# Applying Type-Level and Generic Programming in Haskell

Summer School on Generic and Effectful Programming

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#### Plan for the week

#### Monday:

- ► Learn about *n*-ary products.
- ► Along the way, discuss nearly everything we need in terms of Haskell type-level programming features.

#### Today:

- Recap and cover the missing parts of Monday's lecture.
- Introduce n-ary sums and the generics-sop view.
- Representing datatypes using generics-sop.
- More combinators and simple applications.

#### Friday:

- ► Metadata.
- More applications.



Recap: NP

### Environments, NP

```
data NP (f :: k -> *) (xs :: [k]) where
  Nil :: NP f '[]
  (:*) :: f x -> NP f xs -> NP f (x ': xs)
infixr 5 :*
```

### Building and combining environments

```
hpure :: SListI xs => (forall a . f a) -> NP f xs
```



### Building and combining environments

```
hpure :: SListI xs => (forall a . f a) -> NP f xs
```

```
hap :: NP (f -.-> g) xs -> NP f xs -> NP g xs

newtype (f -.-> g) a = Fn {apFn :: f a -> g a}
```



### Mapping and zipping environments

```
hmap :: SListI xs
     => (forall a . f a -> g a)
     -> NP f xs -> NP g xs
hmap f xs = hpure (Fn f) 'hap' xs
```



### Collapsing environments

```
hcollapse :: NP (K a) xs -> [a]
```

```
newtype I a = I a
newtype K a b = K a
```



Abstracting from classes, type functions

### Mapping constrained functions?

```
hmap (K . show . unI) group'
```

fails, because

```
K . show . unI :: forall x . Show x \Rightarrow I x \rightarrow K String x
```

does not match

forall 
$$x$$
 . f  $x \rightarrow g$ 

Х

# Constraints are types of kind Constraint

```
GHCi> :kind Eq
Eq :: * -> Constraint
GHCi> :kind Functor
Functor :: (* -> *) -> Constraint
GHCi> :kind MonadReader
MonadReader :: * -> (* -> *) -> Constraint
```

### Constraints are types of kind Constraint

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GHCi> :kind Eq
Eq :: * -> Constraint
GHCi> :kind Functor
Functor :: (* -> *) -> Constraint
GHCi> :kind MonadReader
MonadReader :: * -> (* -> *) -> Constraint
```

#### Overloaded tuple syntax:

```
type NoConstraint = (() :: Constraint)
type SomeConstraints a = (Eq a, Show a)
type MoreConstraints f a = (Monad f, SomeConstraints a)
```



## The All type family

## The All type family

#### Example:

```
GHCi> :kind! All Eq '[Int, Bool]
All Eq '[Int, Bool] :: Constraint
= (Eq Int, (Eq Bool, ()))
```

(Constraints are flattened.)



#### We want:

```
hpure :: SListI xs

=> (forall a . f a) -> NP f xs

hcpure :: (SListI xs, All c xs)

=> (forall a . c a => f a) -> NP f xs
```

#### Then:

However, this does not work.



### Limitations in GHC's type inference

#### Assume:

```
hcpure :: (SListI xs, All c xs)
=> (forall a . c a => f a) -> NP f xs
hcpure = undefined
```

#### Then

```
minBound :: Bounded a => a
I minBound :: Bounded a => I a
```

```
GHCi> hcpure (I minBound) :: NP I '[Char, Bool]
```

is a type error.



#### **Proxies**

```
data Proxy (a :: k) = Proxy
```

#### Examples:

```
pBounded :: Proxy Bounded
```

pBounded = Proxy

pShow :: Proxy Show

pShow = Proxy



### Using proxies to define hcpure

### Using proxies to define hcpure

#### Example:

```
GHCi> hcpure pBounded (I minBound) :: NP I '[Char, Bool]

GHCi> hcpure pShow (Fn (K . show . unI)) 'hap' group'
```



Generalizing choice

### Choosing from a list

data LChoice a = LCZero a | LCSuc (LChoice a)

An index into a list paired with the element at that position.



### Choosing from a list

```
data LChoice a = LCZero a | LCSuc (LChoice a)
```

An index into a list paired with the element at that position.

Equivalently in GADT syntax:

```
data LChoice (a :: *) where
  LCZero :: a -> LChoice a
  LCSuc :: LChoice a -> LChoice a
```



### Choosing from a vector

```
data LChoice (a :: *) where
  LCZero :: a -> LChoice a
  LCSuc :: LChoice a -> LChoice a
```

### Choosing from a heterogeneous list

```
data VChoice (a :: *) (n :: Nat) where
  VCZero :: a -> VChoice a (Suc n)
  VCSuc :: VChoice a n -> VChoice a (Suc n)
```

```
data HChoice (xs :: [*]) where
  HCZero :: x -> HChoice (x ': xs)
  HCSuc :: HChoice xs -> HChoice (x ': xs)
```



### Choosing from an environment

```
data VChoice (a :: *) (n :: Nat) where
  VCZero :: a -> VChoice a (Suc n)
  VCSuc :: VChoice a n -> VChoice a (Suc n)

data HChoice (xs :: [*]) where
  HCZero :: x -> HChoice (x ': xs)
  HCSuc :: HChoice xs -> HChoice (x ': xs)
```

```
data NS (f :: k -> *) (xs :: [k]) where
Z :: f x -> NS f (x ': xs)
S :: NS f xs -> NS f (x ': xs)
```



#### An example

```
data NS (f :: k -> *) (xs :: [k]) where
Z :: f x -> NS f (x ': xs)
S :: NS f xs -> NS f (x ': xs)
```

```
type ExampleChoice = NS I '[Char, Bool, Int]
```

```
c0, c1, c2 :: ExampleChoice
c0 = Z (I 'x')
c1 = S (Z (I True))
c2 = S (S (Z (I 3)))
```



Representing types as sums of products

### Representing expressions

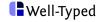
#### Values are of form:

$$C_i x_0 \dots x_{n_i-1}$$



### Representing expressions – contd.

```
data Expr = NumL Int
    | BoolL Bool
    | Add Expr Expr
    | If Expr Expr
```



### Representing expressions – example

```
exampleExpr :: Expr
exampleExpr = If (BoolL True) (NumL 1) (NumL 0)
```



### Representing expressions – example

```
exampleExpr :: Expr
exampleExpr = If (BoolL True) (NumL 1) (NumL 0)
```

#### Beautified syntax:

```
exampleRepExpr :: RepExpr
exampleRepExpr =
  C<sub>3</sub> [I (BoolL True), I (NumL 1), I (NumL 0)]
```



```
fromExpr :: Expr -> RepExpr
fromExpr (NumL n) =
 Z (In:*Nil)
fromExpr (BoolL b) =
 S(Z(Ib:*Nil))
fromExpr (Add e1 e2) =
 S (S (Z (I e1 :* I e2 :* Nil)))
fromExpr (If e1 e2 e3) =
 S (S (S (Z (I e1 :* I e2 :* I e3 :* Nil))))
```

Similarly toExpr.



# The Generic class

#### Monday:

```
class Generic a where
  type Rep a
  from :: a -> Rep a
  to :: Rep a -> a
```

### The Generic class

#### Monday:

```
class Generic a where
  type Rep a
  from :: a -> Rep a
  to :: Rep a -> a
```

#### Wednesday:



#### Instance for expressions



# Instance for lists

Shows how parameters are handled.



# On defining Generic instances

# The role of <a href="Generic">Generic</a>

If a type is an instance of **Generic**, then lots of generic functions will be available for that type.

However, the Generic instance must still be written.



# **Options**

- ► Define by hand.
- ▶ Use Template Haskell.
- Extend the compiler (GHC).
- ▶ Use "Generic Generic Programming".



# Using Template Haskell

```
data Expr = ...
deriveGeneric ''Expr
```

This is implemented.



# Direct compiler support

