Lecture 1: Causal Identification

POL-GA 1251
Quantitative Political Analysis II
Prof. Cyrus Samii
NYU Politics

January 23, 2022

Plan for the week

Mon:

- Discuss overview of class and syllabus.
- Explain what "causal identification" means.
- Introduce potential outcomes and causal graphs.

Wed:

- Randomized experiments.
- Explain *estimation* concepts (estimand, estimators, bias, consistency, efficiency).
- Explain *statistical inference* concepts (sampling distribution, randomization distribution, CLT, confidence intervals, p-value).

Where this class fits in

Model of quantitative research process:

- Theory motivates causal hypothesis or target of inference:
  - *H: manipulating X results in (...) effect on Y.*
- Hypothesis, statistical theory, and substantive theory motivate a research design:
  - Operationalize X and Y.
  - Define ways to get necessary variation in X and Y given constraints.
- Research design and statistical theory motivate analysis plan:
  - Optimal estimation strategy, given constraints.
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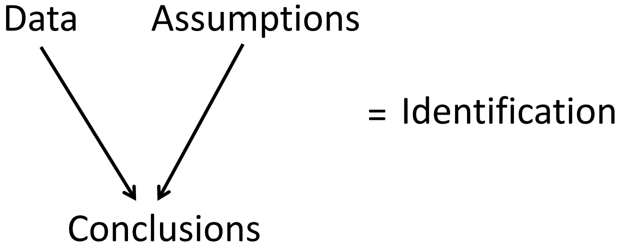

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In this class we focus on causal identification. This is a specific application of the general idea of "identification," distinct from some other applications:

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Alternative application of “identification” (I):

Suppose in the NYC mayoral primary someone said...

...they preferred Adams over Wiley, and

...they preferred Garcia over Adams.

Does this information (data) identify the person’s preference ordering over these three candidates?

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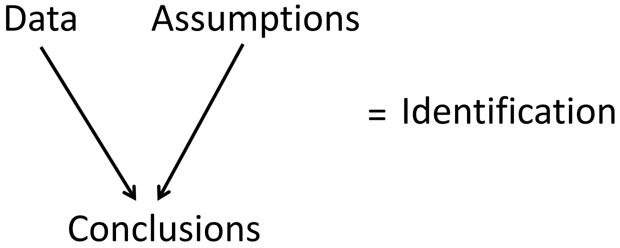

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Alternative application of "identification" (II):

Suppose none of the coefficients below are equal to zero but the error terms (last ones) are iid mean zero draws. Which system of simultaneous equations identifies its coefficients?

$$x_t = \alpha_1^a + \alpha_2^a y_t + \nu_t^a$$
$$y_t = \beta_1^a + \beta_2^a x_t + \varepsilon_t^a$$

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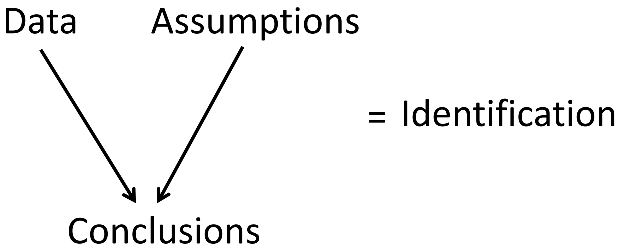

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Angrist and Krueger (1999):

> *The combination of a clearly labeled source of identifying variation in a causal variable and the use of a particular econometric[/statistical] technique to exploit this information is what we call an identification strategy.*

