The Digital Image

Image Formation

- Geometry of image formation
  - camera model, etc.

- Physics of image formation
  - light, material, appearance, etc.

Topics Covered

- Basic Camera Model (Image formation)
- Pin-hole camera
- Lens camera
  - exposure, motion blur, DOF
- Coordinate systems:
  - object, world, camera, image plane, pixel (image)

Outline

- What is an image?
- What is a pixel?
- How do we store pixels?

What is an Image?

Image as 2D Signal

- **Signal:** function depending on some variable with physical meaning
- **Image:** continuous function
  - 2 variables: xy - coordinates
  - 3 variables: xy + time (video)
- Brightness is usually the value of the function
- But can be other physical values too: temperature, pressure, depth, …

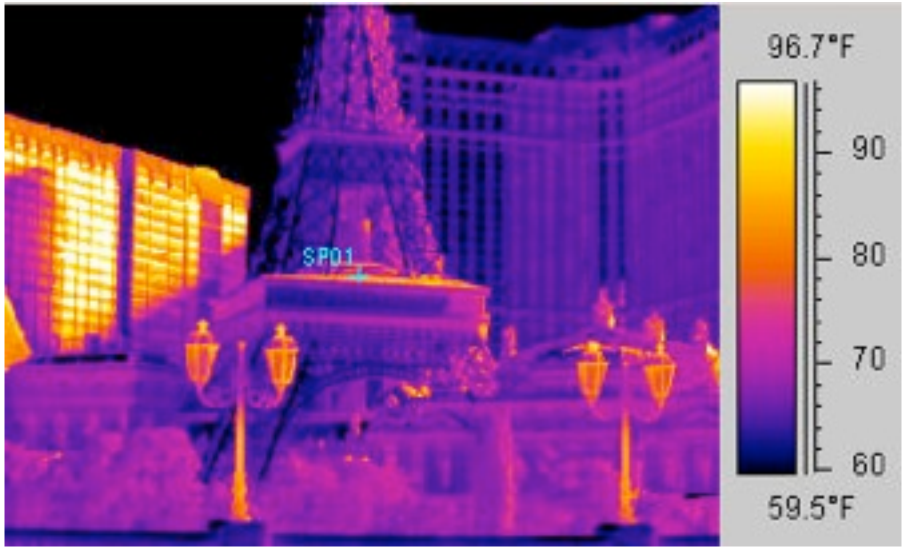
Examples of 2D Signal

Examples of 2D Signal

ultrasound

Examples of 2D Signal

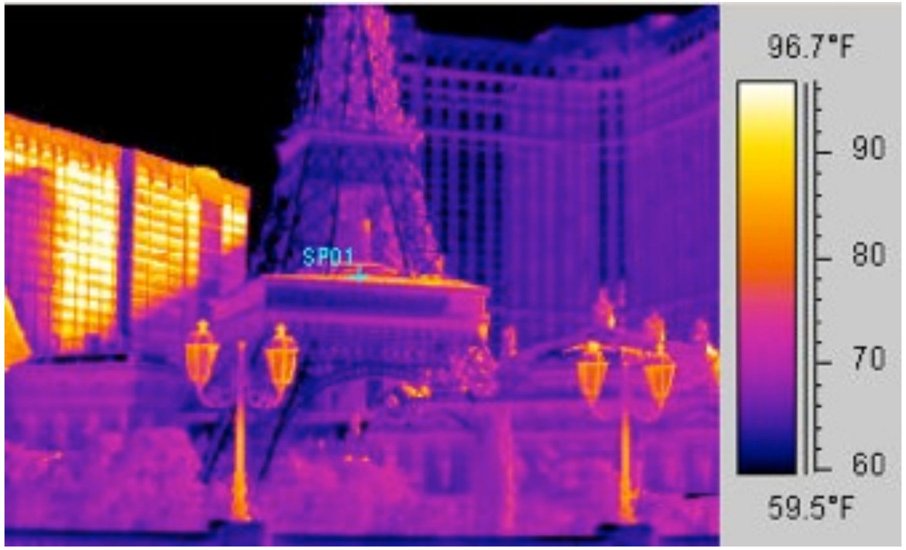
ultrasound



temperature

Examples of 2D Signal

ultrasound

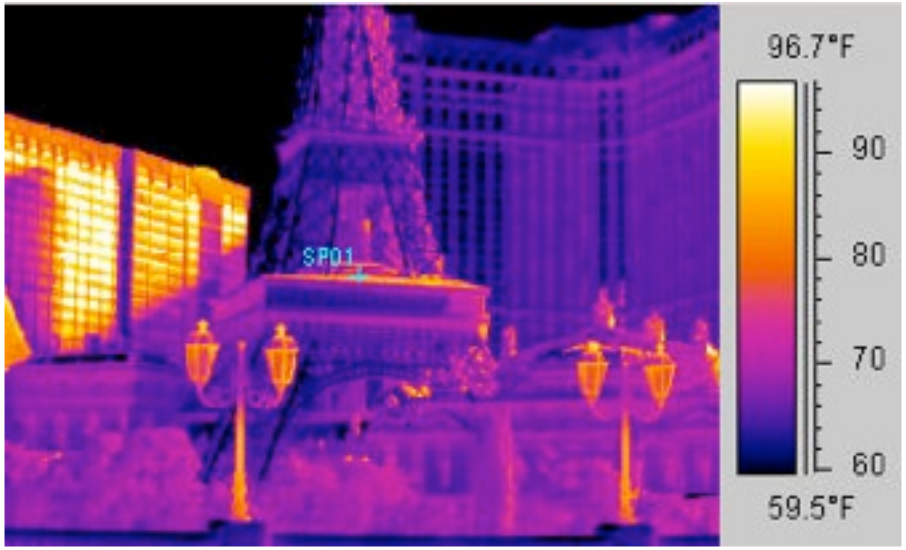


temperature

camera image

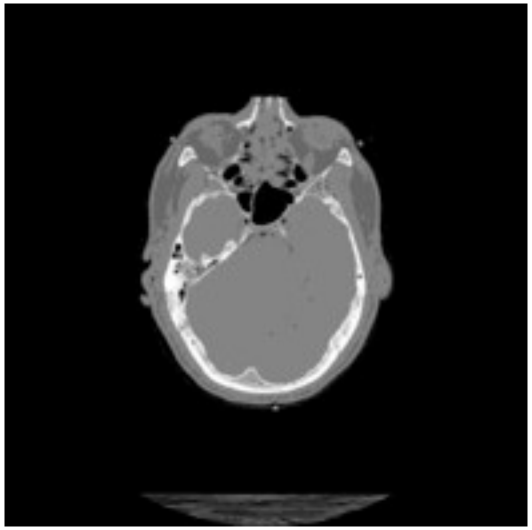
Examples of 2D Signal

ultrasound



temperature

camera image



CT

Image?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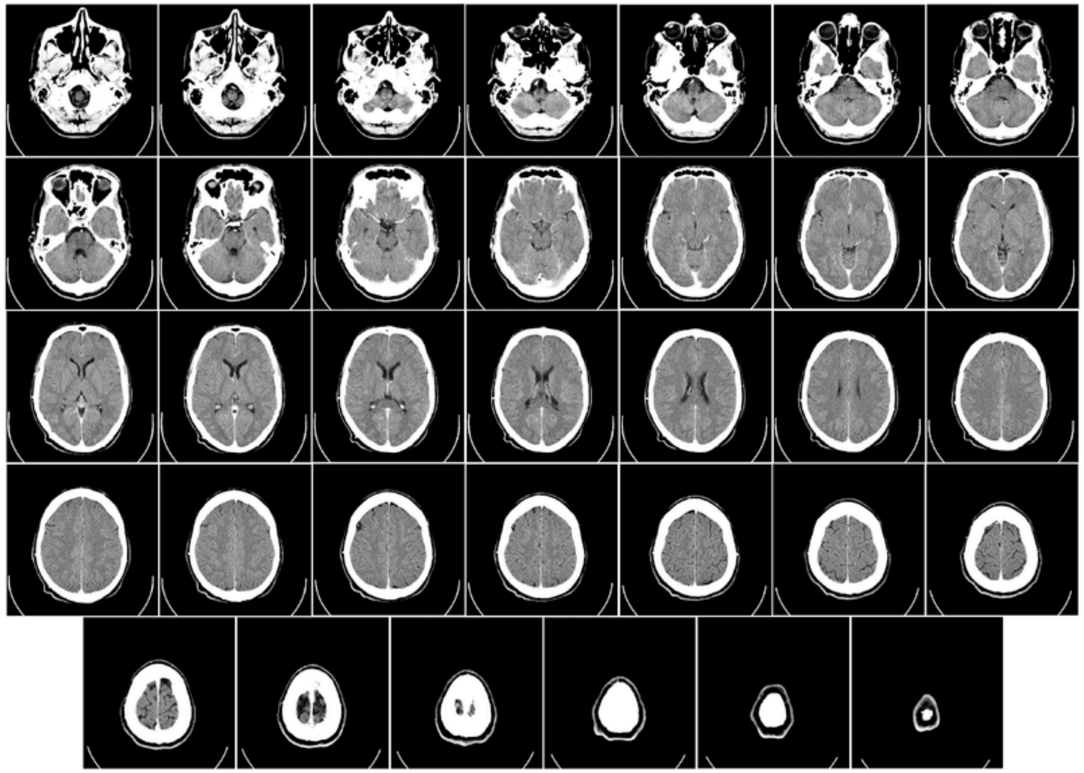
Image?



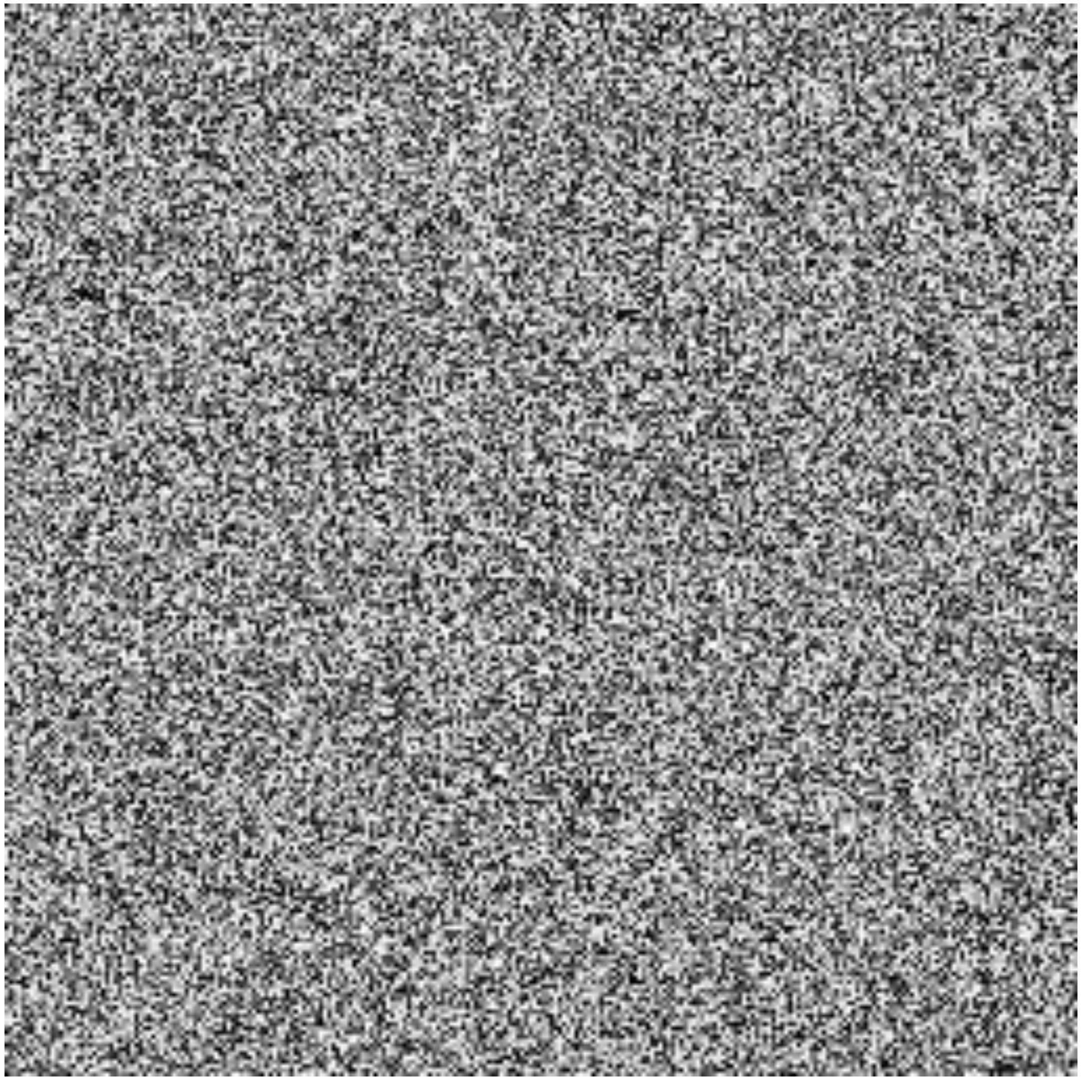
Image?


KEEP YOUR
HELP FIGHT NECK ROT
FUTURAMA™
R.U.R.
REHABILITATE UNSANITARY ROBOTS
ROBOT CLEAN

Microsoft Kinect

Image?

```
>> t=rand(256,256);
>> imshow(t)
```



What is an Image?

