



System-adapted Quantum Chemistry with Tequila

Jakob S. Kottmann

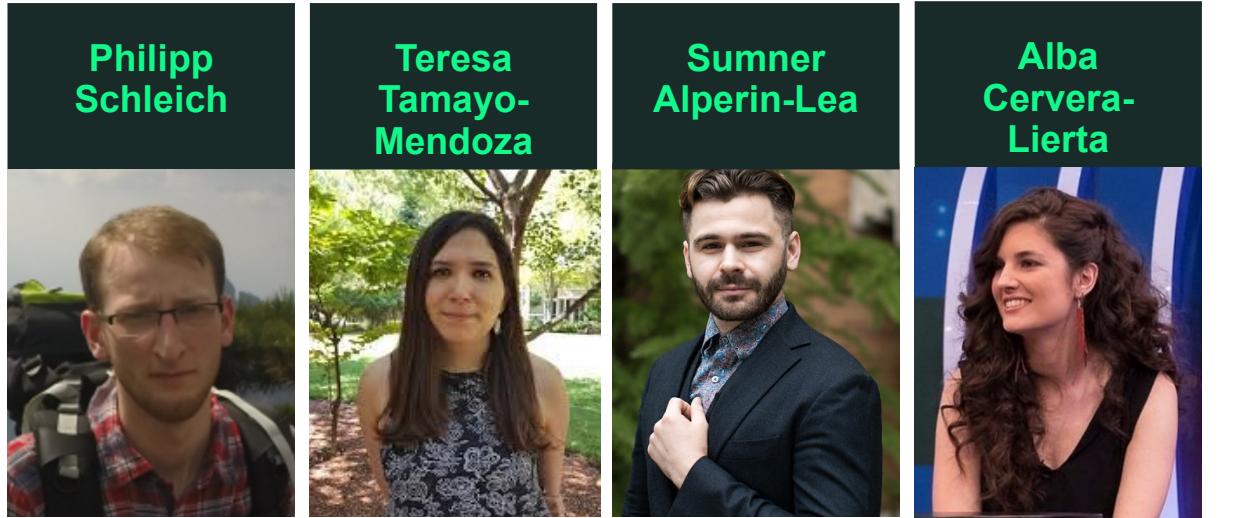
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Acknowledgement



Tzu-Ching (Thomson) Yen
Vladyslav Verteletskyi
Artur Izmaylov



Quantum Chemistry on Quantum Computers

Generators: Hermitian Operators

$$G_{abkl} = i(a_a^\dagger a_i a_b^\dagger a_j - h.c.)$$

Circuits from unitaries

$$U(\theta) = e^{-i\frac{\theta}{2}G}$$

Fermionic operators are mapped to paulistrings

$$a_k^\dagger = 1^{\otimes k-1} \sigma_k^- \sigma_Z^{\otimes n-k}$$

send instructions

$$\begin{aligned} L &= \langle H \rangle^2 \\ \frac{dL}{d\theta} &= 2\langle H \rangle \frac{d\langle H \rangle}{d\theta} \end{aligned}$$

