

PLOTS OF CLOSED FORMS

PETRO KOLOSOV

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1. INTRODUCTION

$$P(m, X, N) = \sum_{r=0}^m \sum_{k=1}^N \mathbf{A}_{m,r} k^r (X - k)^r$$

$$Q(m, X, N) = \sum_{r=0}^m \sum_{k=0}^{N-1} \mathbf{A}_{m,r} k^r (X - k)^r$$

$$P(m, N, N) = N^{2m+1}$$

$$Q(m, N, N) = N^{2m+1}$$

$$P(m, N + 1, N) = (N + 1)^{2m+1} - 1 \quad (\textit{verified})$$

$$Q(m, N - 1, N) = (N - 1)^{2m+1} + 1 \quad (\textit{verified})$$

1.1. Polynomials $P(1,X,N)$.

$$P(1, X, 0) = 0$$

$$P(1, X, 1) = 6X - 5$$

$$P(1, X, 2) = 18X - 28$$

$$P(1, X, 3) = 36X - 81$$

$$P(1, X, 4) = 60X - 176$$

$$P(1, X, 5) = 90X - 325$$

$$P(1, X, 6) = 126X - 540$$

$$P(1, X, 7) = 168X - 833$$

$$P(1, X, 8) = 216X - 1216$$

$$P(1, X, 9) = 270X - 1701$$

$$P(1, X, 10) = 330X - 2300$$

$$P(1, X, 11) = 396X - 3025$$

$$P(1, X, 12) = 468X - 3888$$

$$P(1, X, 13) = 546X - 4901$$

$$P(1, X, 14) = 630X - 6076$$

$$P(1, X, 15) = 720X - 7425$$

$$P(1, X, 16) = 816X - 8960$$

$$P(1, X, 17) = 918X - 10693$$

$$P(1, X, 18) = 1026X - 12636$$

$$P(1, X, 19) = 1140X - 14801$$

