Time-Varying Graph Signal Median Filtering Regeneration

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1 Preliminary

In *Median Filtering* [1] approach, basically, a *multiset*[1] is generated based on time and graph neighboring values, and the median of the multiset is used as the filtered value at that time and node index. For a joint time-vertex signal $\mathbf{X}$, indexed by $\mathbf{X}_{i,t}$ for the value at node i and at time t, where $1 \leq i \leq N$ and $1 \leq t \leq T$, two versions of these multisets are proposed, and denoted as $\mathcal{X}^{(1)}_{i,t}$ and $\mathcal{X}^{(2)}_{i,t}$. These multisets are defined as follows:

$$\mathcal{X}^{(1)}_{i,t} = \{\mathbf{X}_{i,t-1}, \mathbf{X}_{i,t}, \mathbf{X}_{i,t+1}\} \cup \{\mathbf{X}_{j,t} \mid j \in \mathcal{N}_1(i)\} \tag{1}$$

$$\mathcal{X}^{(2)}_{i,t} = \mathcal{X}^{(1)}_{i,t} \cup \{\mathbf{X}_{j,t-1} \mid j \in \mathcal{N}_1(i)\} \cup \{\mathbf{X}_{j,t+1} \mid j \in \mathcal{N}_1(i)\} \tag{2}$$

where the values with overflowing indices are excluded. After the generation of these multisets, the filtered value at that node and time instance is denoted as $\mathbf{Y}^{(p)}_{i,t} = \text{median}(\mathcal{X}^{(p)}_{i,t})$, for $p \in \{1, 2\}$, and overall filtering is denoted as $\mathbf{Y}^{(p)} = \mathcal{M}_p(\mathbf{X})$.

2 Full Comparison

Full result comparison between the reported and regenerated median filtering experiments on *sea surface temperature* dataset is provided in Table 1. The underlying graph of reported results are not provided in the original paper [1], the generated results are obtained with k-NN underlying graph structure with several k values. The k-NN structure is used because the papers that use the *sea surface temperature* dataset usually use 5-NN graph for underlying graph [2], [3], which are also cited in [1]. The best overall results are bolded, where the best *generated* results are underlined.

Table 1: Full result comparison of SNR values of denoised signals between reported and generated versions.

	Reported with Unknown Graph			**Generated with k–NN Graph**								
				$\forall k$	$k=1$		$k=3$		$k=5$		$k=10$	
σ	Noisy	$\mathcal{M}_1$	$\mathcal{M}_2$	Noisy	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_1$	$\mathcal{M}_2$
0.05	22.63	**27.69**	27.26	22.63±0.05	$\underline{27.00\pm0.06}$	26.39±0.06	26.58±0.08	24.93±0.06	25.87±0.06	23.40±0.06	22.59±0.04	20.64±0.02
0.10	16.62	23.54	**25.09**	16.61±0.05	21.68±0.06	22.64±0.08	22.30±0.08	$\underline{22.86\pm0.07}$	22.21±0.07	22.02±0.07	20.87±0.08	20.24±0.07
0.15	13.10	20.59	**22.93**	13.08±0.05	18.33±0.06	19.72±0.09	19.38±0.09	$\underline{20.93\pm0.08}$	19.69±0.08	20.71±0.09	19.43±0.08	19.72±0.09
0.20	10.61	18.30	**21.05**	10.59±0.05	15.90±0.07	17.45±0.09	17.15±0.10	19.17±0.09	17.71±0.09	$\underline{19.44\pm0.10}$	18.10±0.07	19.12±0.10
0.25	8.66	16.50	**19.49**	8.65±0.05	13.99±0.07	15.63±0.09	15.34±0.10	17.63±0.10	16.05±0.09	18.24±0.11	16.87±0.08	$\underline{18.45\pm0.12}$
0.30	7.06	14.96	**18.08**	7.06±0.05	12.43±0.07	14.11±0.09	13.83±0.10	16.28±0.10	14.64±0.09	17.12±0.11	15.74±0.08	$\underline{17.77\pm0.13}$
0.35	5.72	13.59	16.78	5.72±0.05	11.10±0.07	12.81±0.09	12.53±0.11	15.10±0.10	13.42±0.09	16.09±0.12	14.71±0.08	$\underline{\mathbf{17.08\pm0.14}}$
0.40	4.57	12.53	15.73	4.57±0.05	9.94±0.07	11.67±0.09	11.40±0.11	14.04±0.11	12.33±0.09	15.15±0.12	13.77±0.08	$\underline{\mathbf{16.39\pm0.15}}$

3 Best Comparison

The best generated results along different underlying graph structures for both $\mathcal{M}_1$ and $\mathcal{M}_2$ filtering methods are compared with the reported ones in Tables 2 and 3, respectively.

Table 2: Best Comparison of $\mathcal{M}_1$

	Reported	Generated	
σ	$\mathcal{M}_1$	$\mathcal{M}_1$	Graph
0.05	**27.69**	27.00±0.06	1-NN
0.10	**23.54**	22.30±0.08	3-NN
0.15	**20.59**	19.69±0.08	5-NN
0.20	**18.30**	18.10±0.07	10-NN
0.25	16.50	**16.87±0.08**	10-NN
0.30	14.96	**15.74±0.08**	10-NN
0.35	13.59	**14.71±0.08**	10-NN
0.40	12.53	**13.77±0.08**	10-NN

Table 3: Best Comparison of $\mathcal{M}_2$

	Reported	Generated	
σ	$\mathcal{M}_2$	$\mathcal{M}_2$	Graph
0.05	**27.26**	26.39±0.06	1-NN
0.10	**25.09**	22.86±0.07	3-NN
0.15	**22.93**	20.93±0.08	3-NN
0.20	**21.05**	19.44±0.10	5-NN
0.25	**19.49**	18.45±0.12	10-NN
0.30	**18.08**	17.77±0.13	10-NN
0.35	16.78	**17.08±0.14**	10-NN
0.40	15.73	**16.39±0.15**	10-NN

[1] A multiset is a set with possible repetition, e.g., $\mathcal{S} = \{1, 1, 2, 3, 3, 3\}$.

References

[1] D. B. Tay and J. Jiang, "Time-varying graph signal denoising via median filters," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 68, no. 3, pp. 1053–1057, 2021. DOI: 10.1109/TCSII.2020.3017800.

[2] K. Qiu, X. Mao, X. Shen, X. Wang, T. Li, and Y. Gu, "Time-varying graph signal reconstruction," *IEEE Journal of Selected Topics in Signal Processing*, vol. 11, no. 6, pp. 870–883, 2017. DOI: 10.1109/JSTSP.2017.2726969.

[3] J. H. Giraldo, A. Mahmood, B. Garcia-Garcia, D. Thanou, and T. Bouwmans, "Reconstruction of time-varying graph signals via sobolev smoothness," *IEEE Transactions on Signal and Information Processing over Networks*, vol. 8, pp. 201–214, 2022. DOI: 10.1109/TSIPN.2022.3156886.