Causal Inference under Interference

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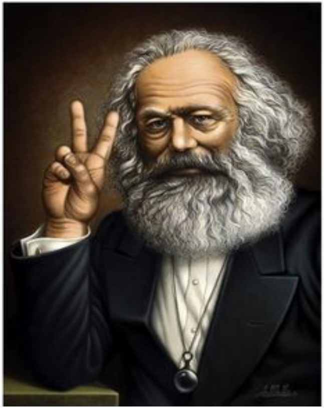
- What would happen to Y if Z changes?
- We call Y the outcome and Z the treatment.
- The better we understand causal relationships, the better we can design policy interventions.

Why do we care about causality?

- "The philosophers have only interpreted the world, in various ways; the point is to change it."



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- It is widely treated as the "golden rule" for causal inference.
- But there are limitations.
- Feasibility, cost, external validity, etc.

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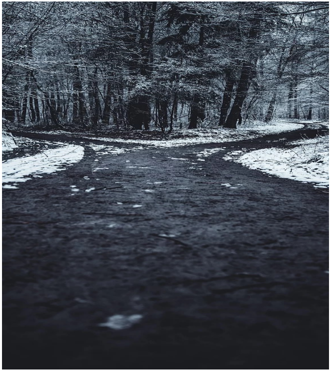
- We follow the current practice and define causality using counterfactuals.
- Ideally, we travel back to the past with a time machine and alter the value of Z.
- We then observe what would happen to Y.

How do we define causality?

- "Two roads diverged in a wood, and I—
  I took the one less traveled by,
  And that has made all the difference."
  — Robert Frost, *The Road Not Taken*



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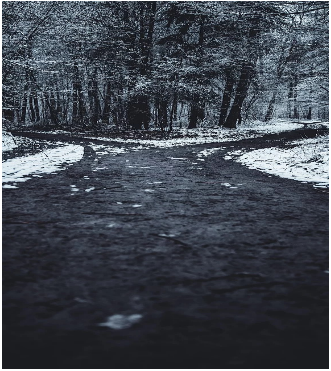

- This simple idea is captured by the Neyman-Rubin model.

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- $Y_i(d)$ is called the "potential outcome."
- $\tau_i = Y_i(1) - Y_i(0)$ is the individualistic treatment effect.

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- When the treatment is randomly assigned, we can estimate the ATE with the group-mean-difference estimator.
- Yet the framework is built upon the Stable Unit Treatment Value Assumption (SUTVA).
- When SUTVA is violated, we have interference.

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- But interference differs from contagion.
- The phenomenon is prevalent in the real world.
- Two questions: Does interference affect the estimation of the ATE? How do we estimate the spillover effects?

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- The group-mean-difference estimator will still be unbiased and consistent under most experimental designs.
- In other words, nothing but the interpretation changes.
- The magical power of randomization!

EATE

- Consider a simple experiment with two subjects and Bernoulli assignment

Treatment status	Prob	Ye	Ran
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- Then,
$$\tau = \frac{1}{2}(4.5 + 1.5) = 3.$$

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- Treated staff members received an invite to an information fair on the plan.

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- How do we estimate the indirect or spillover effects?

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- In this example, it equals to 50% or 0.

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- The estimated indirect effect on untreated staff members is $0.151 - 0.049 = 0.102$.
- We need a large number of departments to estimate the indirect effect precisely.

Application II

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- 28 schools were assigned into the treatment group.
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- Treated students were invited to participate in a bi-weekly meeting to discuss the consequences of conflicts and behavioral strategies.

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- Untreated students may learn about the content of the meetings from their friends.
- Treated students may reinforce each other's commitment to anti-conflict norms.

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- But a school can be big and the indirect effect estimate may not be meaningful.
- Instead, the authors collect information on the social network among the students.
- They ask the students to list all their friends in the school before the treatment.

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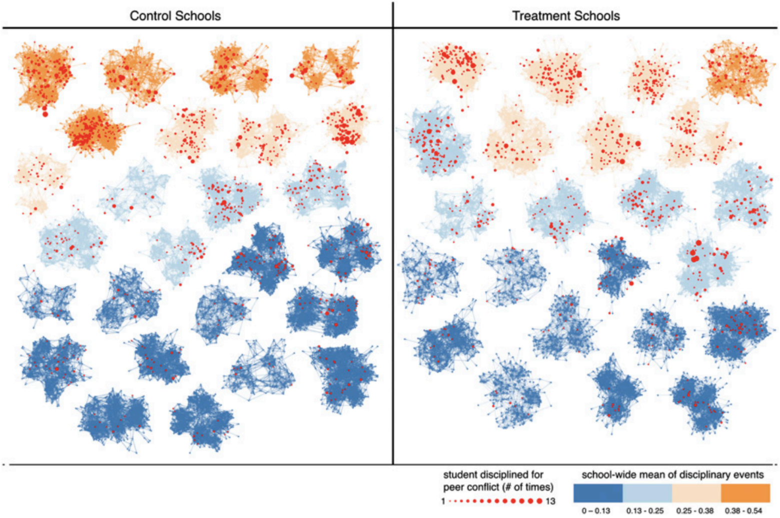

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- We assume that there exists a mapping from treatment assignment to the actual treatment exposure.
- If only one's outcome is only affected by her direct friends in the social network, we can measure the level of exposure with the proportion of treated friends.

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- This method requires the correct specification of the exposure mapping.

