

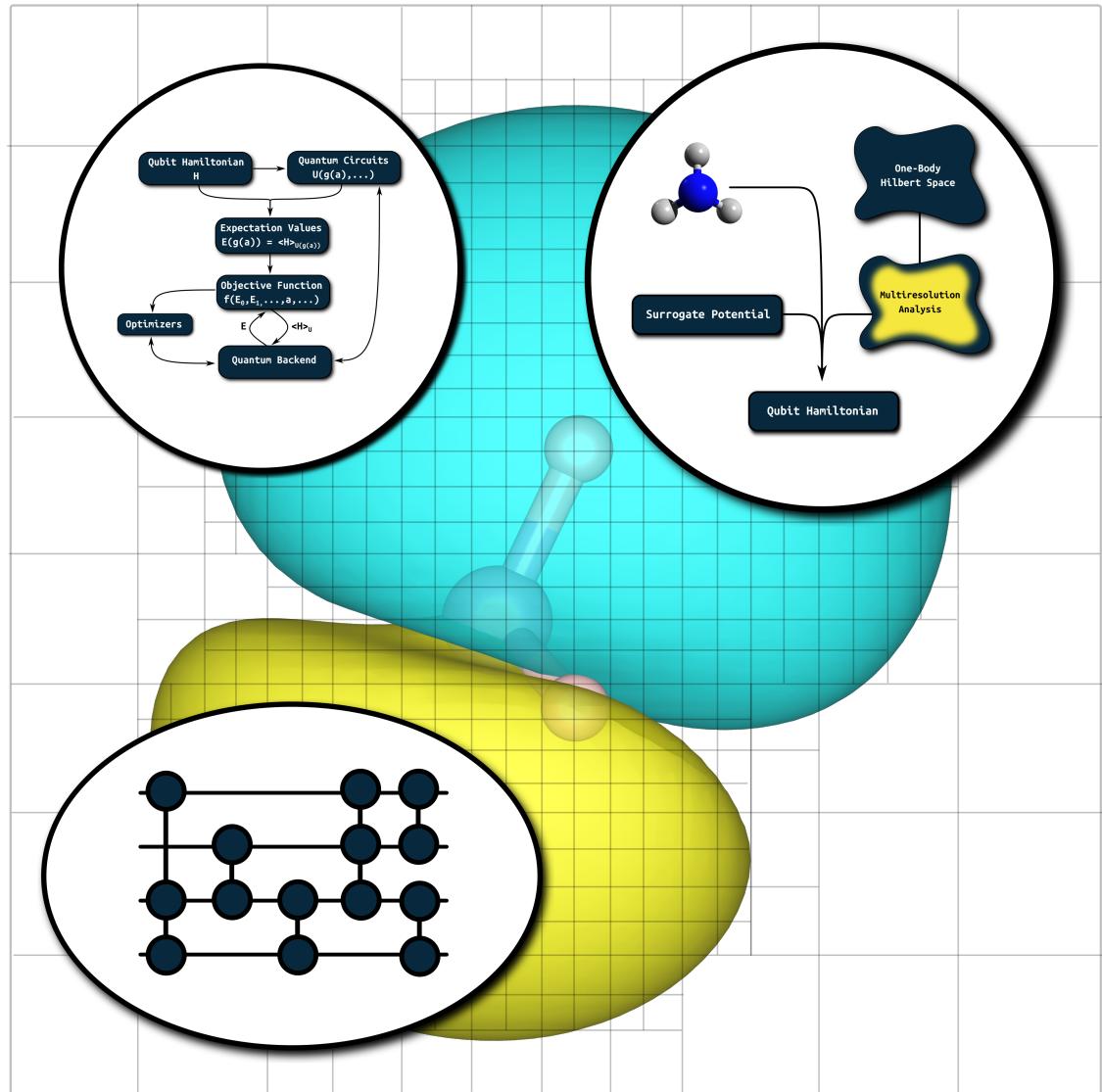
# Getting Started with Tequila

Jakob S. Kottmann

University of Toronto

 @JakobKottmann

 [github/tequilahub](https://github.com/tequilahub)



```
# install from PyPi  
pip install tequila-basic  
  
# install from github  
pip install git+https://github.com/tequillahub/tequila.git  
  
# install with windows (not recommended)  
pip install https://github.com/tequillahub/tequila@windows  
  
# recommended: install fast backend  
pip install qulacs
```

Find Slides here:

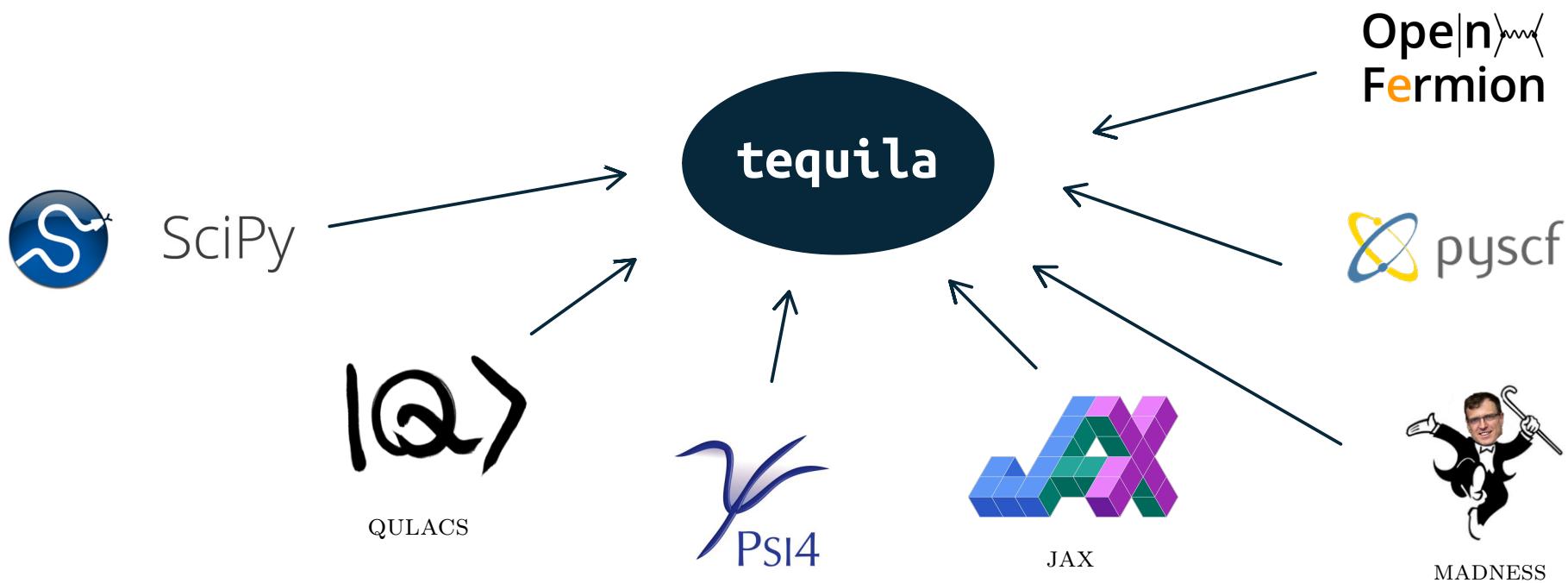
[https://github.com/kottmanj/talks\\_and\\_material/](https://github.com/kottmanj/talks_and_material/)

Will also contain small scripts that reproduce data shown in the talk

Problems/Wishes/Feedback:

firstname.lastname@gmail.com  
twitter pm: @jakobkottmann

- 1. Intuitive Application Programming Interface
- 2. Leverage existing software
- 3. Simplicity before Performance



**github.com/tequilahub**

https://github.com/tequilahub/tequila

README.md

## How to contribute

If you find any bugs or inconveniences in `tequila` please don't be shy and let us know. You can do so either by raising an issue here on github or contact us directly.

If you already found a solution you can contribute to `tequila` over a pull-request. Here is how that works:

1. Make a fork of `tequila` to your own github account.
2. Checkout the `devel` branch and make sure it is up to date with the main [github repository](#).
3. Create and checkout a new branch from `devel` via `git branch pr-my-branch-name` followed by `git checkout pr-my-branch-name`. By typing `git branch` afterwards you can check which branch is currently checked out on your computer.
4. Introduce changes to the code and commit them with git.
5. Push the changes to [your](#) github account
6. Log into github and create a pull request to the main [github repository](#). The pull-request should be directed to the `devel` branch (but we can also change that afterwards).

