Chapter 7

Conclusions and recommendations for future work

7.1 Introduction

This PhD has made advances in several aspects of multi-scale modelling of granular flows and understanding the complex rheology of dry and submerged granular flows. The significant contributions of this PhD are summarised in this chapter.

7.1.1 Multi-scale modelling of dry granular flows

The granular flow problem is modelled using material point method, a continuum approach, and at the grain-scale level using discrete element technique. In the present study, a two-dimensional DEM code is developed in C++ to study the micro-scale rheology of dry granular flows. A Verlet-list algorithm is adopted for neighbourhood detection to improve the computational efficiency. A linear-elastic contact model with a frictional contact is used to model rapid granular flows. A sweep-line Voronoi tesselation algorithm is implemented in the present study to extract continuum properties such as packing density from the local grain-scale simulations.

In order to capture the macro-scale response, a template-based three-dimensional C++ Material Point Method code, an Eulerian-Lagrangian approach, developed at the University of Cambridge is modified and extended to study granular flows as a continuum. In the present study, the Generalised Interpolation Material Point GIMP method is implemented to reduce the cell-crossing noise and oscillations observed during large-deformation problem, when using the standard MPM. The three-dimensional MPM code is parallelised to run on multi-core systems, thus improving the computational efficiency. The algorithm of the MPM code is improved to handle multi-body dynamics and interactions. In the present study, constitutive models, such as NorSand and Bingham fluid were implemented in MPM. Various smoothing/averaging techniques, such as BBar approach were implemented. This dissertation includes only those results from two-dimensional plane-strain granular flow problems. The multi-scale tools developed in the present study is used to understand the capability and limitations of continuum approach in modelling dry granular flows.

Granular column collapse

Multi-scale simulation of dry granular flows are performed to capture the local rheology, and to understand the capability and limitations of continuum models in realistic simulation of granular flow dynamics. For short columns, the run-out distance is found to be proportional to the granular mass above the failure surface. The spreading results from a Coulomb-like failure of the edges. The continuum approach, using a simple frictional dissipation model, is able to capture the flow dynamics of short columns. However, the collapse of tall columns is characterised by an initial collisional regime and a power-law dependence between the run-out and the initial aspect ratio of the granular column is observed. The energy evolution study reveals that the lack of collisional dissipation mechanism in the MPM simulations results in a substantially longer run-out distance for large aspect ratio columns. This shows that continuum approach using frictional laws are able to capture the flow kinematics at small aspect ratios, which is characterised by an inertial number I less than 0.2 indicating a dense granular flow. However, continuum approach like MPM are unable to precisely describe the flow dynamics of tall columns, which is characterised by an initial collisional regime ($I > 0.2$). DEM studies on the role of initial material properties reveal that the initial packing fraction and the distribution of kinetic energy in the system have a significant influence on the flow kinematics and the run-out behaviour. For the same material, a dense granular packing results in a longer run-out distance in comparison to the initially loose granular column. Hence it is important to consider macroscopic parameters like packing fraction, which are due to meso-scale grain arrangements, when modelling the granular system as a continuum.

Granular slopes subjected to impact loading

The ability of MPM in modelling transient flows that does not involve collision is further investigated. The distribution of kinetic energy in the granular mass is found to have a significant effect on the flow kinematics. In the present study, a multi-scale analyses of a granular slope subjected to impact velocities reveals a power-law dependence of the run-out distance and time as a function of the input energy with non-trivial exponents. The power-law behaviour is found to be a generic feature of granular dynamics. Two different regimes are observed depending on the input energy. The low energy regime reflects mainly the destabilisation of the pile, with a run-out time independent of the input energy. The high energy regime involves spreading dynamics, which is characterised by a decay time that is defined as the time required for the input energy to decline by a factor $1/2$. MPM is successfully able to simulate the transient evolution with a single input parameter, the macroscopic friction angle. This study exemplifies the suitability of MPM, as a continuum approach, in modelling large-deformation granular flow dynamics and opens the possibility of realistic simulation of geological-scale flows on complex topographies.

The distribution of the kinetic energy in the system is found to have a significant influence in the low energy regime, where a large fraction of the input energy is consumed in the destabilisation process. However at higher input energy, where most of the energy is dissipated during the spreading phase, the run-out distance has a weak dependence on the distribution of velocity in the granular mass. The material characteristics of the granular slope affect the constant of proportionality and not the exponent in the power-law relation between the run-out and the input energy.

