Lecture 9: Instrumental Variables I

POL-GA 1251
Quantitative Political Analysis II
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NYU Politics

March 3, 2021

For today:

- Basics of IV for causal effects from a semi-parametric linear regression perspective.

For next class:

- IV from a non-parametric, potential outcomes perspective.

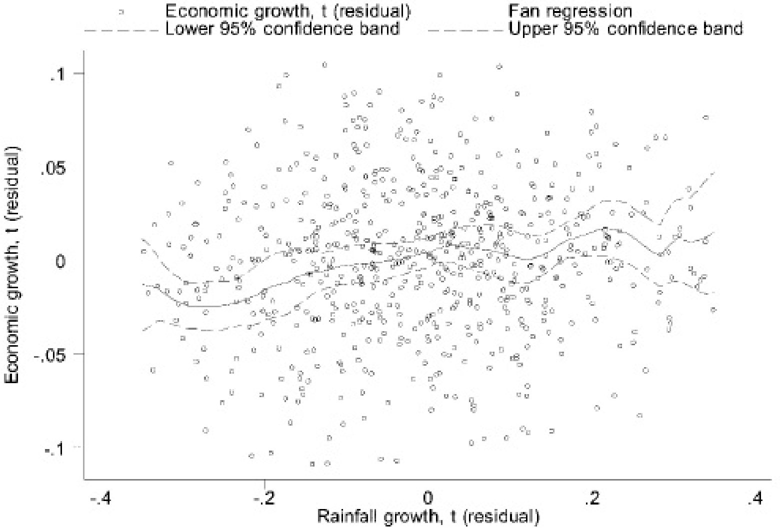


FIG. 1.—Current economic growth rate on current rainfall growth. Nonparametric Fan regression, conditional on country fixed effects and country-specific time trends.

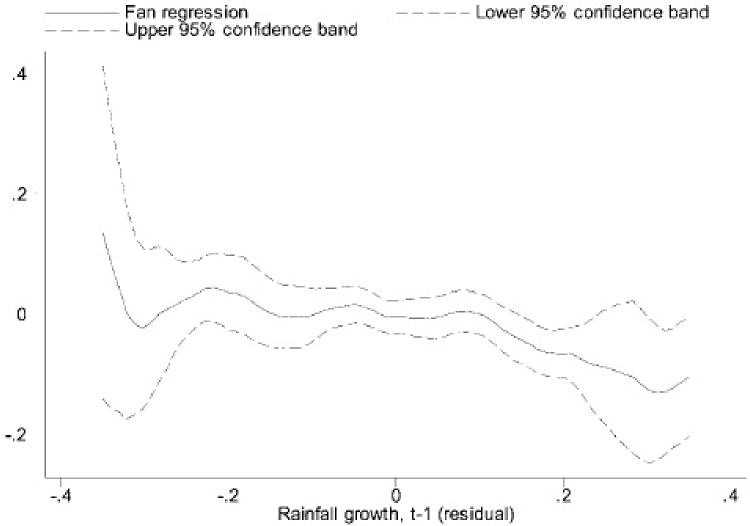
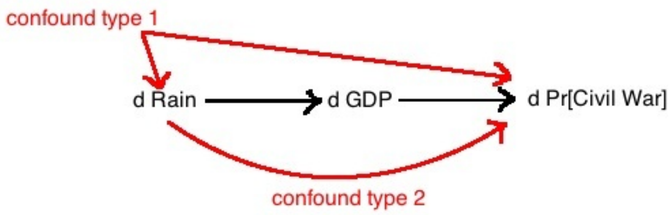


FIG. 2.—Current likelihood of civil conflict (≥25 battle deaths) on lagged rainfall growth. Nonparametric Fan regression, conditional on current rainfall growth, country fixed effects, and country-specific time trends.