If you plan to introduce major changes to the base library it can be beneficial to contact us first. This way we might be able to avoid conflicts before they arise.

If you used `tequila` for your research, feel free to include your algorithms here, either by integrating it into the core libraries or by demonstrating it with a notebook in the tutorials section. If you let us know about it, we will also add your research article in the list of research projects that use `tequila` (see above).

https://github.com/tequilahub/tequila

README.md

## Getting Started

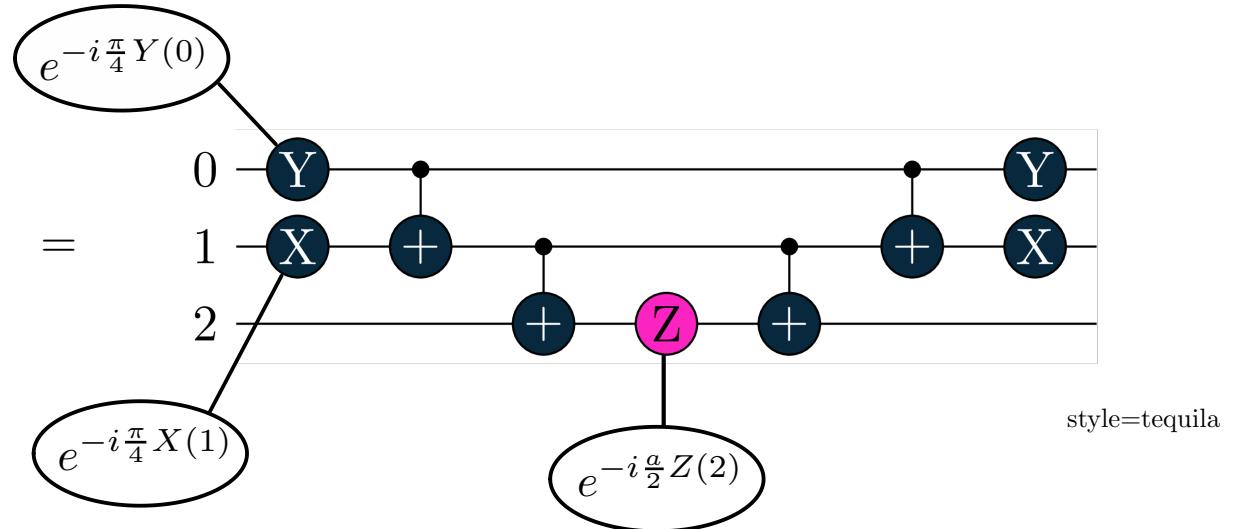
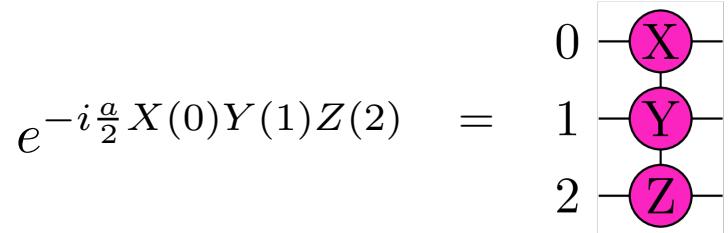
We have a collection of [tutorials](#) covering basic usage of `tequila` as well as cutting edge research content:

- Tutorial on Basic Usage
- Chemistry tutorial with `psi4`: see [here](#)
- Chemistry tutorial with `madness`: see [here](#)
- check the list of research projects below for links to specific examples.
- all tutorials: [github/tequilahub/tequila-tutorials](#).

# Circuits and Notation

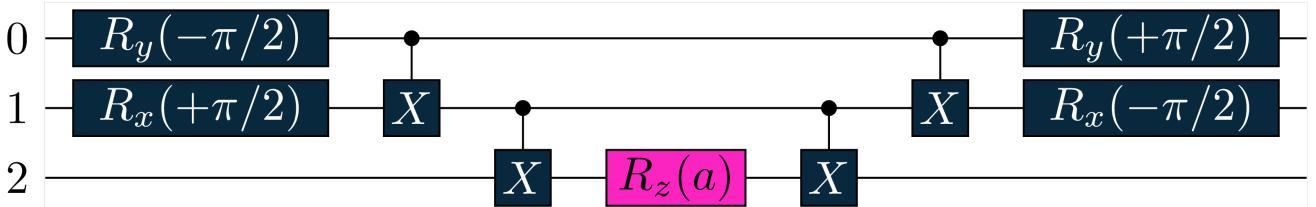
```
U=tq.gates.ExpPauli(paulistring="X(0)Y(1)Z(2)", angle="a")
```

magenta: parametrized gate



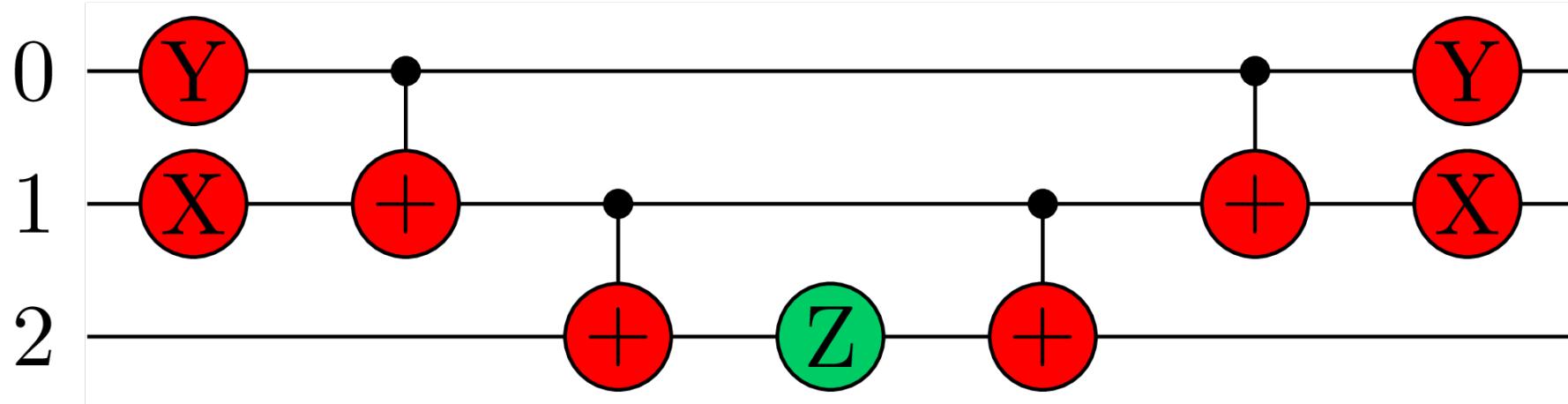
create plots: (png, pdf, tex, qpic)

```
U = tq.compile_circuit(U)
U.export_to("filename1.png", style="tequila")
U.export_to("filename2.png", style="standard")
U.export_to("filename3.png", style="plain")
```



needs qpic and pdf-to-png converter → see qpic docs

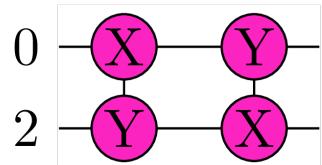
```
U.export_to(filename="asd.png", style="tequila", gatecolor1="red", textcolor1="black", gatecolor2="mine", colors=[{"name": "mine", "rgb":(0.0, 0.8, 0.4)}])
```