7.1.2 Granular flows in fluid

A two-dimensional coupled lattice Boltzmann - DEM technique is developed in C++ to understand the local rheology of granular flows in fluid. A multi-relaxation time LBM approach is implemented in the present study to ensure numerical stability. The coupled LBM–DEM technique offers the possibility to capture the intricate micro-scale effects such as the hydrodynamic instabilities. Coupled LBM-DEM involves modelling interactions of a few thousand soil grains with a few million fluid nodes. Hence, in the present study the LBM-DEM approach is implemented in the General Purpose Graphical Processing Units. The GPGPU implementation of the coupled LBM – DEM technique offers the capability to model large scale fluid – grain systems, which are otherwise impossible to simulate using conventional computational techniques. In the present study, simulations involving up to 5000 soil grains interacting with 9 million LBM fluid nodes are modelled. Efficient data transfer mechanisms that achieves coalesced global memory ensures that the GPGPU implementation scales linearly with the domain size. Granular flows in fluid involves soil grains interacting with fluid resulting in formation of turbulent vortices. In order to model the turbulent nature of granular flows, the LBM-MRT technique is coupled with the Smagorinsky turbulent model.

7.2 Recommendations for future research

This research work involves the following stages: (1) understanding the limits of the continuum and the discrete-element approaches in modelling the granular flow, and investigating the influence of various microscopic parameters on the macroscopic flow behaviour, (2) understanding the differences in the mechanism of the granular flows in dry and submerged conditions, and (3) Modelling the granular flow behaviour using the $\mu(I)$ rheology for dry and submerged flow conditions.

7.2.1 Multi-scale simulations of granular flows

The continuum and discrete-element simulations of the dry and the submerged granular flows will be performed to understand the differences in their flow mechanism. Multi-scale analyses of granular flows enable us to understand the limitations of the continuum approach in modelling large deformation granular flow problems, and help us to identify the micro-scale parameters responsible for the complex macroscopic behaviour. Multi-scale modelling of the collapse of a granular columns on a horizontal surface have been performed. Continuum simulations accurately predict the granular flow behaviour for columns with smaller aspect ratios, however they fail to capture the dynamics of the flow for columns with larger aspect ratios.

In order to understand the difference in the flow dynamics with increasing aspect ratios, further analyses will be performed to study the mechanism of energy dissipation in the particle scale, i.e. the evolution of kinetic energy and potential energy with time. A simple mathematical relationship based on the initial potential energy of the grains lying above the failure surface is being developed to describe the flow dynamics of the granular column collapse problem with different aspect ratios. The relationship will enable us to understand the variation in the flow dynamics in the continuum and the particle-scale. The continuum description of granular column collapse showed non-physical behaviour near the foot of the column, further analyses will be carried out to understand the effect of interface properties on the run-out distance. Further details on the continuum modelling of granular flow are presented in the next section. Micro-scale parameters influencing the flow dynamics will be identified and the influence of these parameters on the deposit morphology will be investigated. Multi-scale analysis of large deformation flow problem such as flow of dry granular materials down an inclined flume will be performed. This analysis will provide an insight on the limits of the continuum approach in modelling large deformation problems, which involve large shear-rates. The results from the analysis will be compared with the experimental results of Denlinger and Iverson (2001) on miniature flume experiments. The influence of parameters, such as particle size, density, packing and dilatancy, on the flow dynamics will be explored. These studies will be useful in describing the granular flow behaviour using the $\mu(I)$ rheology.

In order to understand the differences in the mechanism between the dry and the submerged granular flows, multi-scale analysis of granular flows in fluid will be performed. In particle-scale approach, the Discrete Element Method technique coupled with Lattice Boltzmann approach will be adopted to study the collapse of granular columns in fluids. Dynamic fluid-coupling in the Material Point Method will be developed to study the behaviour of granular flows in fluids. The numerical simulations will be verified with the experimental results of Cassar et al. (2005) on immersed granular flows. The effect of packing density of granular material, frictional parameters and the viscosity of the fluid on the flow dynamics and the phase-transition behaviour will also be investigated. The influence of the fluid viscosity on the flow behaviour will also be investigated. The variation in the flow dynamic and the deposit morphology for dry and submerged granular flows will also be analysed. The parameters that cause a change in the flow dynamic between the dry and the submerged flow conditions will be used in extending the $\mu(I)$ rheology for submerged granular flows.

