Integrating Finite Element Analysis with Systems Engineering Models

Koneksys

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Outline

- Introduction and motivation
- Challenges in FEA standardization
- New proposed FE mathematics description
- Validation
- Next steps and summary

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Integration between Systems Engineering and FEA

PRODUCT

Requirement

Behaviour

Systems Engineering

Structure

Parametric

Software engineering

Electrical engineering

Mechanical engineering

Manufacturing engineering

FEA

Motivation: Communication and archiving

Cross-disciplinary communication

Defining concepts

Are my requirements validated?

What do you want me to simulate?

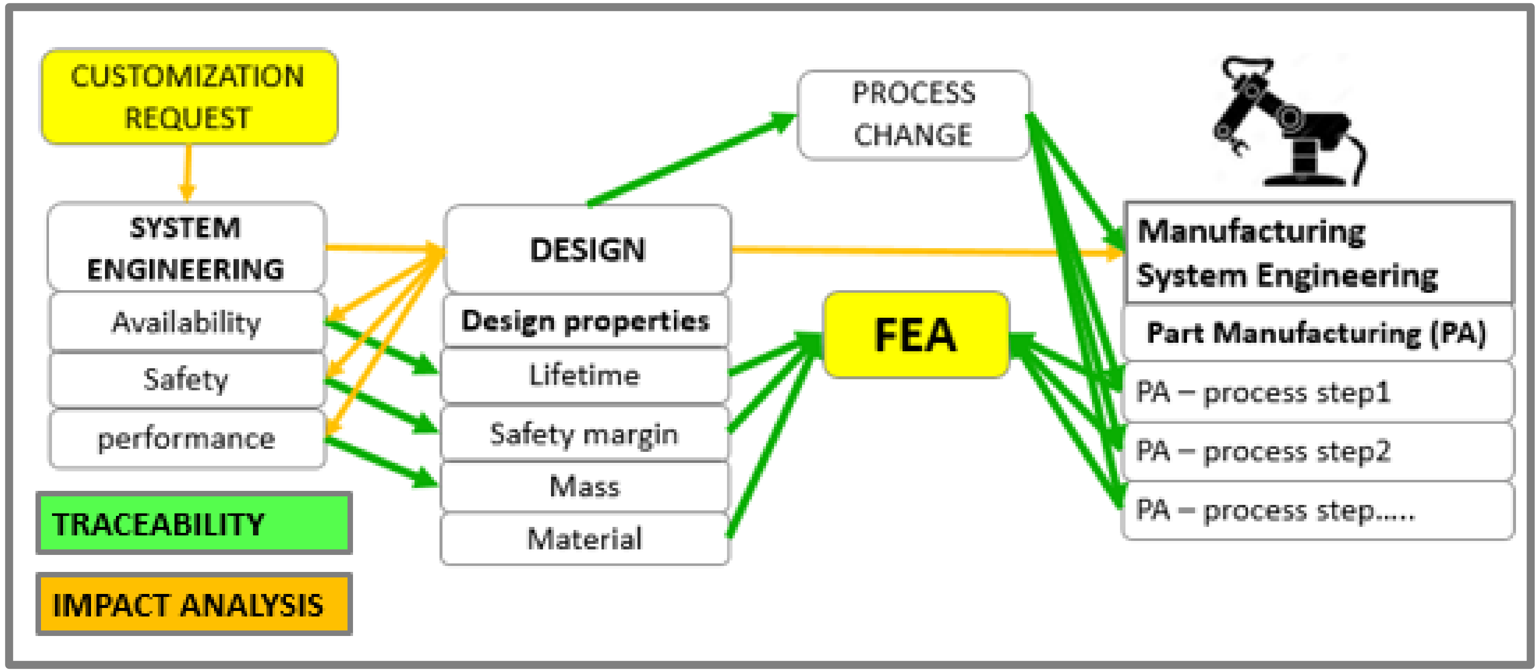
Archiving and reuse

What has been simulated?

Can I reuse the simulation?

Motivation: traceability and impact analysis

Requirement traceability → **Impact analysis**



e.g. Customization request

e.g. Cost reduction program…

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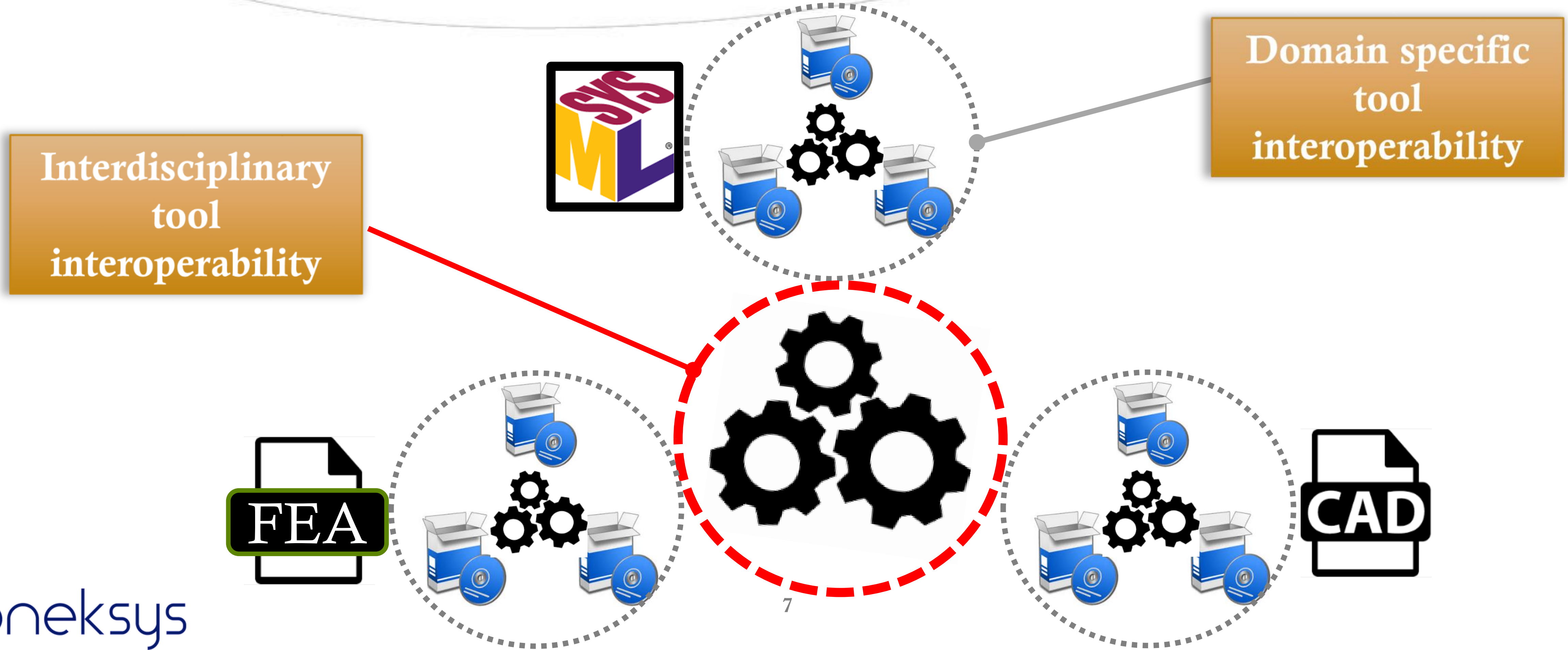
Motivation: Tool interoperability



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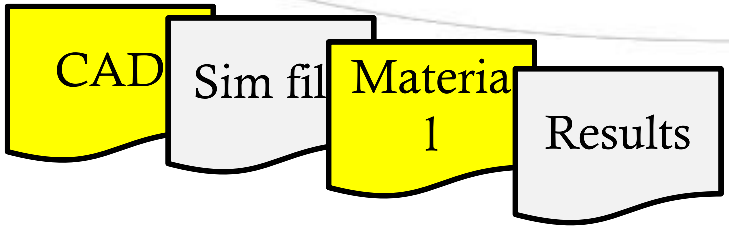
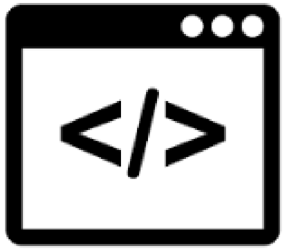
The Challenge