produces qpic file ("asd.qpic") that can also be manipulated manually

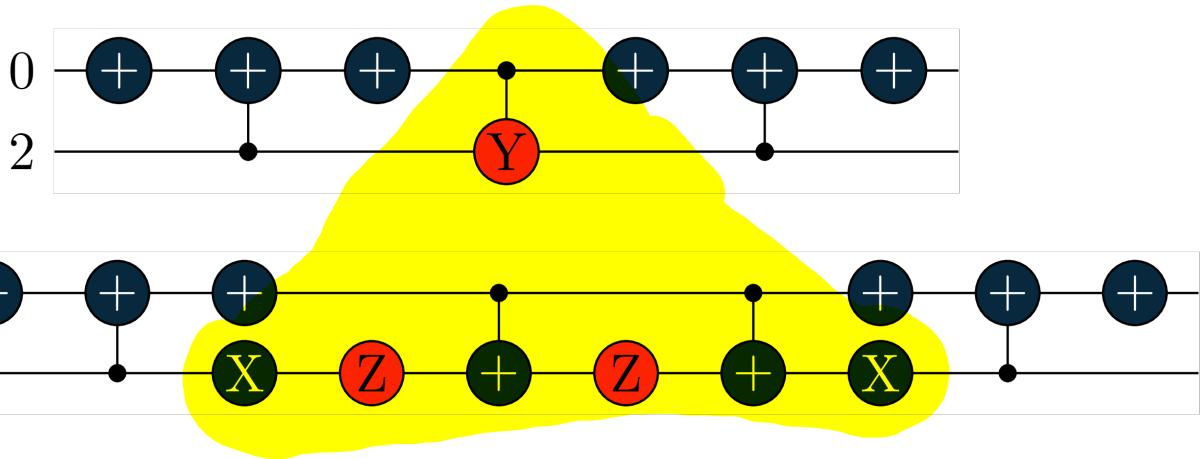
```
linux: pip install qpic # is sufficient when pdflatex is there  
mac: pip install qpic # needs imagemagick installed for pngs  
windows: no qpic (sorry). Can still produce qpic files (filename="whatever.qpic) and convert in virtual machine  
see github.com/qpic for more  
not happy with qpic? Create your own function to export circuits and become a contributor :-)
```

```
U2 = tq.gates.QubitExcitation(target=[0,2], angle="a")
```



=

$$e^{-i\frac{a}{2}(\sigma_+(0)\sigma_-(2)+\sigma_-(0)\sigma_+(2))}$$
$$\sigma_{\pm} = \frac{1}{2}(X \pm Y)$$



```
U = tq.gates.X(0) + U2
U = tq.compile(U, backend=...)
U(variables={"a":1.0})
>>> +0.8776|100> +0.4794|001>
```

backend=qulacs/qiskit/cirq/pyquil/...  
circuit compilation will depend on backend

compiled\_tequila\_circuit = U.abstract\_circuit  
actual\_backend\_circuit = U.circuit

compiled structures are callable like functions  
with their respective variables

```
U4 = tq.gates.QubitExcitation(target=[0,2,1,3], angle="a")
```

**backend=qulacs**

The diagram shows a quantum circuit with four horizontal lines representing qubits. The top line has four X gates. The second line has two Y gates followed by two X gates. The third line has three X gates followed by one Y gate. The bottom line has one Y gate followed by three X gates.

```
U = tq.gates.X([0,1]) + U4
U = tq.compile(U, backend=...)
U(variables={"a":1.0})
>>> +0.8776|1100> +0.4794|0011>
```

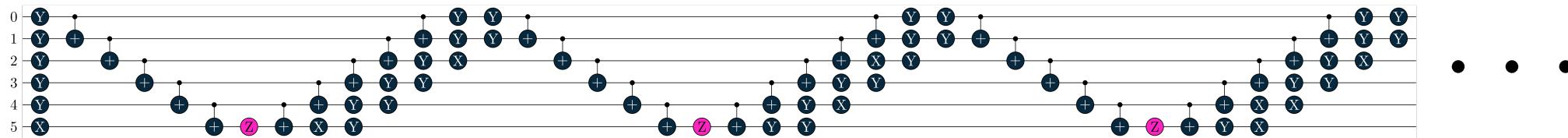
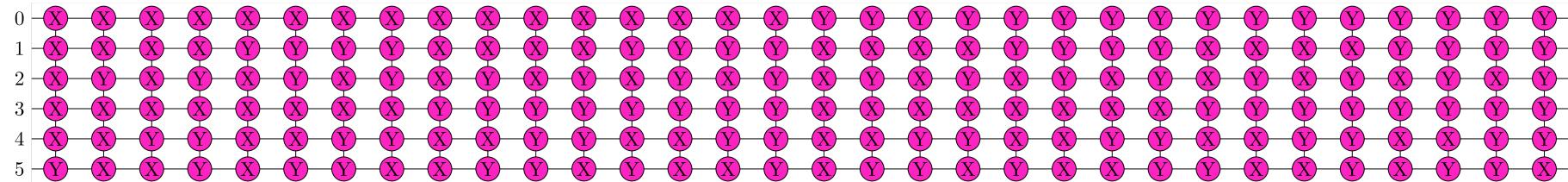
A quantum circuit diagram showing operations on four qubits. The circuit consists of two horizontal layers. The top layer contains a single Y gate on the third qubit. The bottom layer contains CNOT gates between adjacent qubits: (q0, q1), (q1, q2), and (q2, q3). The circuit starts with initial states on all qubits.

→ Yordanov *et.al.*, arxiv:2005.14475  
simila; Anselmetti *et.al.*, arxiv:2104.05695

multicontrol compiling: G. Tsilimigkounakis (qosf project)

`backend=giskit`

```
U4 = tq.gates.QubitExcitation(target=[0,3,1,4,2,5], angle="a")
```



**Triple and higher qubit excitations:**

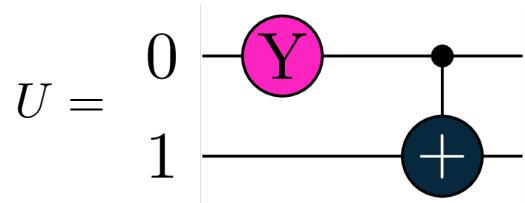
No optimization yet .... (should work analogue to single and double qubit excitations)

**Hamiltonian, ExpectationValue**

$$\langle \psi | H | \psi \rangle \equiv \langle 0 | U^\dagger H U | 0 \rangle \equiv \langle H \rangle_U$$

$$\langle \psi | H | \psi \rangle \equiv \langle 0 | U^\dagger H U | 0 \rangle \equiv \langle H \rangle_U$$

example:  $H = X(0)Y(1) + \frac{1}{2}Z(0)$



```
from tequila.paulis import X,Y,Z
from tequila.gates import Ry, CNOT

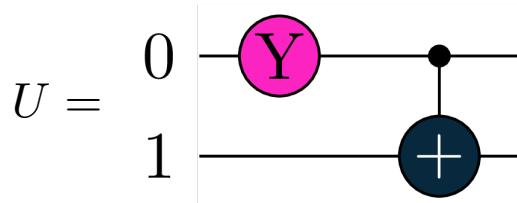
H = X(0)*Y(1) + 0.5*Z(0)
U = Ry(angle="a", target=0)
U+= CNOT(0,1)

E = tq.ExpectationValue(H=H, U=U)

f = tq.compile(E)
```

$$\langle \psi | H | \psi \rangle \equiv \langle 0 | U^\dagger H U | 0 \rangle \equiv \langle H \rangle_U$$

example:  $H = X(0)Y(1) + \frac{1}{2}Z(0)$



```

from tequila.paulis import X,Y,Z
from tequila.gates import Ry, CNOT

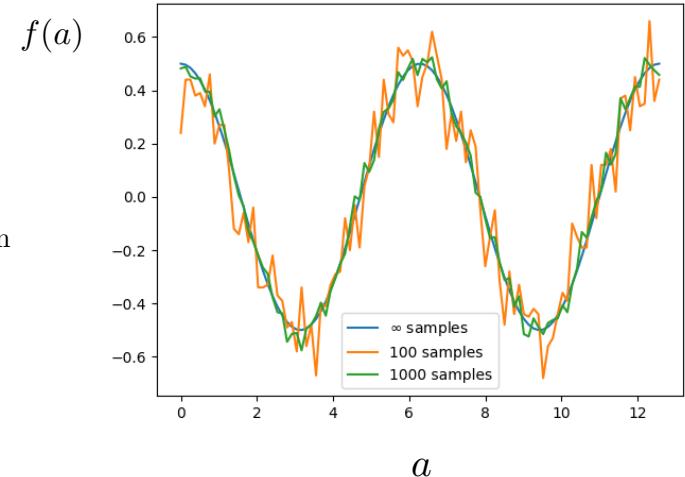
H = X(0)*Y(1) + 0.5*Z(0)
U = Ry(angle="a", target=0)
U+= CNOT(0,1)

E = tq.ExpectationValue(H=H, U=U)

f = tq.compile(E)
  
