

# Some basic graph definitions<sup>1</sup>

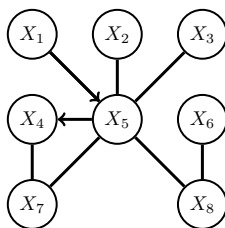
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<sup>1</sup>These slides accompany the book *Bayesian Reasoning and Machine Learning*. The book and demos can be downloaded from [www.cs.ucl.ac.uk/staff/D.Barber/brml](http://www.cs.ucl.ac.uk/staff/D.Barber/brml). Feedback and corrections are also available on the site. Feel free to adapt these slides for your own purposes, but please include a link the above website.

# Graphs



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## Definition

A graph consists of nodes (vertices) and undirected or directed links (edges) between nodes.

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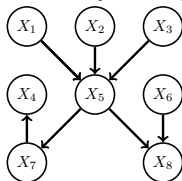
## Path

A path from  $X_i$  to  $X_j$  is a sequence of connected nodes starting at  $X_i$  and ending at  $X_j$ .

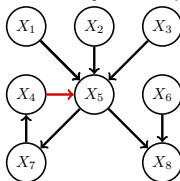
# Directed Graphs

All the edges are directed:

Directed Acyclic Graph



Directed Cyclic Graph



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## DAG

Directed Acyclic Graph: Graph in which by following the direction of the arrows a node will never be visited more than once.

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## Parents and Children:

$X_i$  is a parent of  $X_j$  if there is a link from  $X_i$  to  $X_j$ .  $X_i$  is a child of  $X_j$  if there is a link from  $X_j$  to  $X_i$ .

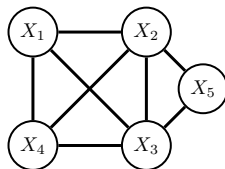
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## Ancestors and Descendants:

The ancestors of a node  $X_i$  are the nodes with a directed path ending at  $X_i$ . The descendants of  $X_i$  are the nodes with a directed path beginning at  $X_i$ .

# Undirected Graph

All the edges are undirected:



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## Clique

A clique is a fully connected subset of nodes.  $(X_1, X_2, X_4)$  forms a (non-maximal) clique.

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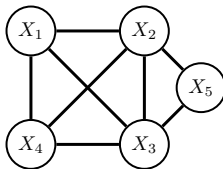
## Maximal Clique

Clique which is not a subset of a larger clique.  $(X_1, X_2, X_3, X_4)$  and  $(X_2, X_3, X_5)$  are both maximal cliques.

# Connectivity

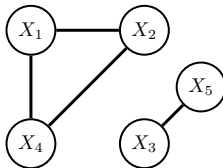
## Connected graph

There is a path between every pair of vertices:



## Connected components

In a non-connected graph, the connected components are the connected-subgraphs:

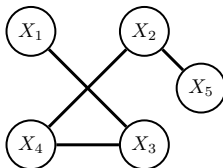


$(X_1, X_2, X_4)$  and  $(X_3, X_5)$  are the two connected components.

# Connectedness

## Singly-connected

There is only one path from any node  $a$  to another other node  $b$



## Multiply-connected

A graph is multiply-connected if it is not singly-connected:

