### Applying Type-Level and Generic Programming in Haskell

Summer School on Generic and Effectful Programming

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#### Datatype-generic programming

Express algorithms that make use of the structure of datatypes



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$$eq_A :: A \rightarrow A \rightarrow Bool$$



#### A class

```
class Generic a where
  type Rep a
  from :: a -> Rep a
  to :: Rep a -> a
```

where from and to are inverses.



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geq :: Generic a => Rep a -> Rep a -> Bool
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#### A class

# class Generic a where type Rep a from :: a -> Rep a to :: Rep a -> a

where from and to are inverses.

```
geq :: Generic a => Rep a -> Bool
```

```
eq :: Generic a => a -> a -> Bool
eq x y = geq (from x) (from y)
```



#### Choices

Much flexibility in the details, in particular the definition of Rep.



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The choice of Rep determines expressive power and flavour of generic programs.



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The choice of Rep determines expressive power and flavour of generic programs.

In this lecture series: generics-sop.



#### **Applications**

- ► (De-)serialization
- Data generation
- Data traversals
- Data navigation
- ▶ ...

The generics-sop view on data, informally

#### Sample datatypes

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- Choice between constructors,
- each with a sequence of arguments.



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- Choice between constructors,
- each with a sequence of arguments.

```
C_i x_0 \dots x_{n_i-1}
```



#### The plan

$$C_i x_0 \dots x_{n_i-1}$$

- ► Choice between constructors modelled as an *n*-ary sum.
- Sequence of fields modelled as an n-ary product.

We'll need Haskell type-level programming concepts along the way.



#### Extensions, extensions

DataKinds GADTs TypeOperators TypeFamilies RankNTypes ConstraintKinds MultiParamTypeClasses **UndecidableInstances** StandaloneDeriving ScopedTypeVariables PolyKinds FlexibleInstances FlexibleContexts DefaultSignatures



#### Plan for the week

#### Today:

- ▶ Learn about *n*-ary products.
- Along the way, discuss everything we need in terms of Haskell type-level programming features.

#### Wednesday:

- ▶ Introduce *n*-ary sums and the generics-sop view.
- Representing datatypes using generics-sop.
- Simple applications.

#### Friday:

More applications.



### Kinds and data kinds

#### Types and kinds

- ► Values / terms have types.
- ► Types have kinds.

#### Example:

```
GHCi> :type 'x'
'x' :: Char
GHCi> :kind Char
Char :: *
```

#### Stars and functions

```
Int :: *
Double :: *
Bool :: *
Char :: *
() :: *
Void :: *
```

```
data () = () -- one value
data Void -- no values
```



#### Stars and functions – contd.

```
Maybe :: * -> *

[] :: * -> *

IO :: * -> *

(,) :: * -> * -> *

Either :: * -> * -> *
```

#### Stars and functions – contd.

```
Maybe :: * -> *
[] :: * -> *
IO :: * -> *
(,) :: * -> * -> *
Fither :: * -> * -> *
Maybe Int
IO [Bool]
                     :: *
Either Char
                    :: * -> *
Either Char (Maybe Int) :: *
IO Maybe -- kind error
```



#### Data kinds and promotion

```
data Bool = False | True
```

Defines a datatype with (data) constructors:

```
Bool :: *
False :: Bool
True :: Bool
```

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```
data Bool = False | True
```

Defines a datatype with (data) constructors:

```
Bool :: *
False :: Bool
True :: Bool
```

Defines also a kind with (type) constructors:

```
Bool :: □
'False :: Bool
'True :: Bool
```

Both False and True are uninhabited.



#### Data kinds and promotion – contd.

Quotes are generally optional; False and True also allowed.