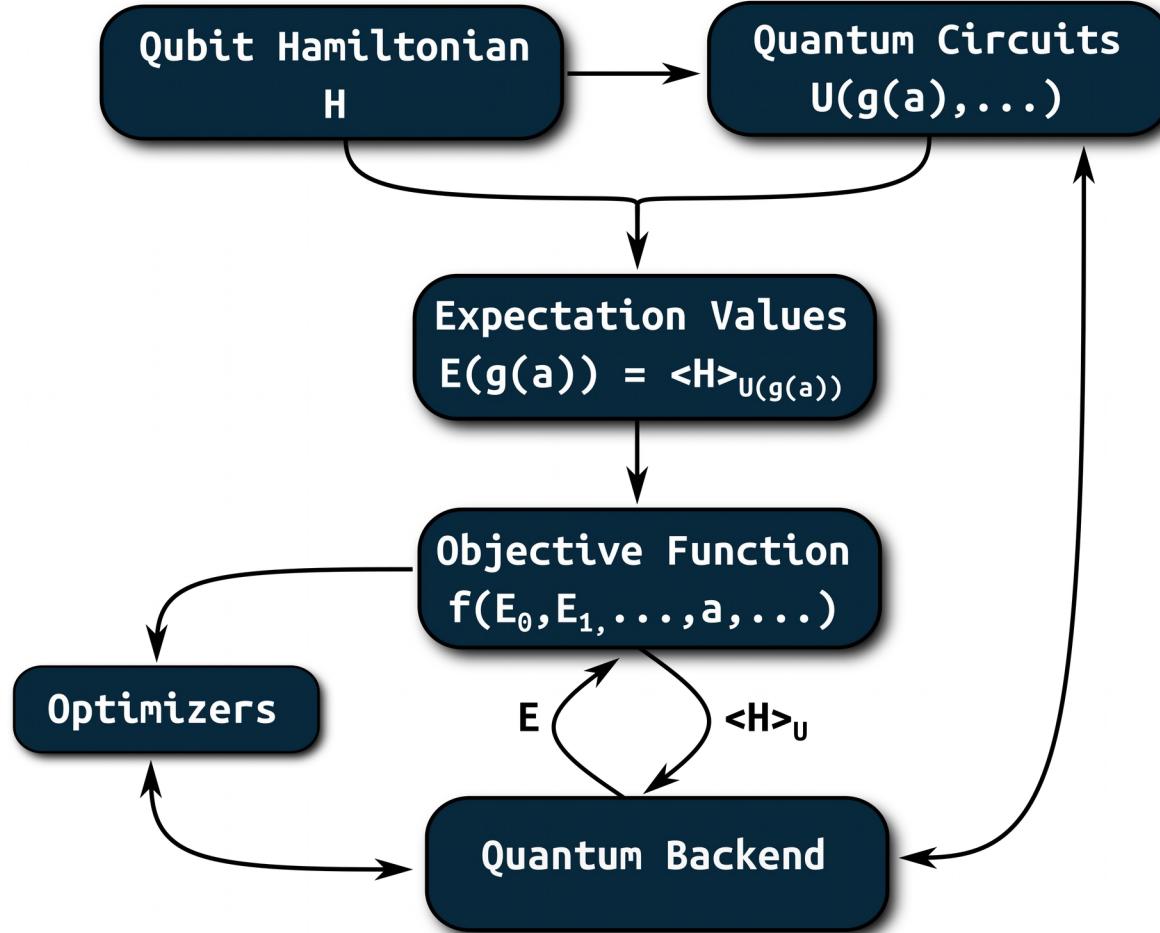
sample
expectation values

Quantum
Computer

Variational optimization

$$\min_{\theta} (\langle H \rangle_{U_\theta}) \equiv \min_{\theta} (\langle 0 | U^\dagger(\theta) H U(\theta) | 0 \rangle)$$

Tequila



The Meta-Variational Quantum Eigensolver (Meta-VQE): Learning energy profiles of parameterized Hamiltonians for quantum simulation

Alba Cervera-Lierta,^{1,2} Jakob S. Kottmann,^{1,2} and Alán Aspuru-Guzik^{1,2,3,4}

Quantum Computer-Aided design of Quantum Optics Hardware

Jakob S. Kottmann,^{1,2} Mario Krenn,^{1,2,3} Thi Ha Kyaw,^{1,2} Sumner Alperin-Lea,^{1,2} and Alán Aspuru-Guzik^{1,2,3,4}

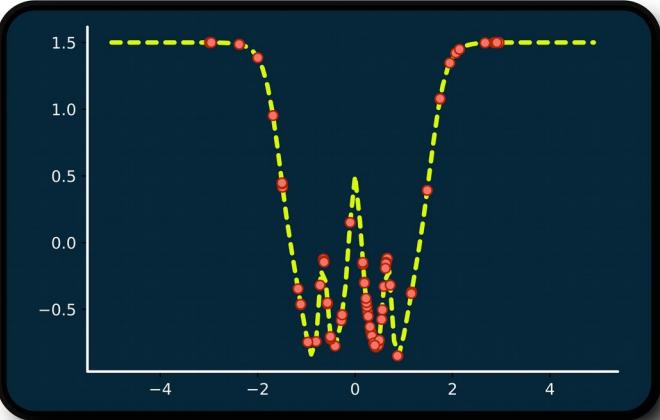
Reducing qubit requirements while maintaining numerical precision for the Variational Quantum Eigensolver: A Basis-Set-Free Approach

Jakob S. Kottmann,^{1,2,*} Philipp Schleich,³ Teresa Tamayo-Mendoza,^{4,1,2} and Alán Aspuru-Guzik^{1,2,5,6,†}