Potential outcomes

The Road Not Taken

Two roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

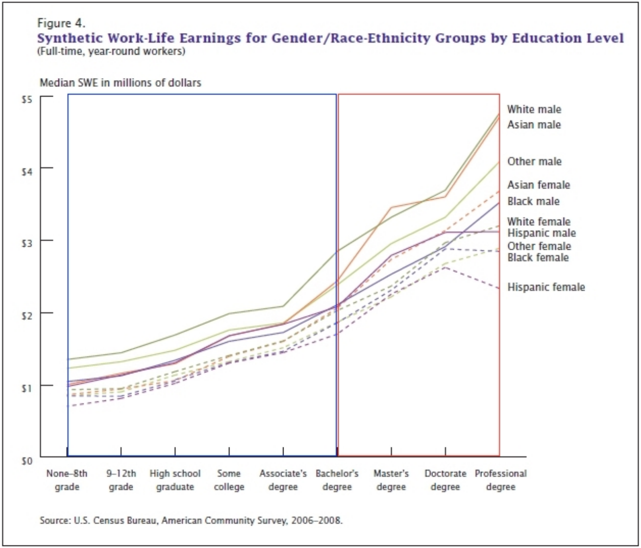
I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.

Modern frameworks for causal analysis:

- Potential outcomes (Neyman, 1923; Rubin, 1974, 1978).
- Causal graphs (Pearl, 2009).

Both rely on “counterfactual” logic.

Running Example: Effect of College on Earnings*



*SWE = expected total earnings over 25-64.

Potential outcomes

A causal effect can be defined as
a contrast between "potential outcomes."

Potential outcomes

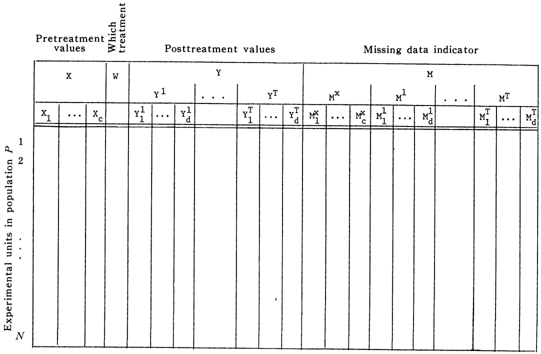


FIG. 1. All values in a study of T treatments.

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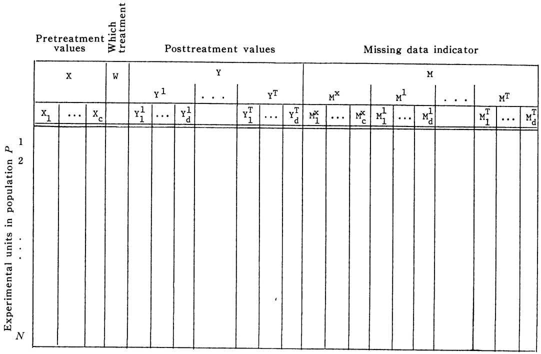

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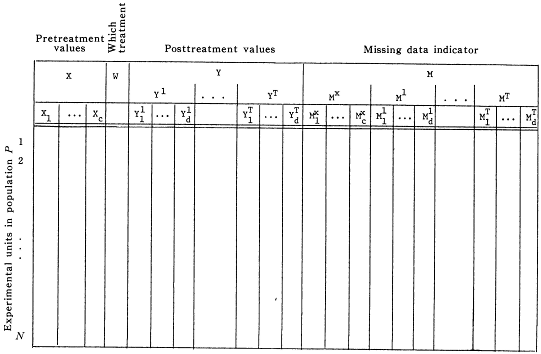

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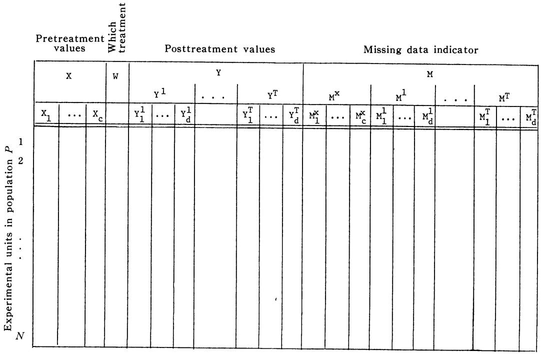