```
GHCi> :kind Bool
Bool :: *
GHCi> :tvpe True
True :: Bool
GHCi> :type False
False :: Bool
GHCi>:kind 'True
'True :: Bool
GHCi>:kind 'False
'False :: Bool
GHCi> :kind True
True :: Bool
GHCi>:kind False
False :: Bool
```



Generalized algebraic data types (GADTs)

#### Generalizing lists in several steps

```
type T1 = [Int]
type T2 = Vec Int (Suc (Suc Zero)))
type T3 = HList '[Char, Bool, Int]
type T4 = NP Maybe '[Char, Bool, Int]
```

#### Generalizing lists in several steps

[Just 'x', Nothing, Just 3] :: T4



#### Vectors – promoting natural numbers

```
data Nat = Zero | Suc Nat
```

#### As a type:

```
Nat :: *
Zero :: Nat
```

Suc :: Nat -> Nat

#### As a kind:

```
Nat :: □
'Zero :: Nat
```

'Suc :: Nat -> Nat

#### From lists to vectors

```
data [a] = [] | a : [a]
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Renaming constructors:

```
data List a = LNil | LCons a (List a)
```



#### From lists to vectors

```
data [a] = [] | a : [a]
```

Renaming constructors:

```
data List a = LNil | LCons a (List a)
```

#### GADT syntax:

```
data List (a :: *) where
  LNil :: List a
  LCons :: a -> List a -> List a
```



#### Defining vectors

#### Lists:

```
data List (a :: *) where
  LNil :: List a
  LCons :: a -> List a -> List a
```

#### Vectors:

```
data Vec (a :: *) (n :: Nat) where
  VNil :: Vec a Zero
  VCons :: a -> Vec a n -> Vec a (Suc n)
infixr 5 'VCons' -- for infix use
deriving instance Show a => Show (Vec a n)
```



#### Vector examples



#### Pattern matching on GADTs

```
vtail :: Vec a (Suc n) -> Vec a n
vtail (VCons x xs) = xs
```

No case for VNil needed (or possible).

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```
vtail :: Vec a (Suc n) -> Vec a n
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```

No case for VNil needed (or possible).

Example:

```
GHCi> vtail ('x' 'VCons' VNil)
VNil
```

```
GHCi> vtail (vtail ('x' 'VCons' VNil))
```

results in a type error!



### Pattern matching on GADTs – contd.



### Pattern matching on GADTs – contd.

```
vmap :: (a -> b) -> Vec a n -> Vec b n
vmap f VNil = VNil
vmap f (x 'VCons' xs) = f x 'VCons' vmap f xs
```

#### First case:

```
n ~ Zero
```



### Pattern matching on GADTs – contd.

```
vmap :: (a -> b) -> Vec a n -> Vec b n
vmap f VNil = VNil
vmap f (x 'VCons' xs) = f x 'VCons' vmap f xs
```

#### First case:

```
n ~ Zero
```

#### Second case:

```
n ~ Suc n'
```



An applicative interface for vectors

# The Applicative class

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b

class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

We've already seen that we can map over vectors . . .



# Generalizing replicate

What to do with the Int for vectors?



## Generalizing replicate

What to do with the Int for vectors?

```
vreplicate :: a -> Vec a n
```

won't work.



# Generalizing replicate – using a class

```
class VReplicate (n :: Nat) where
  vreplicate :: a -> Vec a n
```

## Generalizing replicate – using a class

```
class VReplicate (n :: Nat) where
  vreplicate :: a -> Vec a n
```

```
instance VReplicate Zero where
  vreplicate _ = VNil
instance VReplicate n => VReplicate (Suc n) where
  vreplicate x = x 'VCons' vreplicate x
```

# Generalizing replicate – using a class

```
class VReplicate (n :: Nat) where
  vreplicate :: a -> Vec a n

instance VReplicate Zero where
  vreplicate _ = VNil
instance VReplicate n => VReplicate (Suc n) where
  vreplicate x = x 'VCons' vreplicate x
```

#### Example:

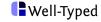
```
type Three = Suc (Suc (Suc Zero))
```

```
GHCi> vreplicate 'x' :: Vec Char Three
VCons 'x' (VCons 'x' (VCons 'x' VNil))
```