- A picture or pattern of a value varying in space and/or time.
- Representation of a function

$$f : \mathbb{R}^n \to S$$

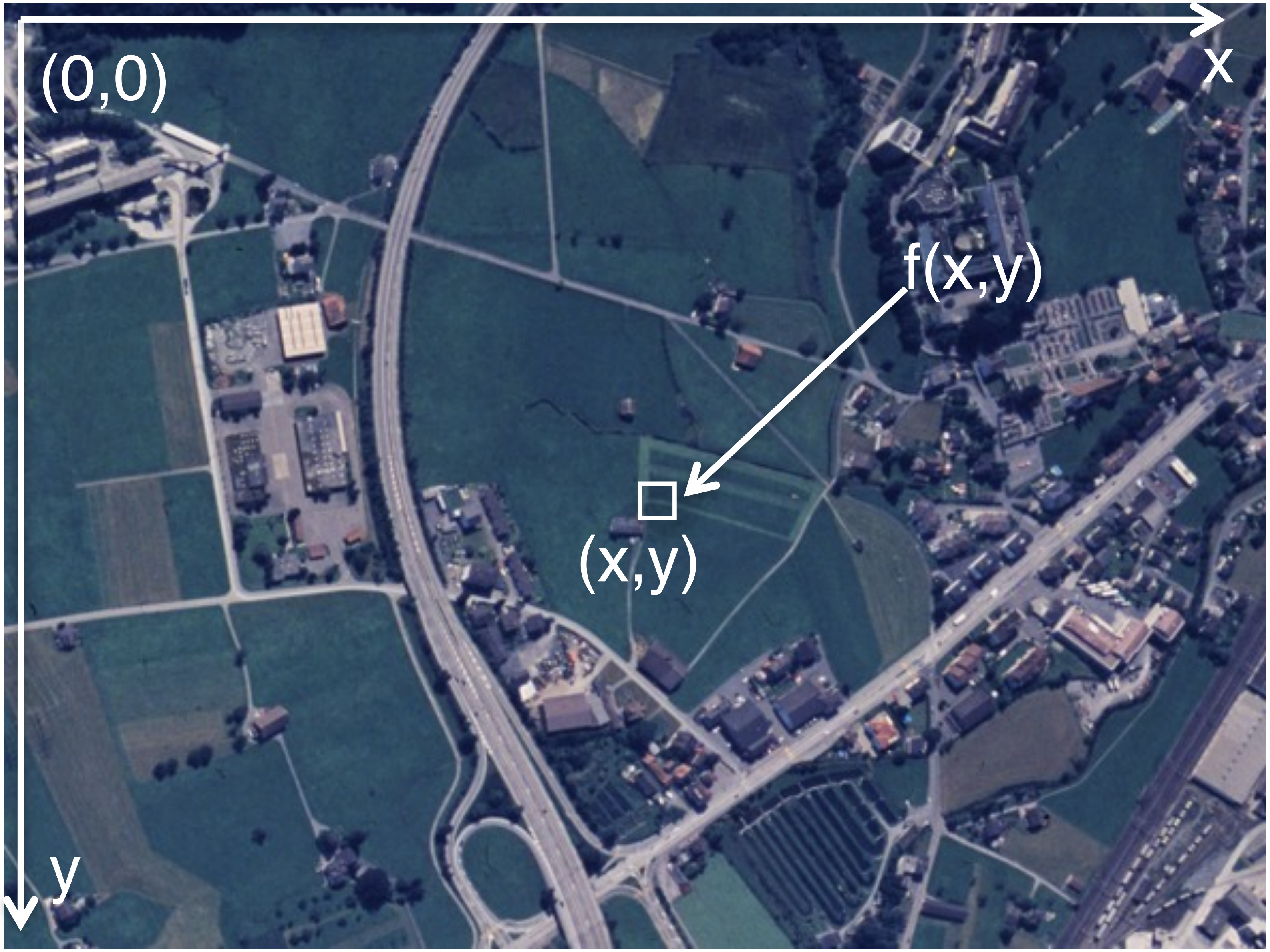
- In digital form, e.g.,

$$I : \{1, \ldots, M\} \times \{1, \ldots, N\} \to S$$

- For greyscale CCD images,

$$n = 2,\ S = \mathbb{R}^+$$

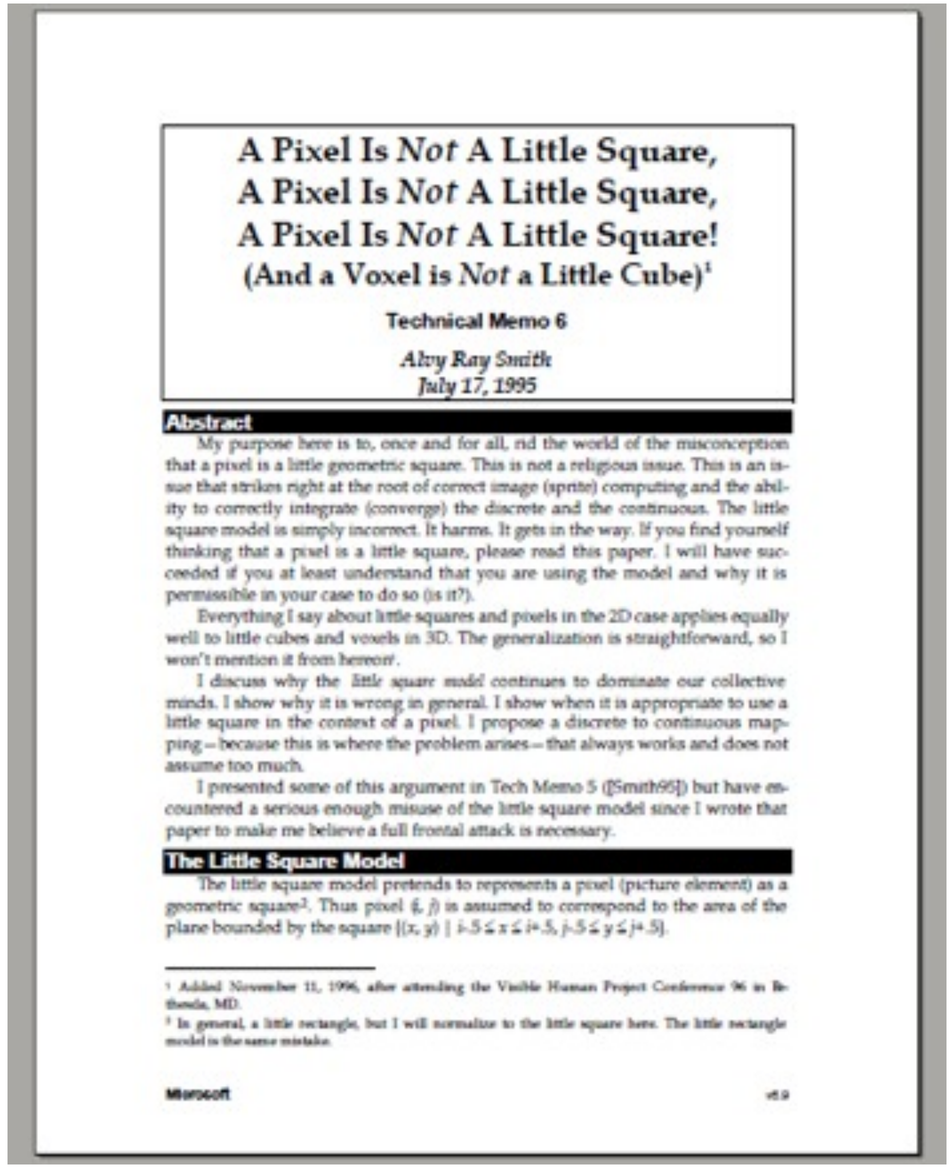
What is a Pix-el?



Not a little Square!

- ***A Pixel Is* Not *A Little Square, A Pixel Is* Not *A Little Square, A Pixel Is* Not *A Little Square! (And a Voxel is* Not *a Little Cube),***
  - **Alvy Ray Smith,**

  MS Tech Memo 6, Jul 17, 1995



A Pixel Is *Not* A Little Square,
A Pixel Is *Not* A Little Square,
A Pixel Is *Not* A Little Square!
(And a Voxel is *Not* a Little Cube)[1]

Technical Memo 6

Alvy Ray Smith
July 17, 1995

Abstract

My purpose here is to, once and for all, rid the world of the misconception that a pixel is a little geometric square. This is not a religious issue. This is an issue that strikes right at the root of correct image (sprite) computing and the ability to correctly integrate (converge) the discrete and the continuous. The little square model is simply incorrect. It harms. It gets in the way. If you find yourself thinking that a pixel is a little square, please read this paper. I will have succeeded if you at least understand that you are using the model and why it is permissible in your case to do so (is it?).

Everything I say about little squares and pixels in the 2D case applies equally well to little cubes and voxels in 3D. The generalization is straightforward, so I won't mention it from hereon.

I discuss why the *little square model* continues to dominate our collective minds. I show why it is wrong in general. I show when it is appropriate to use a little square in the context of a pixel. I propose a discrete to continuous mapping—because this is where the problem arises—that always works and does not assume too much.

I presented some of this argument in Tech Memo 5 ([Smith95]) but have encountered a serious enough misuse of the little square model since I wrote that paper to make me believe a full frontal attack is necessary.

The Little Square Model

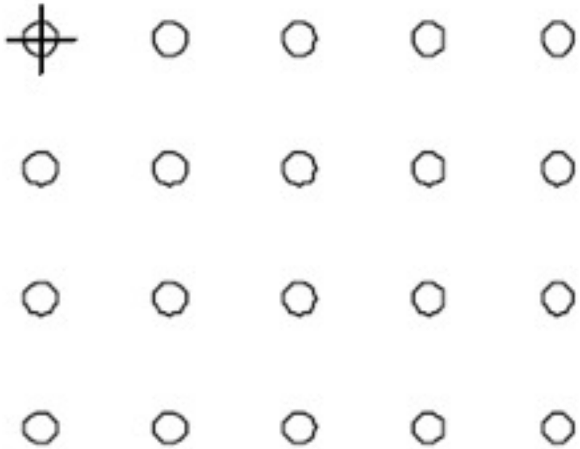
The little square model pretends to represents a pixel (picture element) as a geometric square[2]. Thus pixel (i, j) is assumed to correspond to the area of the plane bounded by the square $\{(x, y) \mid i\text{-}.5 \le x \le i\text{+}.5, j\text{-}.5 \le y \le j\text{+}.5\}$.

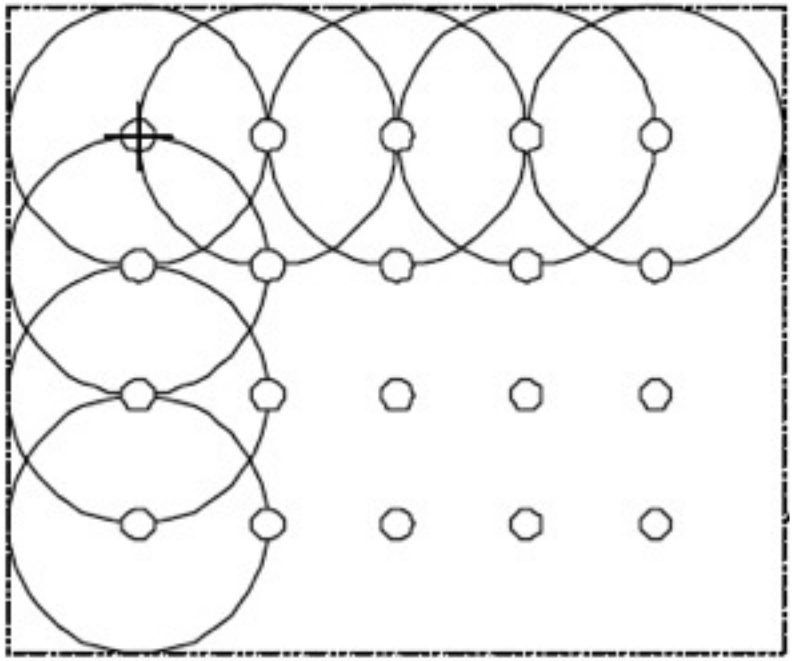
[1] Added November 11, 1996, after attending the Visible Human Project Conference 96 in Bethesda, MD.

[2] In general, a little rectangle, but I will normalize to the little square here. The little rectangle model is the same mistake.

Microsoft v5.9

Not a Little Square!

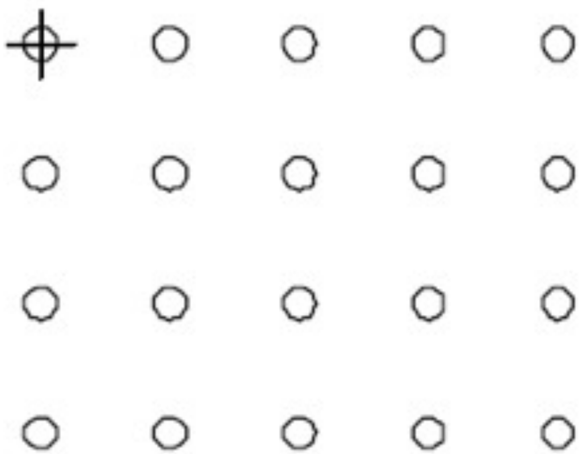


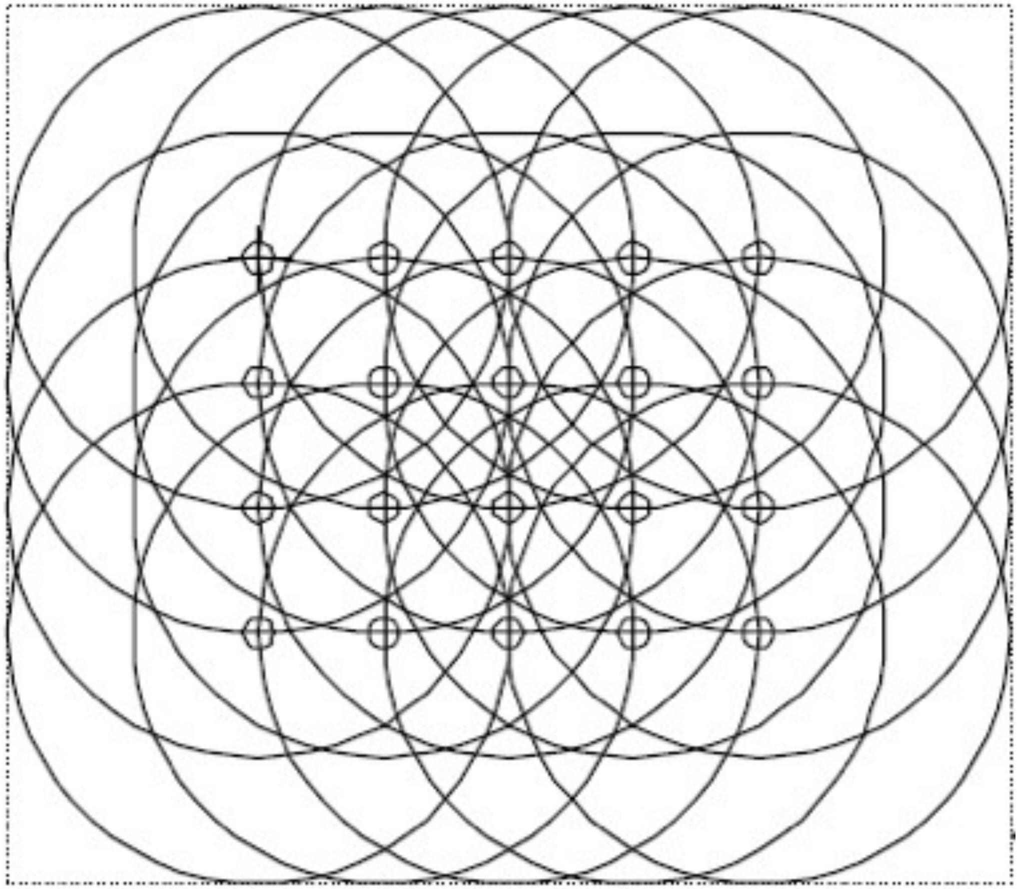


Gaussian reconstruction filter

Illustrations: Smith, MS Tech Memo 6, Jul 17, 1995

Not a Little Square!





cubic reconstruction filter

Illustrations: Smith, MS Tech Memo 6, Jul 17, 1995

Not a Little Square!