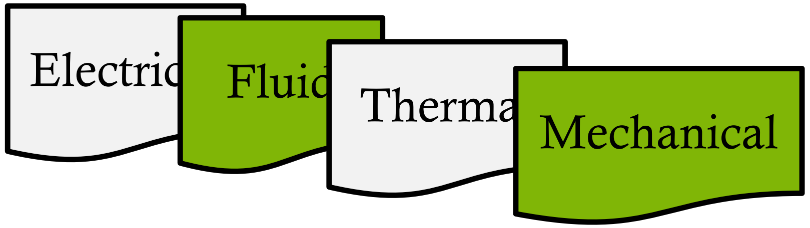


Many Artifacts

Many Vendors



Custom code



Multiphysics

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

FEA is complex

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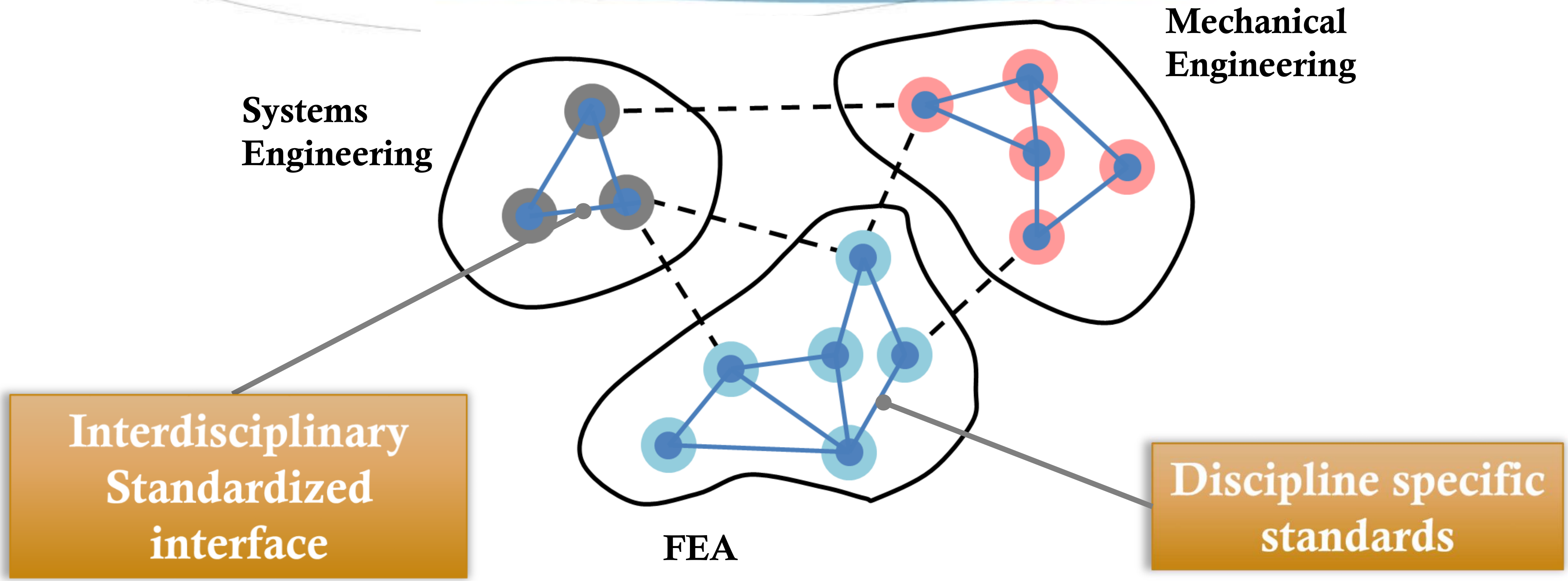
Requirement to success



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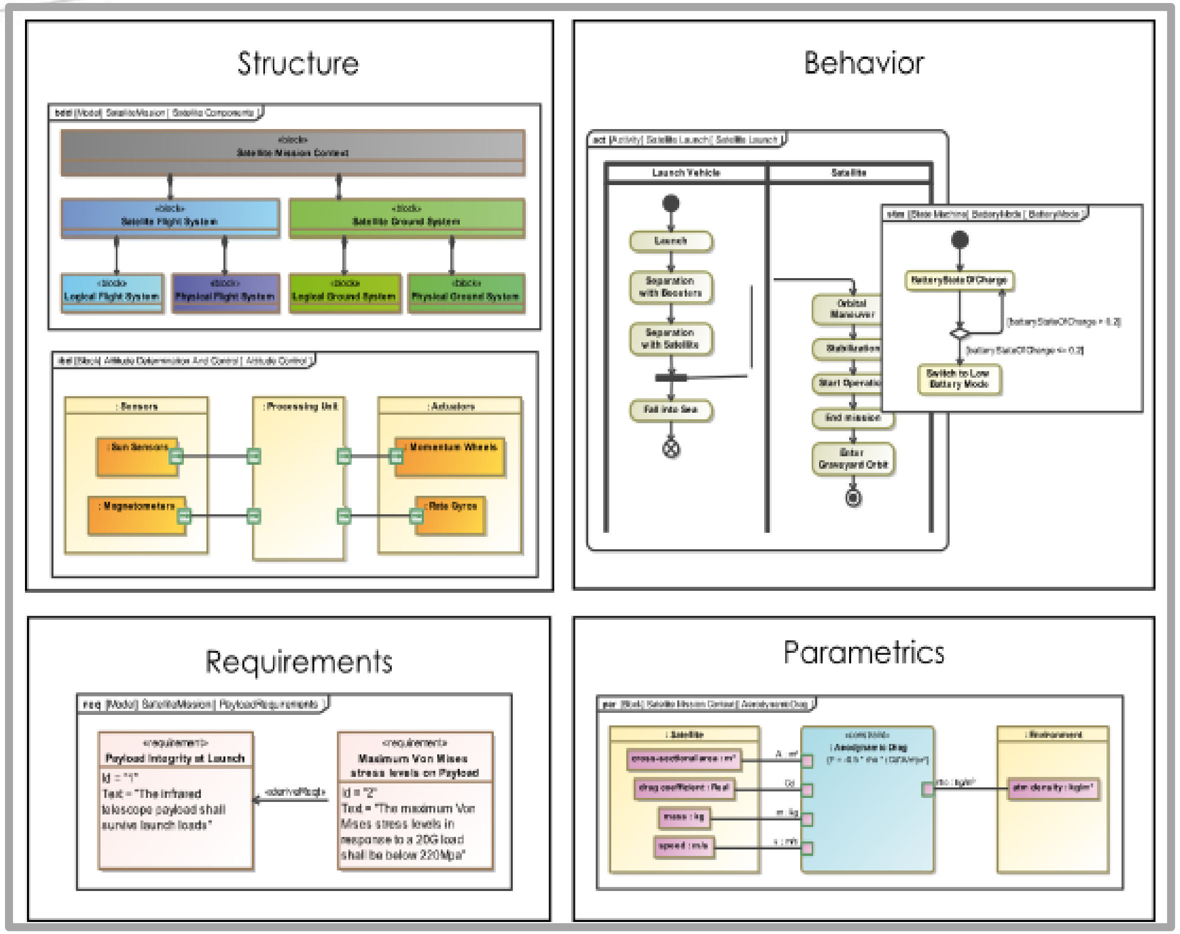
SysML – Standard for Systems Engineering

- SysML: Systems Modeling Language
- Defined by the OMG as standard in 2007
- Widely adopted for Model-Based Systems Engineering (MBSE)
- Current version: 1.4 (2015)