Tequila

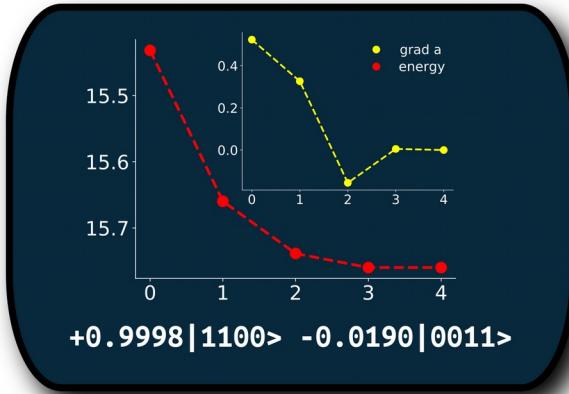
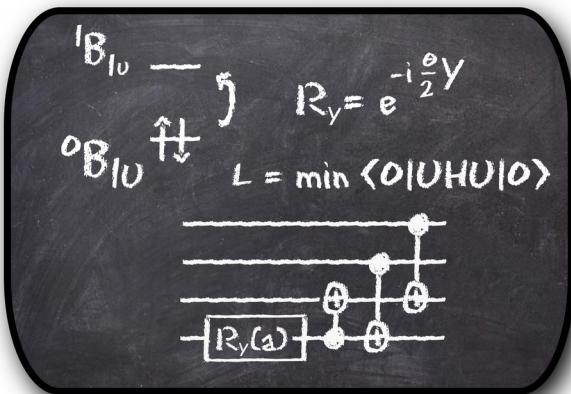
$$H = -X(0)X(1) + \frac{1}{2}Z(0) + Y(1)$$

$$G = e^{-i \frac{t}{2} e^{-i^2} Y}$$
$$L = \langle H \rangle_{U(t)} + e^{-\left(\frac{d}{dt} \langle H \rangle_{U(t)}\right)^2}$$



```
a = tq.Variable("a")
U = tq.gates.Ry(angle=(-a**2).apply(tq.numpy.exp)*pi, target=0)
U += tq.gates.X(target=1, control=0)
H = tq.QubitHamiltonian.from_string("-1.0*X(0)X(1)+0.5Z(0)+Y(1)")
E = tq.ExpectationValue(H=H, U=U)
dE = tq.grad(E, "a")
objective = E + (-dE**2).apply(tq.numpy.exp)
result = tq.minimize(method="phoenics", objective=objective)
```

Tequila



```
active = {"b1u": [0, 1]}
mol = tq.chemistry.Molecule("beh2.xyz", "6-31g", active)
H = mol.make_hamiltonian()
U = tq.gates.Ry("a", 0)
U += tq.gates.CNOT(0, 1) + tq.gates.CNOT(0, 2)
U += tq.gates.CNOT(1, 3) + tq.gates.X([2, 3])
expv = tq.ExpectationValue(U, H)
result = tq.optimizer_scipy.minimize(expv, "bfgs")
wfn = tq.simulate(U, variables=result.angles)
```



Quantum Chemistry on Quantum Computers

Generators: Hermitian Operators

$$G_{abkl} = i(a_a^\dagger a_i a_b^\dagger a_j - h.c.)$$

```
mol.make_excitation_generator([(a,i),(b,j)])
```

