Lecture 2: Estimation and Inference for a Randomized Experiment

POL-GA 1251
Quantitative Political Analysis II
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NYU Politics

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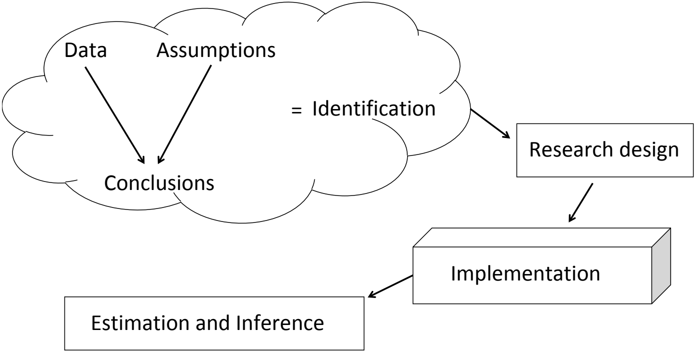
From identification to estimation and inference



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We will look at an idealized randomized experiment to illustrate key estimation and inference ideas.

The population and target

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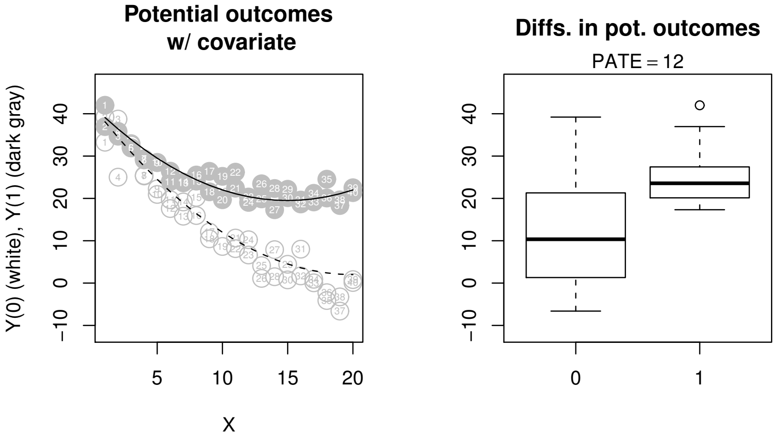
- Each unit in $j \in U$ has potential outcomes, $\{y_{dj}\}_{d \in \mathscr{D}}$.
- Each unit is also characterized by a continuous covariate, x_j.
- The population average treatment effect (PATE) is given by the expected value of difference in potential outcomes for an arbitrary unit drawn from U,

$$PATE = \rho = \mathrm{E}[Y_1 - Y_0].$$

- The PATE is our target of estimation and inference.

The population and target



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- We take a random sample S indexed by $i = 1, ..., n$.
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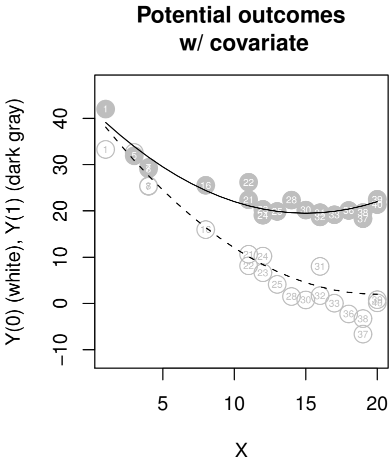
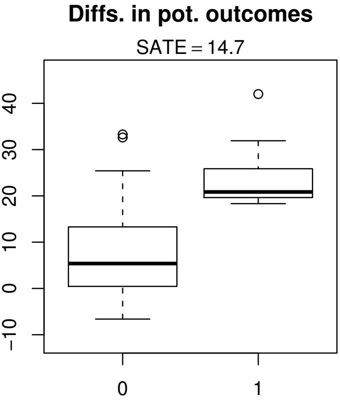
- We take a random sample S indexed by $i = 1, ..., n$.
- An arbitrary member of S is characterized by potential outcomes, (Y_{1i}, Y_{0i}), and a covariate, X_i.
- Another possible target of inference is thus the sample average treatment effect (SATE), given by the average difference in potential outcomes over members of the sample,

$$SATE = \rho_S = \frac{1}{n} \sum_{i \in S} Y_{1i} - Y_{0i}$$

- The SATE is a random quantity whose distribution is characterized by the sampling design.
- For any given sample, the SATE may not equal the PATE, introducing one source of variation for which we may want to account.



Potential outcomes
w/ covariate
Y(0) (white), Y(1) (dark gray)
X



Diffs. in pot. outcomes
SATE = 14.7

Simple randomized experiment

- We randomly assign $1 < n_1 < n - 1$ units to treatment ($D_i = 1$), in which case $n_0 = n - n_1$ are assigned to control ($D_i = 0$).
- We observe $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$.

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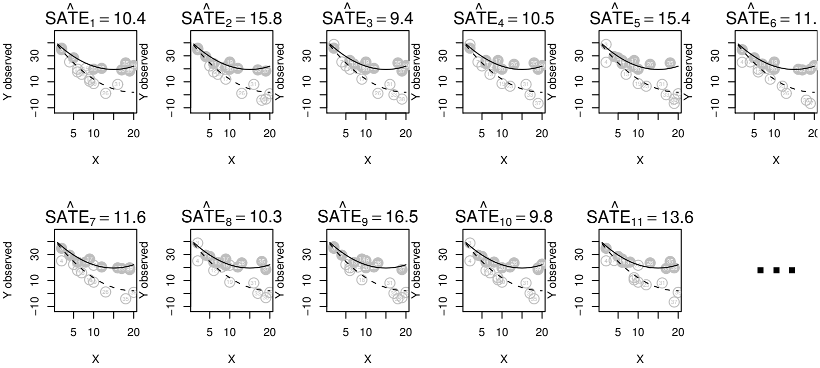
- We observe $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$.
- Consider an intuitive estimator for the SATE,

$$\hat{SATE} = \frac{1}{n_1} \sum_{i:D_i=1} Y_i - \frac{1}{n_0} \sum_{i:D_i=0} Y_i = \bar{Y}_1 - \bar{Y}_0.$$

- For any given experiment, this quantity will not equal SATE exactly. Assignment presents another source of variation for which we may want to account.


$\hat{SATE}_1 = 10.4$
$\hat{SATE}_2 = 15.8$
$\hat{SATE}_3 = 9.4$
$\hat{SATE}_4 = 10.5$
$\hat{SATE}_5 = 15.4$
$\hat{SATE}_6 = 11.$
$\hat{SATE}_7 = 11.6$
$\hat{SATE}_8 = 10.3$
$\hat{SATE}_9 = 16.5$
$\hat{SATE}_{10} = 9.8$
$\hat{SATE}_{11} = 13.6$
Y observed
X
...

Simple randomized experiment

- By random assignment,

$$
\begin{aligned}
\mathrm{E}_D[\hat{SATE}|S] &= \frac{1}{n_1} n_1 \mathrm{E}_D[Y_{1i}|S] - \frac{1}{n_0} n_0 \mathrm{E}_D[Y_{0i}|S] \\
&= \sum_{y_1 \in \{Y_{1i}: i \in S\}} y_1 \Pr[Y_{1i} = y_1] - \sum_{y_0 \in \{Y_{0i}: i \in S\}} y_0 \Pr[Y_{0i} = y_0] \\
&= \frac{1}{n} \sum_{i \in S} Y_{1i} - Y_{0i} = SATE,
\end{aligned}
$$

implying unbiasedness with respect to the SATE.