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FEA-related Standards

No useful standards

Standards supporting FEA

- STEP-AP209-ED1
- STEP-AP209-ED2
- IGES V6
- Finite element module
- Finite element shape analysis
- Finite element shape analysis
- JT 9.5 ISO/FDIS 14306

Incomplete
Informal

Many data formats

Common format mesh and visualization format

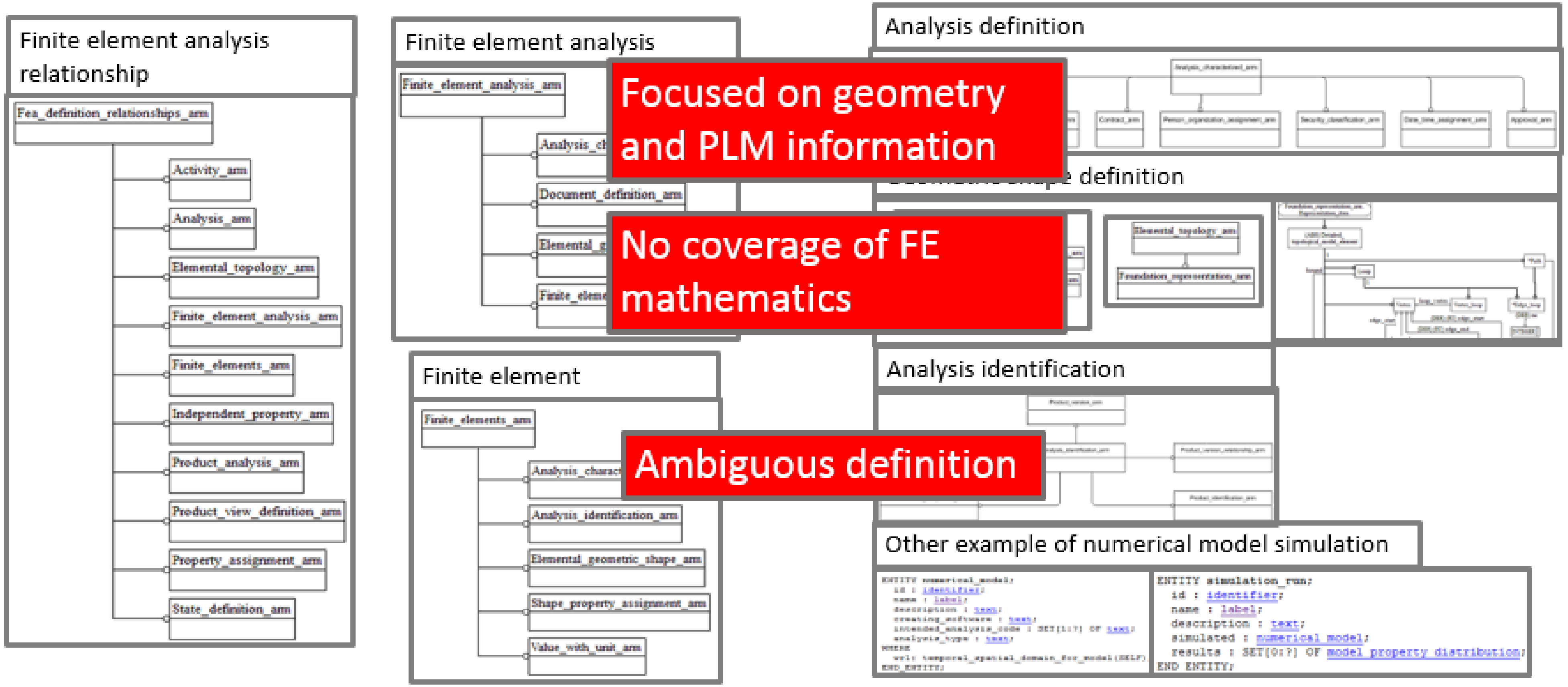
- HDF5
- UNV
- Gmsh
- VTK

No standard for mesh data

Workflow support

- Pro-Step Sim PDM 4.2 (xDM for PDM)
- FEDES
- CAE-Pronet

AP209 (v2014)-based FEA model description



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Ref: ISO 10303-209:2014(E) - Application protocol: Multidisciplinary analysis and design

Impact of missing FEA standard

- Interoperability is compromised
- Impact on reusability (custom code)
- Communication between system and FEA engineers is not efficient
- No open-standard

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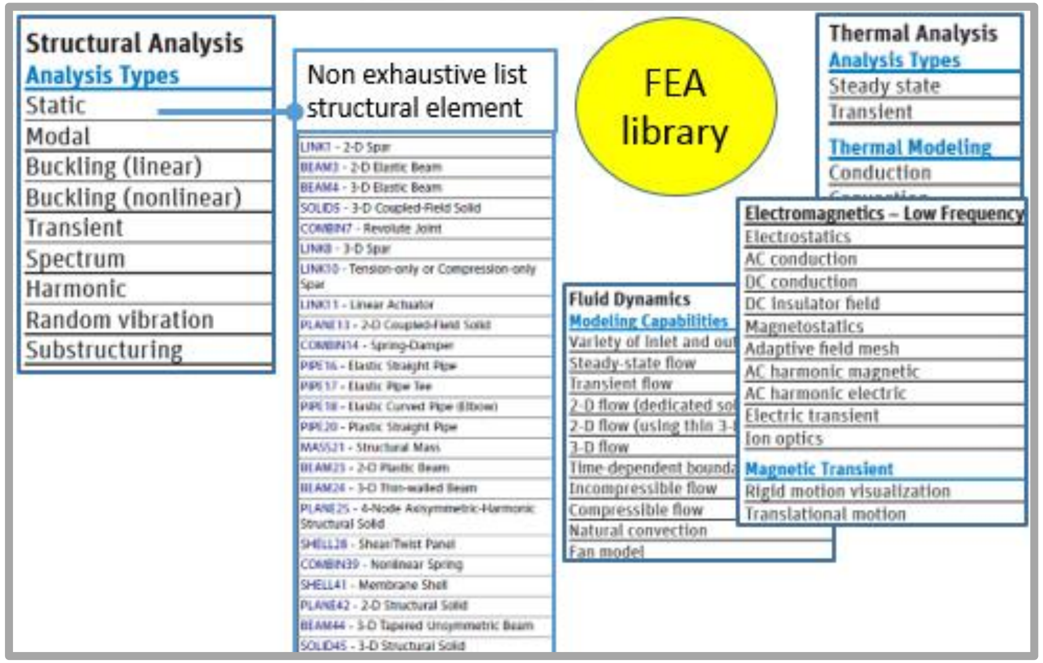
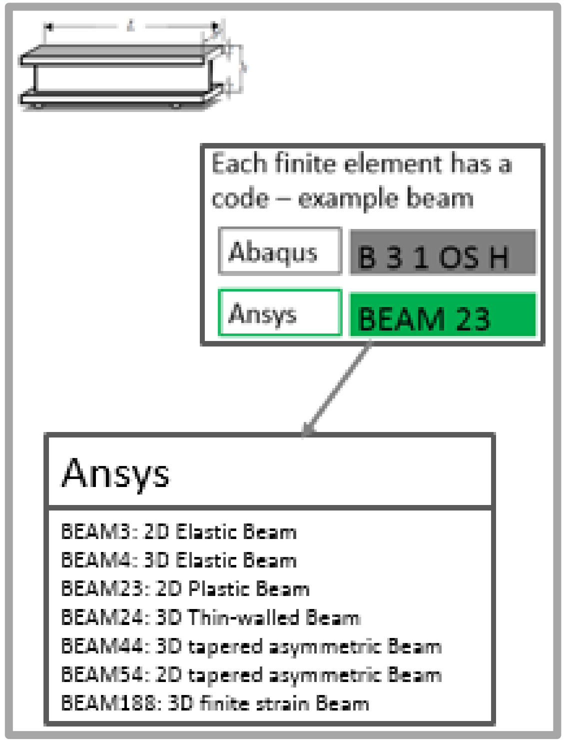
Challenge 1: Capturing model information