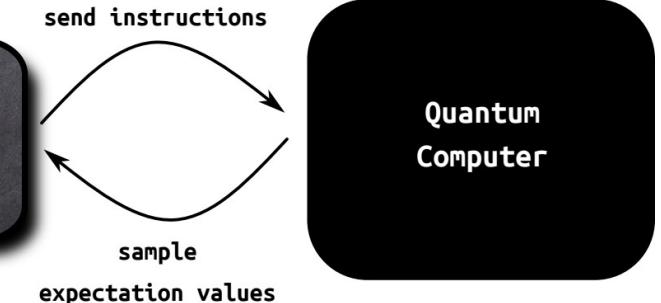
Circuits from unitaries

$$U(\theta) = e^{-i\frac{\theta}{2}G}$$

```
tq.gates.Trotterized(G, "a", 1)
```

Fermionic operators are mapped to paulistrings

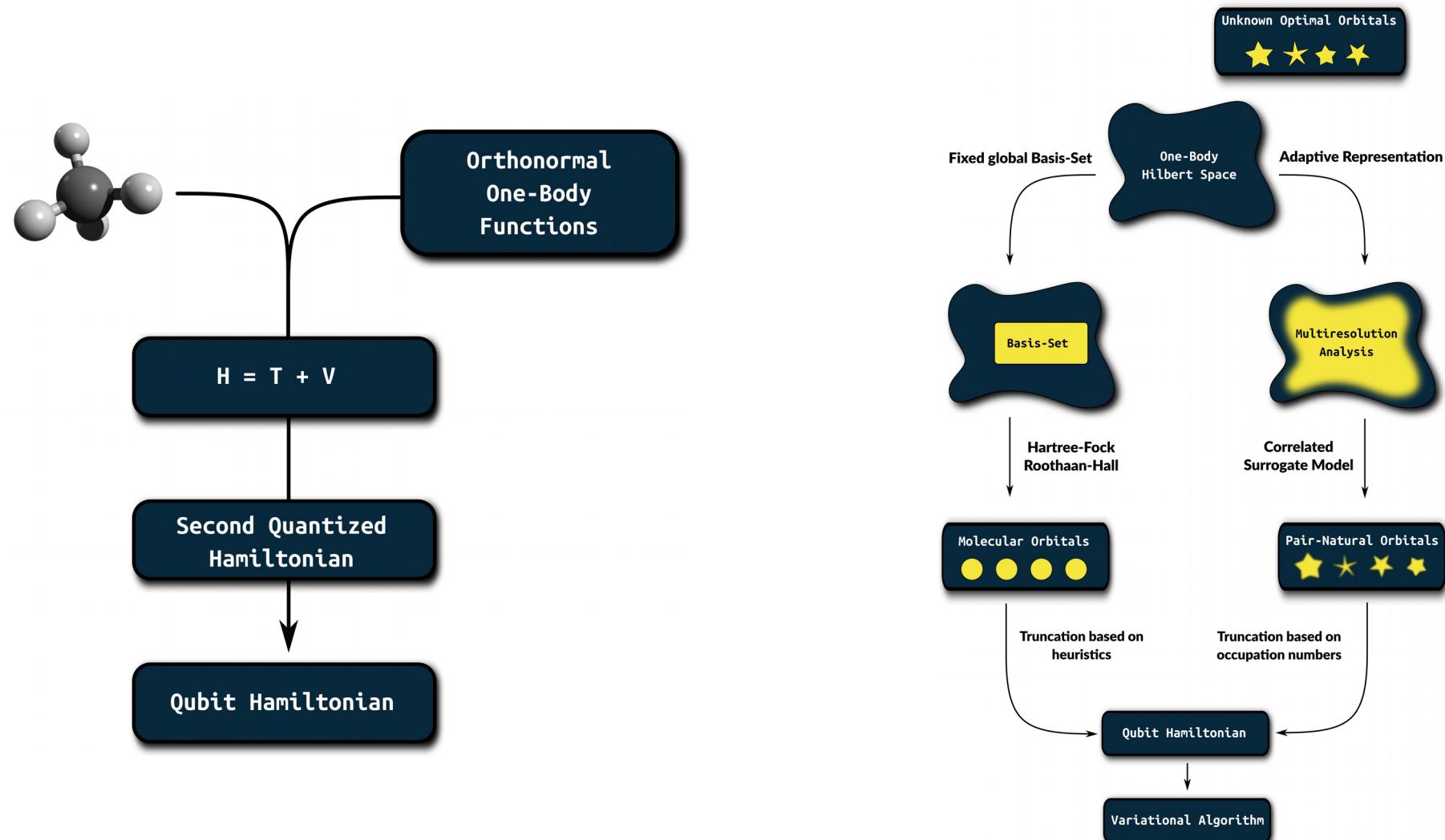
$$a_k^\dagger = 1^{\otimes k-1} \sigma_k^- \sigma_Z^{\otimes n-k}$$