Graphics: Dick Lyon, 2006

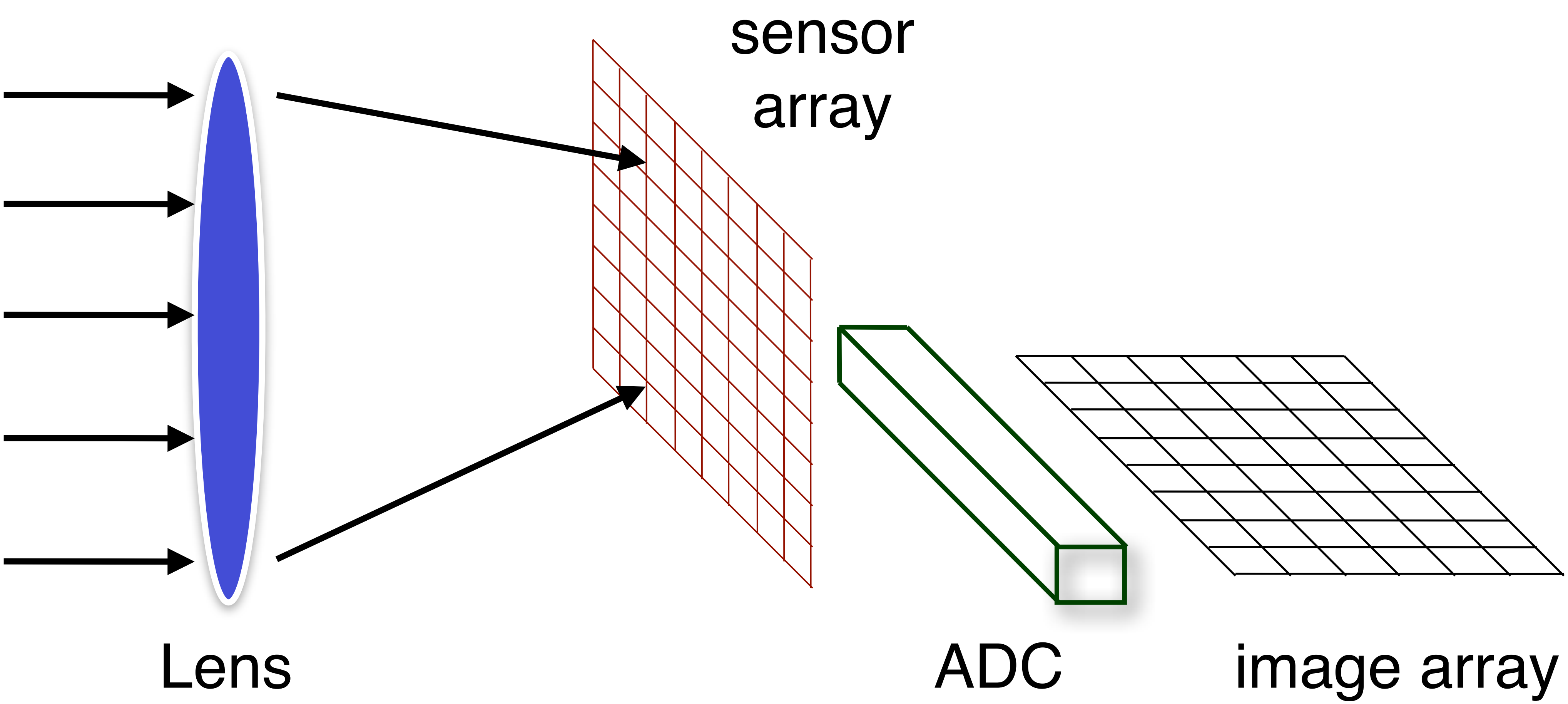
Image Sources

- Digital cameras
- MRI scanners
- Computer graphics packages
- Body scanners
- Laser range finders

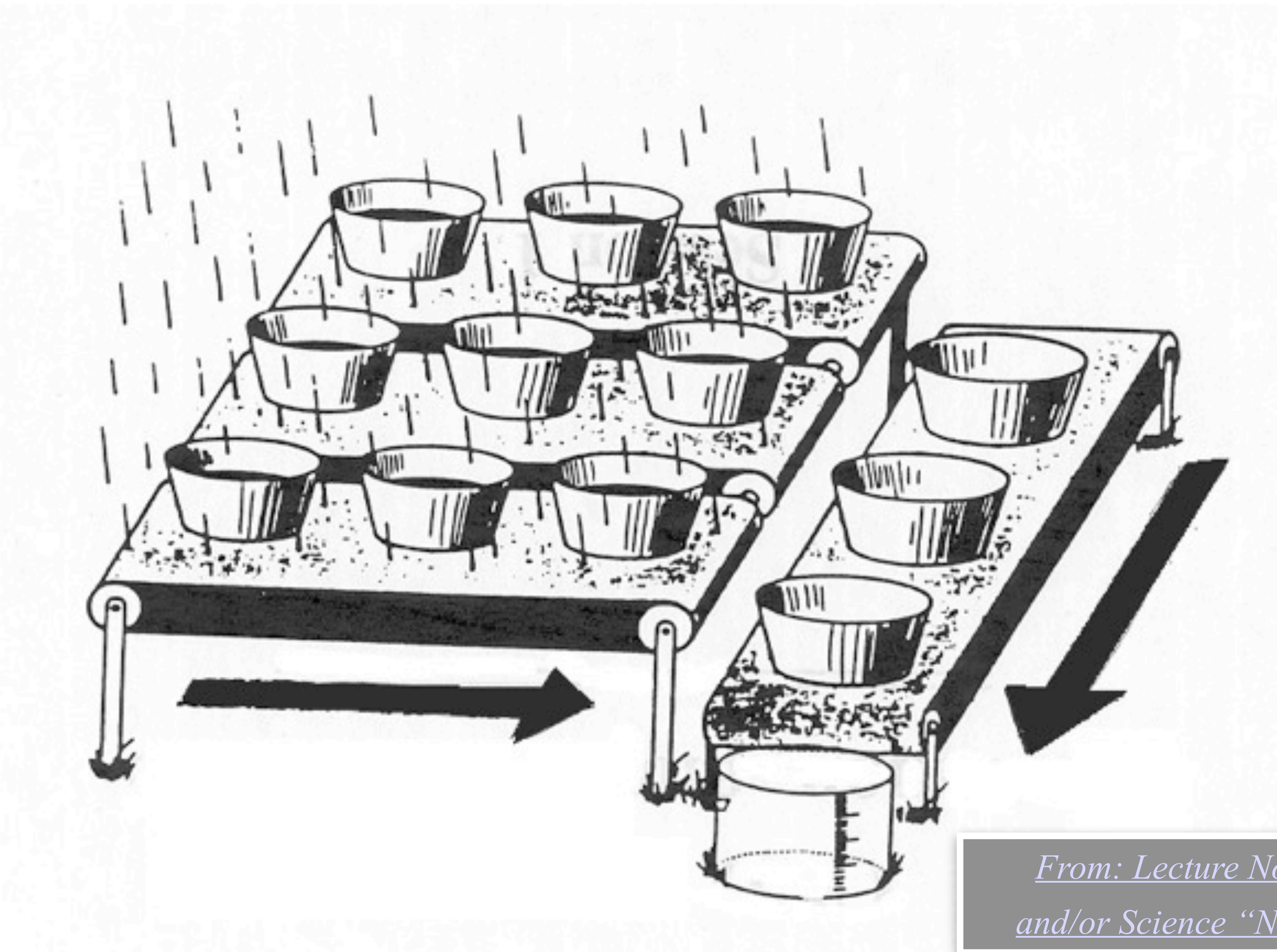
- Many more …

Camera Setup

Charge Coupled Device (CCD)

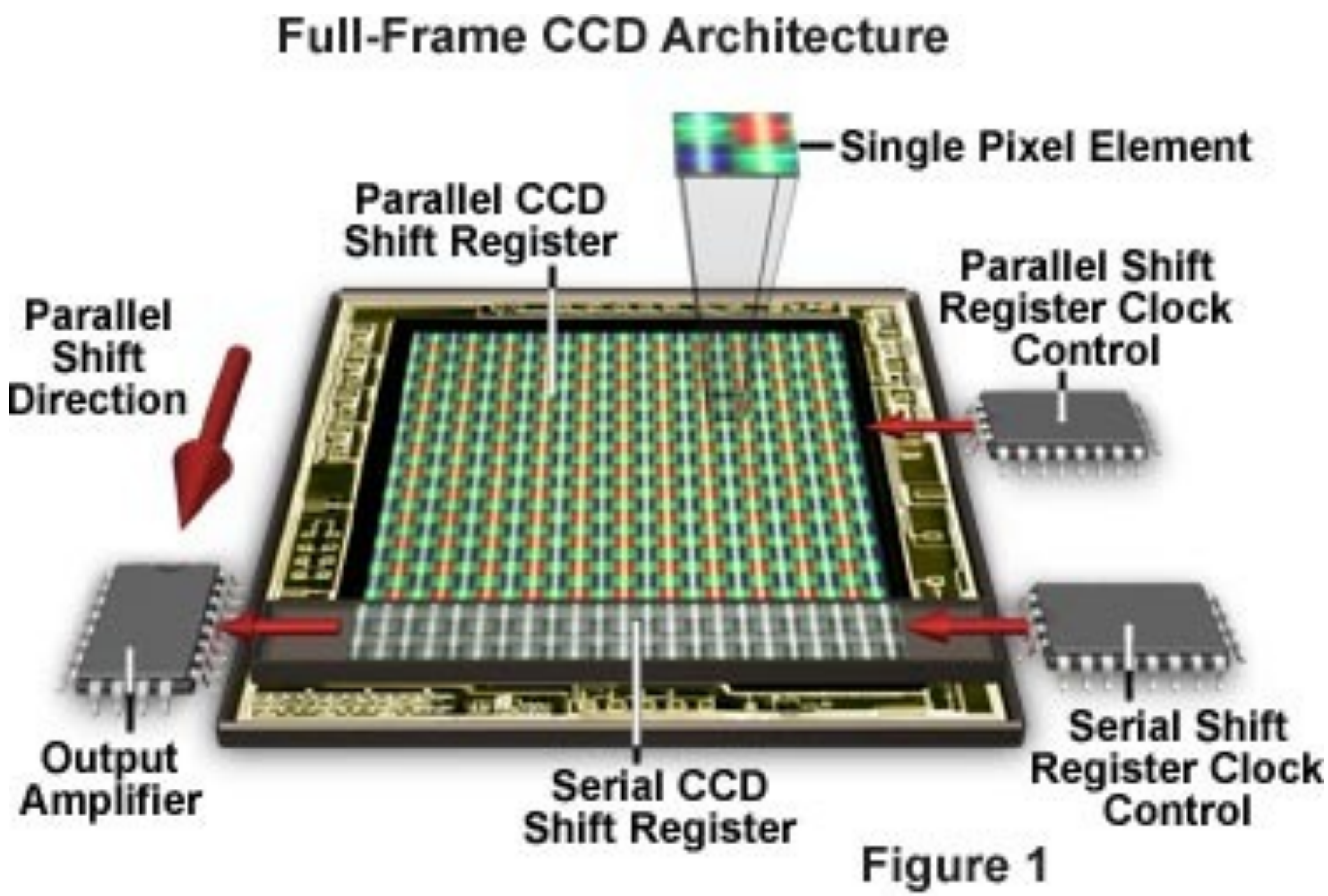


Capturing Photons



[From: Lecture Notes – EAAE and/or Science “Nuggets” 2000]()

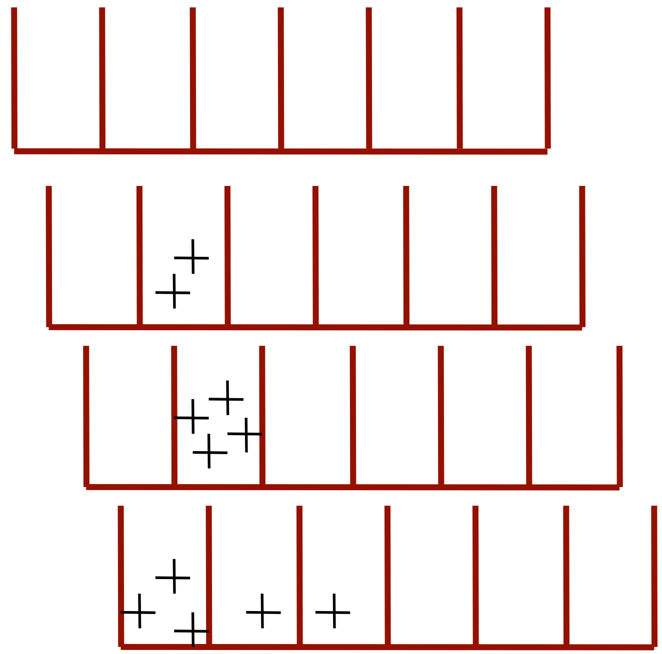
CCD Array



http://www.astro.virginia.edu/class/oconnell/astr121/im/CCD-fullframearc-FSU.jpg

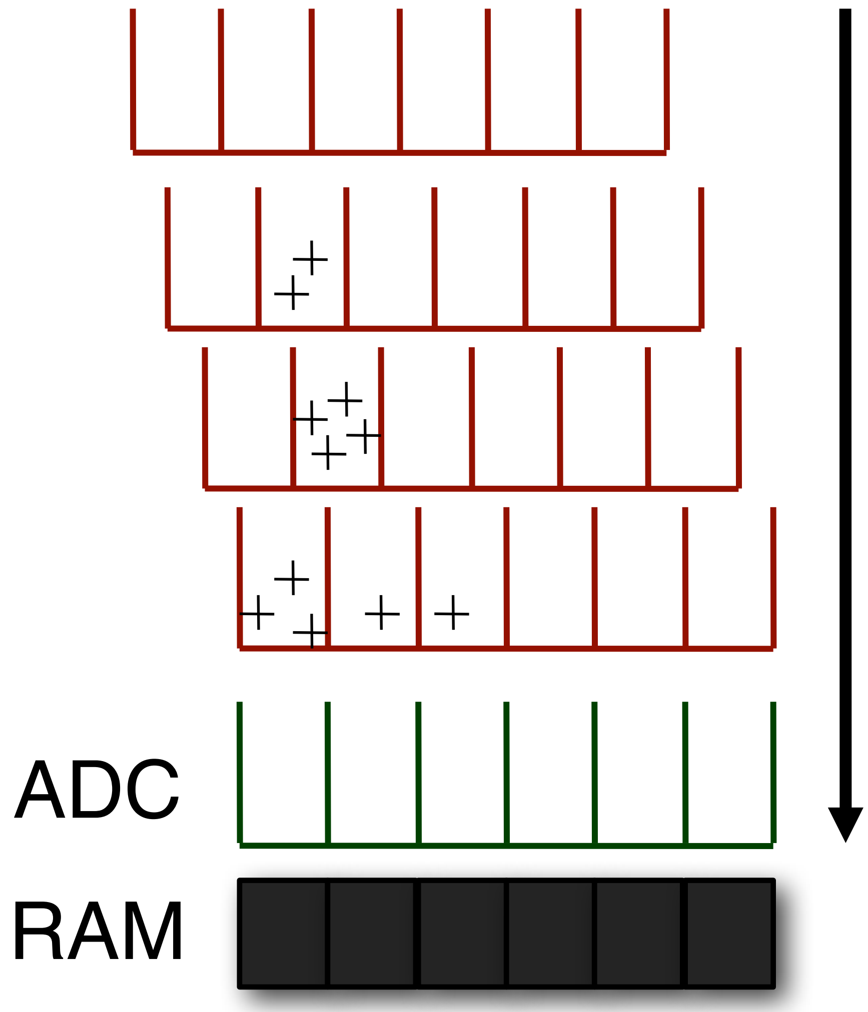
Sensor Array

- Can be $< 1\text{cm}^2$.
- An array of *photosites*.
- Each photosite is a bucket of electrical charge.
- They contain charge proportional to the incident light intensity during exposure.