$$P(1, X, 20) = 1260X - 17200$$



Figure 1. Polynomials $P(1, X, N)$ for $N=1..4$

1.2. Polynomial $P(1, X, N)$ Table of values for $N = 6$.

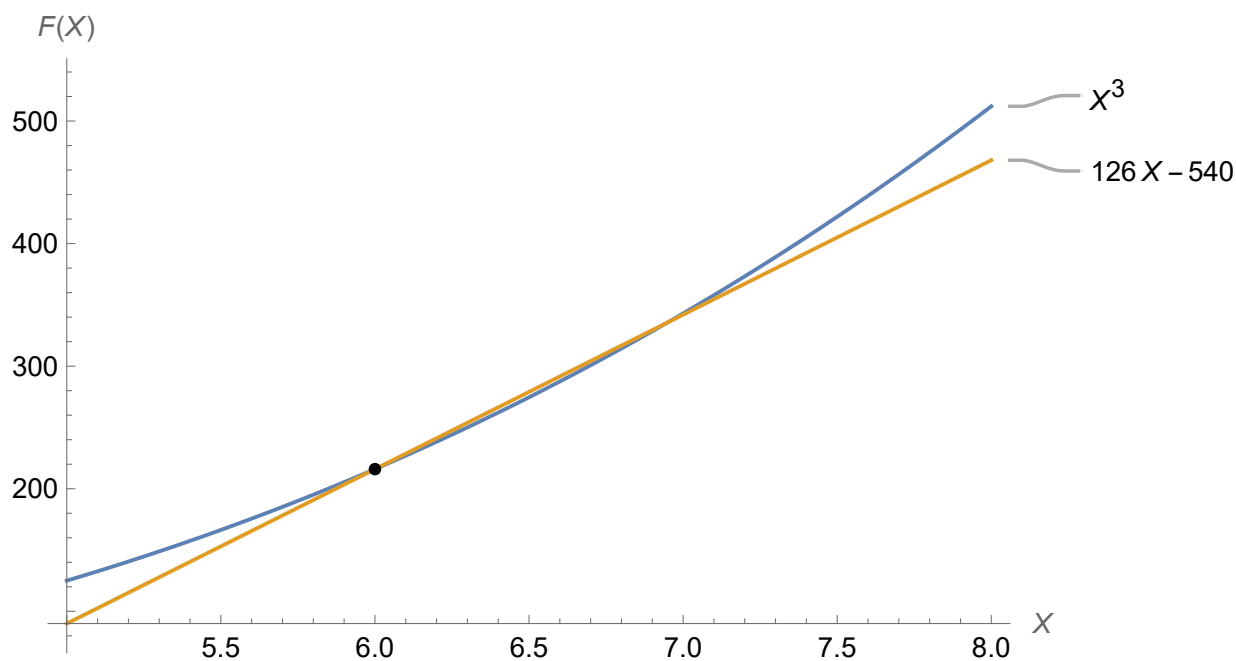


Figure 2. Polynomial plot $P(1, X, 6)$ with cubes

1.3. Polynomial $P(1, X, 6)$ plot with cubes.

1.4. Polynomials $Q(1,X,N)$.

$$Q(1, X, 0) = 0$$

$$Q(1, X, 1) = 1$$

$$Q(1, X, 2) = 6X - 4$$

$$Q(1, X, 3) = 18X - 27$$

$$Q(1, X, 4) = 36X - 80$$

$$Q(1, X, 5) = 60X - 175$$

$$Q(1, X, 6) = 90X - 324$$

$$Q(1, X, 7) = 126X - 539$$

$$Q(1, X, 8) = 168X - 832$$

$$Q(1, X, 9) = 216X - 1215$$

$$Q(1, X, 10) = 270X - 1700$$

$$Q(1, X, 11) = 330X - 2299$$

$$Q(1, X, 12) = 396X - 3024$$

$$Q(1, X, 13) = 468X - 3887$$

$$Q(1, X, 14) = 546X - 4900$$

$$Q(1, X, 15) = 630X - 6075$$

$$Q(1, X, 16) = 720X - 7424$$

$$Q(1, X, 17) = 816X - 8959$$

$$Q(1, X, 18) = 918X - 10692$$

$$Q(1, X, 19) = 1026X - 12635$$

$$Q(1, X, 20) = 1140X - 14800$$

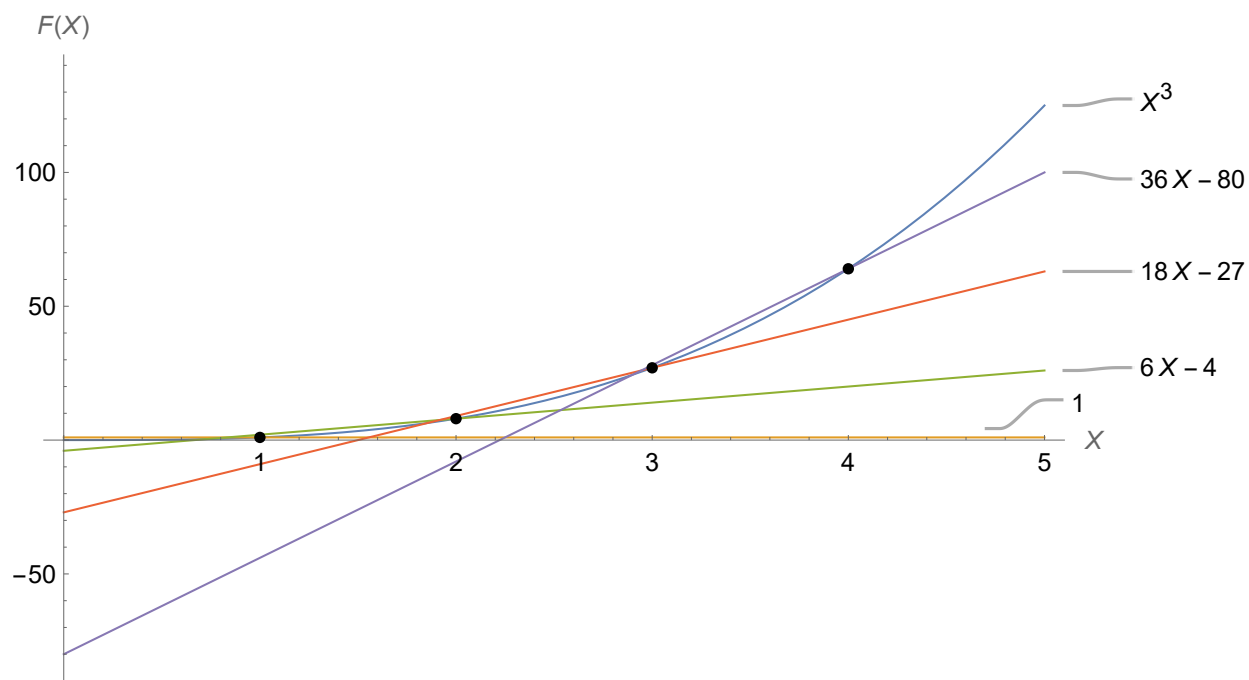


Figure 3. Polynomials $Q(1, n, k)$

1.5. Polynomial $Q(1, X, N)$ Table of values for $N = 6$.