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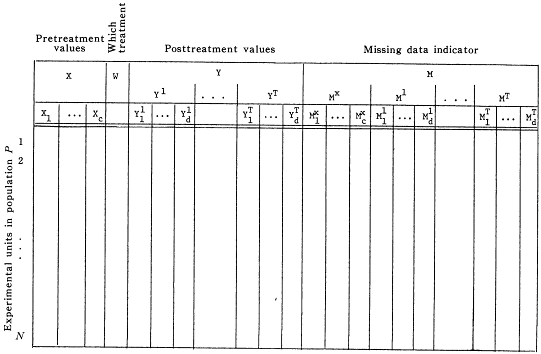

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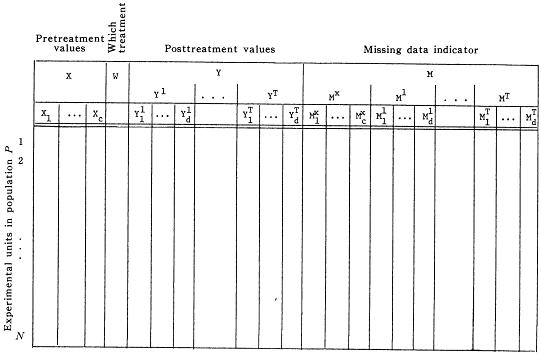

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- Missing data indicators, $M_{i,j}^{(k)}$.

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- *Population causal effects* for compare aggregates of unit level causal effects for members of $\mathscr{P}$.
- Effects are defined in an "agnostic" or "non-parametric" way.
- Potential outcomes and covariates are fixed, treatments and response indicators stochastic.
- Effects are defined by letting only treatments vary, holding units fixed.
- Thus, causal effects are clearly defined for units that can conceivably receive different treatment values.
- A test for the above is "manipulation" (Holland, 1986).

Potential outcomes, causal effects, and manipulability

Holland (1986) : "For causal inference, it is critical that each unit be potentially exposable to any one of the causes."

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Issues arise when trying to interpret things like race or gender. See VanderWeele and Robinson (2014) for a formal treatment of ways to interpret "race effects."

Potential outcomes and fundamental problem of causal inference

Recall, a unit level causal effect compares y_{wi} to $y_{\tilde{w}i}$ for $w \neq \tilde{w}$.

"Fundamental problem of causal inference" (Holland, 1986) : For each i potential outcomes for all w exist, but we only observe the potential outcome for the treatment value that i receives.

- "Scientific solution": Use theory to determine when units are interchangeable.
- "Statistical solution": Study features of conditional distributions, such as averages.

Causal identification under the potential outcomes model

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- Consider a random draw, i, from $\mathscr{P}$, countable but large.
- Each draw is characterized by
  - a covariate vector, X_i,
  - potential outcomes that under SUTVA are characterized as Y_{di} for all $d \in \mathscr{D}$, as well as
  - treatment assignments, $D_i \in \mathscr{D}$.

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- Suppose $\mathscr{D} = \{0, 1\}$. Then under SUTVA, potential outcomes for an arbitrary draw from $\mathscr{P}$ are Y_{1i} and Y_{0i}.

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$$\rho_i = Y_{1i} - Y_{0i},$$

for which,

$$E[\rho_i] = E[Y_{1i} - Y_{0i}] = \rho, \tag{1}$$

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- For our running example, we have $D_i = 1$ if college, $D_i = 0$ if not. Outcome of interest is income. ρ is the average income benefit of college.

Causal identification under the potential outcomes model

- Consider simple difference in mean college grad incomes vs mean no college incomes:

$$E[Y_i|D_i = 1] - E[Y_i|D_i = 0] = E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 0]$$

- Then what does this difference equal?

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- Could do similar wrt to ATC or effect heterogeneity (cf. CCI).