```

corresponds to  $\infty$  samples. Perfect simulation

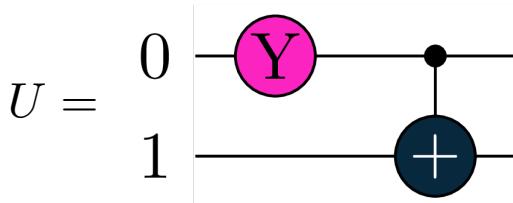
`f(variables={"a":1.0})`



device noise etc .... see tutorials (Sumner Alperin-Lea)

$$\langle \psi | H | \psi \rangle \equiv \langle 0 | U^\dagger H U | 0 \rangle \equiv \langle H \rangle_U$$

example:  $H = X(0)Y(1) + \frac{1}{2}Z(0)$



```
from tequila.paulis import X,Y,Z
from tequila.gates import Ry, CNOT

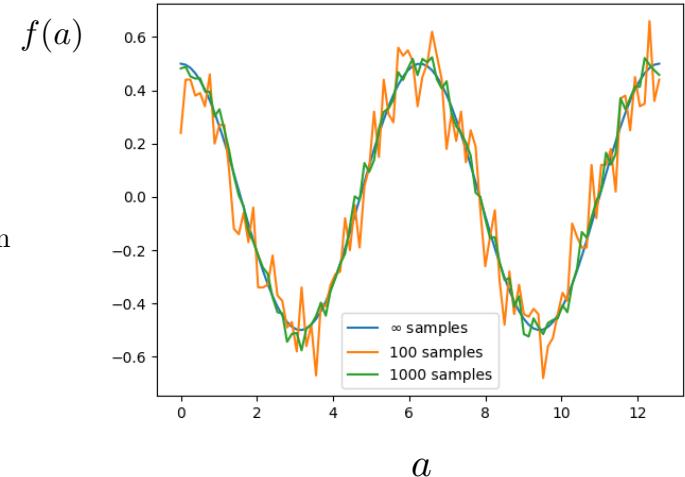
H = X(0)*Y(1) + 0.5*Z(0)
U = Ry(angle="a", target=0)
U+= CNOT(0,1)

E = tq.ExpectationValue(H=H, U=U)

f = tq.compile(E)
```

corresponds to  $\infty$  samples. Perfect simulation

`f(variables={"a":1.0})`

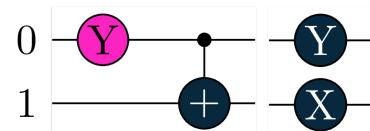


`f(variables={"a":1.0}, samples=100)`

shift into  $XY$  basis

measure both qubits 100 times

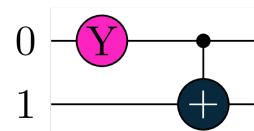
accumulate expectation value



$\langle Z(0) \rangle_U$

measure qubit 0 100 times

accumulate expectation value



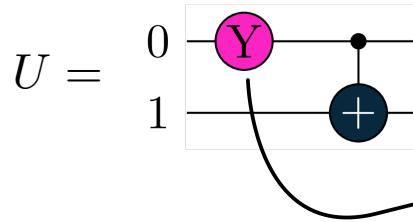
device noise etc .... see tutorials (Sumner Alperin-Lea)

`example_expectationvalue.py`

$$L(a) = \langle H \rangle_{U(a)} + e^{-\left(\frac{\partial}{\partial a} \langle H \rangle_{U(a)}\right)^2}$$

← how does this function look?

$$H = X(0)X(1) + \frac{1}{2}Z(0) + Y(1)$$



```

a = tq.Variable("a")
f = (-a**2).apply(tq.numpy.exp)

U = tq.gates.Ry(angle=f*tq.numpy.pi, target=0)
U += tq.gates.CNOT(0,1)

H = tq.paulis.from_string("-1.0*X(0)*X(1)+0.5*Z(0)+Y(1)")

E = tq.ExpectationValue(H=H, U=U)
dE = tq.grad(E, "a")

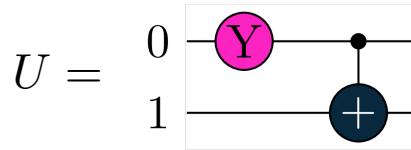
L = E + (-dE**2).apply(tq.numpy.exp)
    
```

$$e^{-i\frac{f(a)}{2}Y(0)}$$

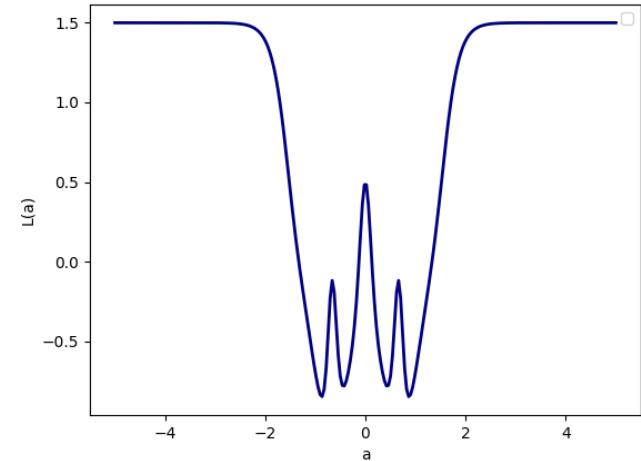
$$f(a) = e^{-a^2}$$

$$L(a) = \langle H \rangle_{U(a)} + e^{-\left(\frac{\partial}{\partial a} \langle H \rangle_{U(a)}\right)^2}$$

$$H = X(0)X(1) + \frac{1}{2}Z(0) + Y(1)$$



`tq.compile(L)`



```

a = tq.Variable("a")
f = (-a**2).apply(tq.numpy.exp)

U = tq.gates.Ry(angle=f*tq.numpy.pi, target=0)
U += tq.gates.CNOT(0,1)

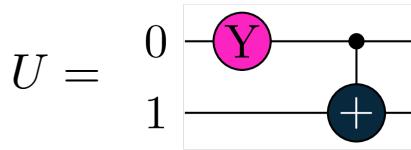
H = tq.paulis.from_string("-1.0*X(0)X(1)+0.5*Z(0)+Y(1)")

E = tq.ExpectationValue(H=H, U=U)
dE = tq.grad(E, "a")

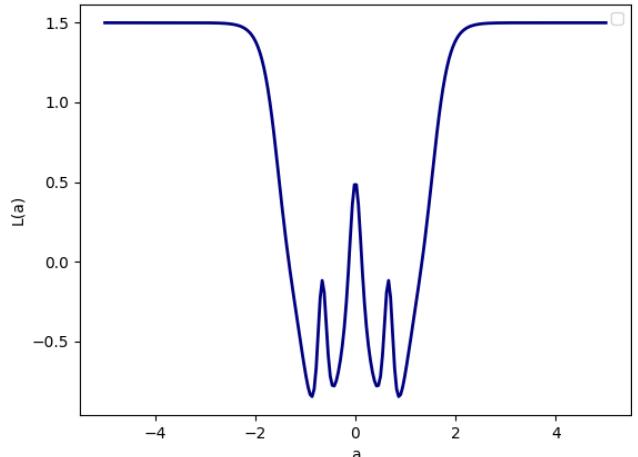
L = E + (-dE**2).apply(tq.numpy.exp)
    
```

$$L(a) = \langle H \rangle_{U(a)} + e^{-\left(\frac{\partial}{\partial a} \langle H \rangle_{U(a)}\right)^2}$$

$$H = X(0)X(1) + \frac{1}{2}Z(0) + Y(1)$$



`tq.compile(L)`



```

a = tq.Variable("a")
f = (-a**2).apply(tq.numpy.exp)

U = tq.gates.Ry(angle=f*numpy.pi, target=0)
U += tq.gates.CNOT(0,1)

H = tq.paulis.from_string("-1.0*X(0)X(1)+0.5*Z(0)+Y(1)")

E = tq.ExpectationValue(H=H, U=U)
dE = tq.grad(E, "a")

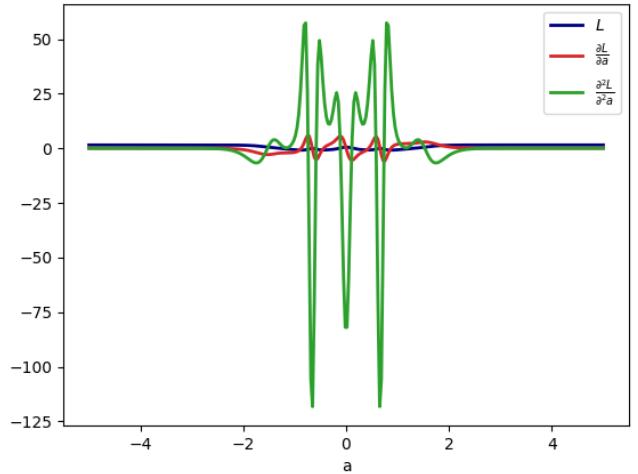
L = E + (-dE**2).apply(tq.numpy.exp)
    
```