Ref: Ansys capabilities overview



Problem of encoding one model

Ambiguity

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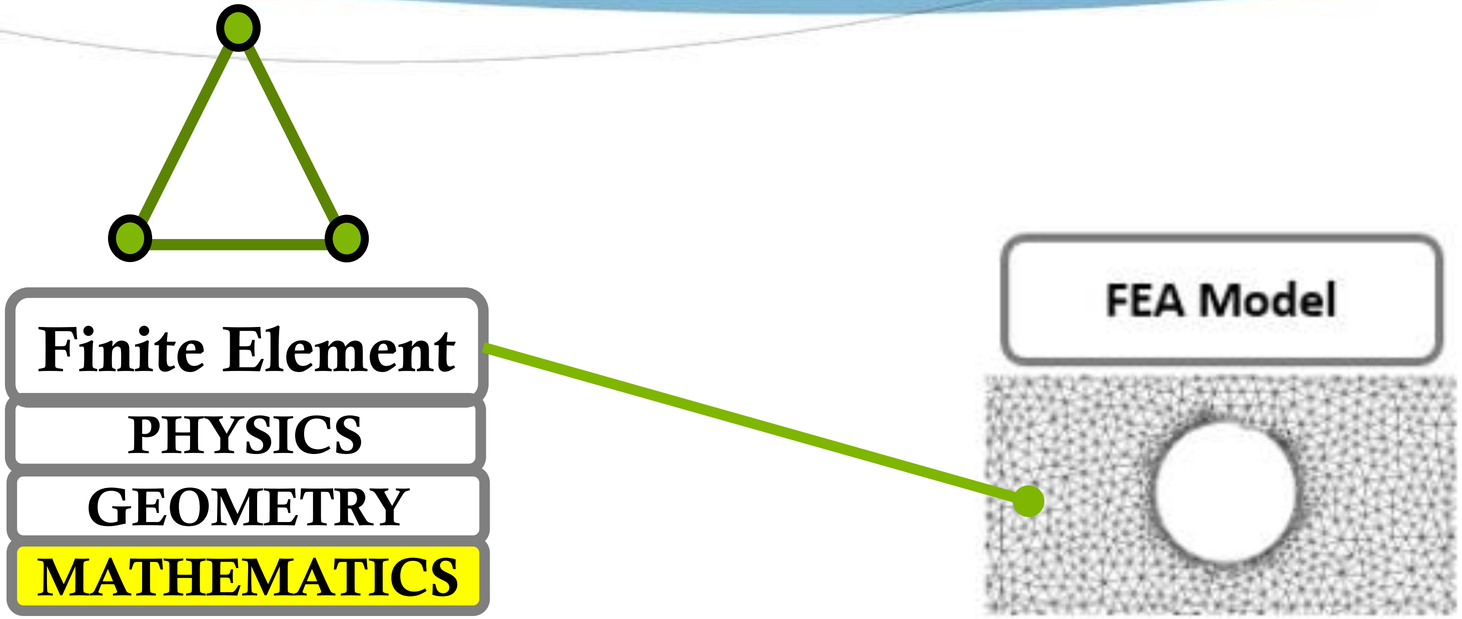
The method: decomposition and reuse



Start with the definition of finite element mathematics

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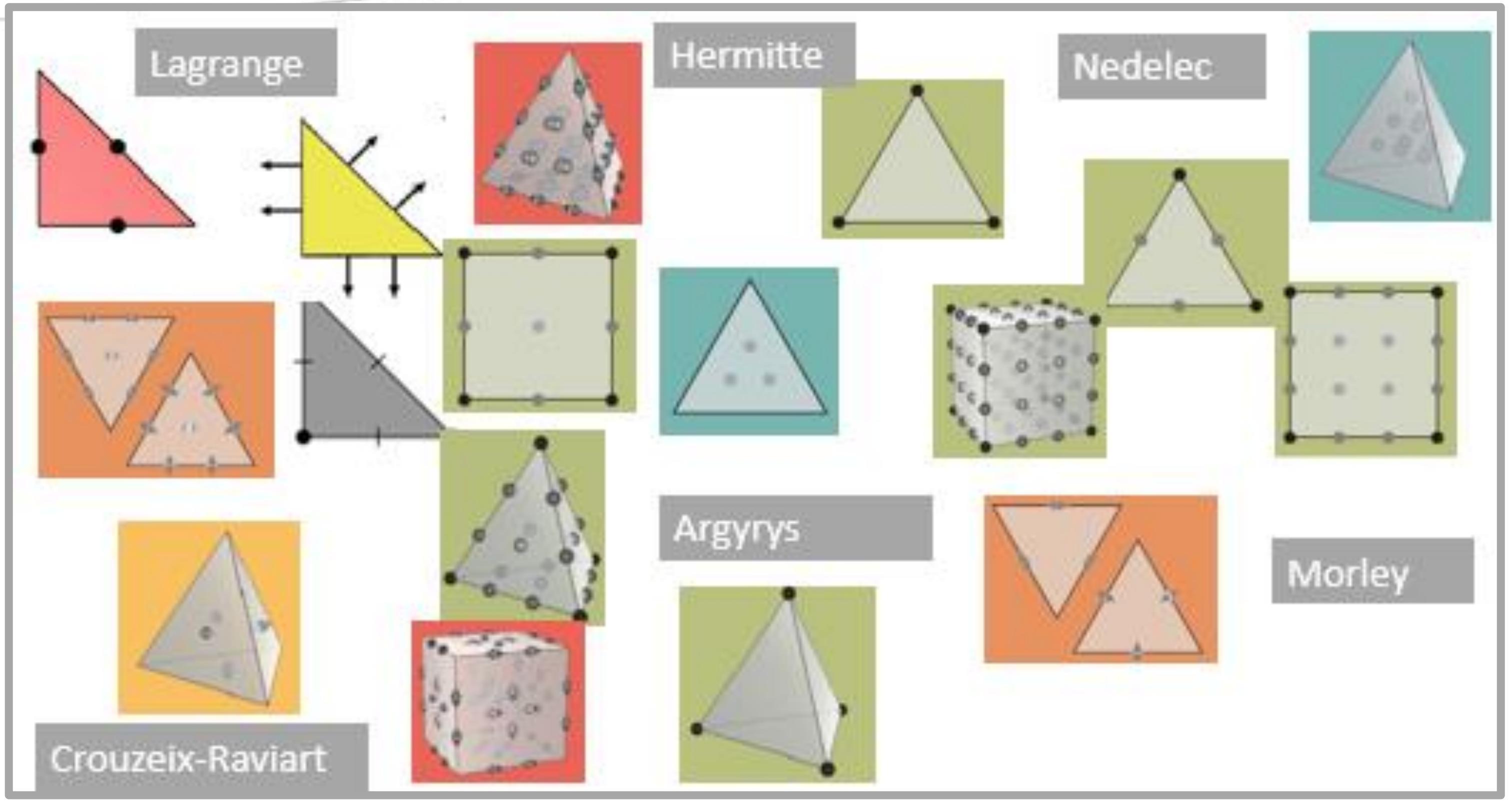
Challenge 2: Describing finite element mathematics

Literature names are not descriptive

Difficult to create an ontology



Logg A. et Al., Automated solution of differential equation by the finite element method, 2012, Springer
Logg A., Arnold D., periodic table of finite elements, 2014, Siam News

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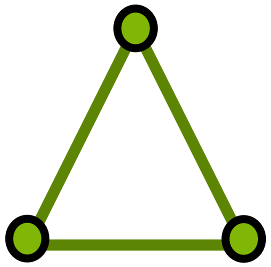
Removing ambiguity?