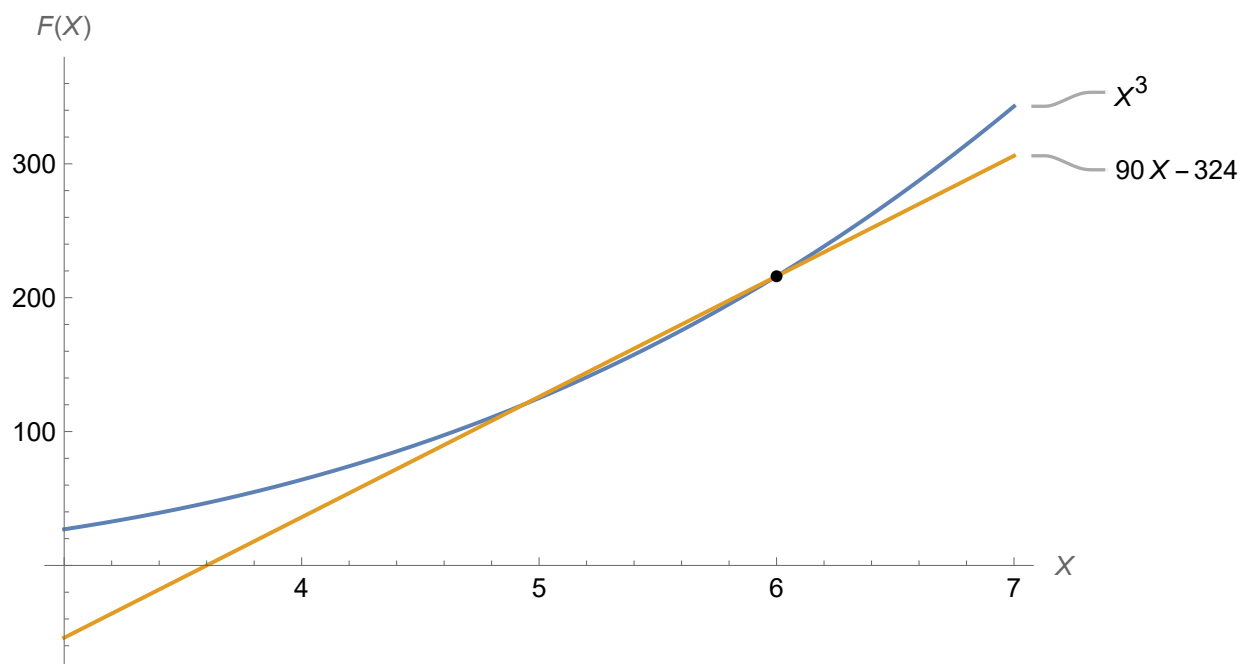


Figure 4. Polynomial plot $Q(1, X, 6)$ with cubes

1.6. Polynomial $Q(1,X,6)$ plot with cubes.

1.7. Polynomials $P(2, X, N)$.

$$P(2, X, 0) = 0$$

$$P(2, X, 1) = 30X^2 - 60X + 31$$

$$P(2, X, 2) = 150X^2 - 540X + 512$$

$$P(2, X, 3) = 420X^2 - 2160X + 2943$$

$$P(2, X, 4) = 900X^2 - 6000X + 10624$$

$$P(2, X, 5) = 1650X^2 - 13500X + 29375$$

$$P(2, X, 6) = 2730X^2 - 26460X + 68256$$

$$P(2, X, 7) = 4200X^2 - 47040X + 140287$$

$$P(2, X, 8) = 6120X^2 - 77760X + 263168$$

$$P(2, X, 9) = 8550X^2 - 121500X + 459999$$

$$P(2, X, 10) = 11550X^2 - 181500X + 760000$$

$$P(2, X, 11) = 15180X^2 - 261360X + 1199231$$

$$P(2, X, 12) = 19500X^2 - 365040X + 1821312$$

$$P(2, X, 13) = 24570X^2 - 496860X + 2678143$$

$$P(2, X, 14) = 30450X^2 - 661500X + 3830624$$

$$P(2, X, 15) = 37200X^2 - 864000X + 5349375$$

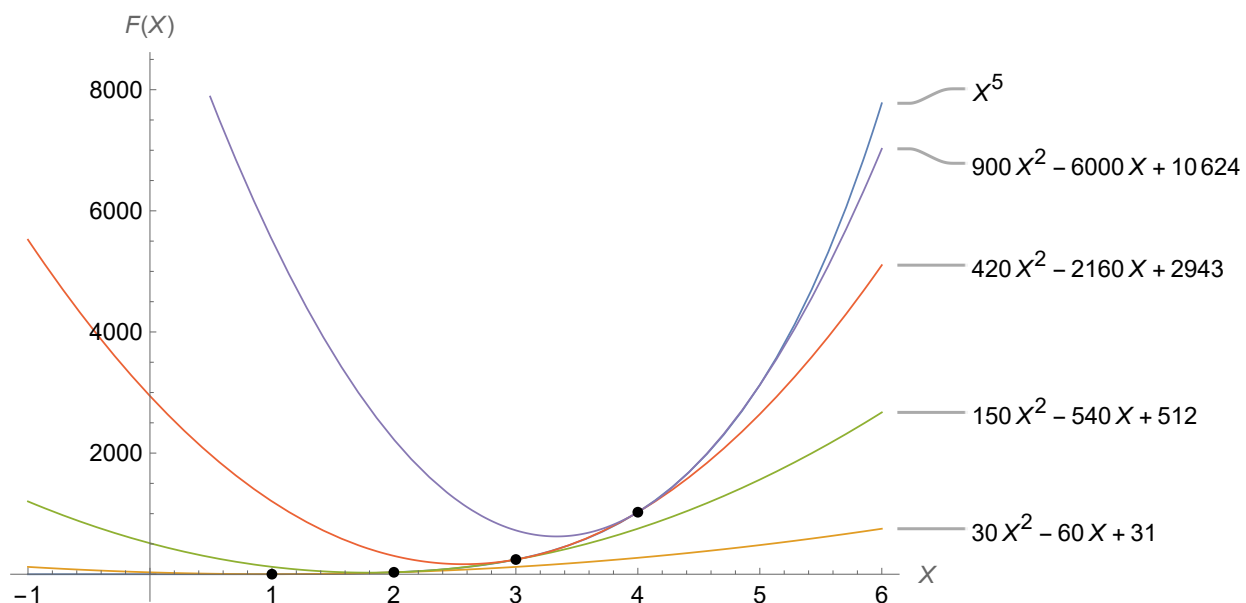
$$P(2, X, 16) = 44880X^2 - 1109760X + 7315456$$

$$P(2, X, 17) = 53550X^2 - 1404540X + 9821087$$

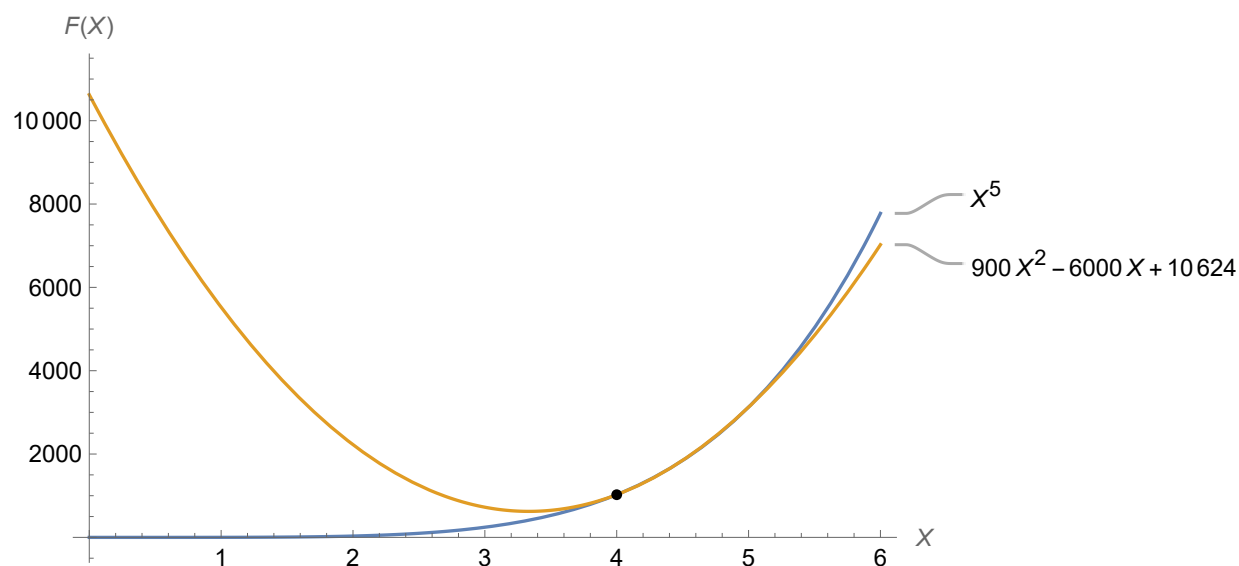
$$P(2, X, 18) = 63270X^2 - 1754460X + 12970368$$