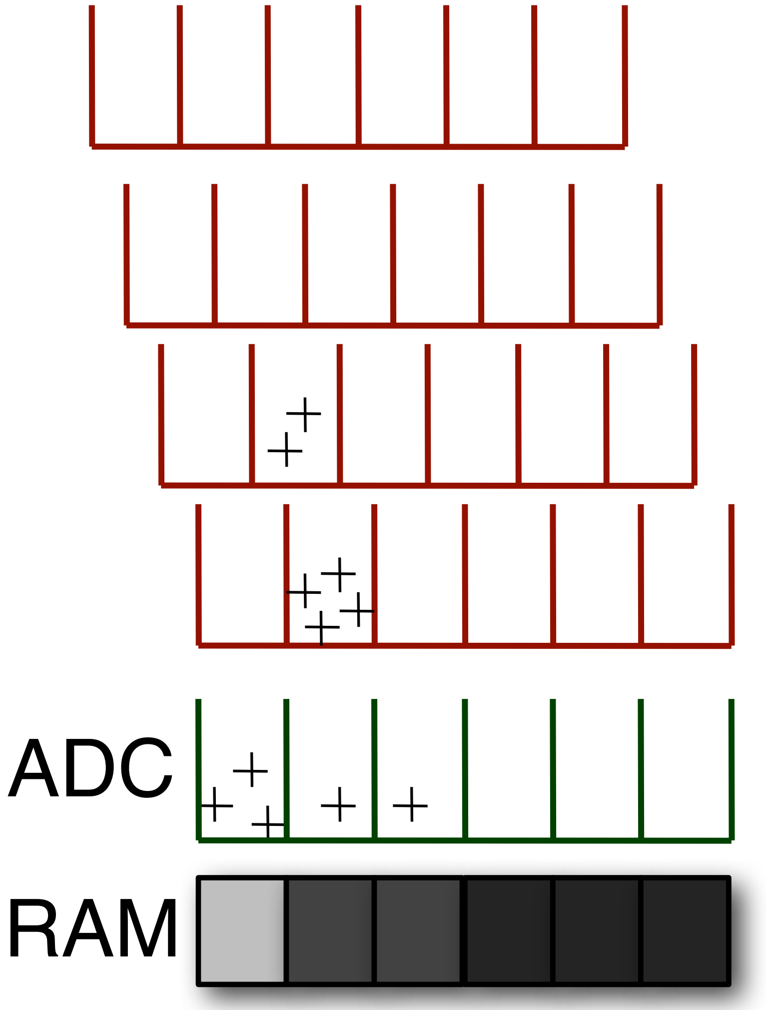
AD Conversion

- The ADC measures the charge and digitizes the result.
- Conversion happens line by line.
- The charges in each photosite move down through the sensor array.

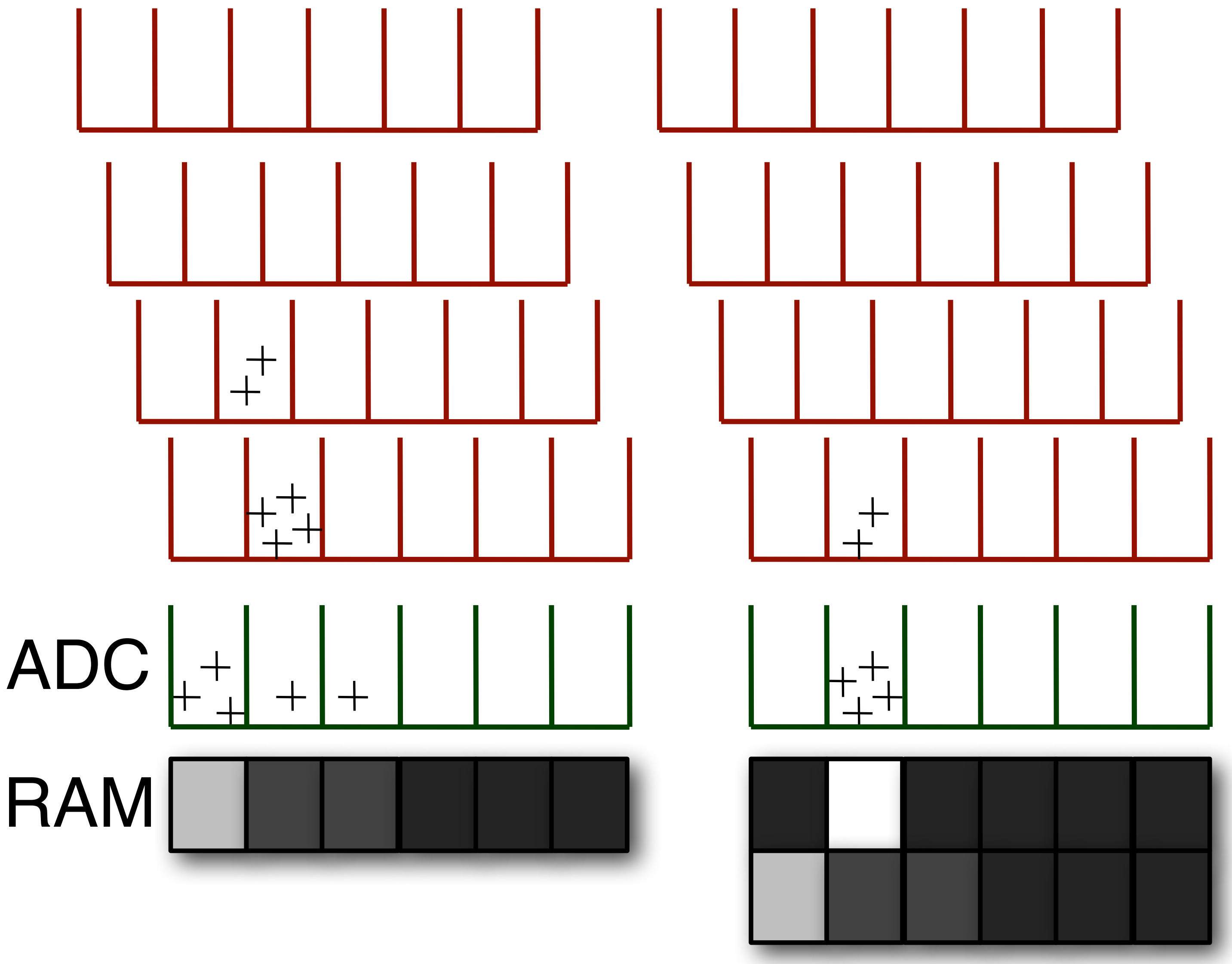


AD Conversion

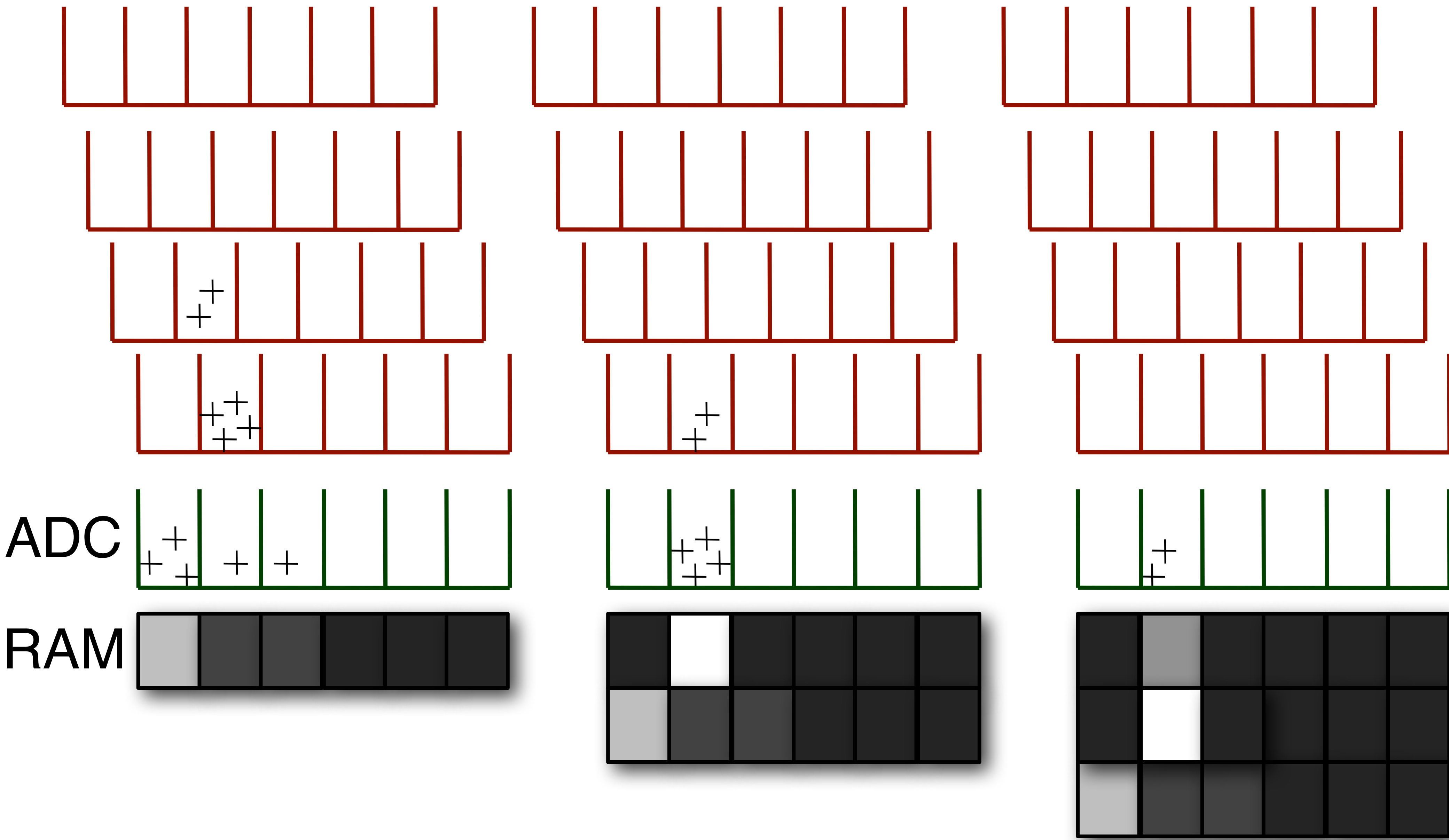
AD Conversion



AD Conversion



AD Conversion

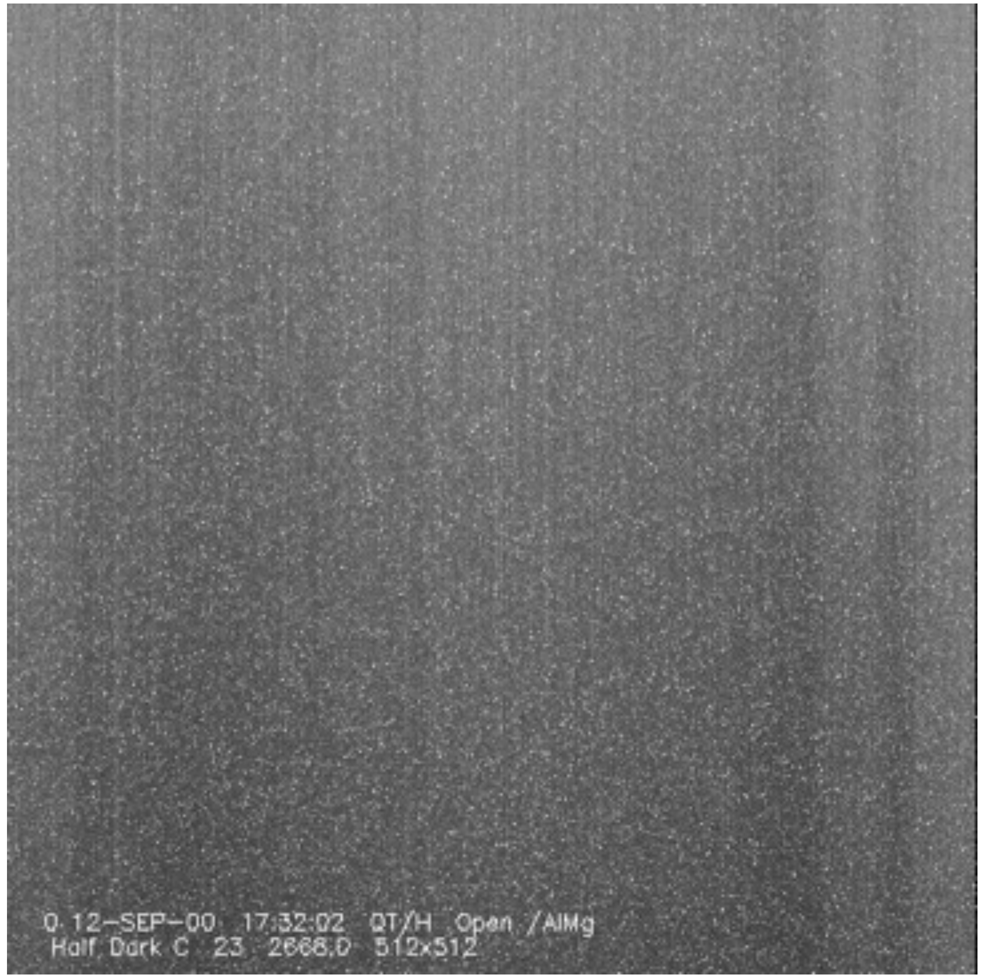


Blooming/Vertical Smear

- The buckets have finite capacity
- Photosite saturation causes blooming

Dark Current



Dark Current

- CCDs produce thermally-generated charge.
- They give non-zero output even in darkness.
- Partly, this is the *dark current*.
- Fluctuates randomly.

- How can we reduce dark current?

Dark Current

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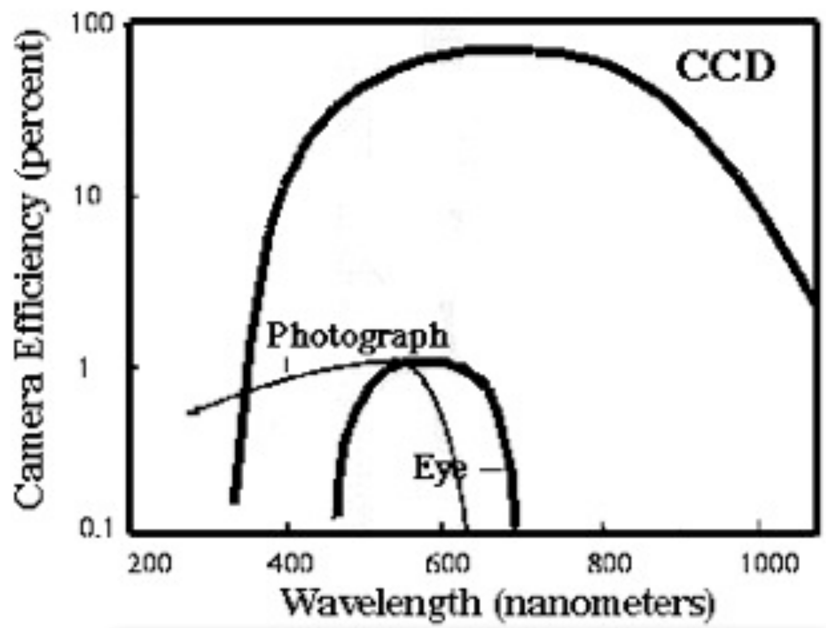
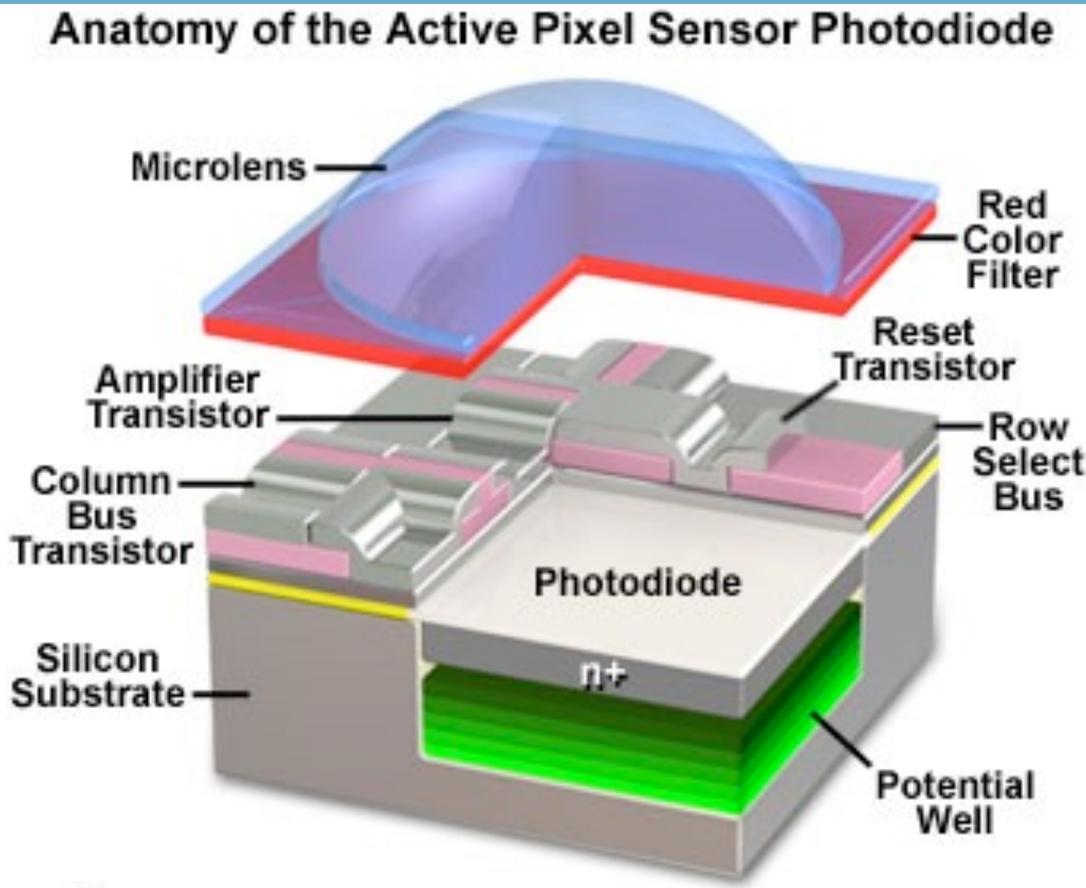


From: Lecture Notes - EAAE

- How can we reduce dark current?