```
data Expr = ...
deriving (..., Generic)
```

This is not implemented (and unlikely to be).



# On compiler support

GHC has first-class support for (at least) two approaches:

- Scrap your boilerplate (SYB) via the Data class
- ► Generic deriving via (another) Generic class

In essence, **Data** and **Generic** are different structural representations of datatypes, encoding similar or even the same information as we are trying to.



# On compiler support

GHC has first-class support for (at least) two approaches:

- Scrap your boilerplate (SYB) via the Data class
- ► Generic deriving via (another) Generic class

In essence, **Data** and **Generic** are different structural representations of datatypes, encoding similar or even the same information as we are trying to.

What if we could translate one representation into the other?



# Generic Generic Programming

- Use Haskell libraries to translate between different representations.
- ▶ Built-in representation can aim to be as informative as possible; no need for it to be (directly) practical.
- Makes it easier to use several approaches in a single program.
- Encourages specialized representations for specific domains, rather than trying to find the "one true generic programming approach".



# Generic Generic Programming in practice

```
import qualified GHC.Generics as GHC
data Expr = ...
  deriving (..., GHC.Generic)
instance Generic Expr
```

This is implemented.



# Generic Generic Programming in practice

```
import qualified GHC.Generics as GHC
data Expr = ...
  deriving (..., GHC.Generic)
instance Generic Expr
```

This is implemented.

In the future:

```
data Expr = ...
deriving (..., GHC.Generic, Generic)
```



Generic equality

# Equality for products

We are going to fall back on the Eq class for the components of the product.



# Equality for sums of products

```
geqNS :: ... => NS (NP I) xss -> NS (NP I) xss -> Bool
geqNS (Z np1) (Z np2) = geqNP np1 np2
geqNS (S ns1) (S ns2) = geqNS ns1 ns2
geqNS _ _ = False
```

# Equality for sums of products

```
geqNS :: ... => NS (NP I) xss -> NS (NP I) xss -> Bool
geqNS (Z np1) (Z np2) = geqNP np1 np2
geqNS (S ns1) (S ns2) = geqNS ns1 ns2
geqNS _ _ = False
```

We need All (All Eq) xss - but we can't do that.



To avoid partial applications, we have to "copy" the All family for two-dimensional structures:



# Equality for sums of products – contd.



# Completing the definition

```
geqSOP :: All2 Eq xss => SOP I xss -> SOP I xss -> Bool
geqSOP (SOP sop1) (SOP sop2) = geqNS sop1 sop2
```



# Completing the definition

```
geqSOP :: All2 Eq xss => SOP I xss -> SOP I xss -> Bool
geqSOP (SOP sop1) (SOP sop2) = geqNS sop1 sop2
```

```
geq :: (Generic a, All2 Eq (Code a)) \Rightarrow a \Rightarrow Bool geq x y = geqSOP (from x) (from y)
```



# Using generic equality

```
instance Eq Expr where
  (==) = geq
```



# Using generic equality

```
instance Eq Expr where
  (==) = geq
```

# Example:

```
GHCi> geq (Add (NumL 3) (NumL 5)) (Add (NumL 3) (NumL 5))
GHCi> geq (Add (NumL 3) (NumL 5)) (Add (NumL 3) (NumL 6))
```



# Another look at the product case

# Another look at the product case

It's a hczipWith!



# Redefining the product case



# Generic producers

# Default values

Provided by the data-default package:

```
class Default a where
  def :: a
```



# Default values

Provided by the data-default package:

```
class Default a where
  def :: a
```

We will try to define:

Note the rather precise type.



# Completing the definition

# Will generate

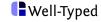
```
C<sub>0</sub> [def, ...]
```



# Using the function

```
instance Default Int where
  def = 0
instance Default Bool where
  def = False
```

```
GHCi> gdef :: Expr
```



# Using default signatures

# Using default signatures

Then:

```
instance Default Expr
```

Or in the near future:

```
data Expr = ...
deriving (..., Default)
```

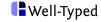


# Generating one value for each constructor

```
gdefAll :: (Generic a, All2 Default (Code a)) => [a]
gdefAll = map (to . SOP) gdefAllNS
```

# Generating one value for each constructor