$$P(2, X, 19) = 74100X^2 - 2166000X + 16879999$$

$$P(2, X, 20) = 86100X^2 - 2646000X + 21680000$$

Figure 5. Polynomials $P(2, n, k)$

1.8. Polynomial $P(2, X, N)$ Table of values for $N = 4$.

Figure 6. Polynomial plot $P(2, X, 4)$ with fifth

1.9. Polynomial $P(2, X, 4)$ plot with fifth.

1.10. **Polynomials $Q(2,X,N)$.**

$$Q(2, X, 0) = 0$$

$$Q(2, X, 1) = 1$$

$$Q(2, X, 2) = 30X^2 - 60X + 32$$

$$Q(2, X, 3) = 150X^2 - 540X + 513$$

$$Q(2, X, 4) = 420X^2 - 2160X + 2944$$

$$Q(2, X, 5) = 900X^2 - 6000X + 10625$$

$$Q(2, X, 6) = 1650X^2 - 13500X + 29376$$

$$Q(2, X, 7) = 2730X^2 - 26460X + 68257$$

$$Q(2, X, 8) = 4200X^2 - 47040X + 140288$$

$$Q(2, X, 9) = 6120X^2 - 77760X + 263169$$

$$Q(2, X, 10) = 8550X^2 - 121500X + 460000$$

$$Q(2, X, 11) = 11550X^2 - 181500X + 760001$$

$$Q(2, X, 12) = 15180X^2 - 261360X + 1199232$$

$$Q(2, X, 13) = 19500X^2 - 365040X + 1821313$$

$$Q(2, X, 14) = 24570X^2 - 496860X + 2678144$$

$$Q(2, X, 15) = 30450X^2 - 661500X + 3830625$$

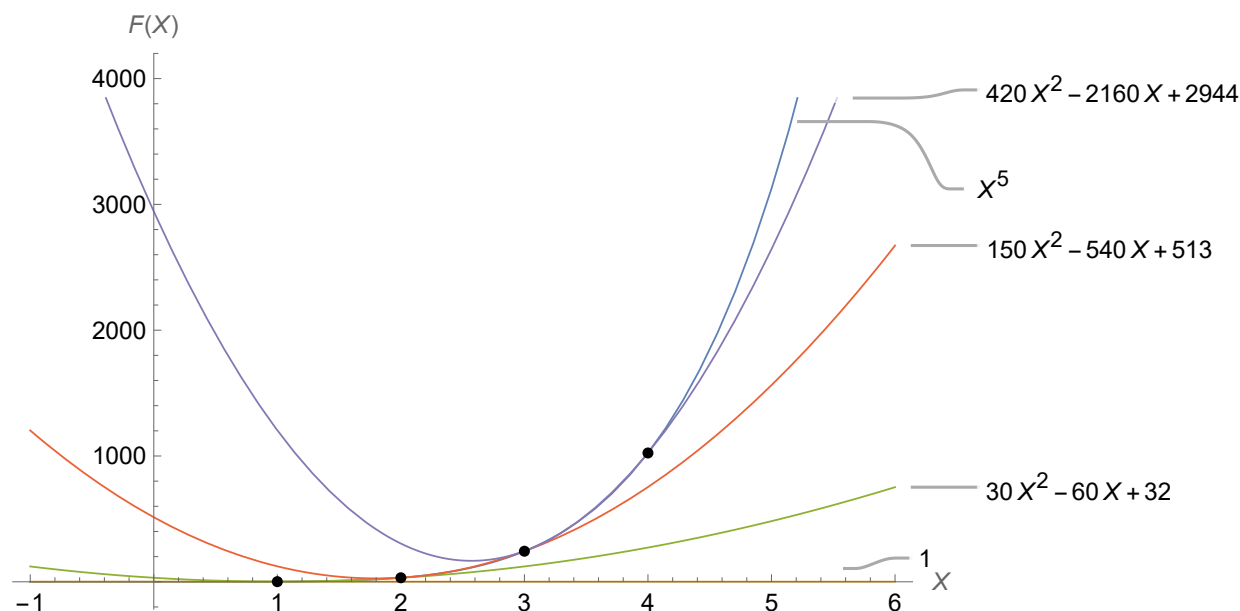
$$Q(2, X, 16) = 37200X^2 - 864000X + 5349376$$

$$Q(2, X, 17) = 44880X^2 - 1109760X + 7315457$$

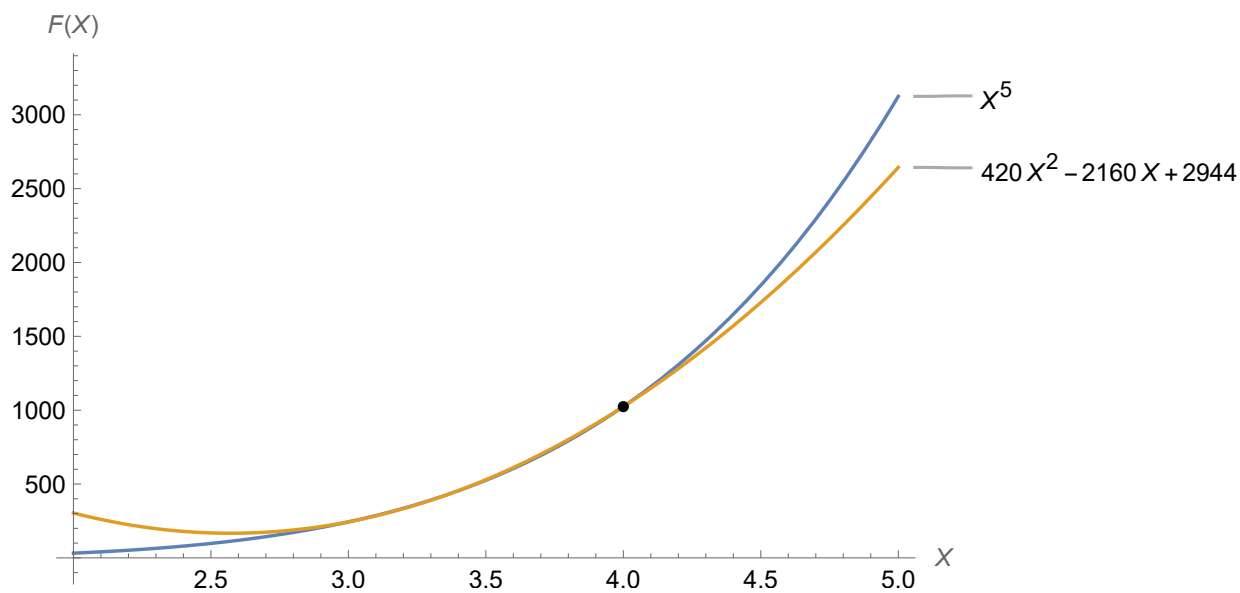
$$Q(2, X, 18) = 53550X^2 - 1404540X + 9821088$$