1 Finite Element



Many Names

Linear simplex
Linear triangle
Linear Lagrange element…

1 Reference

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Characteristics of new FE mathematics description

- Covering FE mathematics
- Non ambiguous
- Using new mathematical concepts based on algebraic topology
- Understandable to engineers who are not mathematicians
- Simple and precise definition of finite element mathematics
- Covering information for implementing FE mathematics in FEA code
- Can describe more FE elements than in literature

Positioning of new finite element mathematics description

Ciarlet's description

Implements

Periodic table

Implements

Ciarlet's definition

Implements

New Finite element description

New finite element mathematics specification facilitates the description of a finite element

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Ciarlet's definition of FE mathematics

- Ciarlet's definition provides the most complete definition of a finite element.

- A finite element is defined by a triple composed of a
  - (1) geometry,
  - (2) functional requirements called also degree of freedoms
  - (3) mathematical representation (polynomial, vector polynomial…)

Ciarlet's definition of FE mathematics

FE maths fully defined as a triple

Geometry

+

PE: Point evaluation
FD: First derivative
SD: second derivative....

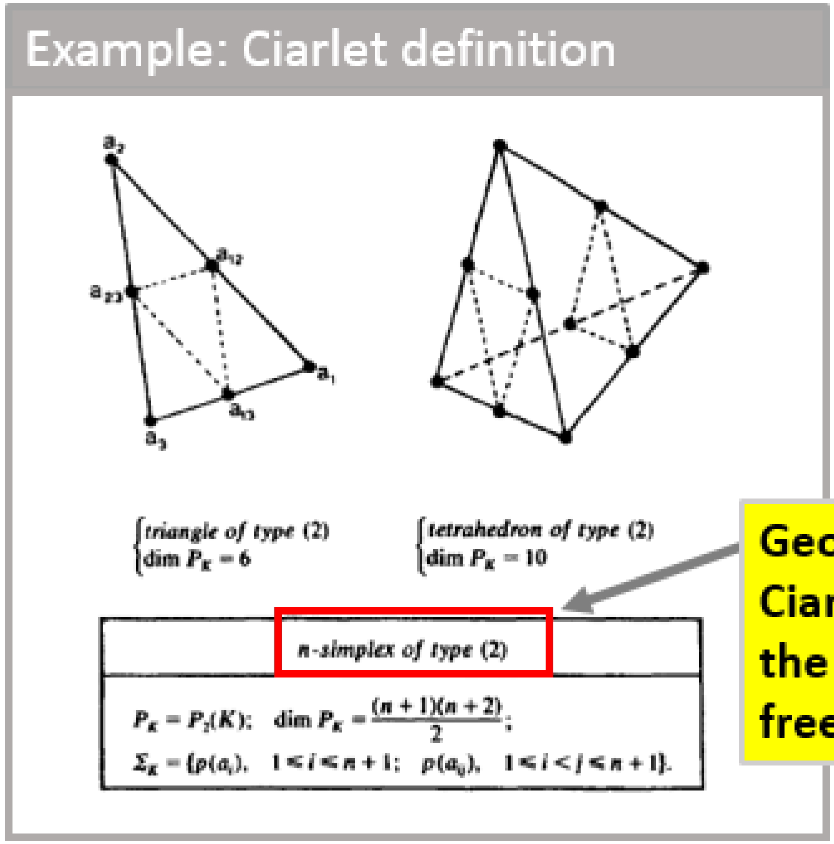
Functional requirement or degrees of freedom

+

$ax^2 + bx + c$

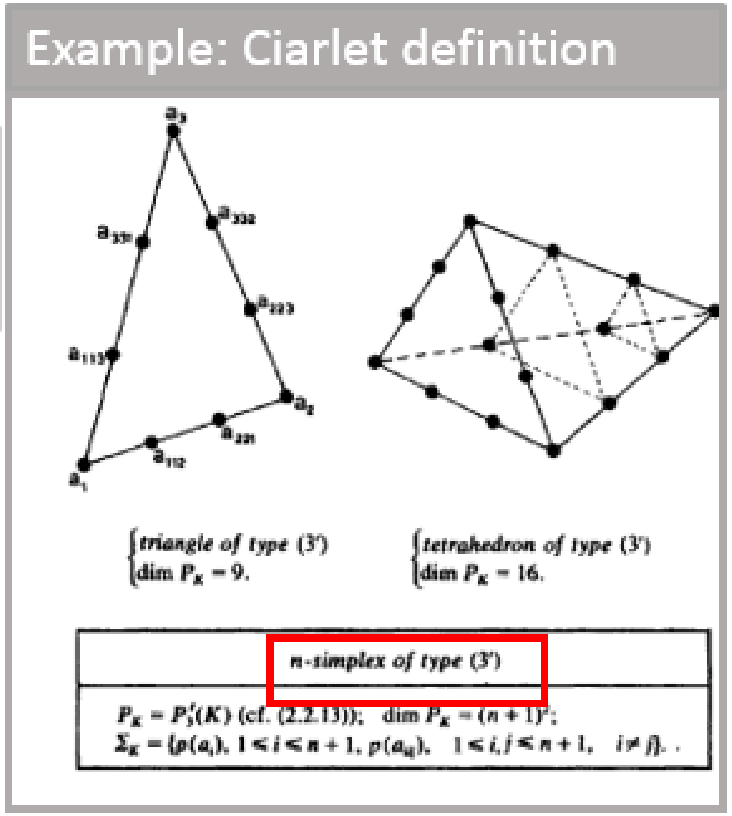
Polynomial

Problem of Ciarlet's description



Non-intuitive definition of finite element mathematics

Geometric definition of Ciarlet is dependent on the degrees of freedoms definition



Ciarlet P., the finite element method for elliptic problems, 2002, SIAM

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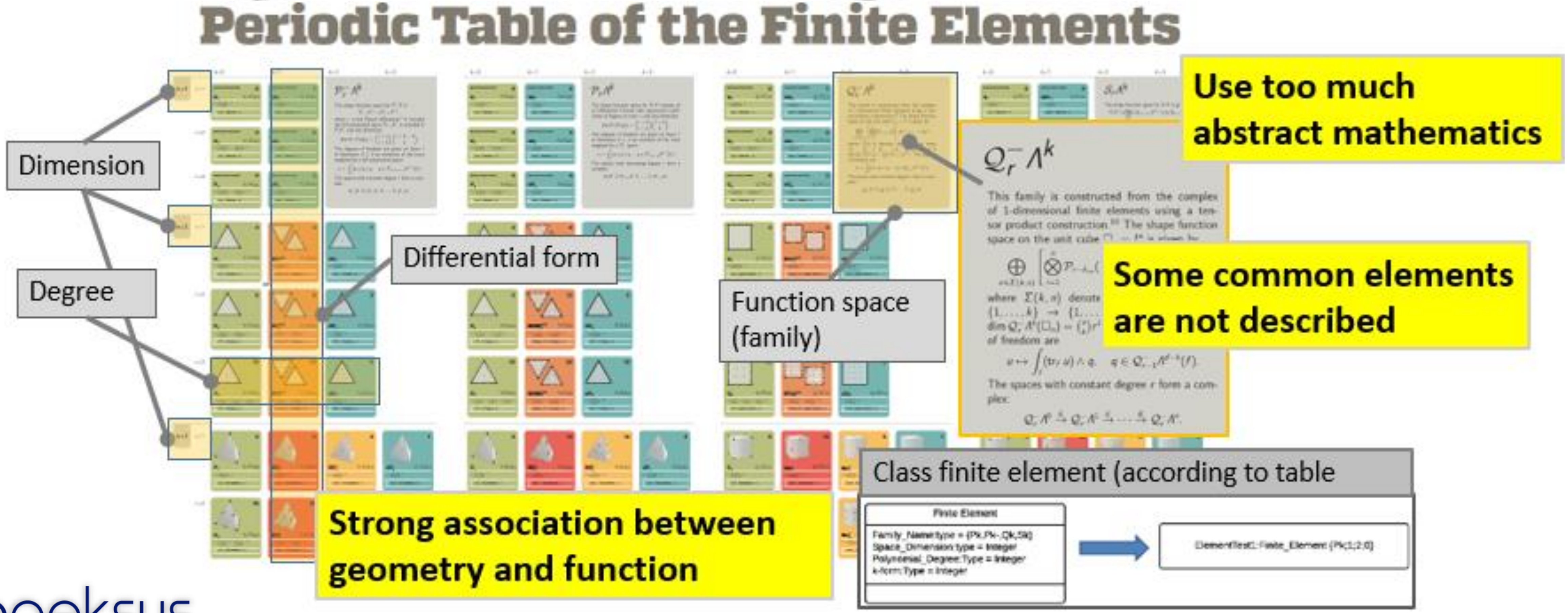
FE periodic table