d Rain $\longrightarrow$ d GDP $\longrightarrow$ d Pr[Civil War]


confound type 1
d Rain
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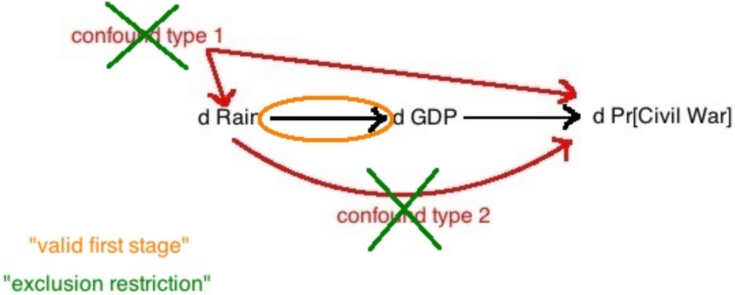
confound type 2


confound type 1
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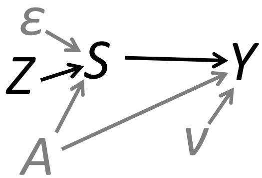
confound type 2
"valid first stage"
"exclusion restriction"

Formal presentation follows MHE (Ch. 4). We start with a simplified regression framework to convey some key intuitions.

"An initial focus on constant effects allows us to explain the mechanics of IV with a minimum of fuss" (115).

Setting



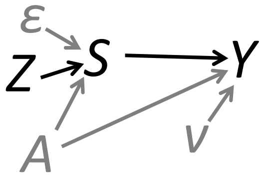
- Suppose the following structural model,

$$S_i = \gamma + A_i'\delta + \lambda Z_i + \varepsilon_i$$
$$Y_i = \alpha + \rho S_i + \underbrace{A_i'\gamma + \nu_i}_{\eta},$$

where ε_i and ν_i are mean zero, and $\mathrm{E}[A_i\varepsilon_i] = \mathrm{E}[A_i\nu_i] = 0$ and $\mathrm{E}[Z_i\varepsilon_i] = \mathrm{E}[S_i\nu_i] = 0$.

- If observe A_i, use OLS on Y_i equation to estimate ρ.
- But what if we do not observe A_i?

Identification with an instrument



- Suppose we observe some pre-treatment variable Z_i that is correlated with the treatment S_i *but not* A_i:

$$\text{Cov}\left[S_i, Z_i\right] \neq 0, \text{ but } \text{Cov}\left[A_i, Z_i\right] = 0$$

- Implication:

$$\begin{aligned} S_i &= \gamma + \lambda Z_i + (A_i'\delta + \varepsilon_i) \\ Y_i &= \alpha + \rho(\gamma + A_i'\delta + \lambda Z_i + \varepsilon_i) + A_i'\gamma + \nu_i \\ &= (\alpha + \rho\gamma) + \rho\lambda Z_i + (A_i'(\delta + \gamma)\rho + \rho\varepsilon_i + \nu_i), \end{aligned}$$

where we can estimate λ and $\rho\lambda$ with OLS, and then $\rho = \rho\lambda/\lambda$.

Identification with an instrument

- Another way to see it:
  Given
  $$\text{Cov}[S_i, Z_i] \neq 0, \text{ but } \text{Cov}[\eta_i, Z_i] = 0,$$
- we have

$$\begin{aligned}\text{Cov}[Y_i, Z_i] &= \text{Cov}[\alpha + \rho S_i + \eta_i, Z_i] = \rho \text{Cov}[S_i, Z_i] \\ \Rightarrow \rho &= \frac{\text{Cov}[Y_i, Z_i]}{\text{Cov}[S_i, Z_i]} = \frac{\frac{\text{Cov}[Y_i, Z_i]}{\text{Var}[Z_i]}}{\frac{\text{Cov}[S_i, Z_i]}{\text{Var}[Z_i]}} = \frac{\text{Reduced form}}{\text{First stage}}.\end{aligned}$$

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- For S_i and Z_i binary, we have the "Wald estimator,"
  $\rho = \frac{\mathrm{E}\left[Y_i | Z_i = 1\right] - \mathrm{E}\left[Y_i | Z_i = 0\right]}{\mathrm{E}\left[S_i | Z_i = 1\right] - \mathrm{E}\left[S_i | Z_i = 0\right]}$, which will come back later on.