$$Q(2, X, 19) = 63270X^2 - 1754460X + 12970369$$

$$Q(2, X, 20) = 74100X^2 - 2166000X + 16880000$$

Figure 7. Polynomials $Q(2, n, k)$

1.11. Polynomial $Q(2, X, N)$ Table of values for $N = 4$.

Figure 8. Polynomial plot $Q(2, X, 4)$ with fifth

1.12. Polynomial $Q(2, X, 4)$ plot with fifth.

1.13. **Polynomials $P(3, X, N)$.**

$$P(3, X, 0) = 0$$

$$P(3, X, 1) = 140X^3 - 420X^2 + 406X - 125$$

$$P(3, X, 2) = 1260X^3 - 7140X^2 + 13818X - 9028$$

$$P(3, X, 3) = 5040X^3 - 41160X^2 + 115836X - 110961$$

$$P(3, X, 4) = 14000X^3 - 148680X^2 + 545860X - 684176$$

$$P(3, X, 5) = 31500X^3 - 411180X^2 + 1858290X - 2871325$$

$$P(3, X, 6) = 61740X^3 - 955500X^2 + 5124126X - 9402660$$

$$P(3, X, 7) = 109760X^3 - 1963920X^2 + 12182968X - 25872833$$

$$P(3, X, 8) = 181440X^3 - 3684240X^2 + 25945416X - 62572096$$

$$P(3, X, 9) = 283500X^3 - 6439860X^2 + 50745870X - 136972701$$

$$P(3, X, 10) = 423500X^3 - 10639860X^2 + 92745730X - 276971300$$

$$P(3, X, 11) = 609840X^3 - 16789080X^2 + 160386996X - 524988145$$

$$P(3, X, 12) = 851760X^3 - 25498200X^2 + 264896268X - 943023888$$

$$P(3, X, 13) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774781$$

$$P(3, X, 14) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907076$$

$$P(3, X, 15) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591425$$

$$P(3, X, 16) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398080$$

$$P(3, X, 17) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653693$$

$$P(3, X, 18) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360516$$

$$P(3, X, 19) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778801$$

$$P(3, X, 20) = 6174000X^3 - 303519720X^2 + 5179983060X - 30303773200$$

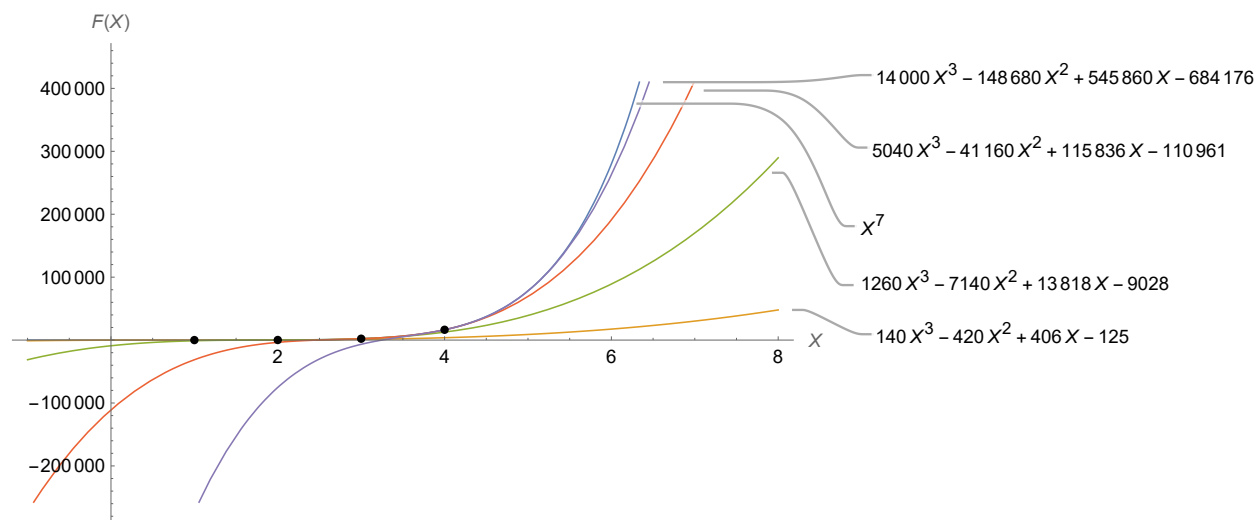


Figure 9. Polynomials $P(3, n, k)$

1.14. Polynomial $P(3, X, N)$ Table of values for $N = 3$.

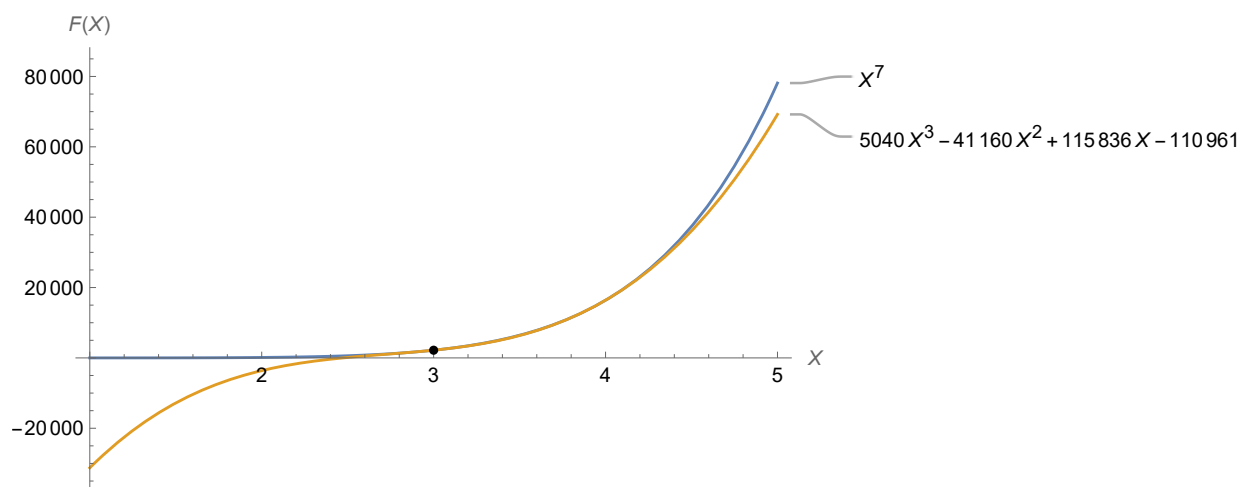


Figure 10. Polynomial plot $P(3, X, 3)$ with fifth

1.15. Polynomial $P(3, X, 3)$ plot with seventh.

1.16. **Polynomials $Q(3,X,N)$.**

$$Q(3, X, 0) = 0$$

$$Q(3, X, 1) = 1$$

$$Q(3, X, 2) = 140X^3 - 420X^2 + 406X - 124$$

$$Q(3, X, 3) = 1260X^3 - 7140X^2 + 13818X - 9027$$

$$Q(3, X, 4) = 5040X^3 - 41160X^2 + 115836X - 110960$$

$$Q(3, X, 5) = 14000X^3 - 148680X^2 + 545860X - 684175$$

$$Q(3, X, 6) = 31500X^3 - 411180X^2 + 1858290X - 2871324$$

$$Q(3, X, 7) = 61740X^3 - 955500X^2 + 5124126X - 9402659$$

$$Q(3, X, 8) = 109760X^3 - 1963920X^2 + 12182968X - 25872832$$