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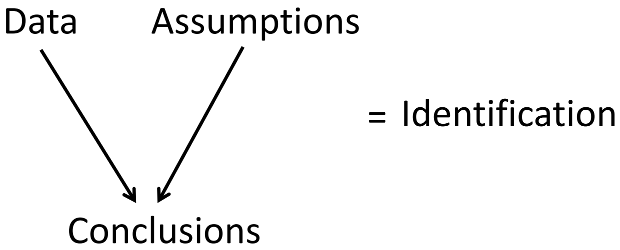

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As such,

$$\underbrace{E[Y_{1i}|D_i = 1] - E[Y_{0i}|D_i = 1]}_{\text{ATT}} = E[Y_{1i} - Y_{0i}],$$

so the simple difference, (2), equals ρ.

Causal identification under the potential outcomes model

Identifying assumption 2 (conditionally independent/unconfounded/strongly ignorable assignment):

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) | X_i \text{ and } 0 < Pr[D_i = 1 | X_i = x] < 1 \text{ for all } x \in \mathscr{X} \quad (4)$$

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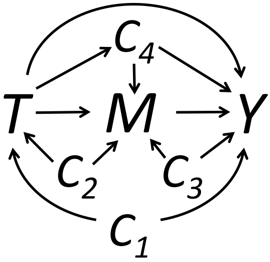
Marginalization over $\mathscr{X}$, the support of X_i, yields,

$$\int_{\mathscr{X}} \rho(x) dF(x) = \rho.$$



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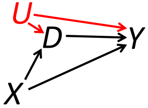

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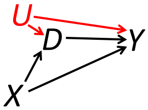

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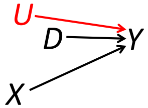

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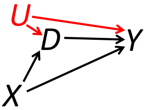

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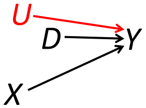

- Difference between $\mathrm{E}[Y|do(D_i = 1)] - \mathrm{E}[Y|do(D_i = 0)]$ and $\mathrm{E}[Y|D_i = 1] - \mathrm{E}[Y|D_i = 0]$ is "backdoor paths" from D to Y.

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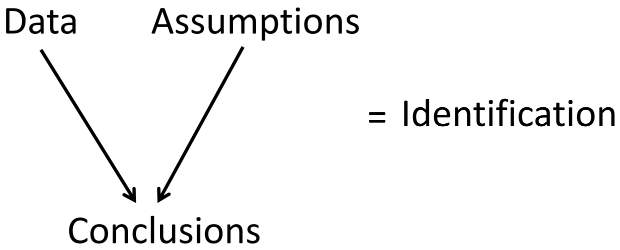

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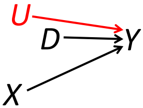

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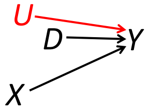
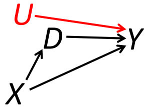


- CIA implies no backdoor path through U:



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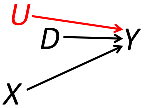
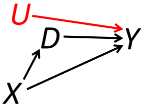
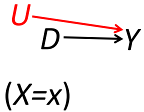
- Random assignment $\Rightarrow$ intervention graph *is* population graph:



- CIA implies no backdoor path through U:



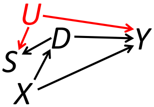
- Conditioning on X removes the other backdoor path:

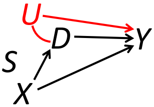


- Marginalize over x to recover the intervention graph.

Causal graphs

- These are examples of "closing" backdoor paths.
- Other operations, e.g., "opening" backdoor paths by conditioning on "colliders":





Looking forward to the rest of the class

- With respect to our example and cases that resemble it:
  - These identifying assumptions rule out confounding by U_i.
  - If true, sufficient for identifying average treatment effect.
  - Plausibility is suspect, so need either other sources of identification or ways to check sensitivity to violations.

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- Time for questions.