CMOS active sensor pixel



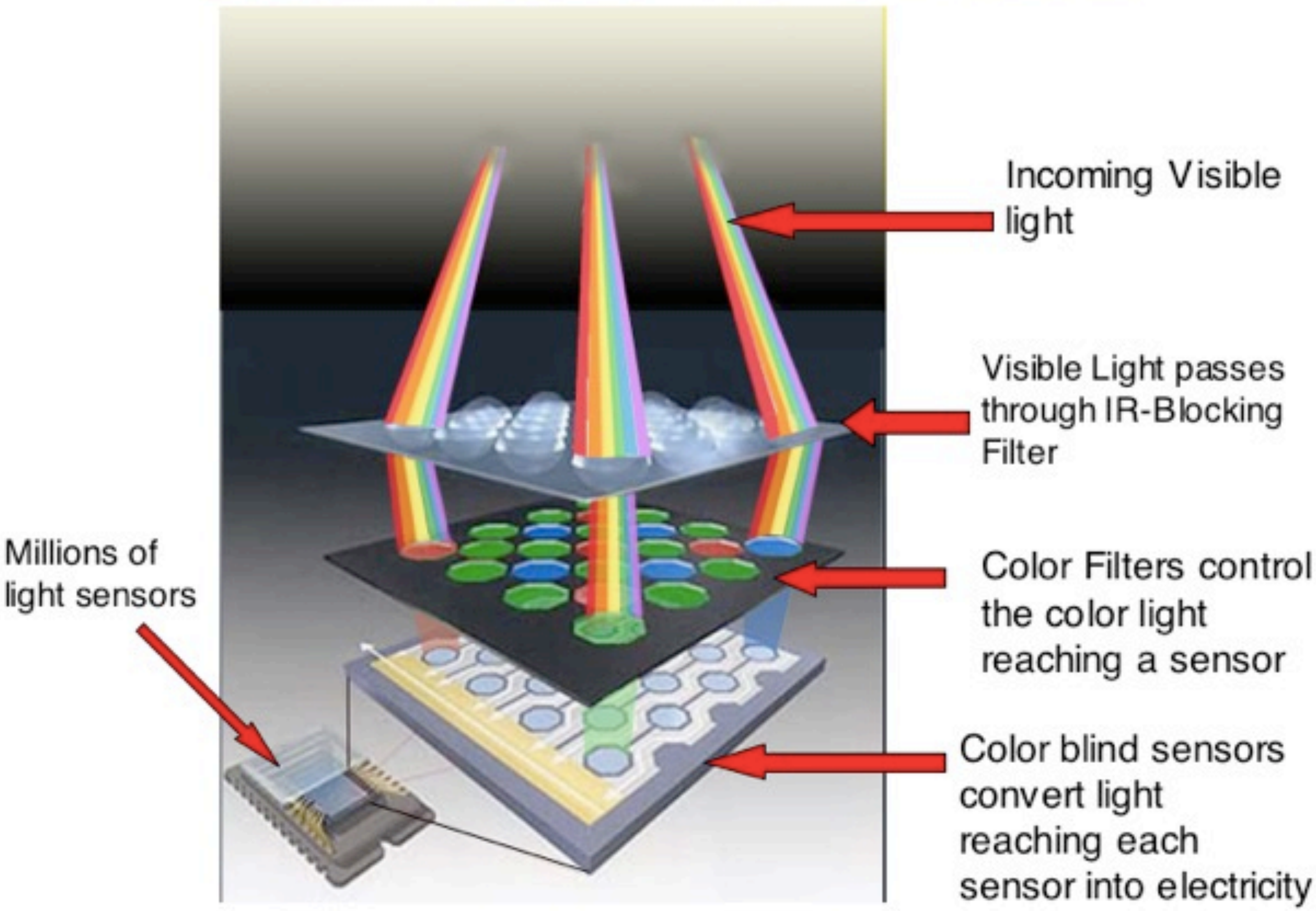
From Abramowitz & Davidson

- CMOS = complementary metal oxide semiconductor (vs. CCD = Charge-Coupled Device)

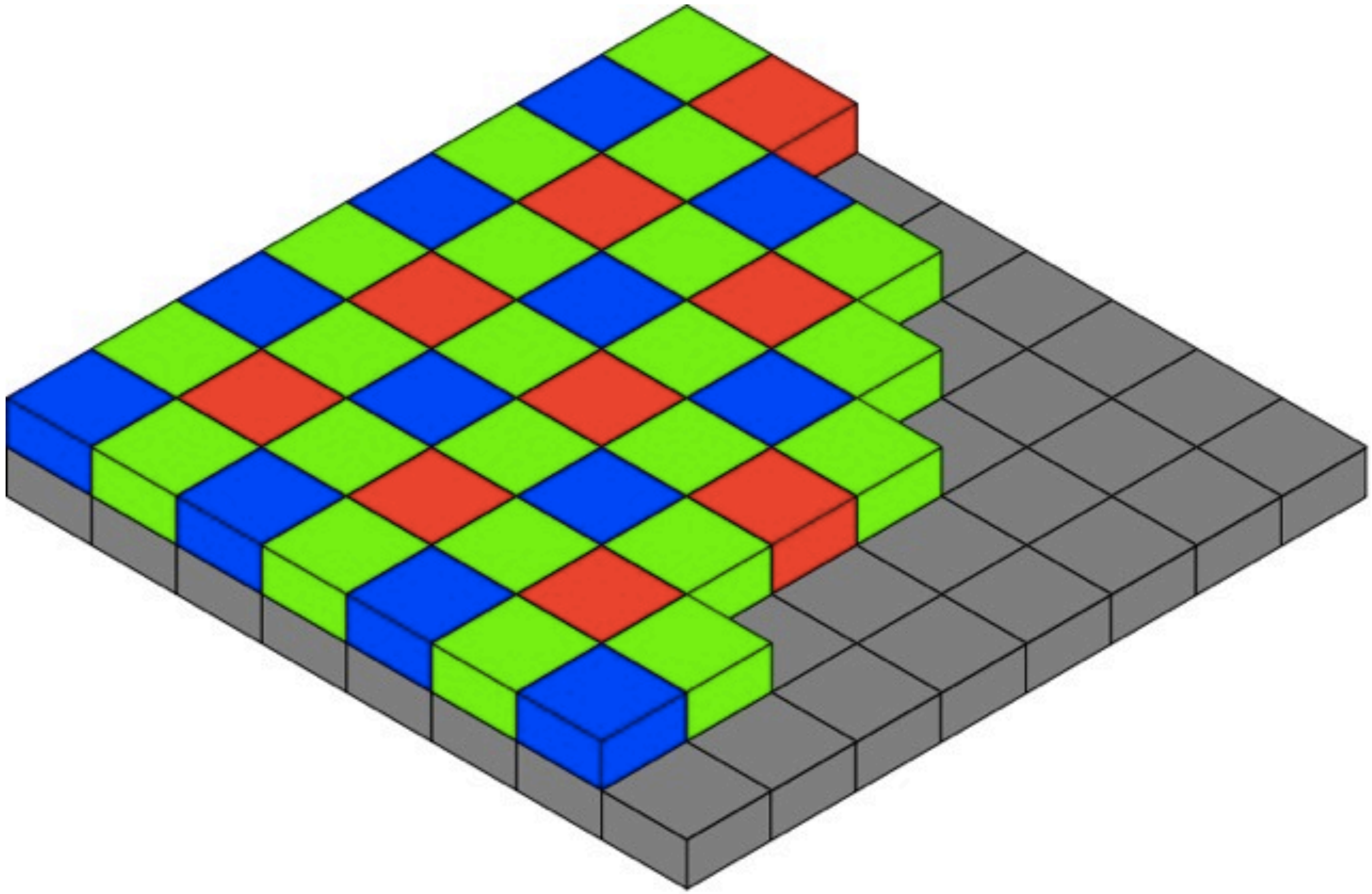
(See Active Pixel Sensors for more about converting light into bits)

Colors

RGB Inside the Camera



Bayer Pattern



CCD

- Well-established
- Low noise
- "Clear view" of light
- Requires up to 5 different voltages, power-hungry
- Typically high(er) resolution
- Interlaced or Progressive Scan (better!)

CMOS

- Only useful since mid/late 90's
- Transistors (can) block photodiode
- Can use 100x less power
- Easy to manufacture (any standard silicon production line)
- Rolling shutter

- Animated explanation

Back-illuminated CMOS

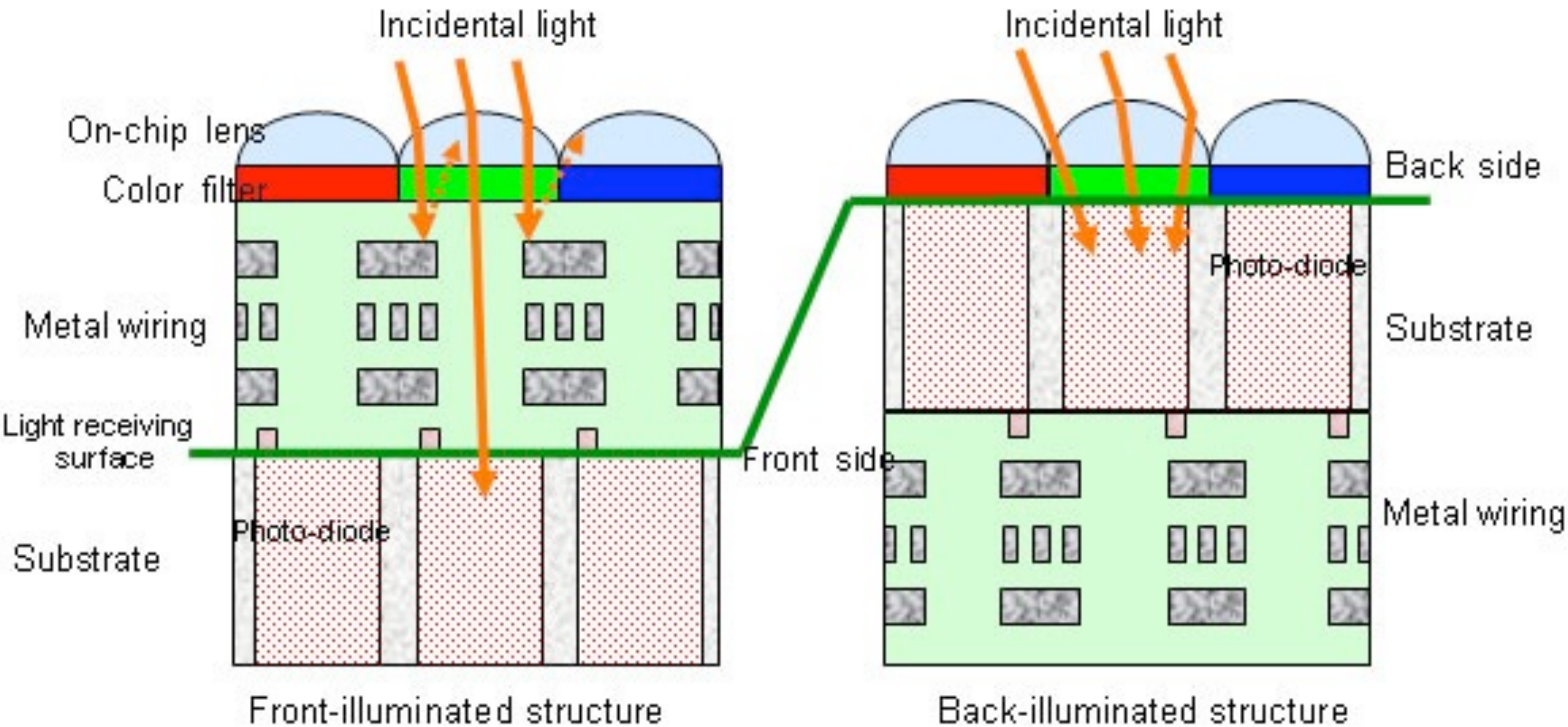
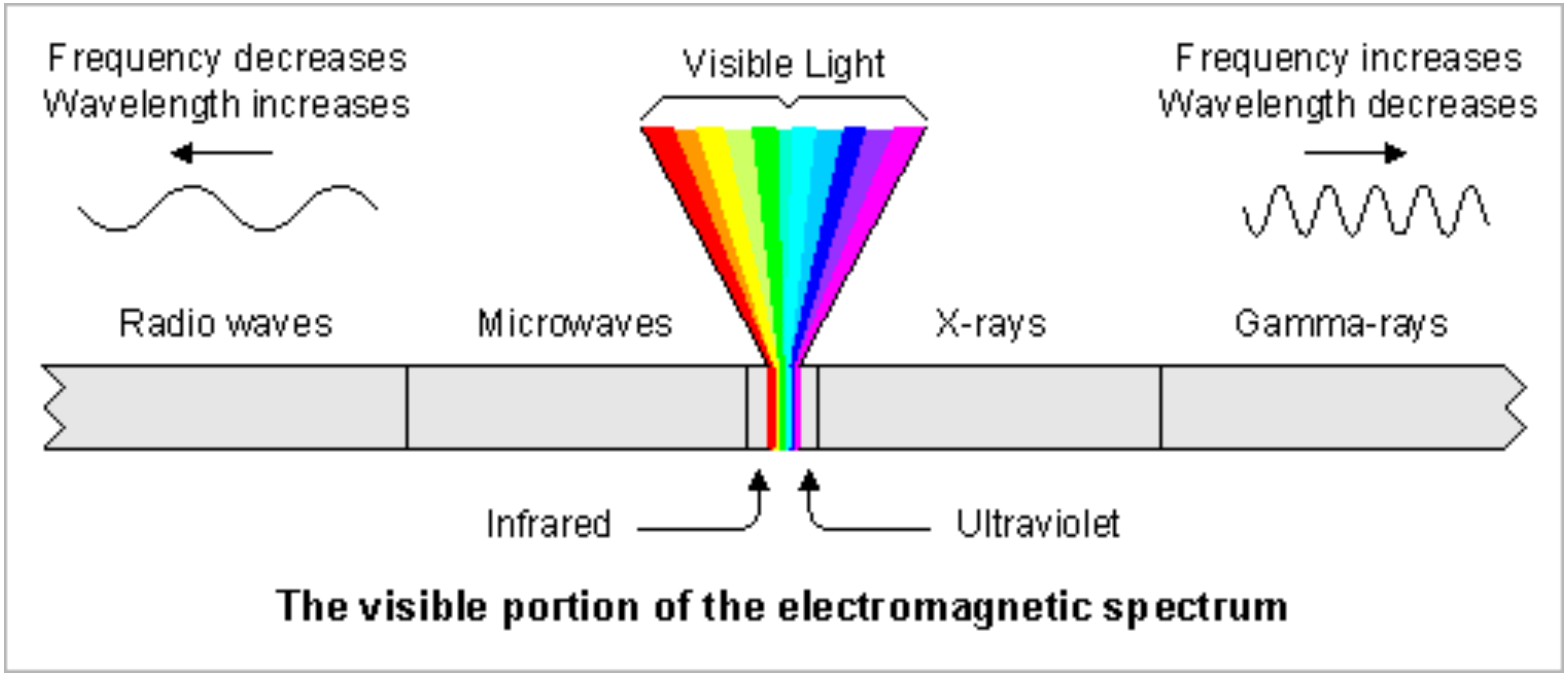