$$Q(3, X, 9) = 181440X^3 - 3684240X^2 + 25945416X - 62572095$$

$$Q(3, X, 10) = 283500X^3 - 6439860X^2 + 50745870X - 136972700$$

$$Q(3, X, 11) = 423500X^3 - 10639860X^2 + 92745730X - 276971299$$

$$Q(3, X, 12) = 609840X^3 - 16789080X^2 + 160386996X - 524988144$$

$$Q(3, X, 13) = 851760X^3 - 25498200X^2 + 264896268X - 943023887$$

$$Q(3, X, 14) = 1159340X^3 - 37493820X^2 + 420839146X - 1618774780$$

$$Q(3, X, 15) = 1543500X^3 - 53628540X^2 + 646725030X - 2672907075$$

$$Q(3, X, 16) = 2016000X^3 - 74891040X^2 + 965662320X - 4267591424$$

$$Q(3, X, 17) = 2589440X^3 - 102416160X^2 + 1406064016X - 6616398079$$

$$Q(3, X, 18) = 3277260X^3 - 137494980X^2 + 2002403718X - 9995653692$$

$$Q(3, X, 19) = 4093740X^3 - 181584900X^2 + 2796022026X - 14757360515$$

$$Q(3, X, 20) = 5054000X^3 - 236319720X^2 + 3835983340X - 21343778800$$

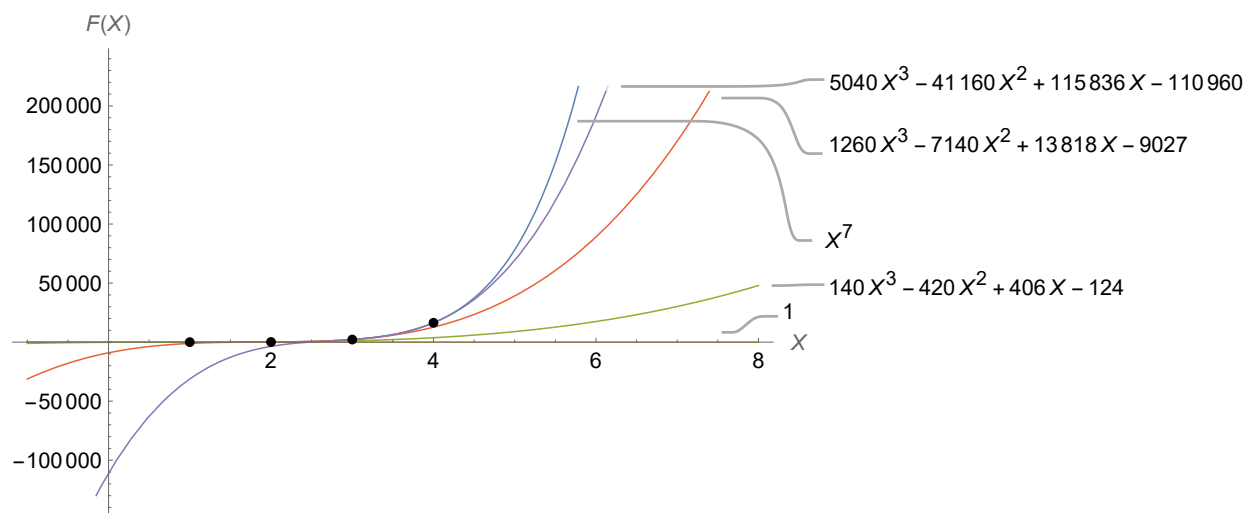


Figure 11. Polynomials $Q(3, n, k)$

1.17. Polynomial $Q(3, X, N)$ Table of values for $N = 3$.

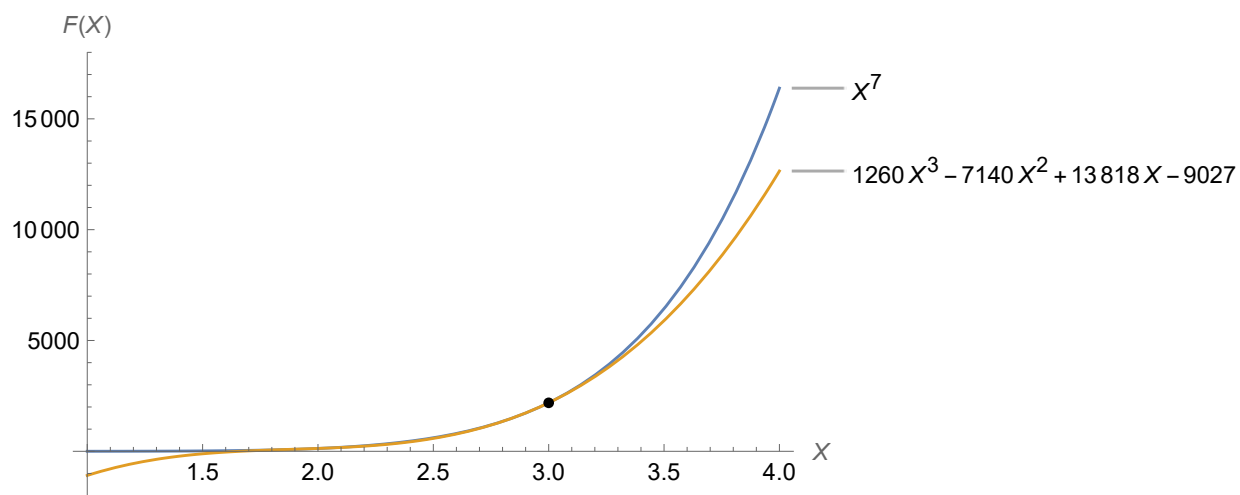


Figure 12. Polynomial plot $Q(3, X, 3)$ with fifth

1.18. Polynomial $Q(3, X, 3)$ plot with seventh.

Table 1. Comparison of X^3 , $P(1, X, 6) = 126X - 540$, Absolute, Relative, and Percentage Error

X	X^3	$126X - 540$	ABS	Relative	% Error
5.3	148.877	127.8	21.077	0.141573	14.1573
5.4	157.464	140.4	17.064	0.108368	10.8368
5.5	166.375	153.0	13.375	0.0803907	8.03907
5.6	175.616	165.6	10.016	0.0570335	5.70335
5.7	185.193	178.2	6.993	0.0377606	3.77606
5.8	195.112	190.8	4.312	0.0221001	2.21001
5.9	205.379	203.4	1.979	0.00963584	0.963584
6.0	216.0	216.0	0.0	0.0	0.0
6.1	226.981	228.6	1.619	0.00713276	0.713276
6.2	238.328	241.2	2.872	0.0120506	1.20506
6.3	250.047	253.8	3.753	0.0150092	1.50092
6.4	262.144	266.4	4.256	0.0162354	1.62354
6.5	274.625	279.0	4.375	0.0159308	1.59308
6.6	287.496	291.6	4.104	0.014275	1.4275
6.7	300.763	304.2	3.437	0.0114276	1.14276
6.8	314.432	316.8	2.368	0.00753104	0.753104
6.9	328.509	329.4	0.891	0.00271225	0.271225
7.0	343.0	342.0	1.0	0.00291545	0.291545
7.1	357.911	354.6	3.311	0.0092509	0.92509
7.2	373.248	367.2	6.048	0.0162037	1.62037
7.3	389.017	379.8	9.217	0.0236931	2.36931
7.4	405.224	392.4	12.824	0.0316467	3.16467
7.5	421.875	405.0	16.875	0.04	4.0
7.6	438.976	417.6	21.376	0.0486951	4.86951
7.7	456.533	430.2	26.333	0.0576804	5.76804
7.8	474.552	442.8	31.752	0.0669094	6.69094
7.9	493.029	455.4	37.629	0.0762408	7.62408

Table 2. Comparison of X^3 , $Q(1, X, 6) = 90X - 324$, Absolute, Relative, and Percentage Error