`DL = tq.grad(L, "a")  
DL2 = tq.grad(DL, "a")`

`print(L)`

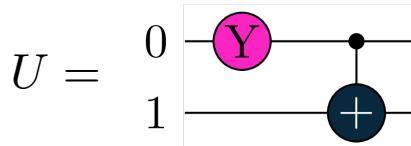
```

>>> Objective with 3 unique expectation values
total measurements = 9
variables          = [a]
types              = not compiled
    
```



$$L(a) = \langle H \rangle_{U(a)} + e^{-\left(\frac{\partial}{\partial a} \langle H \rangle_{U(a)}\right)^2}$$

$$H = X(0)X(1) + \frac{1}{2}Z(0) + Y(1)$$

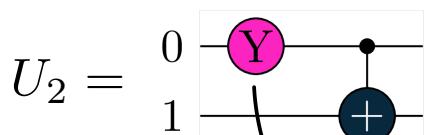


```

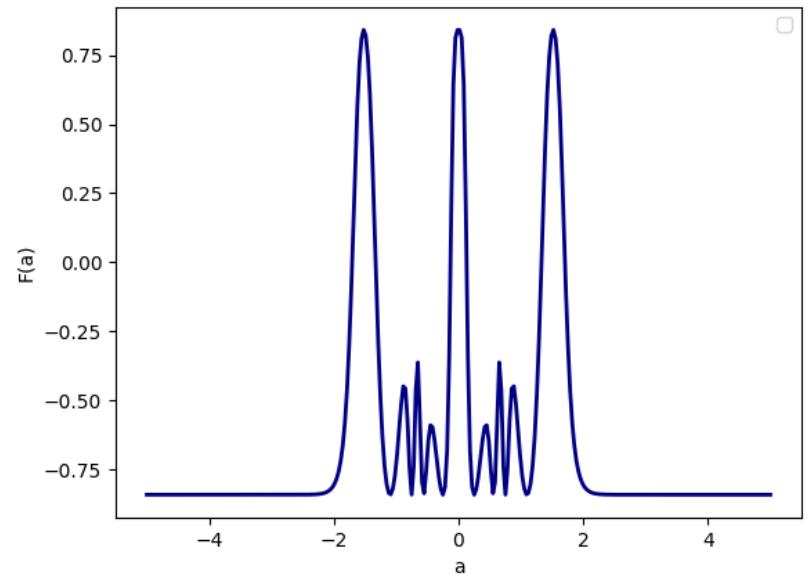
L = tq.compile(L)
U2 = tq.gates.Ry(angle=L, target=0)
U2+= tq.gates.CNOT(0,1)
    
```

$$F(a) = \sin(\langle H_2 \rangle_{U_2})$$

$$H_2 = X(0) + X(1) + X(0)X(1)$$

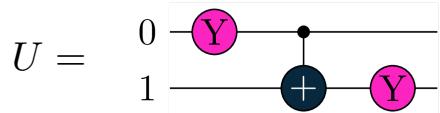


$$e^{-i\frac{L(a)}{2}}Y(0)$$



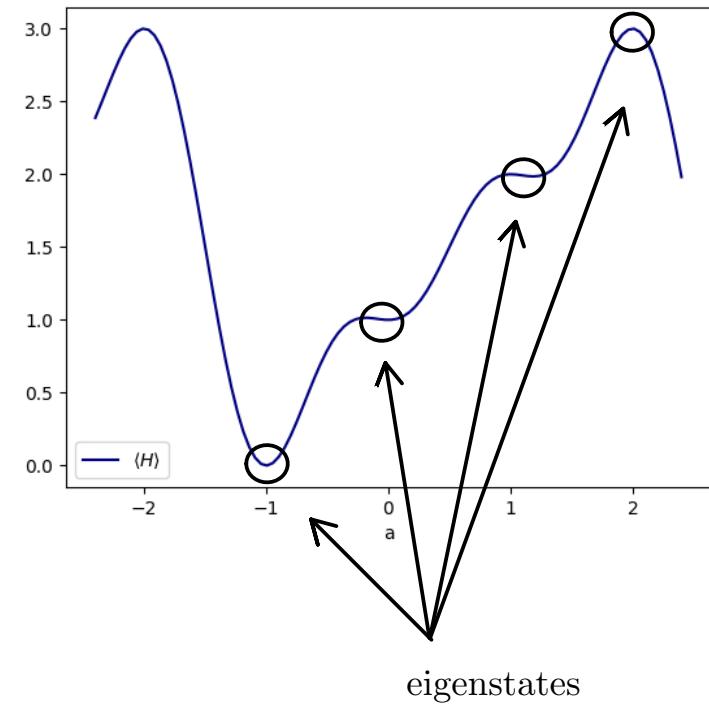
**Example: VQE**

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



$$\min \langle H \rangle_{U(a,b)}$$

Task: Prepare Ground State

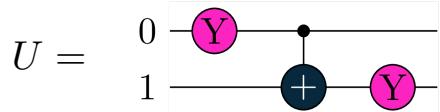


Variational Quantum Eigensolver (VQE):

Peruzzo, McClean *et.al.* Nat. Comm. 2014

McClean *et.al.* NJP, 2016

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



$$\min \langle H \rangle_{U(a,b)}$$

Task: Prepare Ground State

```

import tequila as tq
from tequila.hamiltonian.paulis import X,Y,Z
import numpy

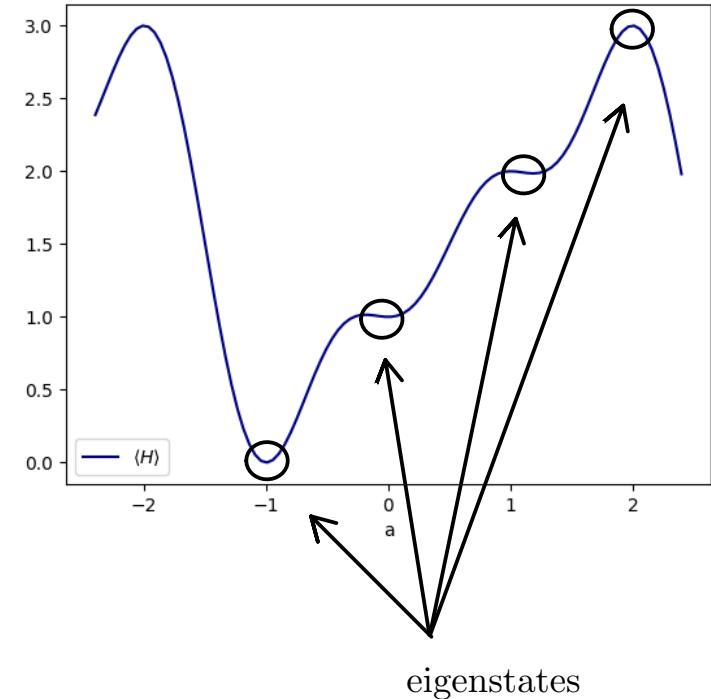
H = 1.5-0.5*(Z(1)-Z(0)+Z(0)*Z(1)+X(1)-Z(0)*X(1))

a = tq.Variable("a")
U = tq.gates.Ry(angle=a*numpy.pi,target=0)
U+= tq.gates.CNOT(0,1)
U+= tq.gates.Ry(angle=(a/2)*numpy.pi, target=1)

E = tq.ExpectationValue(H=H, U=U)

result = tq.minimize(E, initial_values="random")

```



Sumner Alperin-Lea :

Interfaces to SciPy (default), GPyOPT, PHOENICS, Adam etc

more on optimizers: [github/tequilahub/tequila-tutorials/](https://github.com/tequilahub/tequila-tutorials/)

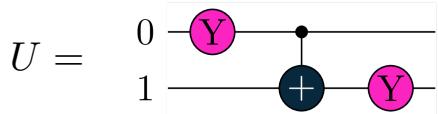
vqe.py

Variational Quantum Eigensolver (VQE):