Simple randomized experiment

- By random assignment,

$$\begin{aligned} \mathrm{E}_D[\hat{SATE}|S] &= \frac{1}{n_1} n_1 \mathrm{E}_D[Y_{1i}|S] - \frac{1}{n_0} n_0 \mathrm{E}_D[Y_{0i}|S] \\ &= \sum_{y_1 \in \{Y_{1i}: i \in S\}} y_1 \Pr[Y_{1i} = y_1] - \sum_{y_0 \in \{Y_{0i}: i \in S\}} y_0 \Pr[Y_{0i} = y_0] \\ &= \frac{1}{n} \sum_{i \in S} Y_{1i} - Y_{0i} = SATE, \end{aligned}$$

  implying unbiasedness with respect to the SATE.
- Over samples, similar calculations show,

$$\mathrm{E}\,[\mathrm{E}_D[\hat{SATE}|S]] = \mathrm{E}\,[SATE] = PATE,$$

  in which case $\hat{SATE}$ is also unbiased for the PATE.

Simple randomized experiment

- By standard sample theoretic results and a bit of algebra,

$$
\begin{aligned}
\mathrm{Var}_D[\widehat{SATE}|S] &= \mathrm{Var}_D[\bar{Y}_1] + \mathrm{Var}_D[\bar{Y}_0] - 2\mathrm{Cov}_D[\bar{Y}_1, \bar{Y}_0] \\
&= \frac{s^2_{Y_1}}{n_1}\left(\frac{n-n_1}{n}\right) + \frac{s^2_{Y_0}}{n_0}\left(\frac{n-n_0}{n}\right) - 2\left[-\frac{s_{Y_1,Y_0}}{n}\right] \\
&= \frac{s^2_{Y_1}}{n_1} + \frac{s^2_{Y_0}}{n_0} - \underbrace{\frac{s^2_{Y_1} + s^2_{Y_0} - 2s_{Y_1,Y_0}}{n}}_{\text{numerator is } \mathrm{Var}[Y_1 - Y_0] = \mathrm{Var}[\rho_i]} \\
&= \frac{s^2_{Y_1}}{n_1} + \frac{s^2_{Y_0}}{n_0} - \frac{s^2_{\rho}}{n},
\end{aligned}
$$

(s^2_W is sample var. for W_i, and $s_{W,V}$ is sample cov. for W_i, V_i).

Simple randomized experiment

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$$\begin{aligned}
\text{Var}_D[S\hat{ATE}|S] &= \text{Var}_D[\bar{Y}_1] + \text{Var}_D[\bar{Y}_0] - 2\text{Cov}_D[\bar{Y}_1, \bar{Y}_0] \\
&= \frac{s^2_{Y_1}}{n_1}\left(\frac{n-n_1}{n}\right) + \frac{s^2_{Y_0}}{n_0}\left(\frac{n-n_0}{n}\right) - 2\left[-\frac{s_{Y_1,Y_0}}{n}\right] \\
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\end{aligned}$$

(s^2_W is sample var. for W_i, and $s_{W,V}$ is sample cov. for W_i, V_i).

- The first two terms are identified, however the third term is not.
- Ignoring third term will overestimate randomization variance.
- Largest when ρ_i is constant over i or, equivalently, when (Y_{1i}, Y_{0i}) are perfectly correlated (linear shift).
  (perfect correlation means high treated values implies low control values.)

Simple randomized experiment

- The variance goes to zero in n_1, n_0.
- As such, the distribution of $\hat{SATE}$ zeroes in on SATE as n_1, n_0 become large.
- Because of this, we say that "$\hat{SATE}$ is consistent for SATE in n_1, n_0."

Simple randomized experiment

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- Because of this, we say that "$\widehat{SATE}$ is consistent for SATE in n_1, n_0."
- Unbiasedness and consistency are two distinct properties:
  - Both are desirable.
  - Consistency is often more important in applied settings, e.g. in cases where a consistent but biased estimator has lower variance than any unbiased estimators. (An example is multiple regression with experimental data.)