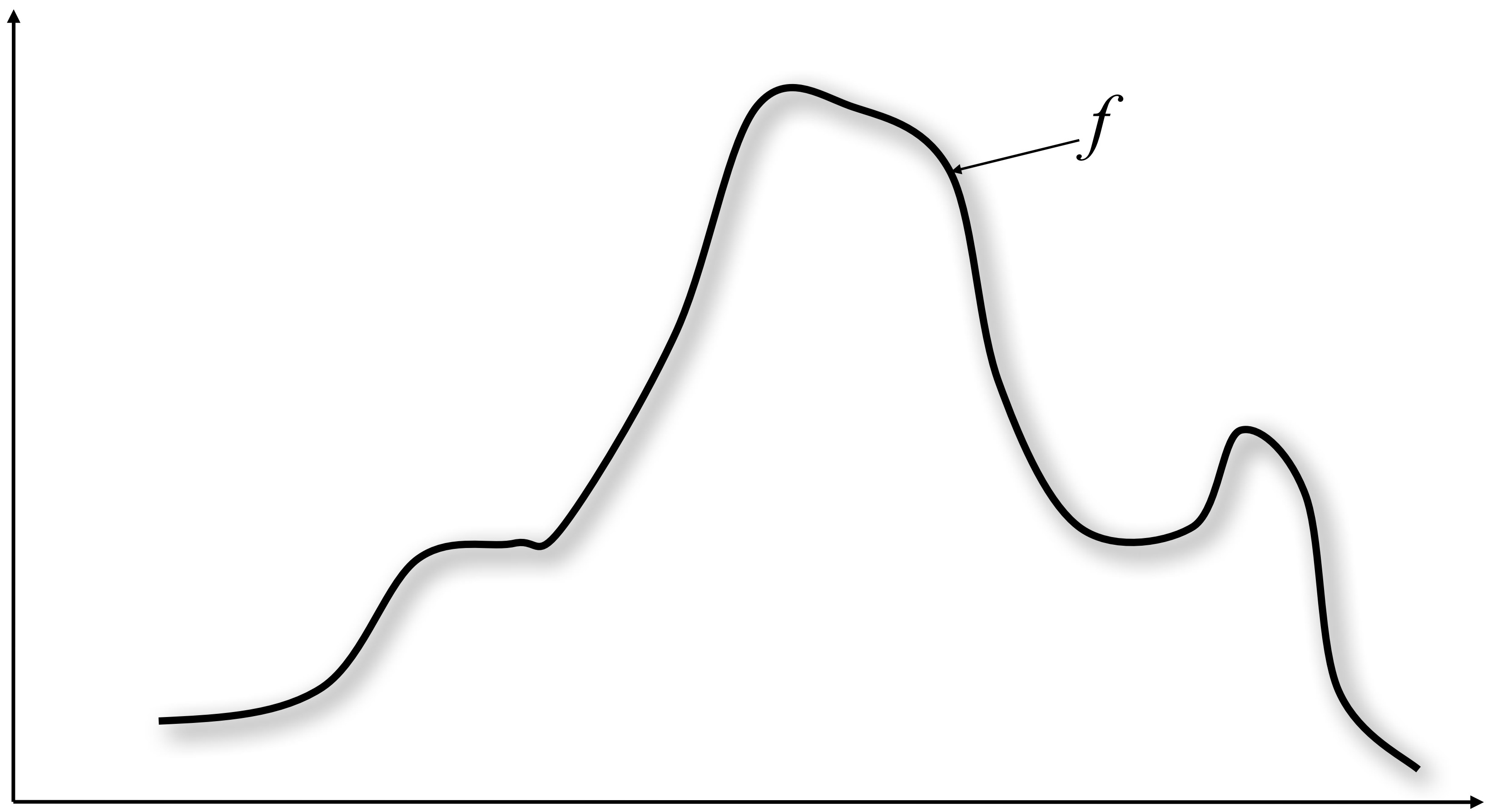
Diagram from Sony article [“Back to Front”]

Light Spectrum

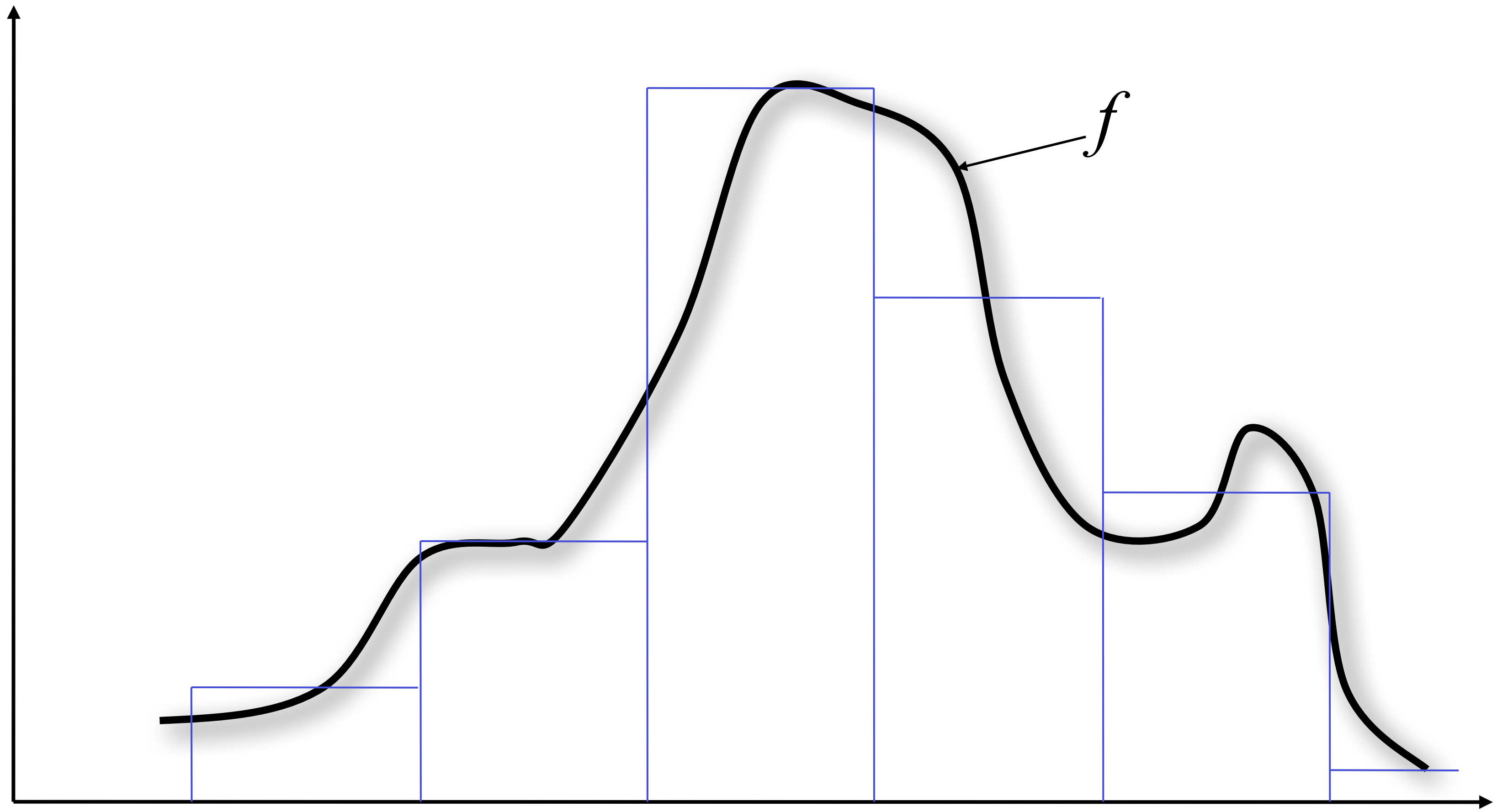


[http://www.diycalculator.com/sp-cvision.shtml]