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Consider some alternative DGPs:

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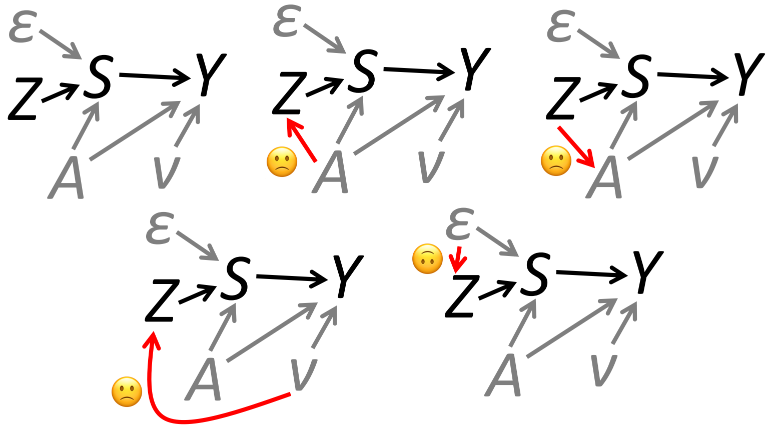
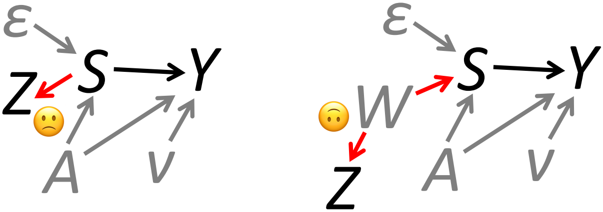


Identification with an instrument

Alternative DGPs:



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- Then $\text{Cov}[Y_i, Z_i] = \rho \text{Cov}[S_i, Z_i] + \text{Cov}[\eta_i, Z_i]$.

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- If we construct $\frac{\mathrm{Cov}[Y_i, Z_i]}{\mathrm{Cov}[S_i, Z_i]}$, we won't recover ρ but rather

$$\frac{\mathrm{Cov}[Y_i, Z_i]}{\mathrm{Cov}[S_i, Z_i]} = \rho + \frac{\mathrm{Cov}[\eta_i, Z_i]}{\mathrm{Cov}[S_i, Z_i]} = \rho + \frac{\mathrm{Cor}[\eta_i, Z_i]}{\mathrm{Cor}[S_i, Z_i]} \frac{\sigma_\eta}{\sigma_S},$$

which is biased for ρ, where bias is large if $\mathrm{Cor}[S_i, Z_i]$ is small.

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- "Strong first stage" is important
  - Old school: "first stage F-stat > 10" (Stock & Watson, 2007, based on bias bounds).
  - New school: "F-stat > 104.7" (McCrary et al. 2020). Need substantial revisions to inference to account for first stage.

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- Population regression equations,

$$\begin{aligned} S_i &= X_i'\pi_{10} + \pi_{11} Z_i + \xi_{1i} && \text{first stage} \\ Y_i &= X_i'\pi_{20} + \pi_{21} Z_i + \xi_{2i} && \text{reduced form} \end{aligned}$$

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- (S_i, Y_i) endogenous, (Z_i, X_i) exogenous.
- Assuming Z_i an instrument,

$$\rho = \frac{\pi_{21}}{\pi_{11}} = \frac{\mathrm{Cov}\left[\tilde{Y}_i, \tilde{Z}_i\right]}{\mathrm{Cov}\left[\tilde{S}_i, \tilde{Z}_i\right]} \quad \text{(by FWL)}$$

First stage:

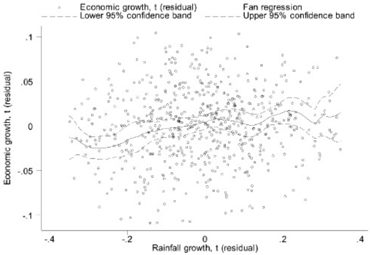

FIG. 1.—Current economic growth rate on current rainfall growth. Nonparametric Fan regression, conditional on country fixed effects and country-specific time trends.