7.2.2 Developments in Material Point Method

The present MPM code is capable of solving the granular flow problems using the Mohr-Coulomb constitutive model. It is also capable of solving seepage problems with static fluid-solid interface. In the present study, the Material Point Method will be extended to solve 3D problems and it will be implemented into a standard framework, similar to the Finite Element framework developed in the University of Cambridge. Constitutive models such as Nor-Sand and $\mu(I)$ rheology will be implemented to model the dense granular flows. Modified Nor-Sand constitutive model (Robert, 2010) implemented in the present study will be validated by performing element testing and the results will be verified with the results of Jefferies and Shuttle (2005). The current MPM code is capable of simulating only small deformation problems, however special attention is required in modelling large deformation problems, especially those involving two-phase systems. Objective stress rate such as Jaumann stress rate has been implemented to study large deformation problems. The dynamic re-meshing technique (Shin, 2010) will be implemented to efficiently solve large deformation problems. The dynamic meshing approach is useful for problems involving motion of a finite size body in unbounded domains, in which the extent of material run-out and the deformation is unknown a priori. The approach involves searching for cells that only contain material points, thereby avoiding unnecessary storage and computation.

The present fluid coupling algorithm in the MPM involves only static boundaries, and conserves the equation of momentum for fluid particles by introducing additional particles in a cell, when the number of particles decreases. A dynamic solid-fluid interface modelling in the Material Point Method will be developed. The approach involves the following steps: identification of the soil particles along the boundary of the current soil domain, obtaining the nodal numbers for the edges of each soil particle and definition of a new boundary (see Figure 7.1). The shape of the boundary is approximated by equivalent rectangular grids, similar to the coupling technique adopted in the discrete-element approach. The procedure is repeated until the entire boundary is defined. This process is repeated for each time step to simulate the dynamic boundary behaviour which is common in the case of granular flows in a fluid.

The MPM code will be extended to include the phase-transition behaviour in a continuum domain for partially fluidized granular flows (Aranson and Tsimring, 2001, 2002; Volfson et al., 2003). The theory is based on the hydrodynamics of the flow, coupled with an order parameter equation, which describes the transition between the flowing and the static components of the granular system. The order parameter is defined as a fraction of static contacts among all contacts between particles. The shear stresses in a partially fluidized granular matter is assumed to have two components: the dynamic part that is proportional to the shear strain rate and the strain-independent or the static part. The ratio of these two parts is a function of the order parameter. The relative magnitude of the static shear stress is controlled by the order parameter which varies from 0 in the liquid phase to 1 in the solid phase.

7.2.3 The I rheology

The $\mu(I)$ rheology is capable of describing the behaviour of dense granular flows. However, it considers the yield strength as an adjustable rheological property, which contradicts the basic understanding that the strength evolves as the debris-flow motion progresses. Also, the $\mu(I)$ rheology uses the Mohr-Coulomb constitutive model to describe the yielding of granular materials. In the present study, the evolution of soil strength with time will be considered using models based on critical state framework. Nor-Sand constitutive model will be implemented in the present study to describe the plastic flow of granular materials. The $\mu(I)$ rheology will be extended to describe the behaviour of granular flows in fluids. In the case of dense granular flows, the parameter I is described as the ratio between the time taken for a particle to fall into the hole, t_{micro}, and the meantime, t_{mean}, which is inversely related to the shear rate. If the velocity of the ambient fluid is low, then the time taken by the particle to fall into a hole, is then controlled by the viscosity of the ambient fluid (Pouliquen et al., 2005). The $\mu(I)$ rheology will be modified to include the effect of fluid viscosity to model granular flows in fluids based on the parameters identified to control the flow dynamics.