X	X^3	$90X - 324$	ABS	Relative	% Error
4.5	91.125	81.0	10.125	0.111111	11.1111
4.6	97.336	90.0	7.336	0.0753678	7.53678
4.7	103.823	99.0	4.823	0.0464541	4.64541
4.8	110.592	108.0	2.592	0.0234375	2.34375
4.9	117.649	117.0	0.649	0.00551641	0.551641
5.0	125.0	126.0	1.0	0.008	0.8
5.1	132.651	135.0	2.349	0.0177081	1.77081
5.2	140.608	144.0	3.392	0.0241238	2.41238
5.3	148.877	153.0	4.123	0.027694	2.7694
5.4	157.464	162.0	4.536	0.0288066	2.88066
5.5	166.375	171.0	4.625	0.0277986	2.77986
5.6	175.616	180.0	4.384	0.0249636	2.49636
5.7	185.193	189.0	3.807	0.0205569	2.05569
5.8	195.112	198.0	2.888	0.0148018	1.48018
5.9	205.379	207.0	1.621	0.00789273	0.789273
6.0	216.0	216.0	0.0	0.0	0.0
6.1	226.981	225.0	1.981	0.0087276	0.87276
6.2	238.328	234.0	4.328	0.0181598	1.81598
6.3	250.047	243.0	7.047	0.0281827	2.81827
6.4	262.144	252.0	10.144	0.0386963	3.86963
6.5	274.625	261.0	13.625	0.0496131	4.96131
6.6	287.496	270.0	17.496	0.0608565	6.08565
6.7	300.763	279.0	21.763	0.0723593	7.23593
6.8	314.432	288.0	26.432	0.0840627	8.40627
6.9	328.509	297.0	31.509	0.0959152	9.59152
7.0	343.0	306.0	37.0	0.107872	10.7872

Table 3. Comparison of X^5 , $P(2, X, 4) = 900X^2 - 6000X + 10624$, Absolute, Relative, and Percentage Error

X	X^5	$900X^2 - 6000X + 10624$	ABS	Relative	% Error
3.6	604.662	688.0	83.3382	0.137826	13.7826
3.7	693.44	745.0	51.5604	0.0743546	7.43546
3.8	792.352	820.0	27.6483	0.034894	3.4894
3.9	902.242	913.0	10.758	0.0119236	1.19236
4.0	1024.0	1024.0	0.0	0.0	0.0
4.1	1158.56	1153.0	5.56201	0.00480079	0.480079
4.2	1306.91	1300.0	6.91232	0.00528905	0.528905
4.3	1470.08	1465.0	5.08443	0.0034586	0.34586
4.4	1649.16	1648.0	1.16224	0.000704746	0.0704746
4.5	1845.28	1849.0	3.71875	0.00201528	0.201528
4.6	2059.63	2068.0	8.37024	0.00406395	0.406395
4.7	2293.45	2305.0	11.5499	0.00503605	0.503605
4.8	2548.04	2560.0	11.9603	0.00469393	0.469393
4.9	2824.75	2833.0	8.24751	0.00291973	0.291973
5.0	3125.0	3124.0	1.0	0.00032	0.032
5.1	3450.25	3433.0	17.2525	0.00500036	0.500036
5.2	3802.04	3760.0	42.0403	0.0110573	1.10573
5.3	4181.95	4105.0	76.9549	0.0184017	1.84017
5.4	4591.65	4468.0	123.65	0.0269294	2.69294
5.5	5032.84	4849.0	183.844	0.0365288	3.65288
5.6	5507.32	5248.0	259.318	0.047086	4.7086
5.7	6016.92	5665.0	351.921	0.0584885	5.84885
5.8	6563.57	6100.0	463.568	0.0706274	7.06274
5.9	7149.24	6553.0	596.243	0.0833995	8.33995
6.0	7776.0	7024.0	752.0	0.0967078	9.67078
6.1	8445.96	7513.0	932.963	0.110463	11.0463