Peruzzo, McClean *et.al.* Nat. Comm. 2014

McClean *et.al* NJP, 2016

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



Task: Prepare Ground State

```

import tequila as tq
from tequila.hamiltonian.paulis import X,Y,Z
import numpy

H = 1.5-0.5*(Z(1)-Z(0)+Z(0)*Z(1)+X(1)-Z(0)*X(1))

a = tq.Variable("a")
U = tq.gates.Ry(angle=a*numpy.pi,target=0)
U+= tq.gates.CNOT(0,1)
U+= tq.gates.Ry(angle=(a/2)*numpy.pi, target=1)

E = tq.ExpectationValue(H=H, U=U)

result = tq.minimize(E, initial_values="random")
  
```

vqe\_excited\_state.py

useful in daily life:

```

v,vv = numpy.linalg.eigh(H.to_matrix())

for i in range(len(v)):
    wfn=tq.QubitWaveFunction(vv[:,i])
    print("E{}={:+2.1f}, wfn=".format(i,v[i]), wfn)
  
```



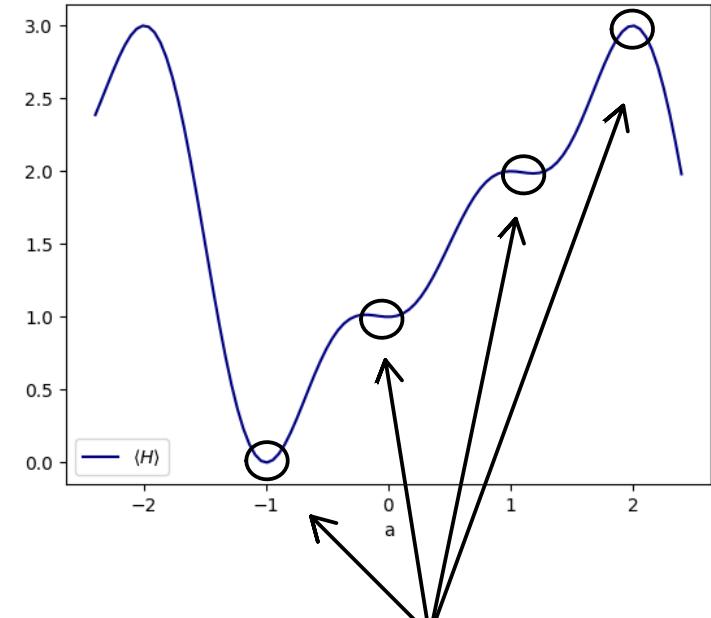
```

>>>E0=+0.0, wfn= -0.7071|10> -0.7071|11>
E1=+1.0, wfn= +1.0000|00>
E2=+2.0, wfn= -0.7071|10> +0.7071|11>
E3=+3.0, wfn= +1.0000|01>
  
```

Sumner Alperin-Lea :

Interfaces to SciPy (default), GPyOPT, PHOENICS, Adam etc

more on optimizers: [github/tequilahub/tequila-tutorials/](https://github.com/tequilahub/tequila-tutorials/)



eigenstates

Variational Quantum Eigensolver (VQE):

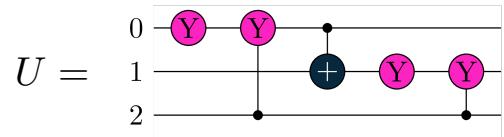
Peruzzo, McClean *et.al.* Nat. Comm. 2014

McClean *et.al.* NJP, 2016

## Example: Circuit Optimization

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$

eigenvalues:  $\{0, 1, 2, 3\}$



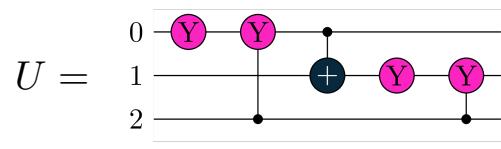
Task: Prepare eigenstates of H depending on qubit 2

qubit 2 in  $|0\rangle$ : Prepare ground state

qubit 2 in  $|1\rangle$ : Prepare highest eigenstate

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$

eigenvalues:  $\{0, 1, 2, 3\}$



Two Strategies:

$$\min (\langle H \rangle_U - \langle H \rangle_{XU})$$

$$\text{or: } \min \left( \langle P_{|000\rangle} \rangle_{UU_0^\dagger} + \langle P_{|001\rangle} \rangle_{XUU_1^\dagger} \right)$$

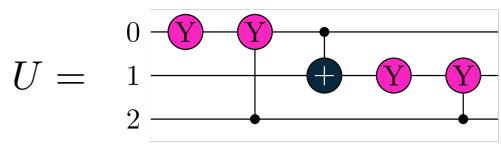
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eigenvalues:  $\{0, 1, 2, 3\}$



Two Strategies:

$$\min (\langle H \rangle_U - \langle H \rangle_{XU})$$

$$\text{or: } \min \left( \langle P_{|000\rangle} \rangle_{UU_0^\dagger} + \langle P_{|001\rangle} \rangle_{XUU_1^\dagger} \right)$$

Task: Prepare eigenstates of H depending on qubit 2

qubit 2 in  $|0\rangle$ : Prepare ground state

qubit 2 in  $|1\rangle$ : Prepare highest eigenstate

```

H = H = 1.5-0.5*(Z(1)-Z(0)+Z(0)*Z(1)+X(1)-Z(0)*X(1))

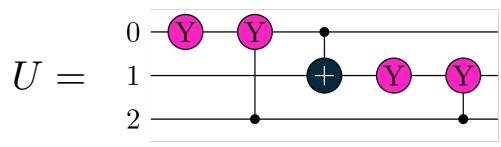
U = Ry("x", 0) + Ry("xx", 0, control=2)
U+= CNOT(0,1) + Ry("y", 1) + Ry("yy", 1, control=2)

E0 = tq.ExpectationValue(H=H, U=U)
E1 = tq.ExpectationValue(H=H, U=tq.gates.X(2)+U)
objective = E0 - E1
result = tq.minimize(objective, initial_values="near_zero")

```

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$

eigenvalues:  $\{0, 1, 2, 3\}$



Two Strategies:

$$\min (\langle H \rangle_U - \langle H \rangle_{XU})$$

$$\text{or: } \min \left( \langle P_{|000\rangle} \rangle_{UU_0^\dagger} + \langle P_{|001\rangle} \rangle_{XUU_1^\dagger} \right)$$

Task: Prepare eigenstates of H depending on qubit 2

qubit 2 in  $|0\rangle$ : Prepare ground state

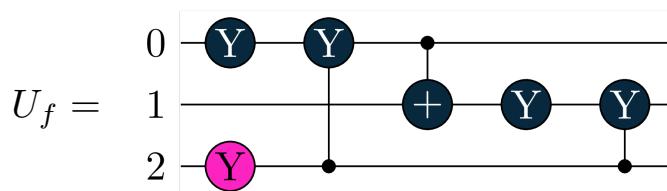
qubit 2 in  $|1\rangle$ : Prepare highest eigenstate

```

H = H = 1.5-0.5*(Z(1)-Z(0)+Z(0)*Z(1)+X(1)-Z(0)*X(1))
U = Ry("x", 0) + Ry("xx", 0, control=2)
U+= CNOT(0,1) + Ry("y", 1) + Ry("yy", 1, control=2)

E0 = tq.ExpectationValue(H=H, U=U)
E1 = tq.ExpectationValue(H=H, U=tq.gates.X(2)+U)
objective = E0 - E1
result = tq.minimize(objective, initial_values="near_zero")

```



```

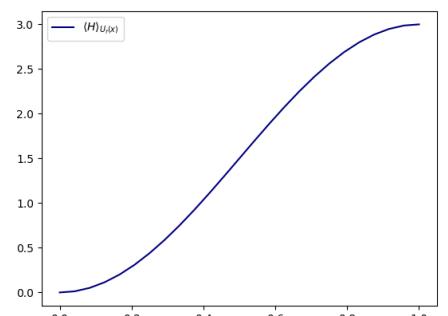
Uf = Ry(tq.Variable("control")*numpy.pi, 2)
Uf+= U.map_variables(result.variables)
f = tq.ExpectationValue(H=H, U=Uf)
f = tq.compile(f)