Reduced form:

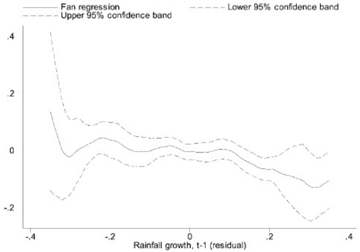

FIG. 2.—Current likelihood of civil conflict (≥25 battle deaths) on lagged rainfall growth. Nonparametric Fan regression, conditional on current rainfall growth, country fixed effects, and country-specific time trends.

Deriving the 2SLS expression

- Substitute first stage into outcome model:

$$\begin{aligned}
Y_i &= X_i'\alpha + \rho(X_i'\pi_{10} + \pi_{11}Z_i + \xi_{1i}) + \eta_i \\
&= X_i'(\alpha + \rho\pi_{10}) + \rho\pi_{11}Z_i + +(\rho\xi_{1i} + \eta_i) \\
&= X_i'\pi_{20} + \pi_{21}Z_i + \xi_{2i} && \text{(from above)}
\end{aligned}$$

So $\xi_{2i} = \rho\xi_{1i} + \eta_i$.

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&= X_i'\pi_{20} + \pi_{21}Z_i + \xi_{2i} \qquad \text{(from above)}
\end{aligned}
$$

  So $\xi_{2i} = \rho\xi_{1i} + \eta_i$.

- Plugging that back into the first line yields,

$$
\begin{aligned}
Y_i &= X_i'\alpha + \rho(X_i'\pi_{10} + \pi_{11}Z_i) + \xi_{2i} \\
&= X_i'\alpha + \rho\mathrm{E}[S_i|X_i, Z_i] + \xi_{2i}.
\end{aligned}
$$

- Could use Z_i to obtain predicted values of S_i, then use predicted values as regressors to estimate ρ.

Deriving the 2SLS expression

- Estimate $\mathrm{E}[S_i|X_i, Z_i]$ with OLS:

$$\hat{S}_i = X_i'\hat{\pi}_{10} + \hat{\pi}_{11} Z_i$$

- Adding and subtracting $\rho\hat{S}_i$ to outcome model yields,

$$\begin{aligned} Y_i &= X_i'\alpha + \rho S_i + \eta_i + \rho\hat{S}_i - \rho\hat{S}_i \\ &= X_i'\alpha + \rho\hat{S}_i + [\eta_i + \rho(S_i - \hat{S}_i)]. \end{aligned}$$

- "Two-stage least squares" (2SLS) expression for Y_i.

Deriving the 2SLS expression

- With H instruments, $Z_i = (Z_{1i} \dots Z_{Hi})'$:

Deriving the 2SLS expression

- With H instruments, $Z_i = (Z_{1i} \dots Z_{Hi})'$:
- First stage equation:

$$S_i = X_i'\pi_{10} + Z_i'\pi_Z + \xi_{1i}.$$

- If each instrument captures the same causal effect, yields a more precise estimate of ρ.
- We return to issue of "captures same causal effect" next lecture.

Deriving the 2SLS expression

- In practice, estimate 2SLS in one step.
- Helps to ensure that we estimate the variance of the coefficients correctly.

Deriving the 2SLS expression

...let $\mathbf{S}$ be endogenous variables, $\hat{\mathbf{S}}$ predicted values, $\mathbf{X}$ regressors, and $\mathbf{Z}$ instruments.

$$\hat{\Gamma}_{2SLS} = (\underbrace{[\hat{\mathbf{S}}\mathbf{X}]'[\hat{\mathbf{S}}\mathbf{X}]}_{A})^{-1}\underbrace{[\hat{\mathbf{S}}\mathbf{X}]'Y}_{B},$$

where $\hat{\Gamma}_{2SLS}$ includes estimate of ρ and other coefficients in regression model.

