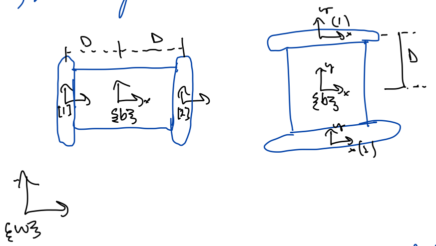
Kinematics Derivation

1) Label body frame & each wheel frame & 2) Find transforms / adjoints ← use ( ) when needed



3) Write body frame twist in each wheel frames

wheel → $\boxed{V_i = A_{ib} V_b}$ for each wheel

$$\begin{bmatrix}\dot\theta \\ V_{xi} \\ V_{yi}\end{bmatrix} = A_{ib}\begin{bmatrix}\dot\theta \\ V_x \\ V_y\end{bmatrix} \Longrightarrow \begin{bmatrix}\dot\theta \\ V_{xi} \\ V_{yi}\end{bmatrix} = T_{bi}\left(\begin{bmatrix}\dot\theta \\ V_x \\ V_y\end{bmatrix}\right)$$

↳ returns a twist2D (V_i)

4) Substitute wheel constraints into equations relating body frame twist to wheel frame twist

$$V_i = \begin{bmatrix}\dot\theta \\ V_{xi} \\ V_{yi}\end{bmatrix} = \begin{bmatrix}\dot\theta \\ r\dot\phi_i \\ 0\end{bmatrix} = \begin{bmatrix}r u_i \\ 0\end{bmatrix} \text{ twist}$$

← non-slippage & conventional wheel

should save this → this changes w/ each update of twist

$\dot\phi_i = u_i$ if wheel is actuated

↑ controlled wheel angular velocit

$$V_i = \begin{bmatrix}\cancel{\dot\theta} \\ r\dot\phi_i \\ 0\end{bmatrix} = A_{ib}\begin{bmatrix}\cancel{\dot\theta} \\ V_x \\ V_y\end{bmatrix} \swarrow V_b$$

Solve for $\dot\phi_i$

$$= \begin{bmatrix}\dot\phi_i \\ 0\end{bmatrix} = \frac{1}{r} A_{ib}\begin{bmatrix}V_x \\ V_y\end{bmatrix} \qquad \begin{bmatrix}00 & 01 & 02 \\ 10 & 11 & 12 \\ 20 & 21 & 22\end{bmatrix}$$

$\dot\phi = H V_b$

$$\begin{bmatrix}\dot\phi_1 \\ \dot\phi_2\end{bmatrix} = \frac{1}{r}\begin{bmatrix}-D & 1 & 0 \\ D & 1 & 0\end{bmatrix}\begin{bmatrix}\dot\theta \\ V_x \\ V_y\end{bmatrix}$$

Wheel 1

$$\begin{bmatrix}\dot\theta \\ r\dot\phi \\ 0\end{bmatrix} = \begin{bmatrix}\dot\theta \\ V_{x1} \\ V_{y1}\end{bmatrix} = \begin{bmatrix}\dot\theta \\ -D\dot\theta + V_x \\ L\dot\theta + V_y\end{bmatrix}$$

$r\dot\phi = -D\dot\theta + V_x$

$\dot\phi = \frac{-D\theta + V_x}{r}$

$\dot\phi = \cancel{rad/s} \times \frac{41.7\ \text{ticks}}{\cancel{rad/s}}$ dynamixel

1 motor command "tick" $= 0.024$ rad/s

so 1 rad/s = 41.7 tic

so $\dot\phi \times$ motor_cmd_per_rad_sec

= velocity in ticks (wheel command)

↓

("normalize" to $-265 \to 265$)

↓

wheel command

↓

nusim: "set wheel velocity until next command is received"

wheel velocity = rad/s

↳ convert back: $\cancel{ticks/s} \times \frac{1\ rad/s}{41.7\ \cancel{ticks}}$ (so $\frac{\text{wheel vel}}{\text{motor\_c\_p\_s}}$)

↓

1st) Update wheel position (tbot3.phi) & publish to: sensor

↳ current phi (tbot3.phi) + Δphi ← must be a time | : try

otherwise: phi + vel ← what is this value?

tbot3.phi = rad × $\frac{tick}{rad}$ (651.899)

2) Use forward kin to update position of red robot & publish APPROPRIATE FRAME

nusim ↓ red robot

correct speed

Pseudo-Inverse Calculation

$$H = \begin{bmatrix}D/r & 1/r & 0 \\ -D/r & 1/r & 0\end{bmatrix}$$

$$H^+ = (H^T H)^{-1} H^T$$

$$H^T H = \overset{3\times 2}{\begin{bmatrix}D/r & -D/r \\ 1/r & 1/r \\ 0 & 0\end{bmatrix}} \overset{2\times 3}{\begin{bmatrix}-D/r & 1/r & 0 \\ D/r & 1/r & 0\end{bmatrix}} \overset{=3\times 3}{=} \begin{bmatrix}D^2/r^2 + D^2/r^2 & \cancel{-D/r^2 + D/r^2} \; 0 & 0 \\ \cancel{-D/r^2 + D/r^2} \; 0 & 1/r^2 + 1/r^2 & 0 \\ 0 & 0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}2D^2/r^2 & 0 & 0 \\ 0 & 2/r^2 & 0 \\ 0 & 0 & 0\end{bmatrix}^{-1}$$

$$= \overset{3\times 3}{\begin{bmatrix}r^2/2D^2 & 0 & 0 \\ 0 & r^2/2 & 0 \\ 0 & 0 & 0\end{bmatrix}} \overset{3\times 2}{\begin{bmatrix}D/r & -D/r \\ 1/r & 1/r \\ 0 & 0\end{bmatrix}}$$

$$= \overset{3\times 2}{\begin{bmatrix}-\frac{r}{2D} & \frac{r}{2D} \\ r/2 & r/2 \\ 0 & 0\end{bmatrix}} \overset{2\times 1}{\begin{bmatrix}u_1 \\ u_2\end{bmatrix}} = V_b = \begin{bmatrix}\dot\theta \\ V_x \\ V_y\end{bmatrix}$$

$$= \begin{bmatrix}00(u_1) + 01(u_2) \\ 10(u_1) + 11(u_2) \\ 0 + 0\end{bmatrix} ✓$$

Integrate Twist

$$\begin{bmatrix}\dot\theta \\ 0 \\ 0\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 \\ y_s & 1 & 0 \\ -x_s & 0 & 1\end{bmatrix}\begin{bmatrix}\dot\theta \\ V.x \\ V.y\end{bmatrix}$$

$\dot\theta = \dot\theta$

$\dot\theta y_s + V.x = 0 \Rightarrow \dot\theta y_s = -V.x$

$\boxed{y_s = \frac{-V.x}{\dot\theta}}$ ✓

$-\dot\theta x_s + V.y = 0$

$\dot\theta x_s = V.y$

$\boxed{x_s = \frac{V.y}{\dot\theta}}$

$$A(\theta, 0, 0) = \overset{3\times 3}{\begin{bmatrix}1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta\end{bmatrix}} \overset{3\times 1}{\begin{bmatrix}\dot\theta \\ \Delta x \\ \Delta y\end{bmatrix}} = \begin{bmatrix}\theta \\ \Delta x\cos\theta - \Delta y\sin\theta \\ \Delta x\sin\theta + \Delta y\cos\theta\end{bmatrix} ✓$$