Back to the population

- Over samples, the ANOVA theorem yields (MHE, p. 33),

$$
\begin{aligned}
\mathrm{Var}\,[\hat{SATE}] &= \mathrm{E}\,[\mathrm{Var}_D[\hat{SATE}|S]] + \mathrm{Var}\,[\mathrm{E}_D[\hat{SATE}|S]] \\
&= \mathrm{E}\left[\frac{s^2_{Y_1}}{n_1} + \frac{s^2_{Y_0}}{n_0} - \frac{s^2_{\rho}}{n}\right] + \frac{\sigma^2_{\rho}}{n} \\
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\text{Var}\,[S\hat{A}TE] &= \text{E}\,[\text{Var}_D[S\hat{A}TE|S]] + \text{Var}\,[\text{E}_D[S\hat{A}TE|S]] \\
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- Shows that $S\hat{A}TE$ is also consistent for PATE in n_1, n_0.

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- Shows that $\hat{SATE}$ is also consistent for PATE in n_1, n_0.
- By sample theoretic results, conventional sample variance estimators are unbiased for these population variances, so

$$
\hat{V} = \frac{\hat{s}^2_{Y_1}}{n_1} + \frac{\hat{s}^2_{Y_0}}{n_0},
$$

is unbiased for $\mathrm{Var}\,[\hat{SATE}]$ (different than $\mathrm{Var}_D[\hat{SATE}|S]$).

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The quantity $\hat{V}$ thus offers two interpretations when we have a randomized experiment with a random sample from the population:

- A conservative approximation of the randomization distribution variance, thus providing a way to make inferences about $\bar{Y}_1 - \bar{Y}_0$ relative to SATE.
- An unbiased estimator for the sampling distribution plus randomization distribution variance, thus providing a way to make inferences about $\bar{Y}_1 - \bar{Y}_0$ relative to PATE.

Inference for SATE or PATE: intervals

- Freedman (2008a) Theorem 1 shows that by a central limit theorem for non-independent random variables, $\bar{Y}_1 - \bar{Y}_0$ is asymptotically normal under well-behaved higher order moments.
- Thus, for large n_0, n_1, $(1-\alpha)100\%$ confidence intervals for SATE or PATE (under interpretations above) are given by,

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- This interval yields an asymptotic coverage rate of $(1-\alpha)100\%$.
- For not-so-large n_0, n_1, the coverage rate may be improved by using critical values from the t-distribution, which has "fatter tails." The t approximation is motivated by the exact finite sample distribution of normally distributed outcomes.

Inference for SATE or PATE: testing

- We first consider testing under the Neyman-Pearson framework, which aims to control error rates in a binary decision problem.
- The "average null" (or "weak null") hypothesis stipulates,

$$H_0^{av} : \rho = 0 \quad \text{versus} \quad H_a^{av} : \rho \neq 0 \text{ (two-sided).}$$

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- Under H_0^{av}, the t-statistic (t = "test"),

$$t = \frac{\bar{Y}_1 - \bar{Y}_0}{\sqrt{\hat{V}}}$$

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- The asymptotic type I error rate is α (if the null is true, you nonetheless reject $\alpha\%$ of the time).
- (The type II error rate is $1 -$ power, which depends on the standing of H_a.)
- The t distribution is used to improve error rates in finite samples.

Inference for SATE or PATE: testing

We may also wish to test the "sharp null" hypothesis,

$$H_0^{sh} : Y_{1i} = Y_{0i} \text{ for all } i \in S \quad \text{versus} \quad H_a^{sh} : Y_{1i} = Y_{0i} \text{ for some } i \in S.$$

This hypothesis is a departure from the estimation and inference framework examined thus far, and gets us into the world of Fisher exact tests (next slide).

Note that $H_0^{sh} \Rightarrow H_0^{av}$, and so rejection of H_0^{av} implies rejection of H_0^{sh} (but not the other way around).

An aside on exact tests

- If H_0^{sh} is true, we actually know all potential outcomes!
- Thus, for those with $D_i = 1$, we impute $Y_{0i} = Y_i$, and for those with $D_i = 0$, we impute $Y_{1i} = Y_i$.