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Logg A., Arnold D., periodic table of finite elements, 2014, Siam News

New description of a finite element mathematics

Geometry name:
(line, polygon, polyhedron, prism…)

Functional requirements definition on geometry:

$$C_n(\Omega) := \{type\ DOF; number\ DOF\}, \dots, C_{n-1}(\Omega) = \{type\ DOF; number\ DOF\}, \dots, C_0(\Omega) = \{type\ DOF; number\ DOF\}$$

where $C_{\mathrm{n}}(\Omega)$ is the set of requirements applied to the geometry n-faces (0-faces: points, 1-faces: lines, 2-faces: faces, 3-faces: volume)
Functional requirements can be of type point evaluation (PE), first derivative (FD), second derivative (SD) and more.

Polynomial ref.:
identification code (only necessary for implementation)

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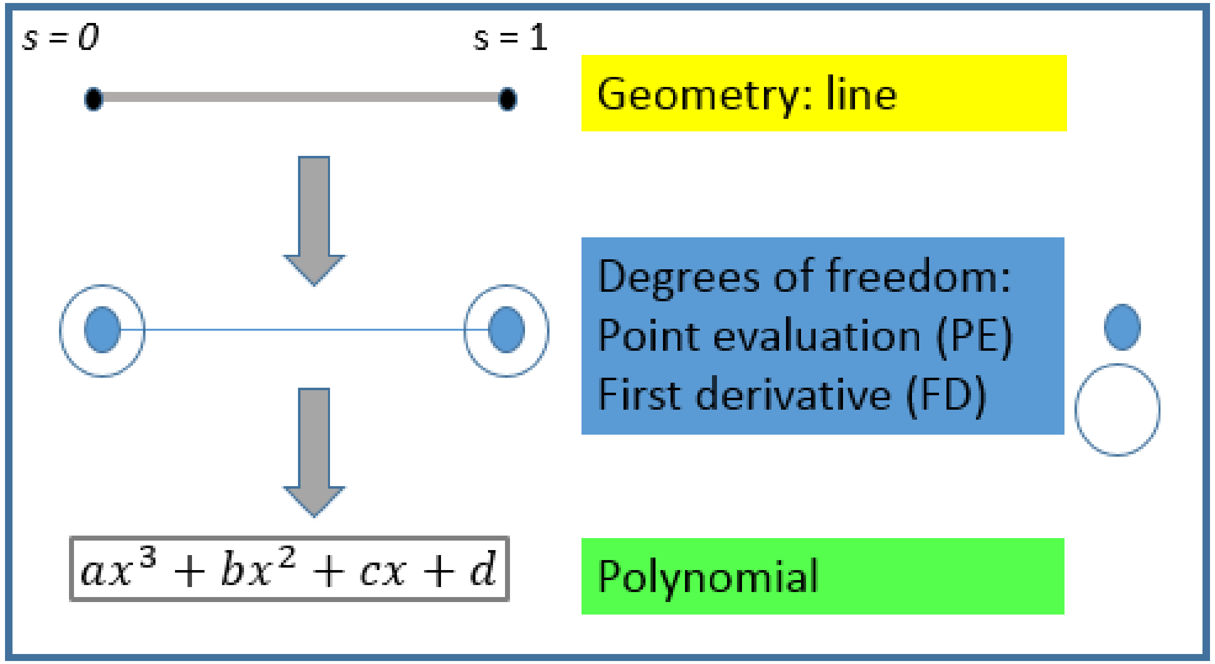
"Beam" according to new description



Name: Element XY

- Line
- $C_1(\Omega) = [\emptyset]$
- $C_0(\Omega) = [\{PE; 1\}, \{FD; 1\}]$
- Polynomial ref.

Element XY is known in the literature as BEAM element, Hermite Element, Cubic element....

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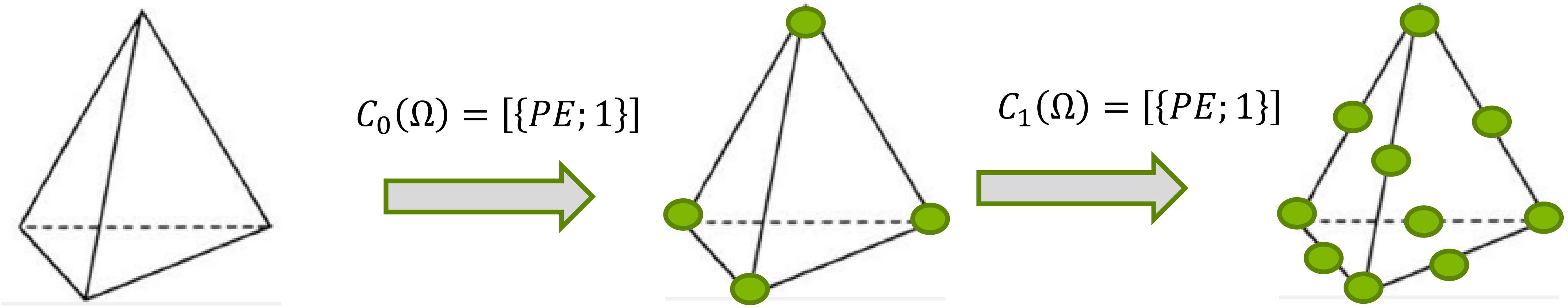
New FE mathematics description: Assigning requirements to the geometry

Name: Element XY

- Tetrahedron
- $C_1(\Omega) = [\{PE; 1\}]$
- $C_0(\Omega) = [\{PE; 1\}]$
- Polynomial ref.



Tetrahedron

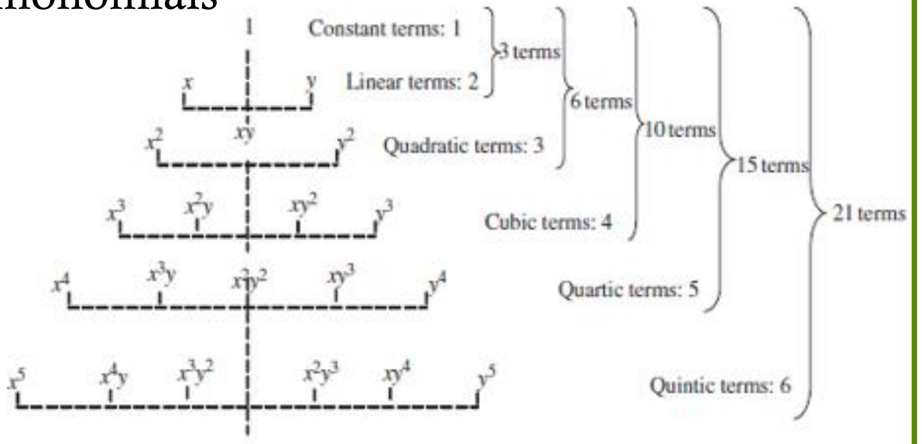
28

Polynomial basis dictionary

A polynomial is composed of monomials

Monomials can be ordered in a dictionary

Pascal triangle of **2-dimensional** monomials

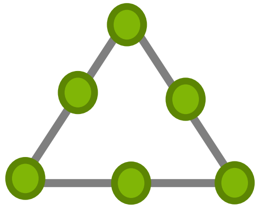


Graded Lexicographic ordering

$$1 < y < x < y^2 < xy < x^2 < y^3 < x^2y \ldots$$

0 1 2 3 4 5 6 7

Encoding FE mathematics

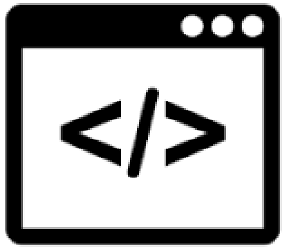


Geometry

Functional requirements

Polynomial

PG3-RC0PE1C1PE1-PFD2-0-1-2-3-4-5



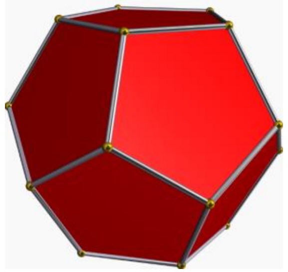
Custom code

This specification provides non ambiguous information for code implementation

New finite element space?