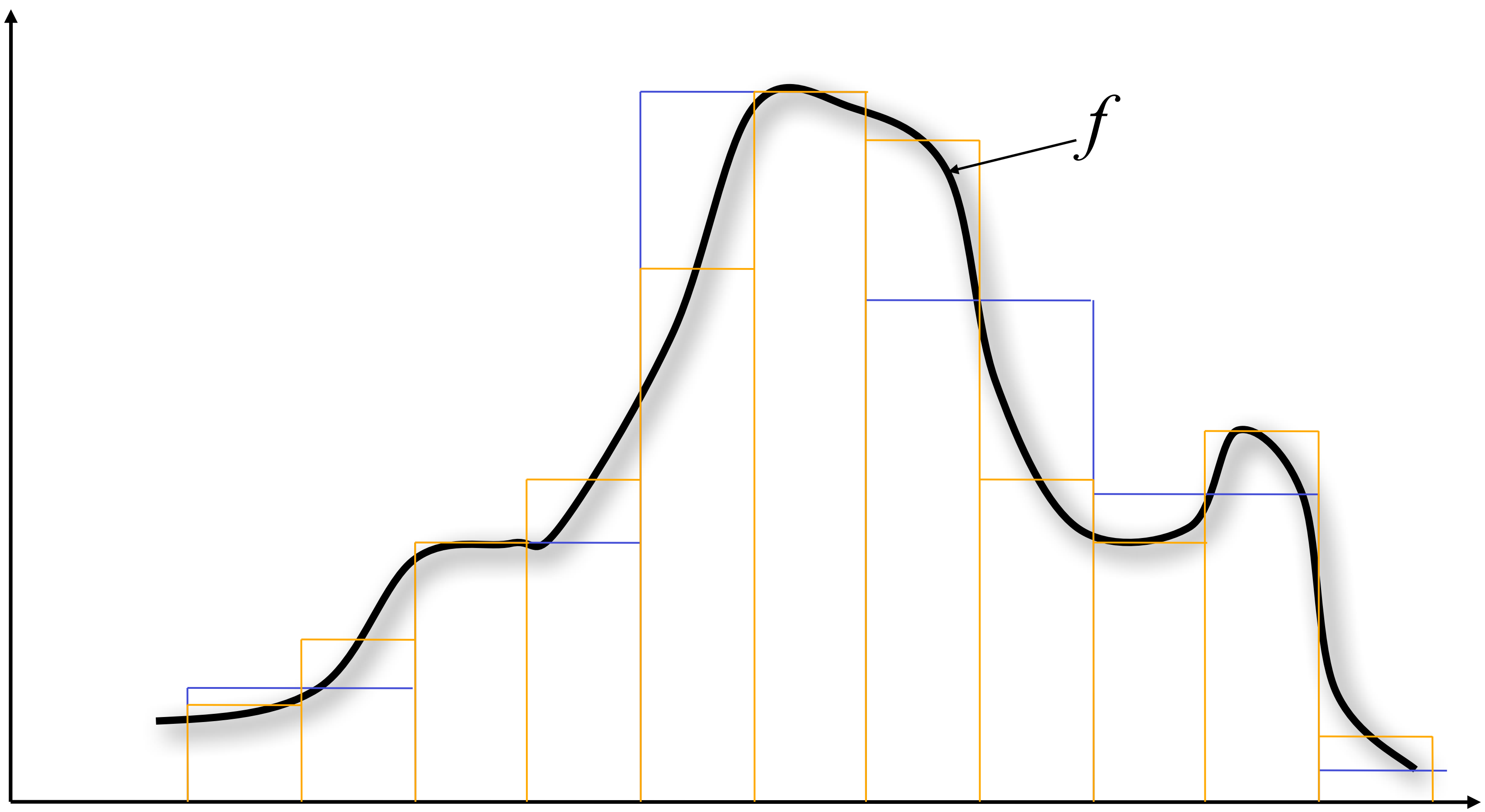
Sampling



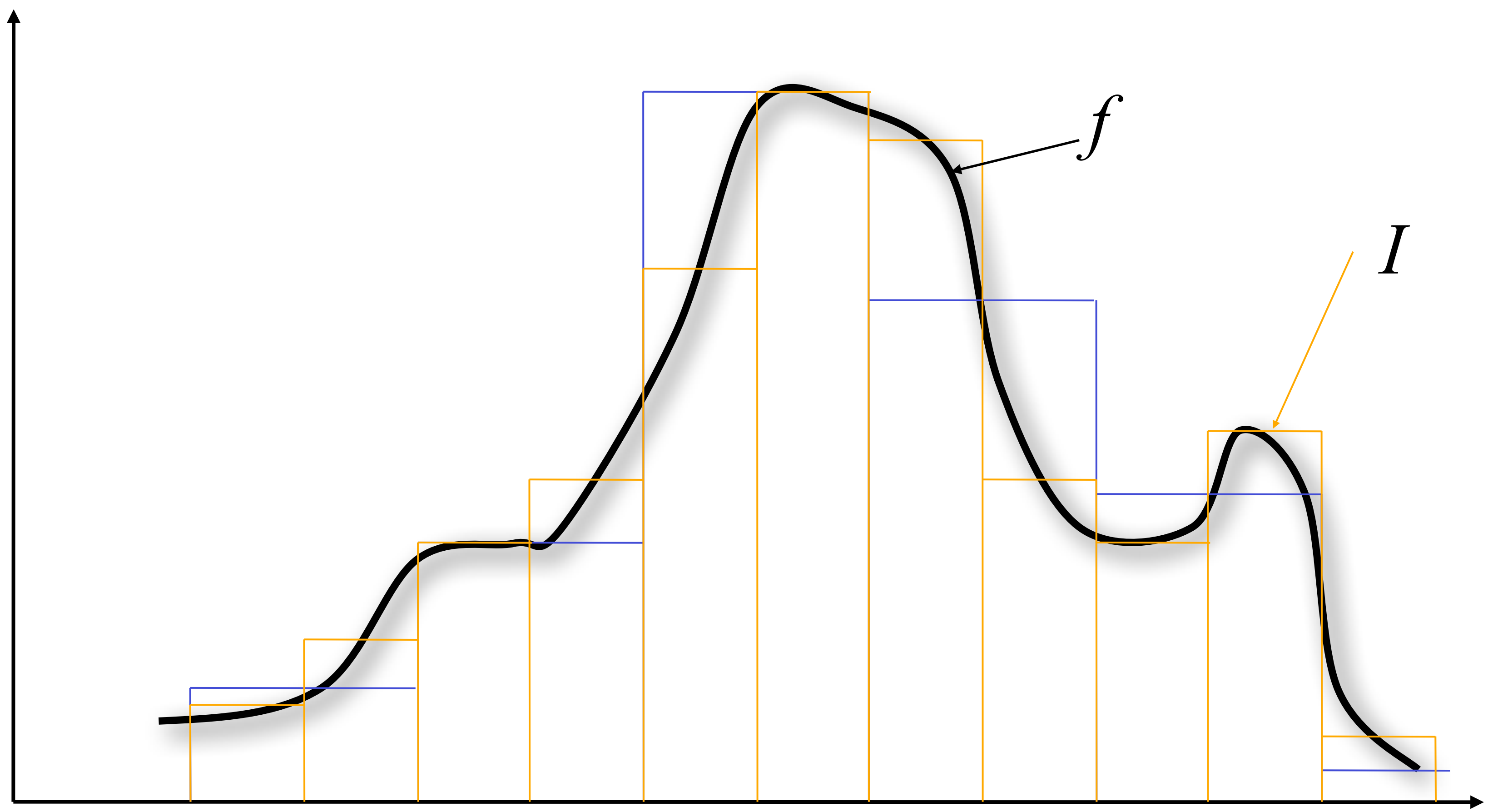
Sampling



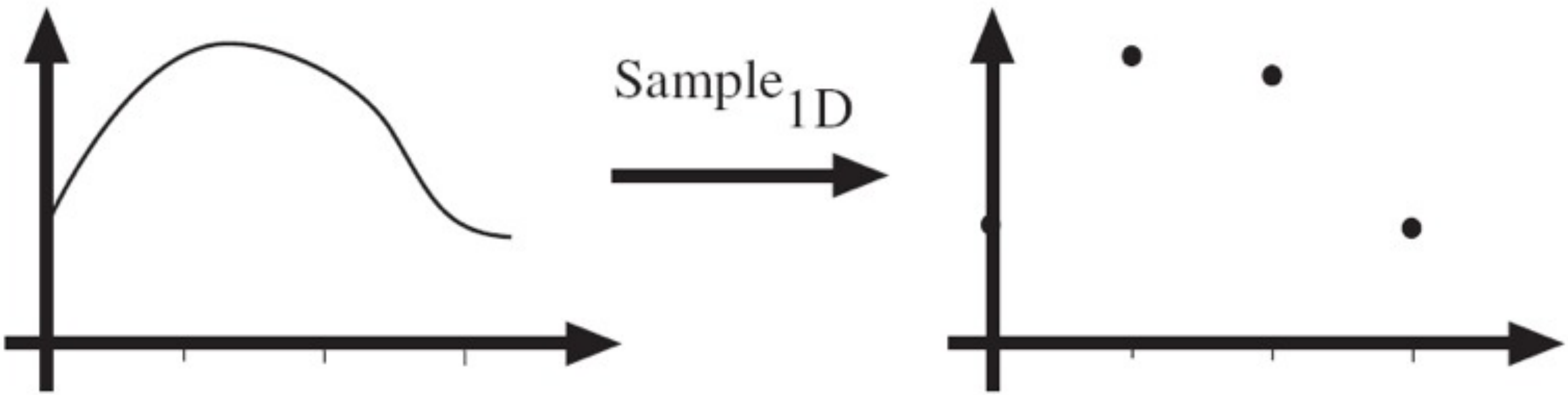
Sampling



Sampling

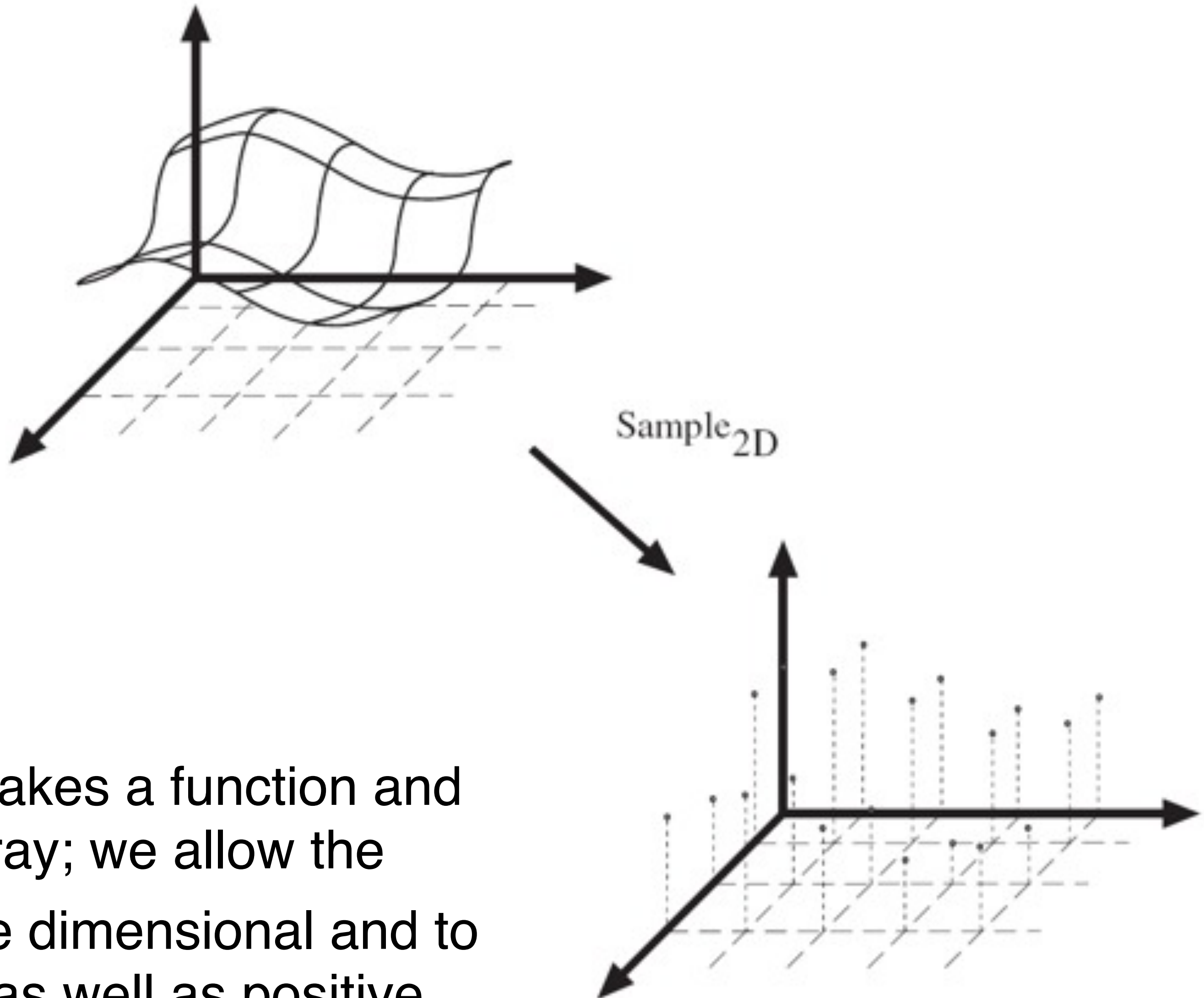


Sampling 1D



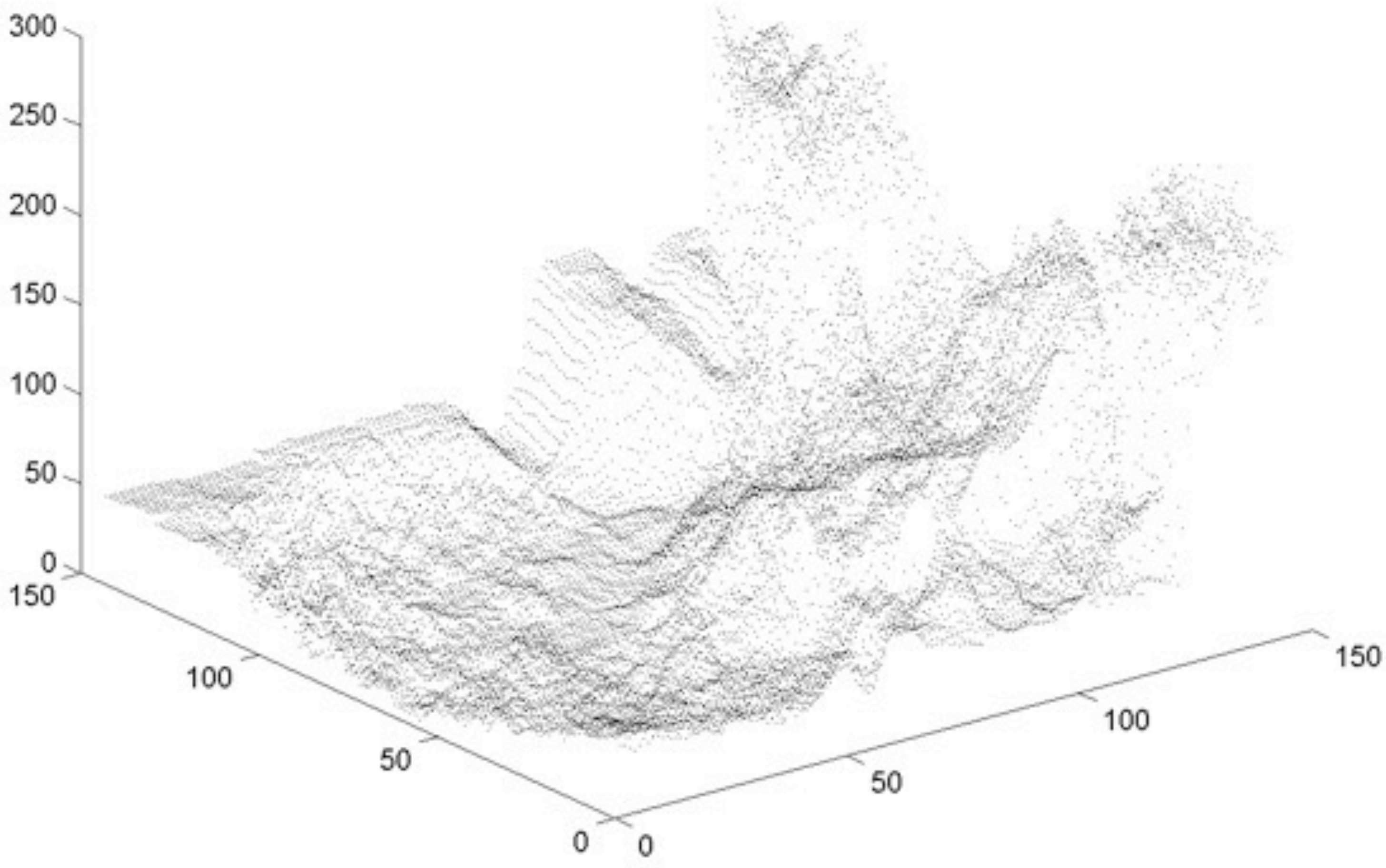
Sampling in 1D takes a function, and returns a vector whose elements are values of that function at the sample points.

Sampling in 2D



Sampling in 2D takes a function and returns an array; we allow the array to be infinite dimensional and to have negative as well as positive indices.