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where $\hat{\Gamma}_{2SLS}$ includes estimate of ρ and other coefficients in regression model.Consider A:

$$\begin{aligned}
[\hat{\mathbf{S}}\mathbf{X}]'[\hat{\mathbf{S}}\mathbf{X}] &= \left[[\mathbf{ZX}]([\mathbf{ZX}]'[\mathbf{ZX}])^{-1}[\mathbf{ZX}]'\mathbf{S} \quad \mathbf{X}\right]' \left[[\mathbf{ZX}]([\mathbf{ZX}]'[\mathbf{ZX}])^{-1}[\mathbf{ZX}]'\mathbf{S} \quad \mathbf{X}\right] \\
&= [\mathbf{M}'\mathbf{S} \quad \mathbf{X}]'[\mathbf{M}'\mathbf{S} \quad \mathbf{X}] \qquad (\mathbf{M}\text{ idempotent}) \\
&= \begin{bmatrix} \mathbf{SMM'S} & \mathbf{S'MX} \\ \mathbf{X'M'S} & \mathbf{X'X} \end{bmatrix} = \begin{bmatrix} \mathbf{SMS} & \mathbf{S'MX} \\ \mathbf{X'M'S} & \mathbf{X'MX} \end{bmatrix} \qquad (\text{by } \mathbf{MX} = \mathbf{X}) \\
&= [\mathbf{SX}]'\mathbf{M}[\mathbf{SX}] = [\mathbf{SX}]'[\mathbf{ZX}]([\mathbf{ZX}]'[\mathbf{ZX}])^{-1}[\mathbf{ZX}]'[\mathbf{SX}].
\end{aligned}$$

Similar for B.

Estimating 2SLS models

Putting it together yields,

$$\hat{\Gamma}_{2SLS} = \Big([\mathbf{SX}]'[\mathbf{ZX}]([\mathbf{ZX}]'[\mathbf{ZX}])^{-1}[\mathbf{ZX}]'[\mathbf{SX}]\Big)^{-1}[\mathbf{SX}]'[\mathbf{ZX}]([\mathbf{ZX}]'[\mathbf{ZX}])^{-1}[\mathbf{ZX}]'Y$$
$$\equiv (\mathbf{W}'\mathbf{M}\mathbf{W})^{-1}\mathbf{W}'\mathbf{M}Y,$$

where $\mathbf{W}$ is the matrix of regressors in the causal model, and $\mathbf{M}$ is the hat matrix from the first stage. This is how regression software actually computes 2SLS.

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- Stata: `ivregress 2sls...` with the usual robust or cluster-robust standard error methods available.
- R: `ivreg(...)` in the `AER` package, and then you can use the `sandwich` or `clubSandwich` packages for robust and cluster-robust standard errors.

Standard errors for $\hat{\Gamma}_{2SLS}$

Following the presentation in MHE, let $V_i \equiv [X_i' \;\; \hat{S}_i]'$. Then,

$$\begin{aligned}
\hat{\Gamma}_{2SLS} = (\mathbf{V}'\mathbf{V})^{-1}\mathbf{V}'Y &= \left(\sum_{i=1}^{N} V_i V_i'\right)^{-1} \sum_{i=1}^{N} V_i Y_i \\
&= \Gamma + \left(\sum_{i=1}^{N} V_i V_i'\right)^{-1} \sum_{i=1}^{N} V_i[\eta_i + \rho(S_i - \hat{S}_i)] \\
&= \Gamma + \left(\sum_{i=1}^{N} V_i V_i'\right)^{-1} \sum_{i=1}^{N} V_i \eta_i, \qquad (1)
\end{aligned}$$

by the fact that $\rho(S_i - \hat{S}_i)$ are orthogonal to V_i by the OLS solution in the first stage.

Standard errors for $\hat{\Gamma}_{2SLS}$

- Asymptotically,

$$\hat{S}_i = X_i'\hat{\pi}_{10} + Z_i'\hat{\pi}_Z \xrightarrow{p} X_i'\pi_{10} + Z_i'\pi_Z,$$

  which is fixed.
- Consistent estimator for $\mathrm{Var}[\hat{\Gamma}_{2SLS}]$ applies the similar results as we used for OLS:
- With no clustering, consistent estimator for the variance:

$$\widehat{\mathrm{Var}}[\hat{\Gamma}_{2SLS}] = \left(\sum_{i=1}^{N} V_i V_i'\right)^{-1} \left(\sum_{i=1}^{N} V_i V_i' \hat{\eta}_i^2\right) \left(\sum_{i=1}^{N} V_i V_i'\right)^{-1}.$$