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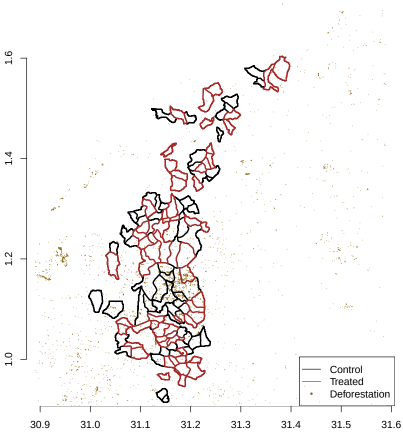
- We use no exposure as the benchmark.

Exposure mapping

Estimator	Estimand	Estimate	S.E.	95% CI
HT	$\tau(d_{001}, d_{000})$	0.057	0.062	(−0.065, 0.179)
	$\tau(d_{011}, d_{000})$	0.154	0.029	(0.097, 0.211)
	$\tau(d_{101}, d_{000})$	0.305	0.141	(0.029, 0.581)
	$\tau(d_{111}, d_{000})$	0.299	0.020	(0.260, 0.338)
Hajek	$\tau(d_{001}, d_{000})$	0.058	0.064	(−0.067, 0.183)
	$\tau(d_{011}, d_{000})$	0.154	0.037	(0.081, 0.227)
	$\tau(d_{101}, d_{000})$	0.292	0.123	(0.051, 0.533)
	$\tau(d_{111}, d_{000})$	0.307	0.049	(0.211, 0.403)
WLS	$\tau(d_{001}, d_{000})$	0.056	0.066	(−0.072, 0.186)
	$\tau(d_{011}, d_{000})$	0.156	0.037	(0.083, 0.229)
	$\tau(d_{101}, d_{000})$	0.295	0.124	(0.050, 0.536)
	$\tau(d_{111}, d_{000})$	0.306	0.049	(0.212, 0.404)

HT = Horvitz–Thompson estimator with conservative variance estimator.
Hajek = Hajek estimator with linearized variance estimator.
WLS = Least squares weighted by exposure probabilities with covariate adjustment for network degree and linearized variance estimator.
S.E. = Estimated standard error; CI = Normal approximation confidence interval.

Application III



Jayachandran et al., (2017): A payments for ecosystem services (PES) program, with effects dissipating in distance from treated villages.

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- Does any exposure mapping exist?

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- How to estimate the spillover effects on forest coverage?
- Can we assume partial interference?
- Does any exposure mapping exist?
- We need new methods for this scenario.

Define the indirect effect in an agnostic way

- Use the previous example

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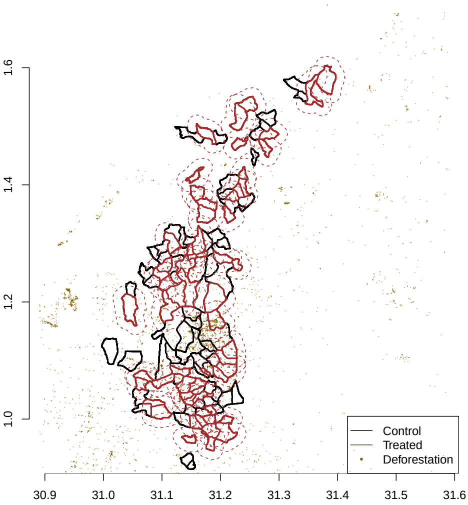
- Instead, we construct a “spillover mapping” μ to aggregate these individual level effects.
- μ captures the influence of one unit on the others.
- The form of μ is decided by the purpose of the study.
- In this example, we choose the “circle mean.”

Application III: estimation

- To estimate the effect at distance d, we draw circles with radius d around each treated village.



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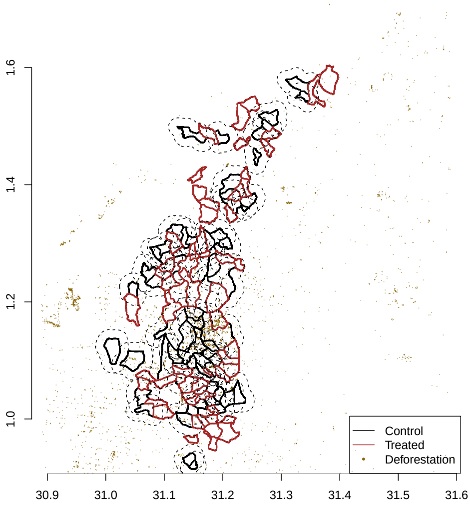
- Then, we do the same for each village in the control group.



Application III: estimation

- At each distance d, we have

$$\hat{\tau}(d) = \frac{1}{60}\sum_{i=1}^{60} Z_i \mu_i(\mathbf{Y}; d) - \frac{1}{61}\sum_{i=1}^{61}(1 - Z_i)\mu_i(\mathbf{Y}; d)$$

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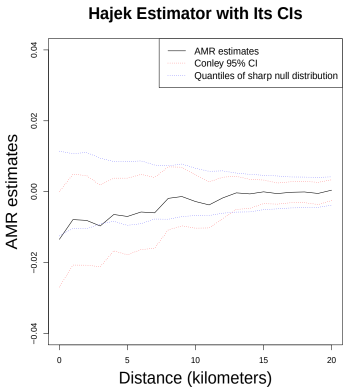
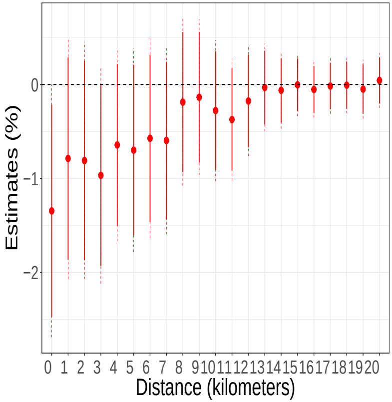
- This is a valid estimate of the EATE at distance d.

Application III: result





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- This is the comparative advantage of the design-based perspective to the outcome-based perspective.

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