### Singleton natural numbers

```
data SNat (n :: Nat) where
 S7ero :: SNat 7ero
 SSuc :: SNatI n => SNat (Suc n)
class SNatI (n :: Nat) where
 sNat :: SNat n
instance SNatI Zero where
 sNat = S7ero
instance SNatI n => SNatI (Suc n) where
 sNat = SSuc
```



# Generalizing replicate – using singletons

```
vreplicate :: forall a n . SNatI n => a -> Vec a n
vreplicate x = case sNat :: SNat n of
    SZero -> VNil
    SSuc -> x 'VCons' vreplicate x
```

This needs "scoped type variables".



## Generalizing replicate – using singletons

```
vreplicate :: forall a n . SNatI n => a -> Vec a n
vreplicate x = case sNat :: SNat n of
   SZero -> VNil
   SSuc -> x 'VCons' vreplicate x
```

This needs "scoped type variables".

With singletons, we need only one class (SNatI) for all functions on natural numbers – but there is potentially more work being done at run-time.



#### Pointwise application

Where vreplicate fills the role of pure, we still need something corresponding to (<\*>):



Heterogeneous lists, *n*-ary products

#### Promoted lists

```
data [a] = [] | a : [a]
```

#### As a type:

```
[] :: * -> *
[] :: forall (a :: *) . [a]
(:) :: forall (a :: *) . a -> [a] -> [a]
```

#### As a kind:

```
[] :: □ -> □
'[] :: forall (a :: □) . [a]
'(:) :: forall (a :: □) . a -> [a] -> [a]
```

Both '[] and '(:) are kind-polymorphic.



#### Promoted lists – examples

```
GHCi> :kind True ': '[]
True ': '[] :: [Bool]
GHCi> :kind '[Zero, Three]
'[Zero, Three] :: [Nat]
GHCi> :kind '[Char, Bool, Int]
'[Char, Bool, Int] :: [*]
GHCi> :kind '[Maybe, [], I0]
'[Maybe, [], I0] :: [* -> *]
```

#### Promoted lists – examples

```
GHCi> :kind True ': '[]
True ': '[] :: [Bool]
GHCi> :kind '[Zero, Three]
'[Zero, Three] :: [Nat]
GHCi> :kind '[Char, Bool, Int]
'[Char, Bool, Int] :: [*]
GHCi> :kind '[Maybe, [], IO]
'[Maybe, [], IO] :: [* -> *]
```

#### Quotes for type-level lists are often not optional:

```
GHCi> :kind '[Bool]
'[Bool] :: [*]
GHCi> :kind [Bool]
[Bool] :: *
```



### Heterogeneous lists

```
data HList (xs :: [*]) where
  HNil :: HList '[]
  HCons :: x -> HList xs -> HList (x ': xs)
infixr 5 'HCons'
```



### Heterogeneous lists

```
data HList (xs :: [*]) where
  HNil :: HList '[]
  HCons :: x -> HList xs -> HList (x ': xs)
infixr 5 'HCons'
```

#### Example:

```
GHCi> :type ('x' 'HCons' False 'HCons' HNil)
'x' 'HCons' False 'HCons' HNil :: HList '[Char, Bool]
```



### Generalizing further

We often need lists of related types.

```
data NP (f :: k -> *) (xs :: [k]) where
  Nil :: NP f '[]
  (:*) :: f x -> NP f xs -> NP f (x ': xs)
infixr 5 :*
```

### Generalizing further

We often need lists of related types.

```
data NP (f :: k -> *) (xs :: [k]) where
  Nil :: NP f '[]
  (:*) :: f x -> NP f xs -> NP f (x ': xs)
infixr 5 :*
```

```
newtype I a = I a
newtype K a b = K a
```

```
NP I xs \approx HList xs
NP (K a) xs \approx Vec a (Length xs)
```



### Collapsing an environment

### Collapsing an environment

Next goal: generalize <a href="mailto:vmap">vmap</a>, <a href="vmap">vreplicate</a> and <a href="mailto:vapply">vapply</a> to environments.



