no significant change in the run-out behaviour was observed for a hydrodynamic radius of up to  $0.8R$ .

With further decrease in permeability ( $r = 0.85R$  and  $0.9R$ ), the run-out distance in the fluid was greater than the run-out observed in the dry condition. At very low permeability ( $r = 0.9R$ ), granular material started to entrain more water at the base, which causes a reduction in the effective stress accompanied by a lubrication effect on the flowing granular media. This can be seen as a significant increase in the peak kinetic energy and the duration of the peak energy, in comparison with dry and high permeable conditions (see Figure 6.11).

The permeability of the granular column did not have an influence on the evolution of height during the flow. However, dry granular column tends to collapse more than the immersed granular column (see Figure 6.10).

Positive pore-pressure generation at the base of the flow was observed for low permeable conditions. Inspection of the local packing density showed entrainment of water at the base of the flow, which can also be observed by the steep decrease in the packing density (see Figure 6.12) for the very low permeability condition ( $r = 0.9R$ ). At the end of the flow ( $t \geq 3 \times \tau_c$ ), the excess pore-pressure dissipates and the granular material, irrespective of their permeability, reaches almost the same packing density.

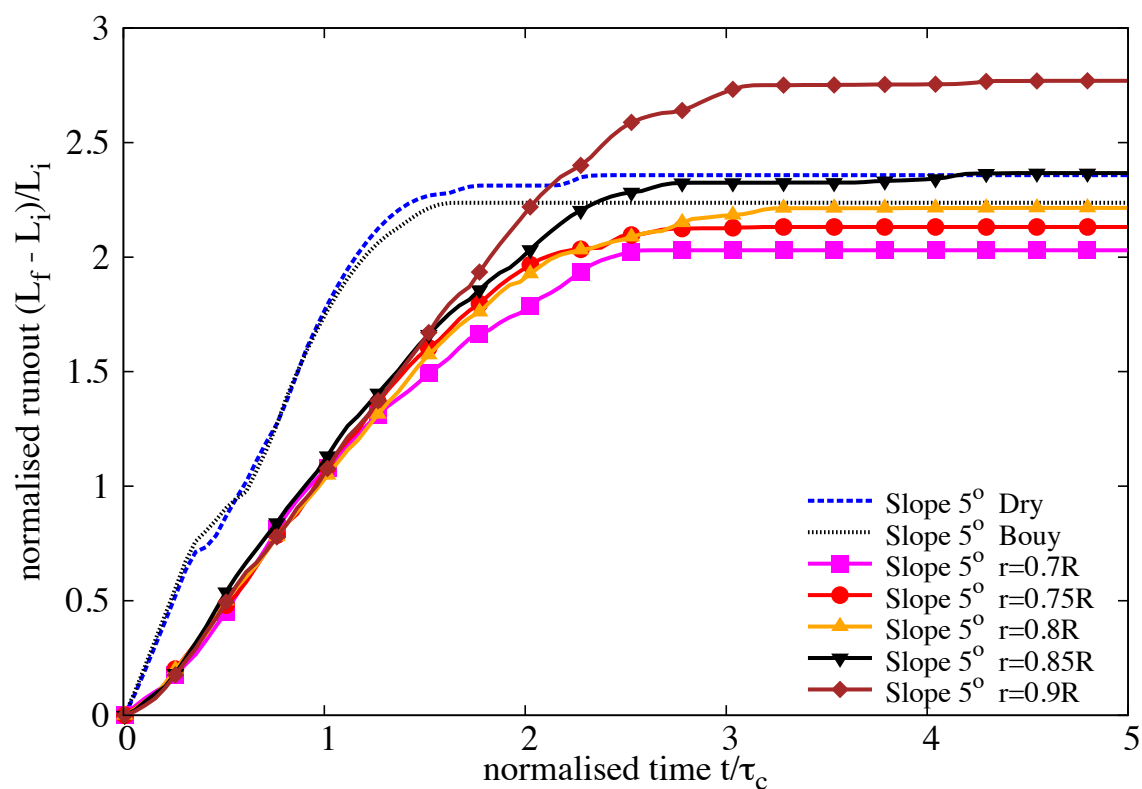


Figure 6.9 Evolution of run-out with time for different permeability (loose slope  $5^\circ$ )