Create your own finite element space

Define geometry



Define requirement

- $C_1(\Omega) = [\{PE; 1\}]$
- $C_0(\Omega) = [\{PE; 1\}]$

Encode

Find polynomial

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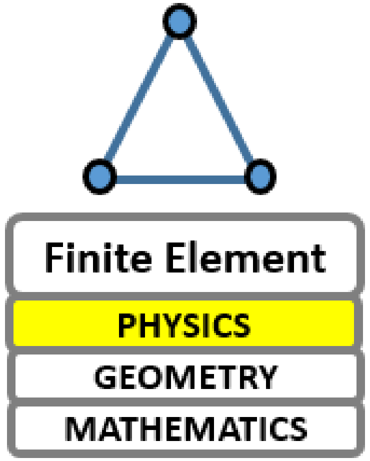
Reusing the new FE description for physics

- line
- $C_0(\Omega) = [\{PE; 1\}]$
- D1-0-1

REUSE



- line
- $C_0(\Omega) = [\{PE; 1\}]$
- D1-0-1
- Temperature
- Type: Scalar

- line
- $C_0(\Omega) = [\{PE; 1\}]$
- D1-0-1
- Displacement
- Type: Vector

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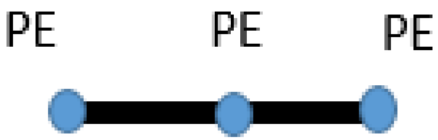
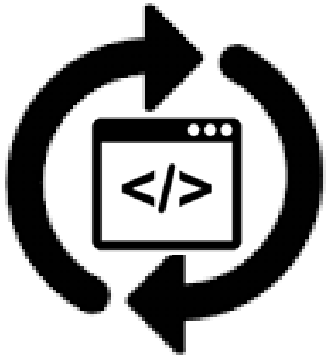
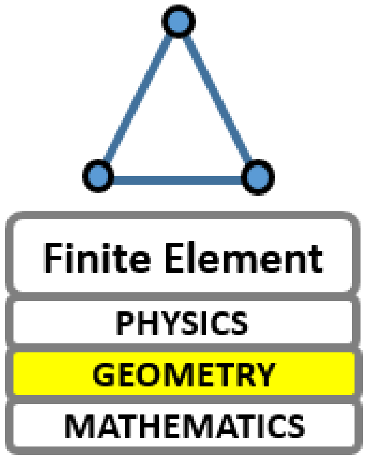
Reusing the new FE specification for geometry description

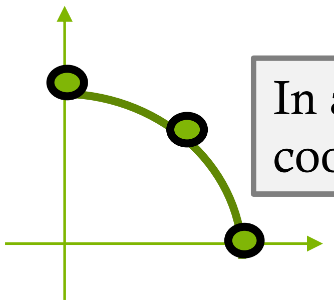


- Line
- $C_1(\Omega) = [\{PE; 1\}]$
- $C_0(\Omega) = [\{PE; 1\}\,]$
- D1-1-2-3



REUSE





In a Cartesian coord. system

- Line
- $C_1(\Omega) = [\{PE; 1\}]$
- $C_0(\Omega) = [\{PE; 1\}\,]$
- D1-1-2-3

- Dimension: 2
- Type: Cartesian

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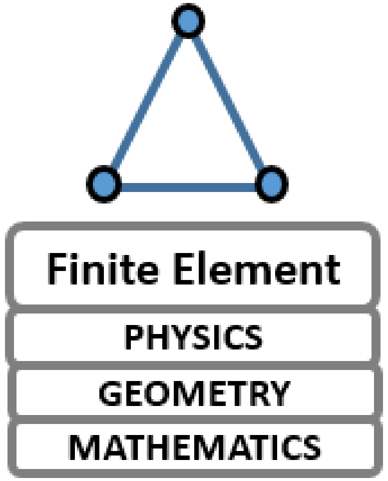
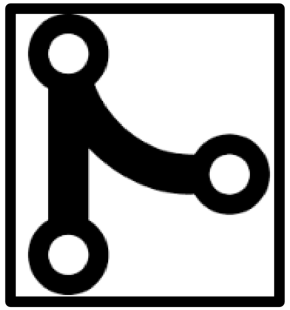
Merging information to describe parametric finite elements