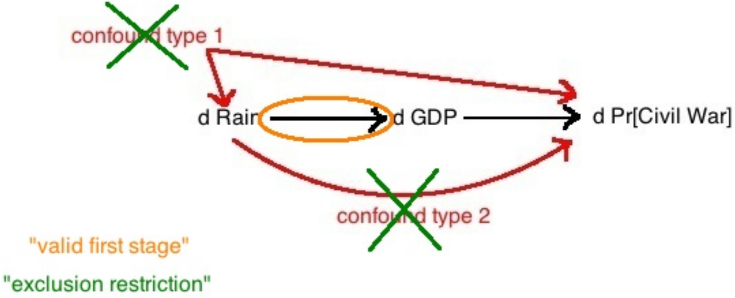
- Like heteroskedasticity-robust estimator for OLS.
- $\hat{\eta}_i$ are estimated by plugging the original S_i back into the outcome equation with the 2SLS coefficients:

$$\hat{\eta}_i = Y_i - (X_i\hat{\alpha}_{2SLS} + \hat{\rho}_{2SLS} S_i).$$

- With clustering, use cluster-robust.

Remarks

- The key assumptions are
  1. exclusion, which embeds both exogeneity and "no-alternative-pathways" assumptions, and
  2. valid first stage.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"valid first stage"
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- For exclusion, MHE (p. 131) provides indirect tests, known as "placebo tests":
  - Look for associations between Z_i and things that predict Y_i but should not be sensitive to Z_i (e.g., things that were determined prior to Z_i).
  - Find subpopulations for which there should be no relationship between Z_i and S_i. If Z_i predicts Y_i in this subsample, then you may have an exclusion violation.

Remarks

- For binary candidate instrument (Z), treatment (S), and outcome (Y), Pearl (1995) shows the following must hold if Z satisfies exclusion as an instrument for S:

$$
\begin{aligned}
P(Y=0,S=0 \mid Z=0)+P(Y=1,S=0 \mid Z=1) &\leq 1 \\
P(Y=0,S=1 \mid Z=0)+P(Y=1,S=1 \mid Z=1) &\leq 1 \\
P(Y=1,S=0 \mid Z=0)+P(Y=0,S=0 \mid Z=1) &\leq 1 \\
P(Y=1,S=1 \mid Z=0)+P(Y=0,S=1 \mid Z=1) &\leq 1.
\end{aligned}
$$

  When violated? When Z is associated ”with significant changes in the response variable Y while the direct cause, S, remains constant” (p. 437).

Remarks

- For binary candidate instrument (Z), treatment (S), and outcome (Y), Pearl (1995) shows the following must hold if Z satisfies exclusion as an instrument for S:

$$P(Y=0,S=0 \mid Z=0)+P(Y=1,S=0 \mid Z=1) \leq 1$$
$$P(Y=0,S=1 \mid Z=0)+P(Y=1,S=1 \mid Z=1) \leq 1$$
$$P(Y=1,S=0 \mid Z=0)+P(Y=0,S=0 \mid Z=1) \leq 1$$
$$P(Y=1,S=1 \mid Z=0)+P(Y=0,S=1 \mid Z=1) \leq 1.$$

  When violated? When Z is associated "with significant changes in the response variable Y while the direct cause, S, remains constant" (p. 437).
  - Y can vary in Z holding S fixed, but it cannot vary *too much*.
- Pearl extends to more general case when S and Z are still discrete, but Y is either discrete or continuous.
- Kedagni & Mourifie (2019) provide further extensions to, e.g., binary outcome and treatment, but unrestricted instrument.
- Other assumptions (e.g., "monotonicity" of effect of instrument on treatment) yield other testable implications of exclusion.

Remarks

Next we will move back to the potential outcomes framework which loosens the constant effects assumption.

We will discover that little changes in terms of how we do estimation, but there are important refinements in how we interpret IV estimates.