Variational optimization

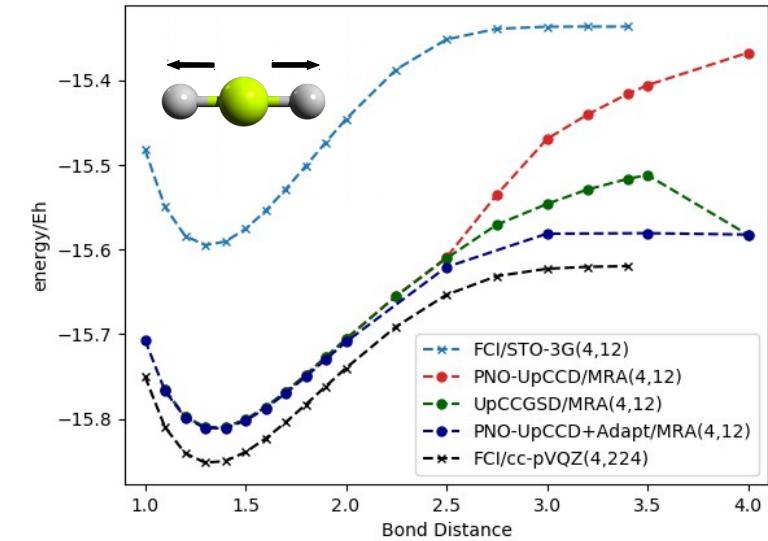
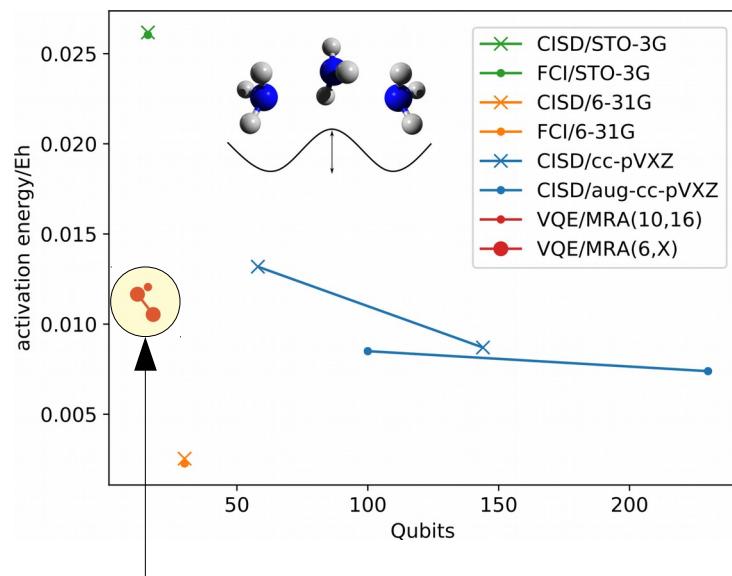
$$\min_{\theta} (\langle H \rangle_{U_\theta}) \equiv \min_{\theta} (\langle 0 | U^\dagger(\theta) H U(\theta) | 0 \rangle)$$

A Basis-Set-Free VQE



A Basis-Set-Free VQE

System	Metric	Qubits/MRA	Qubits/GBS	More
He	MAX	4	4-10	Fig. 3
Be	MAX	10	10-18	Fig. 3
H ₂	NPE	4	20-56	Figs. 5, 4
H ₂	NPE	8	20-56	Figs. 5, 4
H ₂	NPE	20	56-120	Figs. 5, 4
H ₂	MAX	4	8	Figs. 5, 4
H ₂	MAX	8	20-56	Figs. 5, 4
H ₂	MAX	20	56	Figs. 5, 4
LiH	NPE	12	20-38	Figs. 5, 4
LiH	NPE	20	38	Figs. 5, 4
LiH	MAX	12	20-38	Figs. 5, 4
LiH	MAX	20	170-288	Figs. 5, 4
BH	NPE	12-20	38-88	Figs. 5, 4
BH	MAX	12-20	38-88	Figs. 5, 4
NH ₃	ΔE	12-18	58-100	Fig. 2