MERGE

PHYSICS

- line
- $C_0(\Omega) = [\{PE; 1\}]$
- D1-0-1

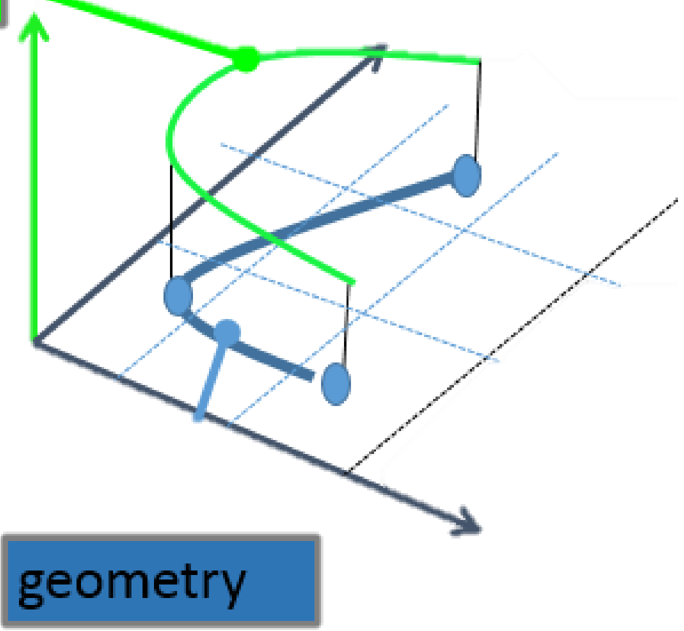
- Temperature
- Type: Scalar

GEOMETRY

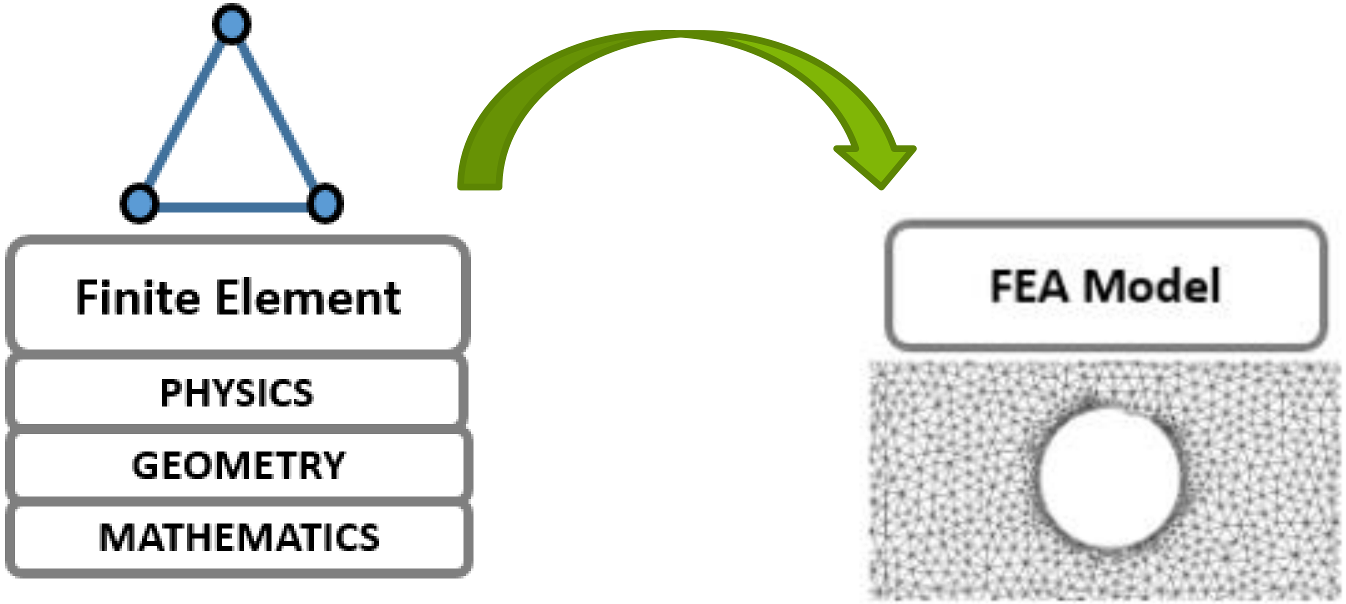
- Line
- $C_1(\Omega) = [\{PE; 1\}]$
- $C_0(\Omega) = [\{PE; 1\}\ ]$
- D1-1-2-3

- Dimension: 2
- Type: Cartesian

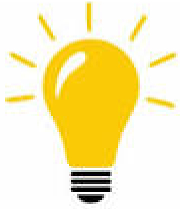
e.g. temperature



Next step of our work:
Specifying the FEA model



Use the same principle:
Decomposition for reusability



Many physics use the same computational model…
To be continued…

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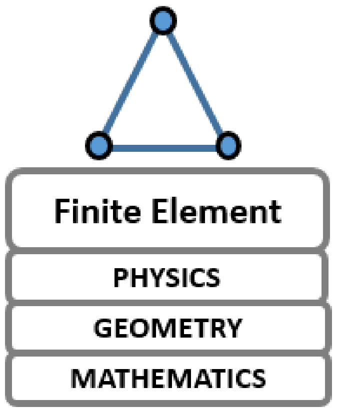
Python code to validate FE mathematics spec

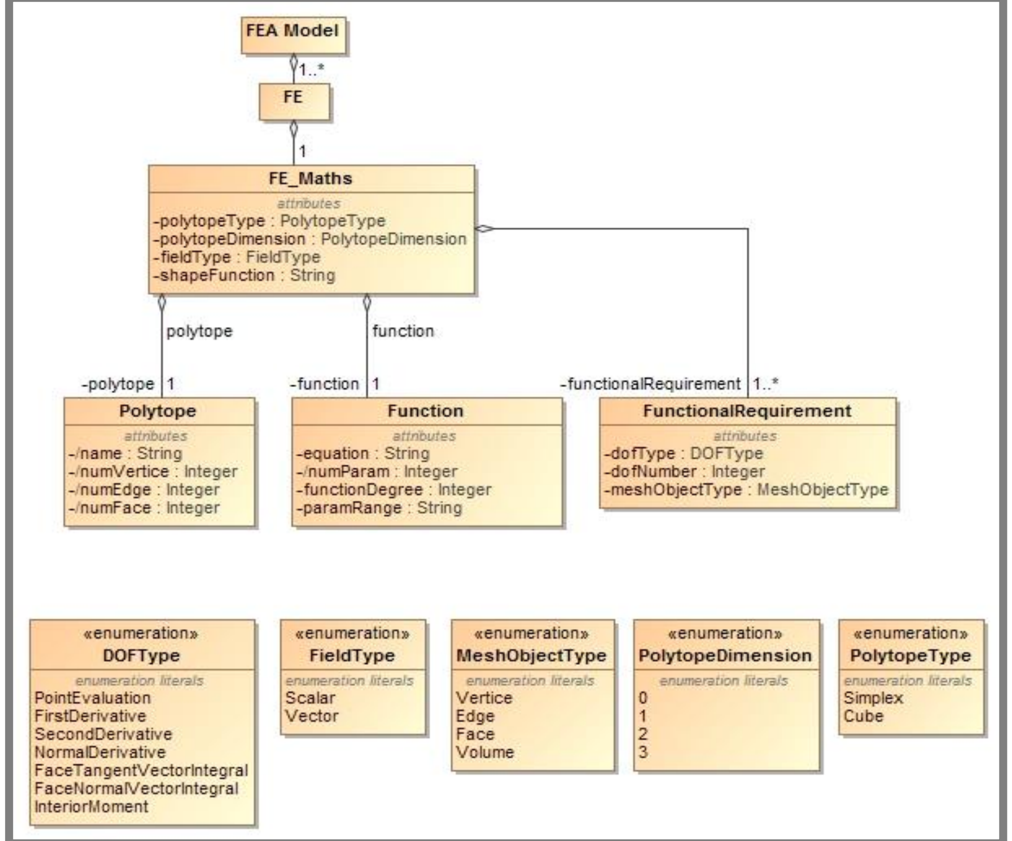
- Model FEA specification in SysML
- FEA code to test the new proposed FEA spec
- FEA implemented in Python using object oriented concepts
- Using symbolic equations for interoperability
- Integration with open source FreeCAD
- Code available on GitHub: https://github.com/koneksys/KFE/

Translating FE description into SysML



New FE-description

SysML model



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Code interoperability

Code uses symbolic equation for interoperability

Symbolic equation program (i.e. Sympy)

HTML specification

Text

Equation image

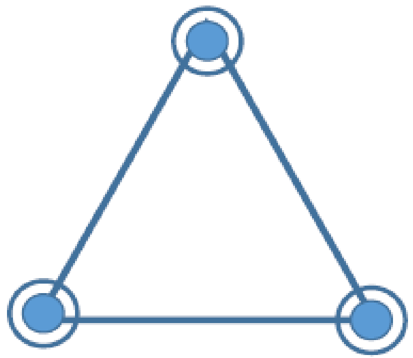
Plot image

MathML info

Code generator

Information aggregation

Aggregation of finite element information in a single python object



Name: T1 element

- triangle
- $C_1(\Omega) = [\emptyset]$
- $C_0(\Omega) = [\{PE;1\},\{FD;1\}]$

```
self = {Femesh} <__main__.Femesh object at 0x04B38090>
  verticelist = {list} [<__main__.Vertice object at 0x04B38030>, <__main__.Vertice object
    __len__ = {int} 3
    0 = {Vertice} <__main__.Vertice object at 0x04B38030>
      coordinates = {list} [0, 0]
      funreq = {list} [<Doftype.pointevaluation: 1>, <Doftype.firstderivative: 2>]
      index = {list} [0]
    1 = {Vertice} <__main__.Vertice object at 0x04B380B0>
      coordinates = {list} [0, 1]
      funreq = {list} [<Doftype.pointevaluation: 1>, <Doftype.firstderivative: 2>]
      index = {list} [1]
    2 = {Vertice} <__main__.Vertice object at 0x04B380D0>
      coordinates = {list} [1, 1]
      funreq = {list} [<Doftype.pointevaluation: 1>, <Doftype.firstderivative: 2>]
      index = {list} [2]
```

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Summary

- **Benefits of new FE mathematics specification based on algebraic topology:**
  - Covering FE mathematics
  - Understandable to engineers who are not mathematicians
  - Simple and precise definition of a finite element
  - Covering information for implementing FE mathematics in FEA code
  - Can describe more FE elements than with descriptions based on Ciarlet/periodic table

- **New FE mathematics specification will benefit integration between systems engineering and FEA**
  - Traceability
  - Consistency/Synchronization
  - Reuse

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Thanks You!

Visit our website : Koneksys.com

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