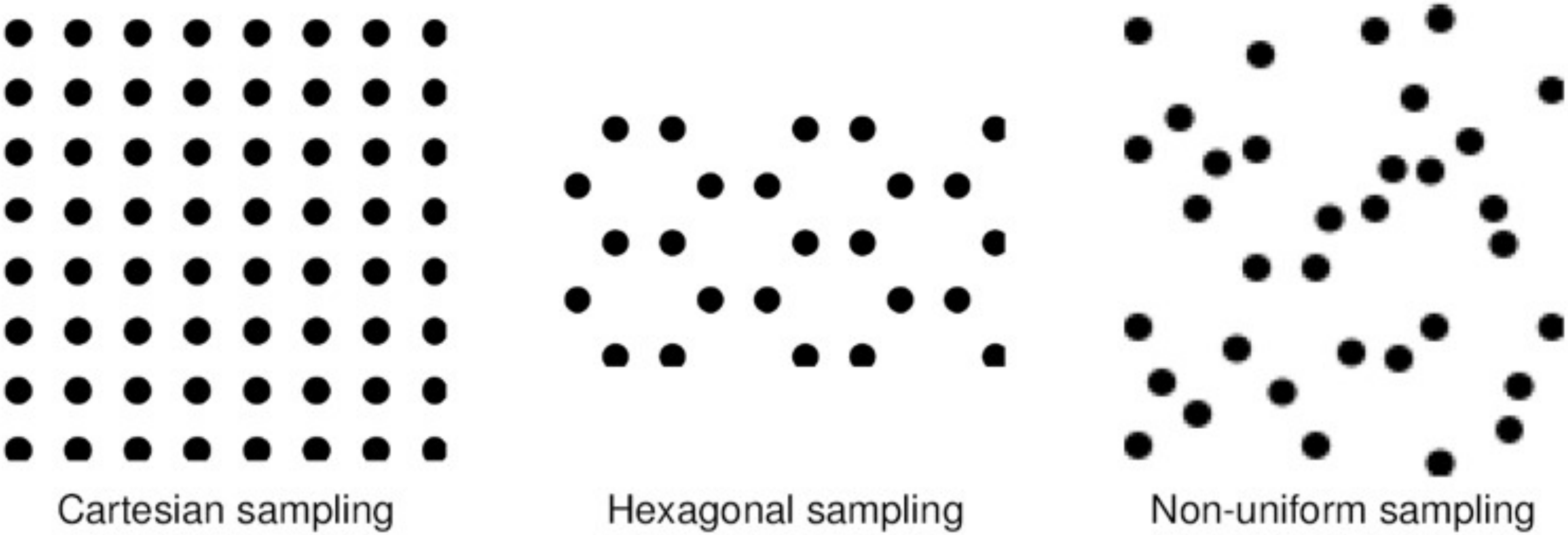
Greyscale Digital Image



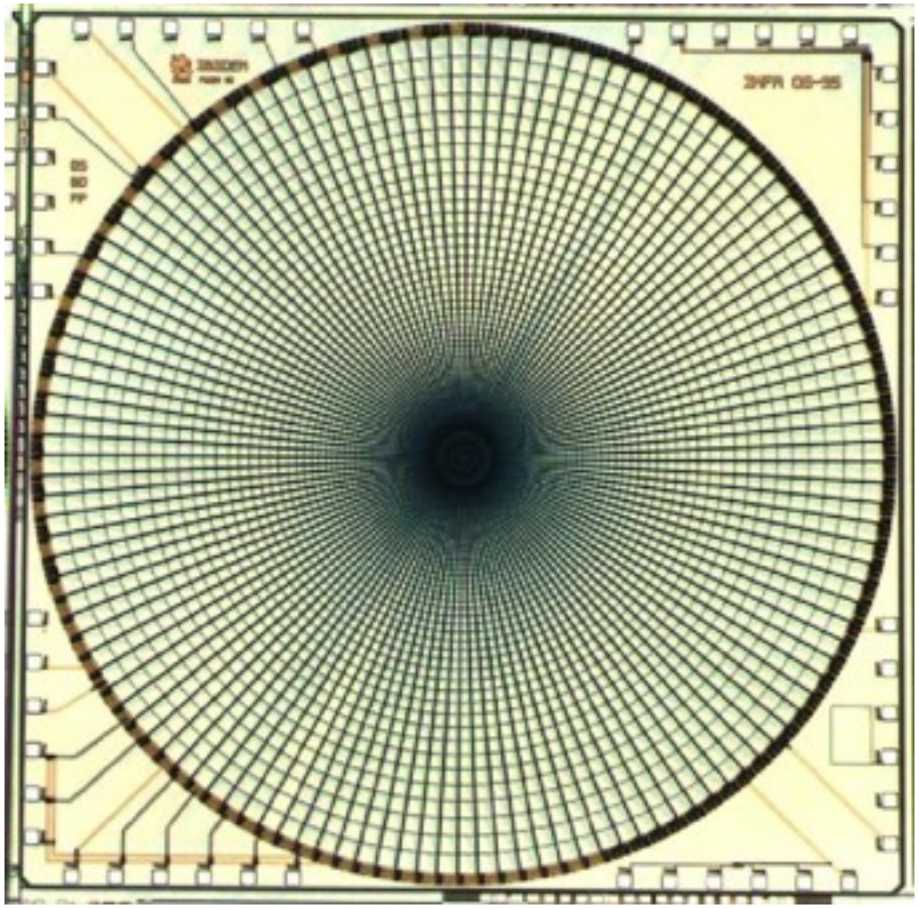
Nyquist Frequency

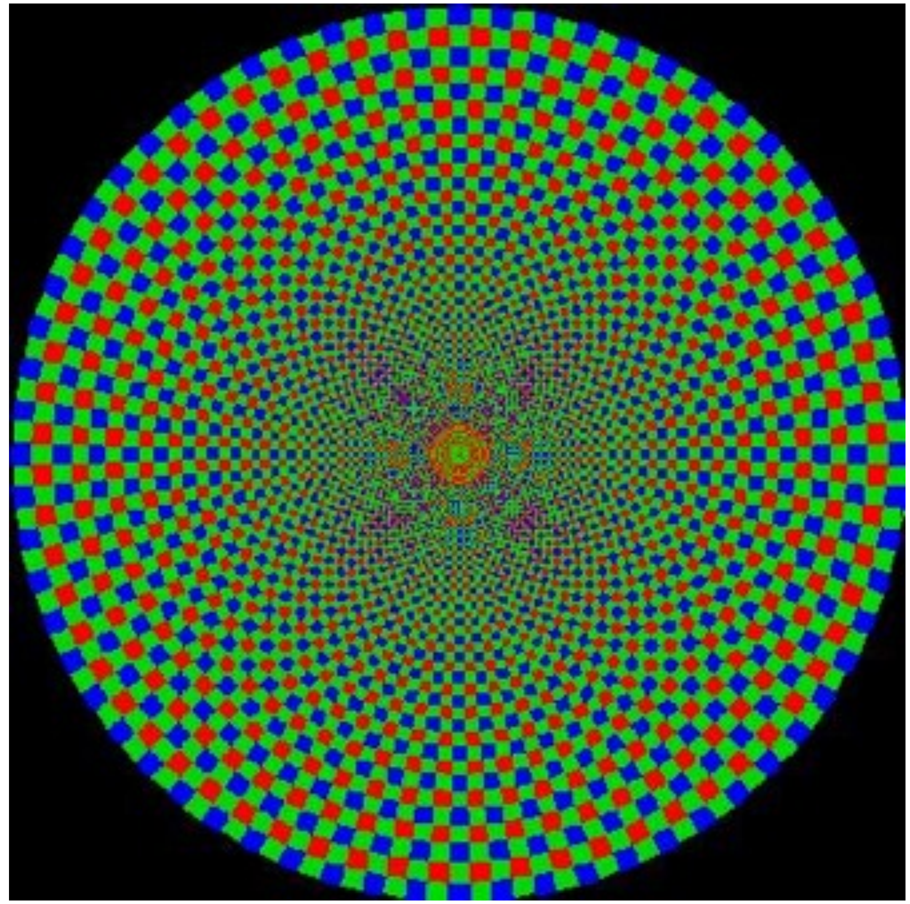
- Half the sampling frequency of a discrete signal processing system

Sampling Grids



Retina-like Sensors



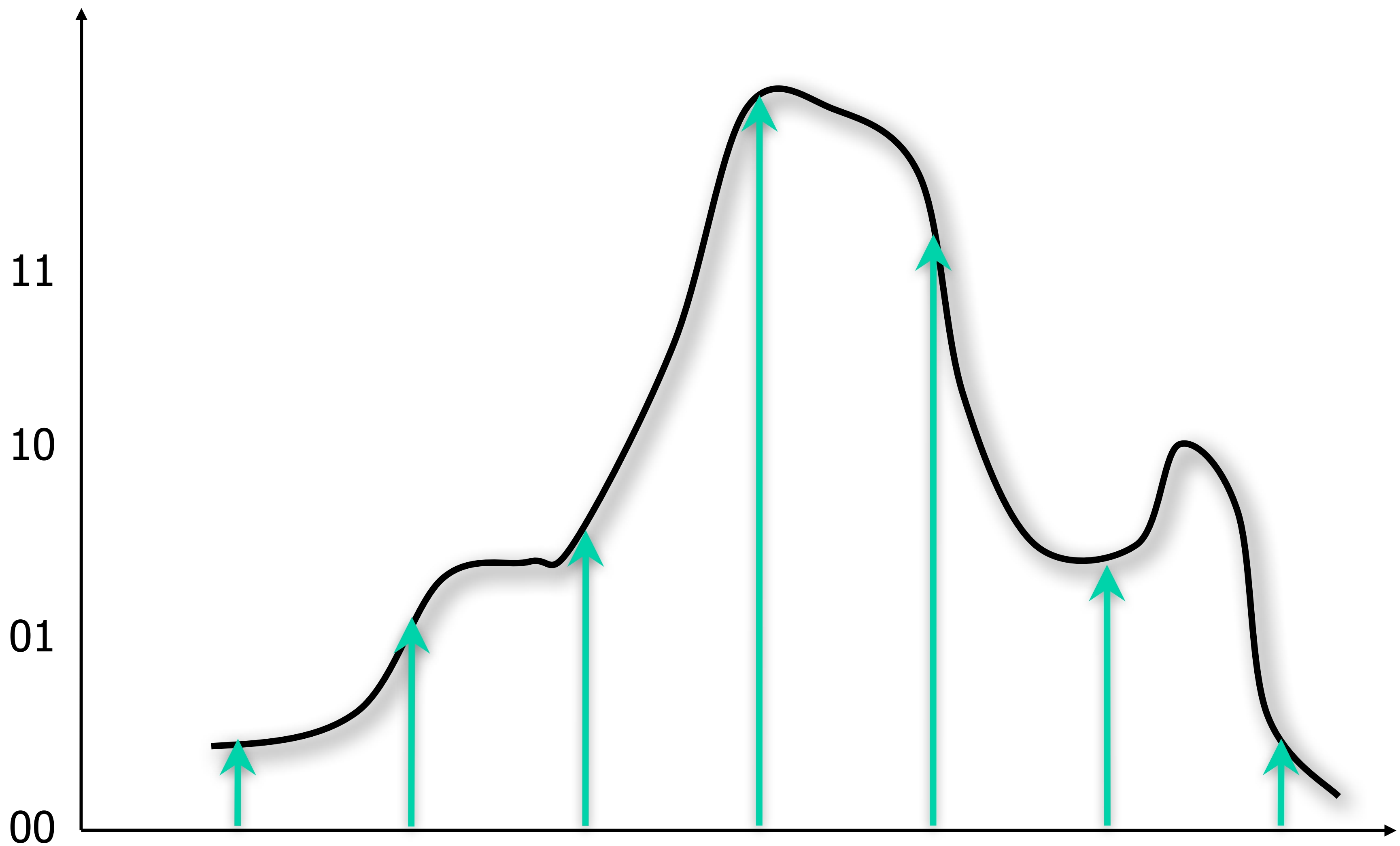


Quantization

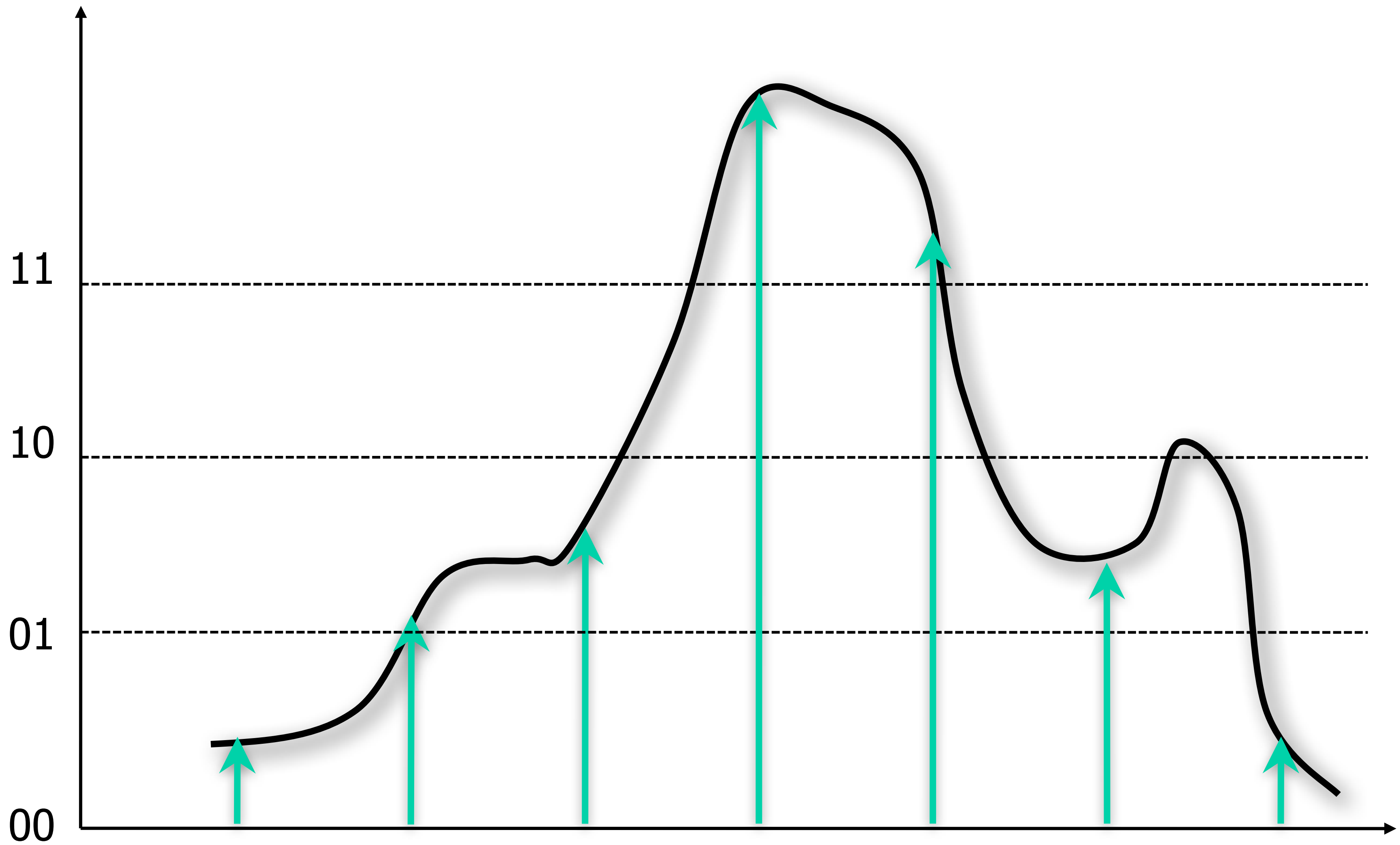
- Real valued function will get digital values – integer values
- Quantization is lossy!!
  - After quantization, the original signal cannot be reconstructed anymore
- This is in contrast to sampling, as a sampled but not quantized signal **can** be reconstructed.
- Simple quantization uses equally spaced levels with k intervals

$$k = 2^d$$

Quantization



Quantization



Usual Quantization Intervals

- Grayvalue image

$$8 \text{ bit} = 2^8 = 256 \text{ grayvalues}$$

- Color image RGB (3 channels)

$$8 \text{ bit/channel} = 2^{24} = 16\text{M colors}$$

- 12bit or 16bit from some sensors
- Nonlinear, for example log-scale

Photo: Paulo Barcellos Jr.

Properties

- Image resolution
- Geometric resolution: How many pixel per area
- Radiometric resolution: How many bits per pixel

Image Resolution

1024x1024

512x1024

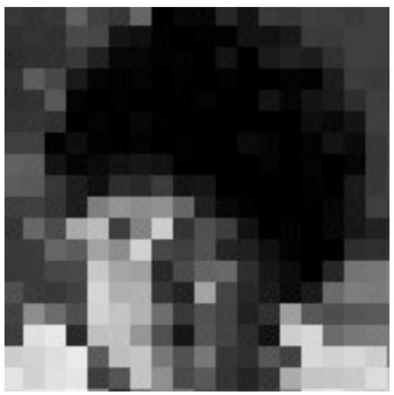
512x512

Geometric Resolution

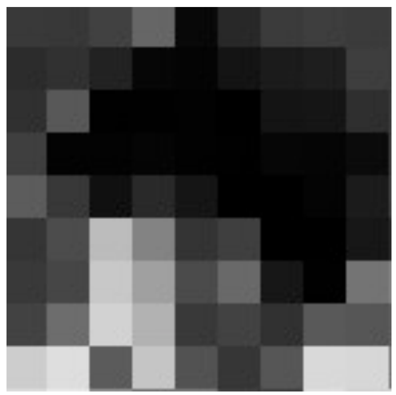
144x144

72x72

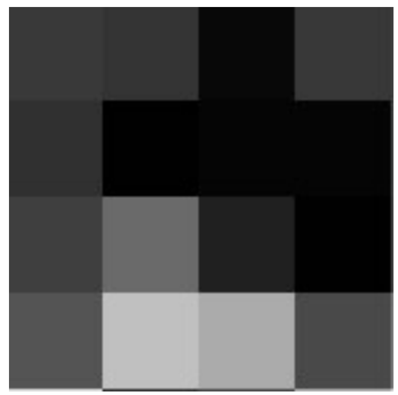
36x36



18x18



9x9



4x4

Radiometric Resolution

256

128

64

32

16

8

4

2

Lossless vs. Lossy

- Name some formats?

Aliasing and SNR

- What is the disadvantage of low sampling resolution?

- What is the disadvantage of high sampling resolution?

Image Noise

- A common model is *additive Gaussian* noise:

$$I(x, y) = f(x, y) + c$$

$$c \sim N(0, \sigma^2) \qquad\qquad p(c) = (2\pi\sigma^2)^{-1} \exp(-c^2/2\sigma^2)$$

- Poisson noise:

$$p(I) = f^I \exp(-f)/I!$$

Image Noise

- Multiplicative noise:

$$I = f + fc$$

- Quantization errors
- Impulse "salt-and-pepper" noise
- The *signal to noise ratio s* is an index of image quality

$$s = \sigma^{-1} F, \text{ where } F = (XY)^{-1} \sum_{x=1}^{X} \sum_{y=1}^{Y} f(x, y)$$

- Other definitions of s.