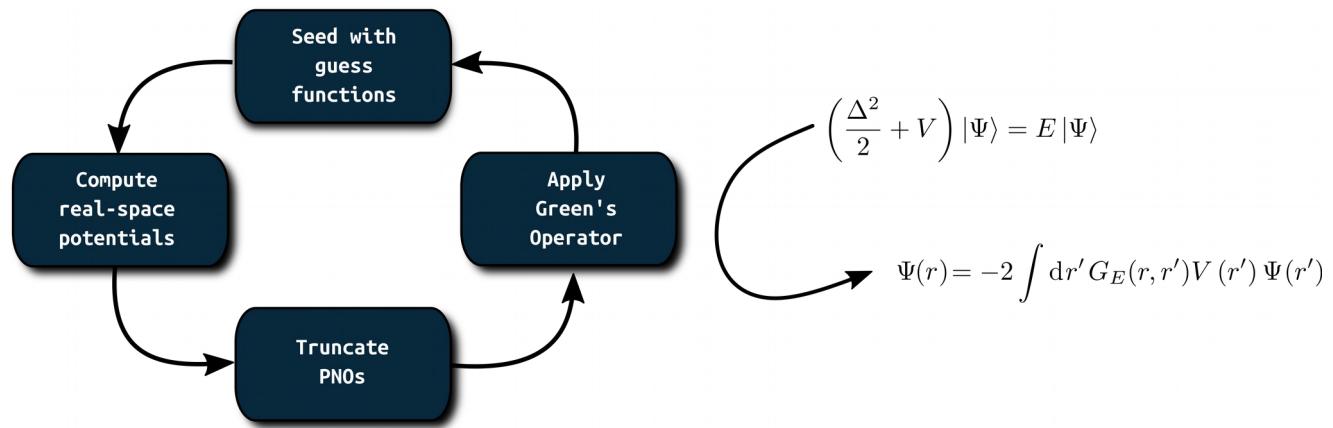


Circuit Sizes (in UCC operators):
PNO-UpCCD: 4
+Adapt : 20 – 100
UpCCGSD : 45

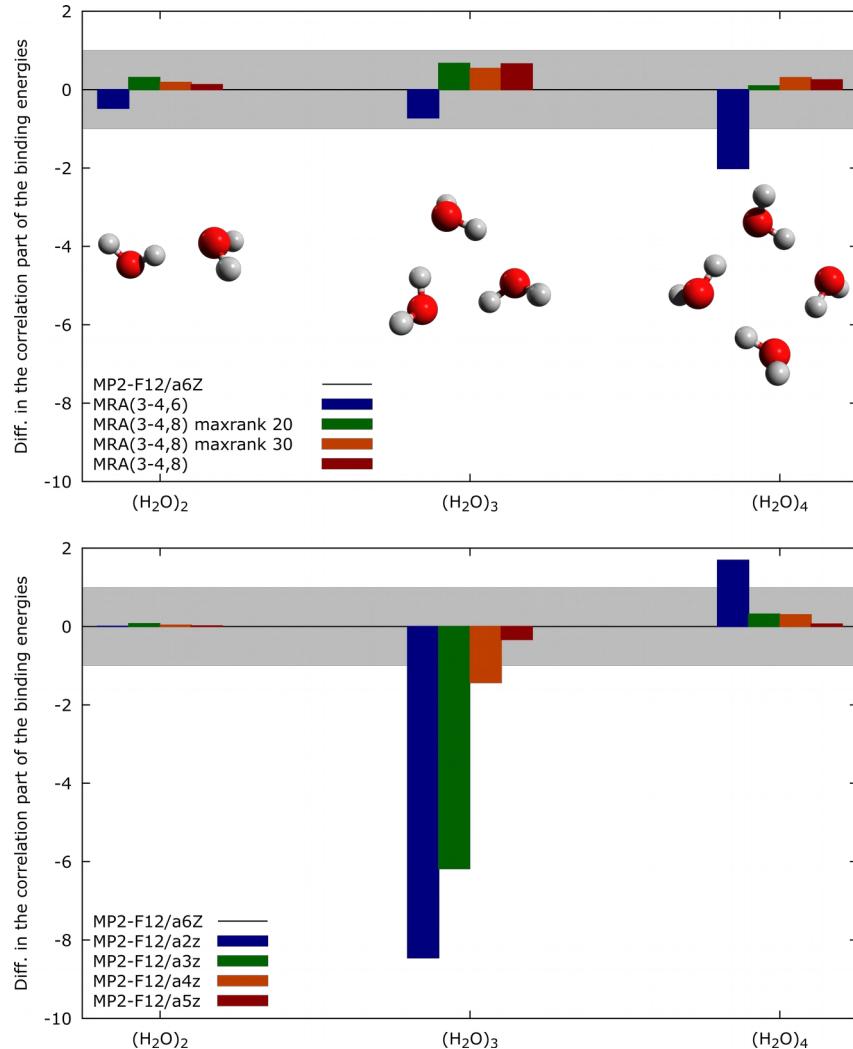


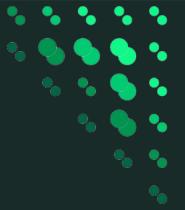
Directly Determined MRA-PNOs

Adaptive and basis-set-free approach



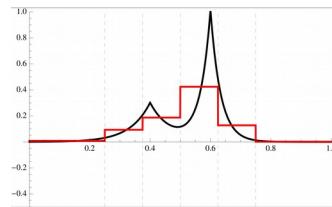
github.com/m-a-d-n-e-s-s/madness



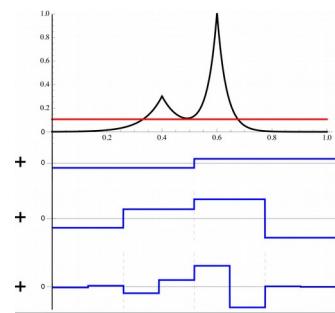
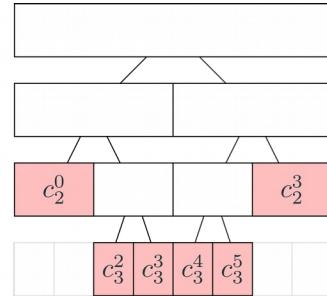


Recap: Basis-Set-Free Quantum Chemistry

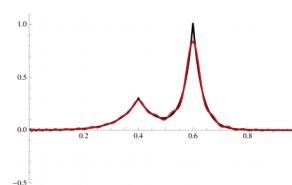
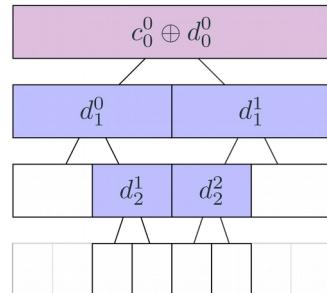