```

```

>>> f(0) = +1.00
      f(1) = +3.00

```

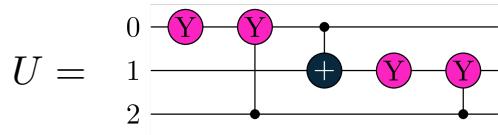


state\_preparation\_hamiltonian.py  
state\_preparation\_fidelity.py

# Measurement Optimization: Izmaylov Group

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$

eigenvalues: {0, 1, 2, 3}



Implementation by Tzu-Ching "Thomson" Yen and Vladislav Verteletskyi

Yen, Verteletskyi, Izmaylov, JCTC, 2020

Verteletskyi, Yen, Izmaylov, JCP, 2020

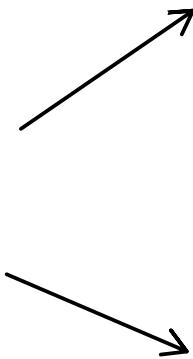
```
E = tq.ExpectationValue(H=H, U=U)
print(E)
E_opt = tq.ExpectationValue(H=H, U=U, optimize_measurements=True)
print(E_opt)
```



```
>>> Objective with 1 unique expectation values
total measurements = 6
variables          = [a,b,c,d]
types              = [not compiled]

Objective with 2 unique expectation values
total measurements = 2
variables          = [a,b,c,d]
types              = not compiled
```

faster when simulated

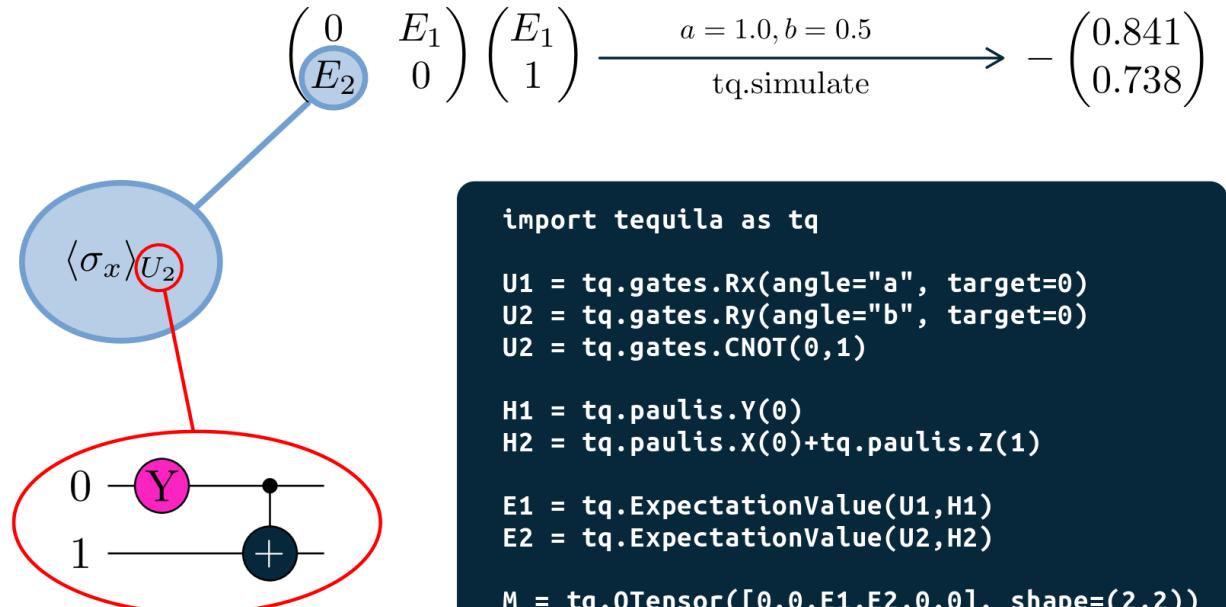


faster on quantum hardware

## More Convenience with QTensor

$$\min (\langle H \rangle_U - \langle H \rangle_{XU})$$

```
E0 = tq.ExpectationValue(H=H, U=U)
E1 = tq.ExpectationValue(H=H, U=tq.gates.X(2)+U)
vector = tq.QTensor([E0, E1], shape=(2,))
weights = numpy.asarray([1.0,1.0])
objective = vector.dot(weights)
```



```
import tequila as tq

U1 = tq.gates.Rx(angle="a", target=0)
U2 = tq.gates.Ry(angle="b", target=0)
U2 = tq.gates.CNOT(0,1)

H1 = tq.paulis.Y(0)
H2 = tq.paulis.X(0)+tq.paulis.Z(1)

E1 = tq.ExpectationValue(U1,H1)
E2 = tq.ExpectationValue(U2,H2)

M = tq.QTensor([0.0,E1,E2,0.0], shape=(2,2))
c = tq.QTensor([E1,1.0], shape=(2,))
b = M.dot(c)

variables={"a":1.0, "b":0.5}
result = tq.simulate(b, variables)
```

QOSF project by Gaurav Saxena (U Calgary)

see [github/tequilahub/tequila-tutorials](https://github.com/tequilahub/tequila-tutorials) for more

$$\min \left( \langle P_{|000}\rangle_{UU_0^\dagger} + \langle P_{|001}\rangle_{XUU_1^\dagger} \right)$$

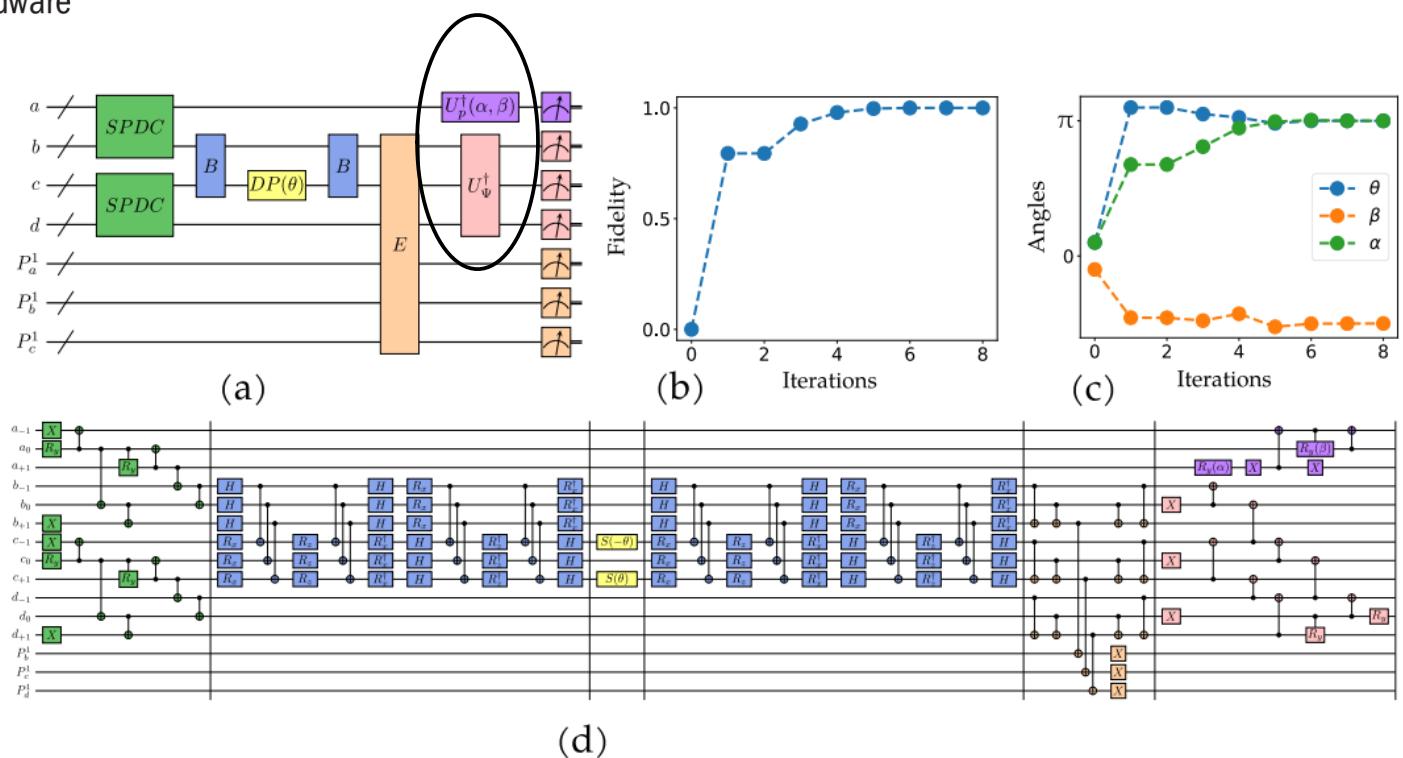
Application: Quantum Optics (same principle as in this example)