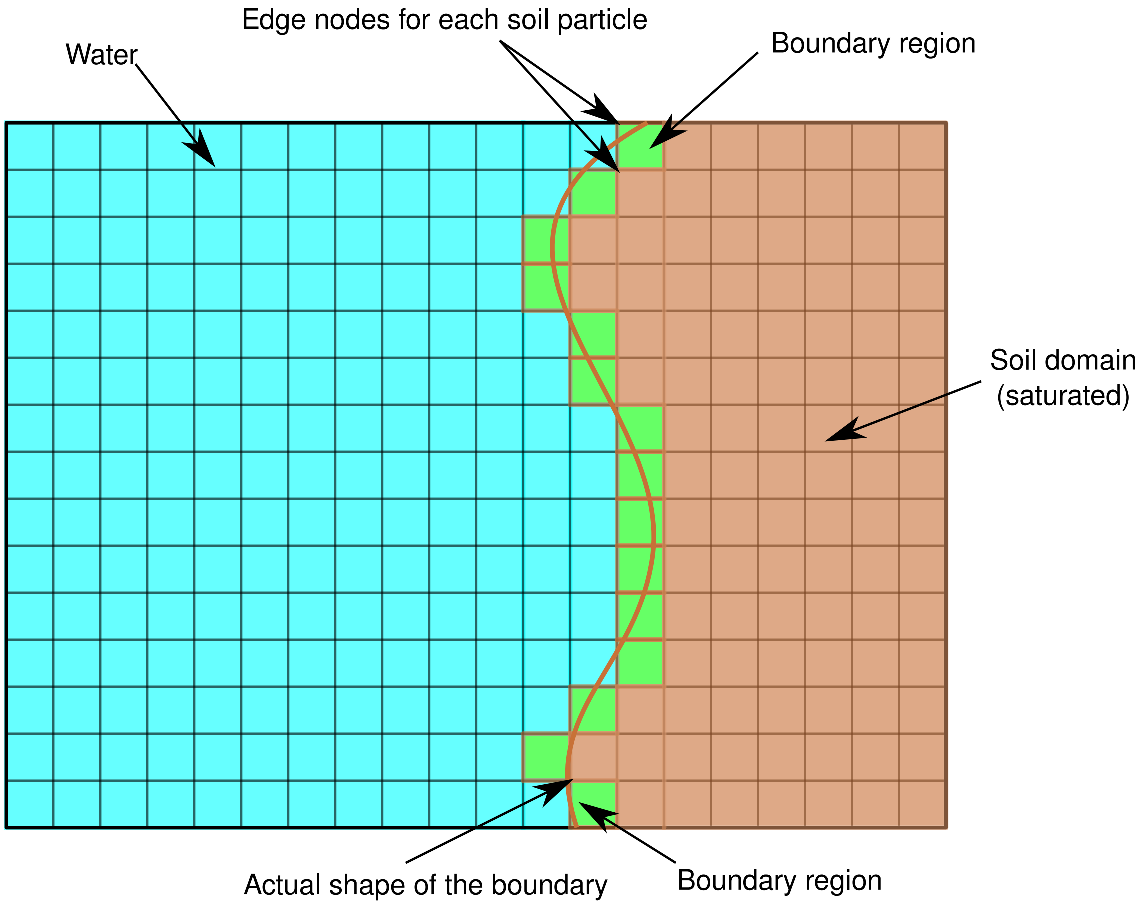

Figure 7.1 Identification of dynamic solid-fluid boundary

7.2.4 Homogenization of granular flows

Granular materials are composed of grains in contact, and are therefore discontinuous and heterogeneous. The macroscopic material properties of these materials are liked to the fabric of the medium. It is interesting to define the behaviour of granular materials at the macroscopic scale from characteristics defined at the local scale. This approach has been widely developed for heterogeneous continuum and is known as the homogenization method. This kind of approach is different from the phenomenological one, in which the constitutive model is derived from the general laws of thermodynamics. These phenomenological models introduce some material parameters whose values are obtained from the simulations performed on representative volume element. The main objective of the homogenization method is to obtain a constitutive relation at the scale of a representative volume element, based the information on the material behaviour at the micro-scale and the micro-structure. For granular media, the micro-scale is generally the grain scale. The scale of the representative volume element is of several orders of magnitude higher. The homogenization process is based on three relations, a localization operator, a local constitutive law and an averaging operator, see Chapter 6. An intrinsic difficulty in the case of granular materials arise from the fact that the variables at the micro-scale and the macro-scale are of different nature. At the micro-scale the material behaviour is described using vectorial variables such as contact forces, grain displacement and grain rotation, where as the macroscopic behaviour law uses tensorial variables (stress tensor, strain tensor). The micro-polar plasticity constitutive formulation (Suiker et al., 2001) that is directly related to micro-scale properties, such as contact stiffness, particle density, particle radius and its micro-structure will be extended to large scale deformation problems which involves loss or gain of contacts between grains. Discrete Element Method simulations provide useful insight on the role of contact forces at micro-scale. The macroscopic stress is related to microscopic contact forces using the virtual work theorem. The microscopic kinematic variables usually considered are the displacement of the centre of mass of the particle and the ration of the particle. The local phenomena occurring at the micro-scale during the deformation of granular material are complex. Different approaches have been proposed in the literature to establish the link between particle-level displacements and macro-scale deformations. The micro-behaviour is converted into a macroscopic model through the conservation of internal work. The local constitutive law gives the value of contact forces and the average over the sample provides the value of the stress tensor.

7.2.5 Slopes subjected to impact

This work may be pursued along two directions: 1) experimental realization of a similar setup with different modes of energy injection and 2) investigating the effect of various particle shapes or the presence of an ambient fluid. Although numerical simulations are generally reliable with realistic results found in the past studies of steady flows, we believe that the transients are more sensitive situations than steady states and the experiments are necessary for checking the validation of the results suggested by the simulations. Provided a convenient method is used for supplying kinetic energy homogeneously into a pile, our configuration is also interesting for the investigation of the behavior of a pile immersed in a viscous fluid.

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