```
gdefAll :: (Generic a, All2 Default (Code a)) => [a]
gdefAll = map (to . SOP) gdefAllNS
gdefAllNS :: forall xss .
             (SListI xss, All SListI xss,
              All2 Default xss)
          => [NS (NP I) xss]
gdefAllNS = case sList :: SList xss of
 SNil -> []
 SCons -> Z (hcpure (Proxy :: Proxy Default) (I def))
         : map S gdefAllNS
```



# Example use of gdefAll

```
GHCi> gdefAll :: [Expr]
[NumL 0,
BoolL False,
Add (NumL 0) (NumL 0),
If (NumL 0) (NumL 0) (NumL 0)]
```

Products of products, injections

# Another idea

A table of recursive calls:

```
[[def], [def], [def, def], [def, def, def]]
```

#### Another idea

A table of recursive calls:

```
[[def], [def], [def, def], [def, def, def]]
```

A list of constructor functions:

$$[C_0, C_1, C_2, C_3]$$

#### Another idea

A table of recursive calls:

```
[[def], [def], [def, def], [def, def]]
```

A list of constructor functions:

```
[C_0, C_1, C_2, C_3]
```

Zip:

```
[C_0 \text{ [def]}, C_1 \text{ [def]}, C_2 \text{ [def, def]}, C_3 \text{ [def, def, def]}]
```



## Products of products

```
newtype POP f a = POP {unPOP :: NP (NP f) a}
```



# An hcpure for POP

Unfortunately, because

```
All2 c xss /= All (All c) xss
```

we cannot just call hcpure twice.



```
hcpure_POP (Proxy :: Proxy Default) (I def) :: POP I (Code Expr)
```

#### yields

```
[[I def]
, [I def]
, [I def, I def]
, [I def, I def, I def]
]
```

# Injections for Either

```
data Either a b = Left a | Right b
```

```
Left :: a -> Either a b
Right :: b -> Either a b
```

## Injections for Either

```
data Either a b = Left a | Right b
Left :: a -> Either a b
Right :: b -> Either a b
eitherInjections :: NP (I -.-> K (Either a b)) '[a, b]
eitherInjections = Fn (K . Left . unI)
                  :* Fn (K . Right . unI)
                  :* Nil
```



### Injections for NS

```
Assume xs = '[x, y, z, ...]:
```

```
Z :: f x \rightarrow NS f xs
S . Z :: f y \rightarrow NS f xs
S . S . Z :: f z \rightarrow NS f xs
```

## Injections for NS

SNil -> Nil

 $SCons \rightarrow Fn (K . Z)$ 

Assume xs = '[x, y, z, ...]:

injections = case sList :: SList xs of

```
Z :: f x -> NS f xs
S . Z :: f y -> NS f xs
S . S . Z :: f z -> NS f xs

injections :: forall xs f . SListI xs
=> NP (f -.-> K (NS f xs)) xs
```

:\* hmap (Fn . ((K . S . unK) .) . apFn) injections

### Putting things together

```
apInjs :: SListI xs => NP f xs -> [NS f xs]
apInjs np = hcollapse (injections 'hap' np)
```

```
apInjs_POP :: SListI xs => POP f xs -> [SOP f xs]
apInjs_POP (POP pop) = map SOP (apInjs pop)
```



### Putting things together

```
apInjs :: SListI xs => NP f xs -> [NS f xs]
apInjs np = hcollapse (injections 'hap' np)
apInjs_POP :: SListI xs => POP f xs -> [SOP f xs]
apInjs_POP (POP pop) = map SOP (apInjs pop)
gdefAll' :: (Generic a, All2 Default (Code a)) => [a]
gdefAll' =
 map to (apInjs_POP (hcpure_POP pDefault (I def)))
```



#### 'Summary

- ► Type a is represented as NS (NP I) (Code a).
- Generic functions can be defined by pattern matching on NS and NP, but also by combining general combinators.



#### **Exercises**

- 1. Generalize equality to comparison.
- 2. Define a variant of generic equality (or comparison) that ignores some constructor arguments.
- 3. Define generic enumeration.