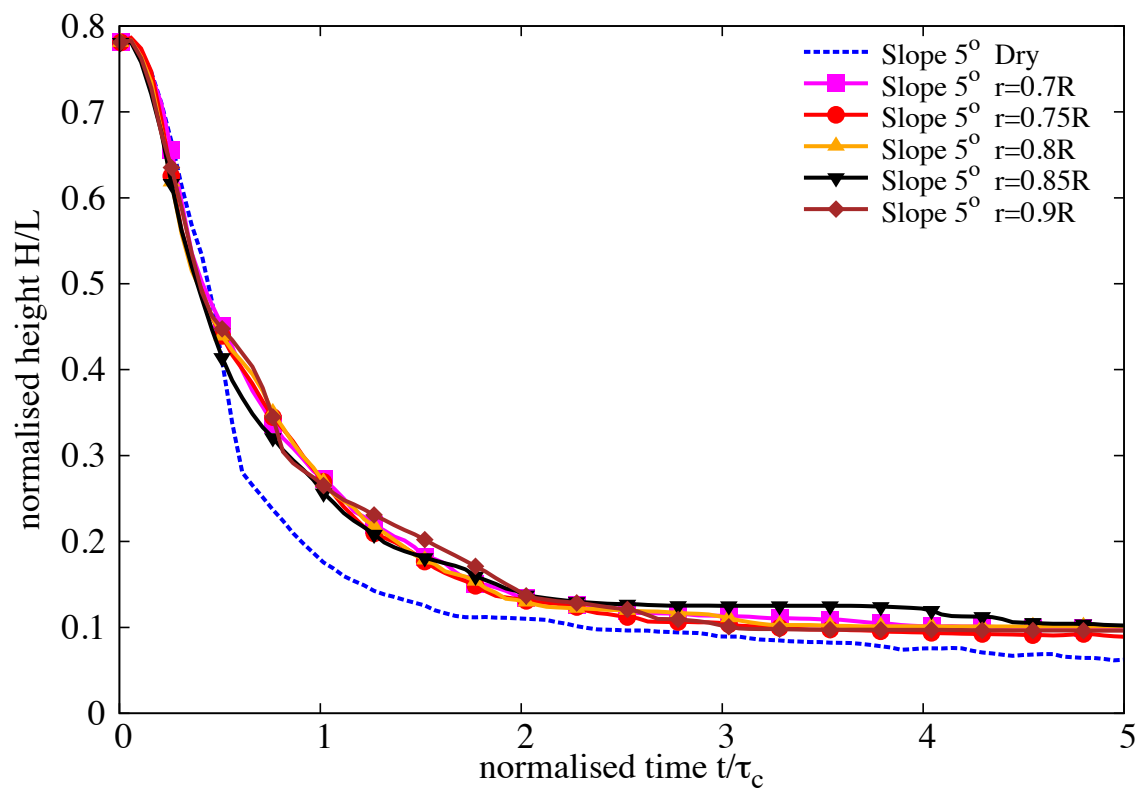


Figure 6.10 Evolution of height with time for different permeability (loose slope  $5^\circ$ )