Table 4. Comparison of X^5 , $Q(2, X, 4) = 420X^2 - 2160X + 2944$, Absolute, Relative, and Percentage Error

X	X^5	$420X^2 - 2160X + 2944$	ABS	Relative	% Error
2.7	143.489	173.8	30.3109	0.211242	21.1242
2.8	172.104	188.8	16.6963	0.0970131	9.70131
2.9	205.111	212.2	7.08851	0.0345593	3.45593
3.0	243.0	244.0	1.0	0.00411523	0.411523
3.1	286.292	284.2	2.09151	0.00730553	0.730553
3.2	335.544	332.8	2.74432	0.00817871	0.817871
3.3	391.354	389.8	1.55393	0.00397065	0.397065
3.4	454.354	455.2	0.84576	0.00186146	0.186146
3.5	525.219	529.0	3.78125	0.00719938	0.719938
3.6	604.662	611.2	6.53824	0.0108131	1.08131
3.7	693.44	701.8	8.36043	0.0120565	1.20565
3.8	792.352	800.8	8.44832	0.0106623	1.06623
3.9	902.242	908.2	5.95801	0.00660356	0.660356
4.0	1024.0	1024.0	0.0	0.0	0.0
4.1	1158.56	1148.2	10.362	0.00894385	0.894385
4.2	1306.91	1280.8	26.1123	0.0199802	1.99802
4.3	1470.08	1421.8	48.2844	0.0328447	3.28447
4.4	1649.16	1571.2	77.9622	0.0472738	4.72738
4.5	1845.28	1729.0	116.281	0.0630155	6.30155
4.6	2059.63	1895.2	164.43	0.0798346	7.98346
4.7	2293.45	2069.8	223.65	0.0975169	9.75169
4.8	2548.04	2252.8	295.24	0.115869	11.5869

Table 5. Comparison of X^7 , $P(3, X, 3) = 5040X^3 - 41160X^2 + 115836X - 110961$,
Absolute, Relative, and Percentage Error

X	X^7	$5040X^3 - 41160X^2 + 115836X - 110961$	ABS	Relative	% Error
2.7	1046.04	942.12	103.915	0.0993421	9.93421
2.8	1349.29	1323.48	25.8129	0.0191307	1.91307
2.9	1724.99	1728.36	3.37237	0.00195501	0.195501
3.0	2187.00	2187.00	0.0	0.0	0.0
3.1	2751.26	2729.64	21.6214	0.00785873	0.785873
3.2	3435.97	3386.52	49.4538	0.014393	1.4393
3.3	4261.84	4187.88	73.9643	0.017355	1.7355
3.4	5252.34	5163.96	88.375	0.0168259	1.68259
3.5	6433.93	6345.00	88.9297	0.013822	1.3822
3.6	7836.42	7761.24	75.1764	0.00959321	0.959321
3.7	9493.19	9442.92	50.2677	0.00529514	0.529514
3.8	11441.6	11420.3	21.2783	0.00185973	0.185973
3.9	13723.1	13723.6	0.459332	0.0000334715	0.00334715
4.0	16384.0	16383.0	1.0	0.0000610352	0.00610352
4.1	19475.4	19428.8	46.5874	0.00239211	0.239211
4.2	23053.9	22891.3	162.613	0.0070536	0.70536
4.3	27181.9	26800.7	381.181	0.0140234	1.40234
4.4	31927.8	31187.2	740.621	0.0231968	2.31968
4.5	37366.9	36081.0	1285.95	0.034414	3.4414
4.6	43581.8	41512.4	2069.33	0.0474815	4.74815
4.7	50662.3	47511.7	3150.59	0.0621881	6.21881
4.8	58706.8	54109.1	4597.75	0.0783172	7.83172
4.9	67822.3	61334.8	6487.55	0.0956551	9.56551
5.0	78125.0	69219.0	8906.0	0.113997	11.3997
5.1	89741.1	77792.0	11949.0	0.13315	13.315

Table 6. Comparison of X^7 , $Q(3, X, 3) = 1260X^3 - 7140X^2 + 13818X - 9027$, Absolute, Relative, and Percentage Error

X	X^7	$1260X^3 - 7140X^2 + 13818X - 9027$	ABS	Relative	% Error
1.7	41.0339	19.38	21.6539	0.527707	52.7707
1.8	61.222	60.12	1.102	0.0180001	1.80001
1.9	89.3872	94.14	4.75283	0.0531712	5.31712
2.0	128.0	129.0	1.0	0.0078125	0.78125
2.1	180.109	172.26	7.84885	0.0435784	4.35784
2.2	249.436	231.48	17.9558	0.0719856	7.19856
2.3	340.483	314.22	26.2625	0.0771333	7.71333
2.4	458.647	428.04	30.6071	0.0667335	6.67335
2.5	610.352	580.5	29.8516	0.0489088	4.89088
2.6	803.181	779.16	24.021	0.0299074	2.99074
2.7	1046.04	1031.58	14.4553	0.0138192	1.38192
2.8	1349.29	1345.32	3.97285	0.0029444	0.29444
2.9	1724.99	1727.94	2.95237	0.00171153	0.171153
3.0	2187.0	2187.0	0.0	0.0	0.0
3.1	2751.26	2730.06	21.2014	0.00770607	0.770607
3.2	3435.97	3364.68	71.2938	0.0207492	2.07492
3.3	4261.84	4098.42	163.424	0.0383459	3.83459
3.4	5252.34	4938.84	313.495	0.0596868	5.96868
3.5	6433.93	5893.5	540.43	0.0839968	8.39968
3.6	7836.42	6969.96	866.456	0.110568	11.0568
3.7	9493.19	8175.78	1317.41	0.138774	13.8774