Multiresolution Analysis (MRA)



$$|f\rangle = \sum_{nl} c_n^l |\varphi_n^l\rangle$$



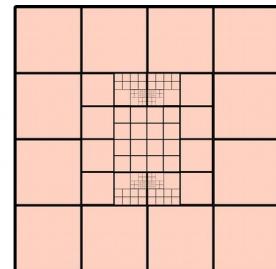
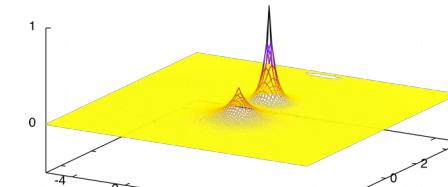
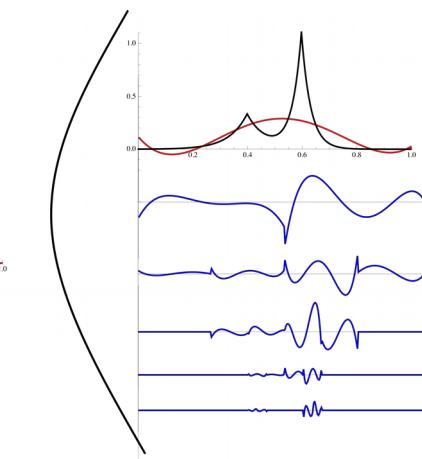
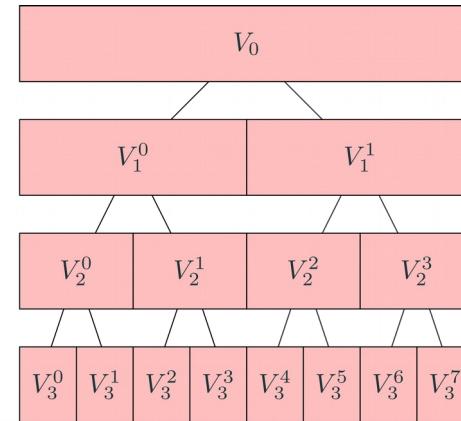
$$|f\rangle = c_0 |\varphi_0\rangle + \sum_{nl} d_n^l |\psi_n^l\rangle$$



$$V_0 \subset V_1 \subset V_2 \subset \dots \subset \mathbf{L}^2$$

$$V_n = \bigoplus_{l=0}^{l=2^n-1} V_n^l$$

$$V_{n+1} = V_n \oplus W_n$$



Acknowledgement



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