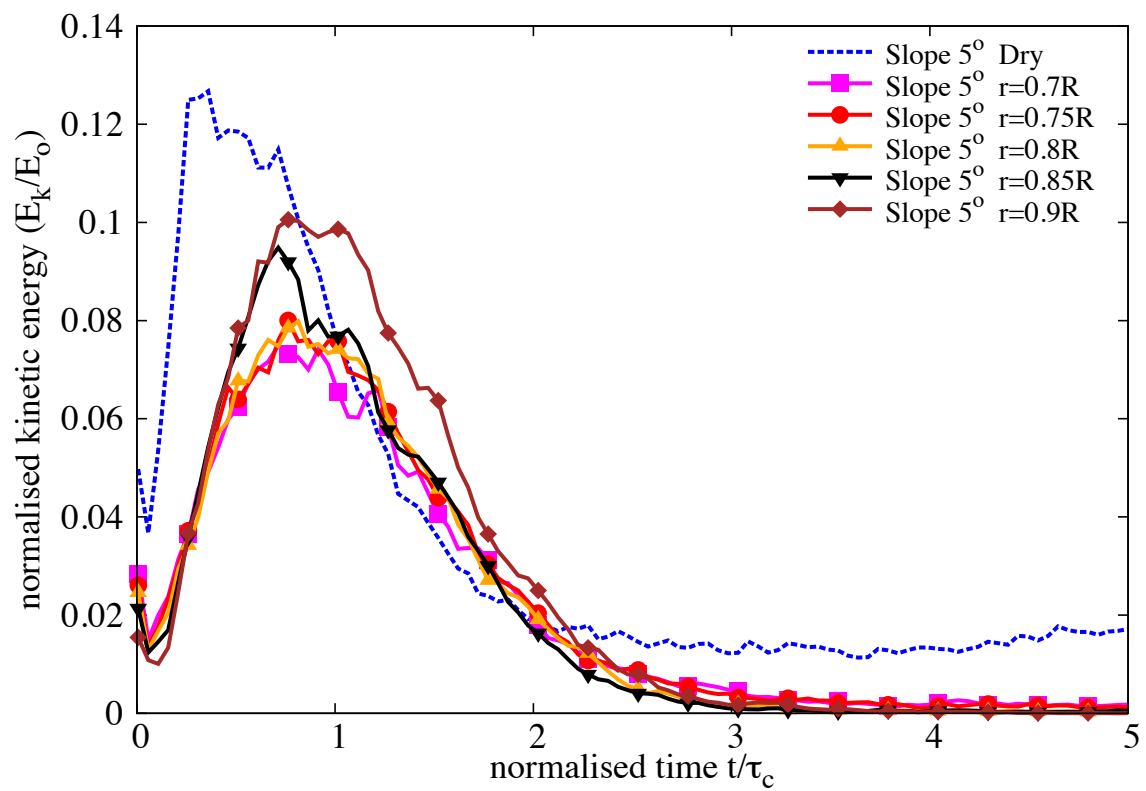


Figure 6.11 Evolution of Kinetic Energy with time for different permeability (loose slope  $5^\circ$ )



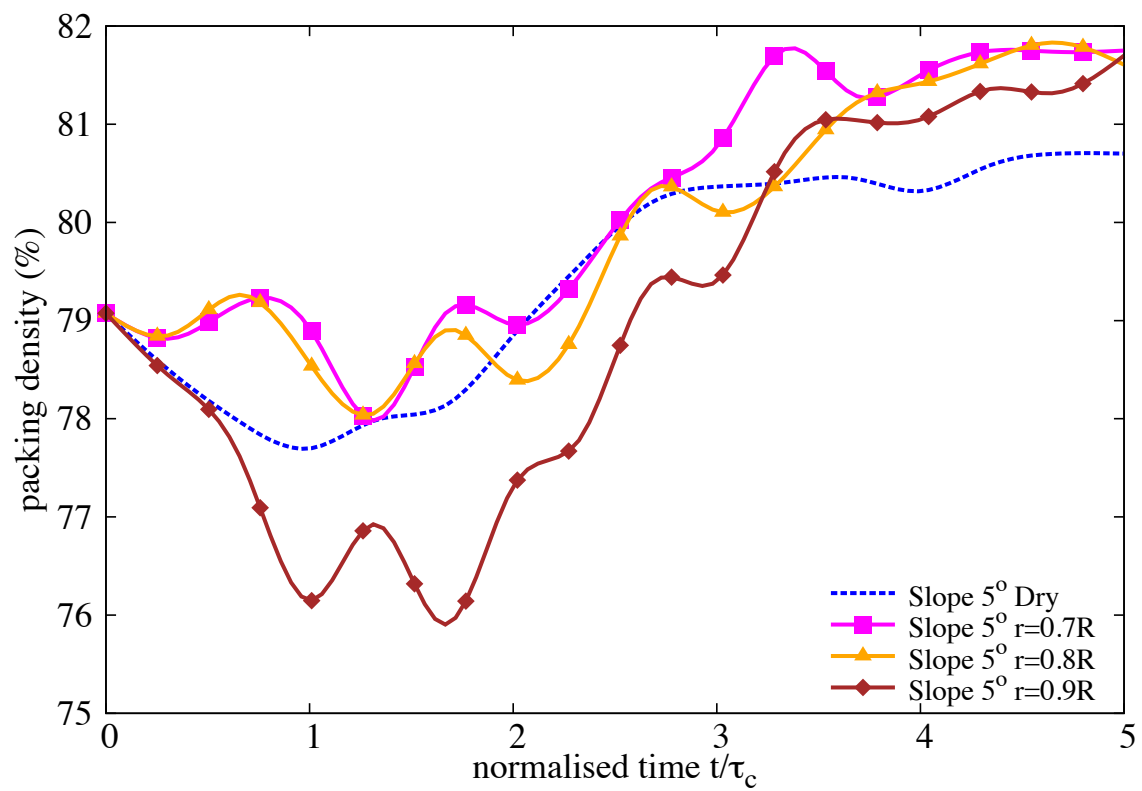


Figure 6.12 Evolution of packing density with time for different permeability (loose slope 5°)

### 6.3.3 Summary

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Two-dimensional LB-DEM simulations were performed to understand the behaviour of submarine granular flows. Unlike dry granular collapse, the run-out behaviour in fluid is dictated by the initial volume fraction. Granular columns with loose packing tend to flow longer in comparison to dense columns, due to entrainment of water at the base resulting in lubrication. The loose column when it starts flowing expands and ejects liquid, leading to a partial fluidization of the material. However, with increase in the slope angle, the run-out in fluid is influenced by the viscous drag on the granular materials. The run-out distance in fluid increases with decrease in permeability. More research work is required to characterise the flow behaviour of granular materials, especially in submerged conditions.

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