Higher-rank types

# Generalizing vmap

#### Compare:

```
Vec a n
NP f xs
```

```
vmap :: (a -> b) -> Vec a n -> Vec b n
```

# Generalizing vmap

#### Compare:

```
Vec a n
NP f xs
```

```
vmap :: (a -> b) -> Vec a n -> Vec b n
```

```
hmap :: ... \rightarrow NP f xs \rightarrow NP g xs
```

# Generalizing <a href="mailto:vmap">vmap</a> – contd.

We apply m to all elements of the list.

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```
m :: forall x . f x -> g x
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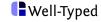
# Generalizing <a href="mailto:vmap">vmap</a> – contd.

```
hmap m Nil = Nil
hmap m (x :* xs) = m x :* hmap m xs
```

We apply m to all elements of the list.

```
m :: forall x . f x -> g x
```

```
hmap :: (forall x . f x \rightarrow g x) \rightarrow NP f xs \rightarrow NP g xs
```



```
group :: NP I '[Char, Bool, Int]
group = I 'x' :* I False :* I 3 :* Nil
unI :: I a -> a
unI (I x) = x
example :: NP Maybe '[Char, Bool, Int]
example = hmap (Just . unI) group
```

```
GHCi> example
Just 'x' :* (Just False :* (Just 3 :* Nil))
```



# Generalizing vreplicate

# Generalizing vreplicate

### Singleton lists

```
data SList (xs :: [k]) where
 SNil :: SList '[]
 SCons :: SListI xs => SList (x ': xs)
class SListI (xs :: [k]) where
 slist :: Slist xs
instance SListI '[] where
 sList = SNil
instance SListI xs => SListI (x ': xs) where
 slist = SCons
```

We ignore the list elements (for now).



# Completing hpure

#### Examples:

```
GHCi> hpure Nothing :: NP Maybe '[Char, Bool, Int]
Nothing :* (Nothing :* (Nothing :* Nil))
GHCi> hpure (K 0) :: NP (K Int) '[Char, Bool, Int]
K 0 :* (K 0 :* (K 0 :* Nil))
```

# Generalizing vapply

```
vapply :: Vec (a \rightarrow b) n \rightarrow Vec a n \rightarrow Vec b n hap :: NP ... xs \rightarrow NP f xs \rightarrow NP g xs
```

What to do with the functions?

## Generalizing vapply

```
vapply :: Vec (a -> b) n -> Vec a n -> Vec b n hap :: NP ... xs -> NP f xs -> NP g xs
```

What to do with the functions?

NP (
$$x \rightarrow (f x \rightarrow g x)) xs$$

## Generalizing vapply

```
vapply :: Vec (a -> b) n -> Vec a n -> Vec b n hap :: NP ... xs -> NP f xs -> NP g xs
```

What to do with the functions?

NP (
$$x - g x$$
) xs

```
newtype (f -.-> g) x = Fn {apFn :: f x -> g x}
infixr 1 -.->
```

## Generalizing vapply

```
vapply :: Vec (a -> b) n -> Vec a n -> Vec b n hap :: NP ... xs -> NP f xs -> NP g xs
```

What to do with the functions?

NP (
$$\x -> (f x -> g x)$$
) xs

hap :: NP 
$$(f -.-> g)$$
 xs -> NP f xs -> NP g xs



```
hap :: NP (f -.-> g) xs -> NP f xs -> NP g xs
hap Nil Nil = Nil
hap (f :* fs) (x :* xs) = apFn f x :* hap fs xs
```

# Using hap - an example

```
lists :: NP [] '[String, Int]
lists = ["foo", "bar", "baz"] :* [1..10] :* Nil
numbers :: NP (K Int) '[String, Int]
numbers = K 2 :* K 5 :* Nil
fn 2 :: (f a -> f' a -> f'' a)
     -> (f -.-> f' -.-> f'') a
fn_2 f = Fn (\langle x - \rangle Fn (\langle y - \rangle f x y))
take' :: (K Int -.-> [] -.-> []) a
take' = fn_2 (\(K n) xs -> take n xs)
```

```
GHCi> hpure take' 'hap' numbers 'hap' lists
["foo", "bar"] :* ([1, 2, 3, 4, 5] :* Nil)
```