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- We can examine how "unusual" is our estimate, $\bar{Y}_1 - \bar{Y}_0$ relative to all the possible values it could take under H_0^{sh}.
- With our imputed potential outcomes, we compute $\bar{Y}_1 - \bar{Y}_0$ for all of the $\binom{n}{n_1}$ treatment assignment possibilities, and see what proportion of those estimates are larger than what we observed.
- This provides an exact one-sided p-value for H_0^{sh}. That is, if we reject against an α threshold, in *finite samples*, the exact type I error rate is α.

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- This provides an exact one-sided p-value for H_0^{sh}. That is, if we reject against an α threshold, in *finite samples*, the exact type I error rate is α.
- Note to test H_0^{sh} we don't have to use $\bar{Y}_1 - \bar{Y}_0$. Imbens and Rubin (CIS, Ch. 5) show that the difference in average *ranks* weakly dominates $\bar{Y}_1 - \bar{Y}_0$ in terms of power.
- This approach to testing is due to Fisher (1935).
- "Inverted test" intervals can be constructed (Rosenbaum, 2002).

Covariates and efficiency

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Covariates and efficiency

- In our examination of estimation and inference concepts thus far, we have not made any use of our covariates.
- Because of random assignment and random sampling, there has been no need to use covariates to obtain unbiased and consistent estimates.
- However, we can use covariate information to improve efficiency (that is, reduce the randomization and sampling distribution variance of our estimate of ρ) while maintaining consistency (though not unbiasedness).

Covariates and efficiency

Consider the interacted regression, allowing X_i to be a vector,

$$Y_i = \alpha + \rho_{reg} D_i + X_i' \beta_0 + D_i (X_i - \bar{X}_D)' \beta_1 + \varepsilon_i$$

where $\bar{X}_D$ is covariate means for the treatment groups.

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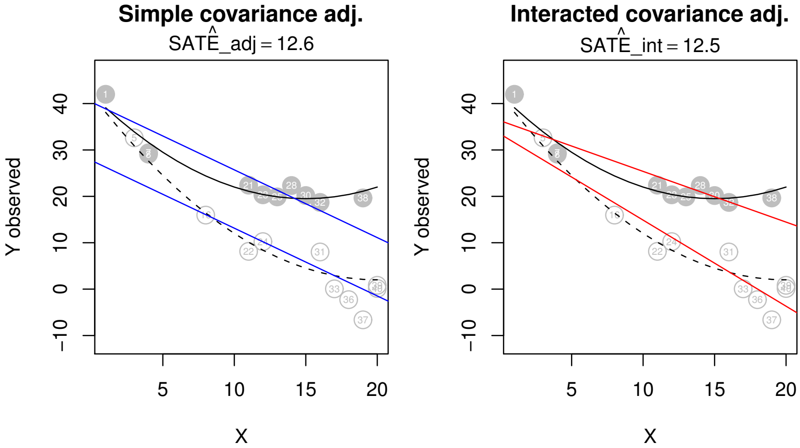
where $\bar{X}_D$ is covariate means for the treatment groups.

- OLS estimation yields $\hat{\rho}_{reg} = \bar{Y}_{1adj} - \bar{Y}_{0adj}$, where

$$\begin{aligned} \bar{Y}_{1adj} &= \bar{Y}_1 + (\bar{X} - \bar{X}_1)'(\hat{\beta}_0 + \hat{\beta}_1) \\ \bar{Y}_{0adj} &= \bar{Y}_0 + (\bar{X} - \bar{X}_0)' \hat{\beta}_0, \end{aligned}$$

- If we exclude the interaction, we would set $\beta_1 = 0$
- No presumption that the regression specification is "correct."