Abstracting from classes, type functions

### Mapping constrained functions?

```
hmap (K . show . unI) group
```

fails, because

```
K . show . unI :: forall x . Show x \Rightarrow I x \rightarrow K String x
```

does not match

forall x . f 
$$x \rightarrow g$$

Х

# Constraints are types of kind Constraint

```
GHCi> :kind Eq
Eq :: * -> Constraint
GHCi> :kind Functor
Functor :: (* -> *) -> Constraint
GHCi> :kind MonadReader
MonadReader :: * -> (* -> *) -> Constraint
```

# Constraints are types of kind Constraint

```
GHCi> :kind Eq
Eq :: * -> Constraint
GHCi> :kind Functor
Functor :: (* -> *) -> Constraint
GHCi> :kind MonadReader
MonadReader :: * -> (* -> *) -> Constraint
```

### Overloaded tuple syntax:

```
type NoConstraint = (() :: Constraint)
type SomeConstraints a = (Eq a, Show a)
type MoreConstraints f a = (Monad f, SomeConstraints a)
```



# The All type family

# The All type family

### Example:

```
GHCi> :kind! All Eq '[Int, Bool]
All Eq '[Int, Bool] :: Constraint
= (Eq Int, (Eq Bool, ()))
```

(Constraints are flattened.)



# Trying to define hcpure

#### We want:

```
hpure :: SListI xs => (forall a . f a)
hcpure :: (SListI xs, All c xs) => (forall a . c a => f a)
```

#### Then:

However, this does not work.

### Limitations in GHC's type inference

### Assume:

```
hcpure :: (SListI xs, All c xs) => (forall a . c a => f a)
hcpure = undefined
```

#### Then

```
minBound :: Bounded a => a
I minBound :: Bounded a => I a
```

```
GHCi> hcpure (I minBound) :: NP I '[Char, Bool]
```

is a type error.



### **Proxies**

```
data Proxy (a :: k) = Proxy
```

### Examples:

```
pBounded :: Proxy Bounded
```

pBounded = Proxy

pShow :: Proxy Show

pShow = Proxy



# Using proxies to define hcpure

# Using proxies to define hcpure

### Example:

```
GHCi> hcpure pBounded (I minBound) :: NP I '[Char, Bool]
I '\NUL' :* (I False :* Nil)
GHCi> hcpure pShow (Fn (K . show . unI)) 'hap' group
K "'x'" :* (K "False" :* (K "3" :* Nil))
```



### Summary

Lots of type system extensions and concepts in action:

- ► GADTs,
- promoted lists,
- singleton lists,
- ▶ higher-rank types,
- ► constraint kinds,
- proxies.

### Summary

Lots of type system extensions and concepts in action:

- GADTs,
- promoted lists,
- singleton lists,
- higher-rank types,
- constraint kinds,
- proxies.

The type NP and a number of useful functions:

- ► hpure, hap, hmap, hzipWith
- hcpure, hcmap, (hczipWith)
- hcollapse

With NP, we can already express one-constructor datatypes nicely.

We'll add NS to express the choice between constructors.



### Exercises

- Define heq :: ... => NP I xs -> NP I xs -> Bool via pattern matching.
- 2. Define hczipWith, i.e., hzipWith with a constrained function.
- 3. Define heq using hczipWith.
- 4. Can you generalize
  heq :: ... => NP f xs -> NP f xs -> Bool ?
- 5. Define hsequence :: Applicative f => NP f xs -> f (NP I xs).
- 6. Try to generalize

  foldr:: (a -> r -> r) -> r -> [a] -> r from lists

  to Vec and NP. Try to redefine e.g. hmap using the
  generalized foldr.