Simple covariance adj.
$\hat{SATE}$_adj = 12.6
Interacted covariance adj.
$\hat{SATE}$_int = 12.5
Y observed
X

Covariates and efficiency

$$Y_i = \alpha + \rho_{reg} D_i + X_i' \beta_0 + D_i (X_i - \bar{X}_D)' \beta_1 + \varepsilon_i$$

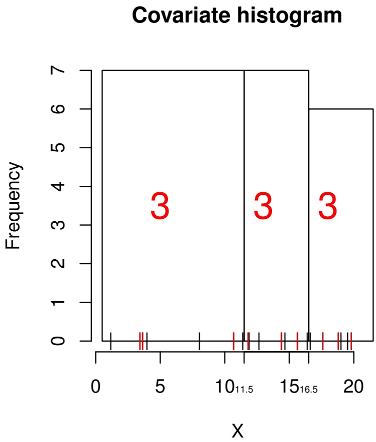
- $\hat{\rho}_{reg}$ is biased but consistent for SATE.
- $\hat{\rho}_{reg}$ has lower asymptotic variance than $\hat{\rho} = \bar{Y}_1 - \bar{Y}_0$ for PATE.
- As such, $\hat{\rho}_{reg}$ is consistent and more efficient than $\hat{\rho}$, albeit biased. The bias diminishes quickly in sample size ($O(1/n)$).
- Simple adjustment (i.e., setting $\beta_1 = 0$) improves efficiency if experiment is not strongly imbalanced ($min[n_1/n, n_0/n] < .25$) and $\text{Cov}(X_i, Y_i)$ is large relative to $\text{Cov}(X_i, Y_i(1) - Y_i(0))$ (Lin, 2013).

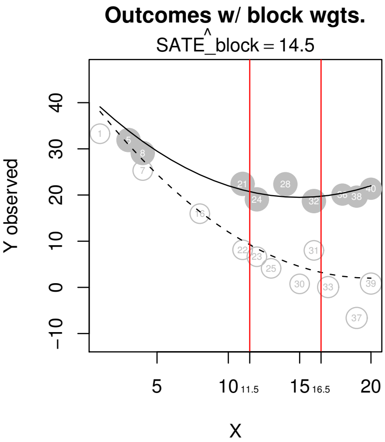
Covariates and efficiency

- Regression in previous slides motivated by efficiency.
- It may be used to address "incidental confounds," with consistency following from usual regression assumptions.

Covariates and efficiency

- A design-based approach (i.e., ex ante approach) to boost efficiency and address covariate imbalance is to incorporate covariates into the randomization.
- One approach is "block randomization."
- Observations are divided into blocks, $b = 1, .., B$, typically by prognostic covariates.
- Random assignment occurs within blocks.


Covariate histogram
Frequency
0
1
2
3
4
5
6
7
3
3
3
0
5
10
11.5
15
16.5
20
X


Outcomes w/ block wgts.
SATE^_block = 14.5
Y observed
−10
0
10
20
30
40
5
10
11.5
15
16.5
20
X

Covariates and efficiency

- By principles of stratified sampling, an unbiased estimator for SATE is, $S\hat{A}TE_{block} = \sum_b (N_b/N) S\hat{A}TE_b$.
- $S\hat{A}TE_{block}$ has lower variance that $S\hat{A}TE$ if outcome variation is reduced within strata.
- $S\hat{A}TE_{block}$ is algebraically equivalent to coefficient from regression with block FEs and inverse propensity score weights.

Conclusion

- The analysis here is “design-based”: we evaluated bias, consistency, coverage rates, and error rates with reference to the *sampling and randomization* distributions.

Conclusion

- The analysis here is “design-based”: we evaluated bias, consistency, coverage rates, and error rates with reference to the *sampling and randomization* distributions.
- For the idealized experiment, these distributions and associated error rates are *created* by the research design.
- They are under the *researcher’s control*.
- It is in this sense that design-based inference for experiments is a “principled basis for inference” (Fisher 1935).
- For observational studies, we can apply this framework in an “as-if” sense (Rubin 2008; Rosenbaum 2002), although it is *nature*, not the researcher, who controls the sampling, randomization, and therefore error rates.