## Quantum Science and Technology

### PAPER

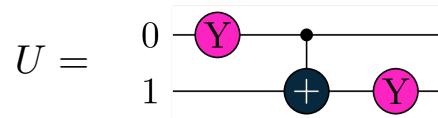
#### Quantum computer-aided design of quantum optics hardware

Jakob S Kottmann<sup>1,2</sup>, Mario Krenn<sup>1,2,3</sup>, Thi Ha Kyaw<sup>1,2</sup>, Sumner Alperin-Lea<sup>1</sup> and Alán Aspuru-Guzik<sup>1,2,3,4,\*</sup>

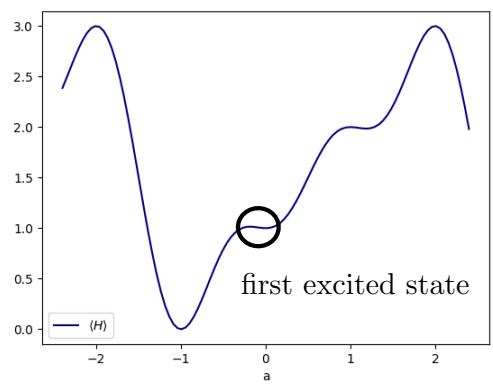


## **Example: Excited States**

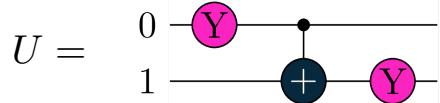
$$H = \frac{3}{2} - \frac{1}{2} (Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



## Task: Prepare First Excited State



$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



Task: Prepare First Excited State

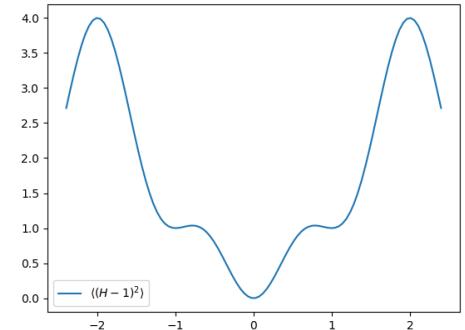
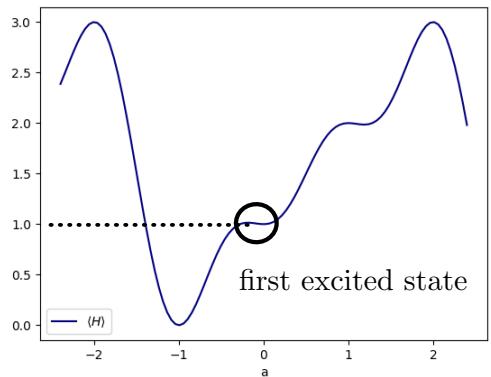
```
result=tq.minimize(blue)
```

$$\langle (H - 1)^2 \rangle_U$$

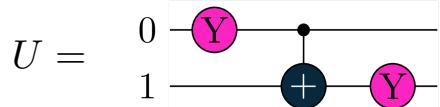
“folded spectrum”

McClean *et.al.* NJP 2016

```
blue=tq.ExpectationValue(H=(H-1)**2, U=U)
```



$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



Task: Prepare First Excited State

$$\langle(H - 1)^2\rangle_U$$

“folded spectrum”

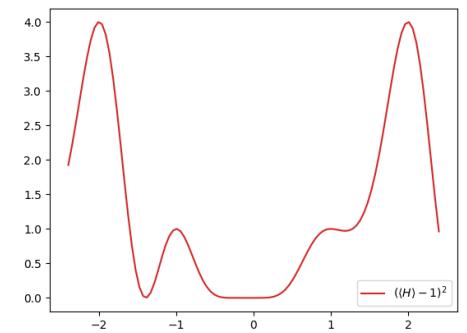
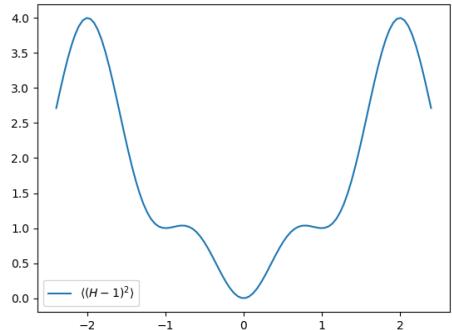
McClean *et.al.* NJP 2016

```
blue=tq.ExpectationValue(H=(H-1)**2, U=U)
```

```
E = tq.ExpectationValue(H=H, U=U)
red = (E - 1.0)**2
```

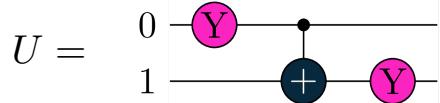
$$(\langle H \rangle_U - 1)^2$$

approximation



```
result=tq.minimize(red)
```

$$H = \frac{3}{2} - \frac{1}{2}(Z(1) - Z(0) + Z(0)Z(1) + X(1) - Z(0)X(1))$$



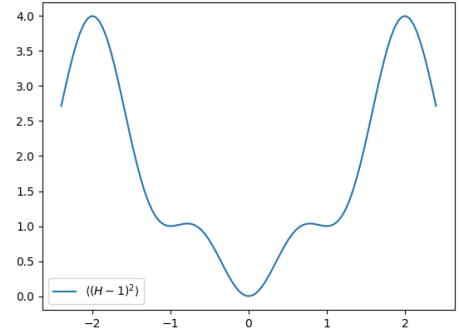
Task: Prepare First Excited State

```
result=tq.minimize(blue)
```

$$\langle(H-1)^2\rangle_U$$

“folded spectrum”

McClean *et.al.* NJP 2016



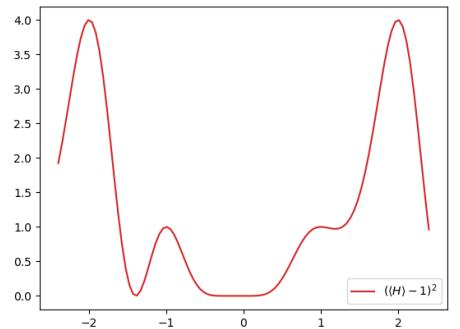
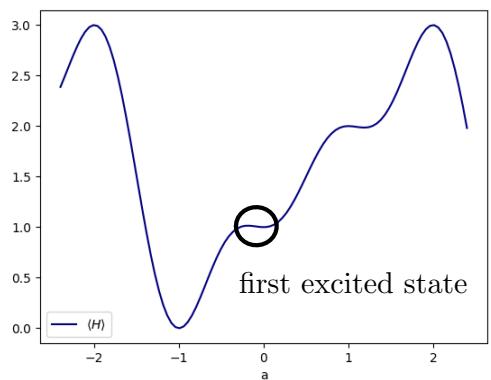
```
blue=tq.ExpectationValue(H=(H-1)**2, U=U)
```

```
E = tq.ExpectationValue(H=H, U=U)
red = (E - 1.0)**2
```

```
U0 = U.map_variables({"a":-1.0})
U1 = U + U0.dagger()
E1 = tq.ExpectationValue(H=P, U=U1)
green = E - 10*E1
```

$$(\langle H \rangle_U - 1)^2$$

approximation



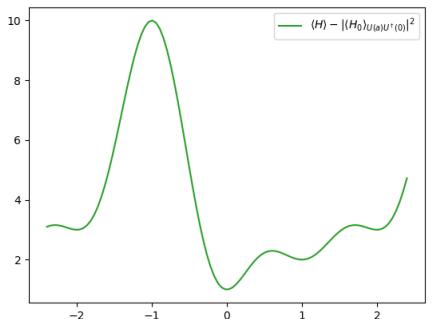
$$\langle H \rangle_U - 10\langle P_{|00}\rangle_{U(a)U^\dagger(-1)}$$

“VQD”, “Orthogonality Constrained”

Lee *et.al.*, JCTC, 2018

Higgott *et.al.* Quantum 2019

JSK, A. Anand, AAG, Chemical Science, 2021  
explicit tequila examples



vqe\_excited\_state.py

$$\langle H \rangle_U - 10 \langle P_{|00\rangle} \rangle_{U(a)U^\dagger(-1)}$$

Application: Excited States in Chemistry

**Chemical  
Science**



**EDGE ARTICLE**

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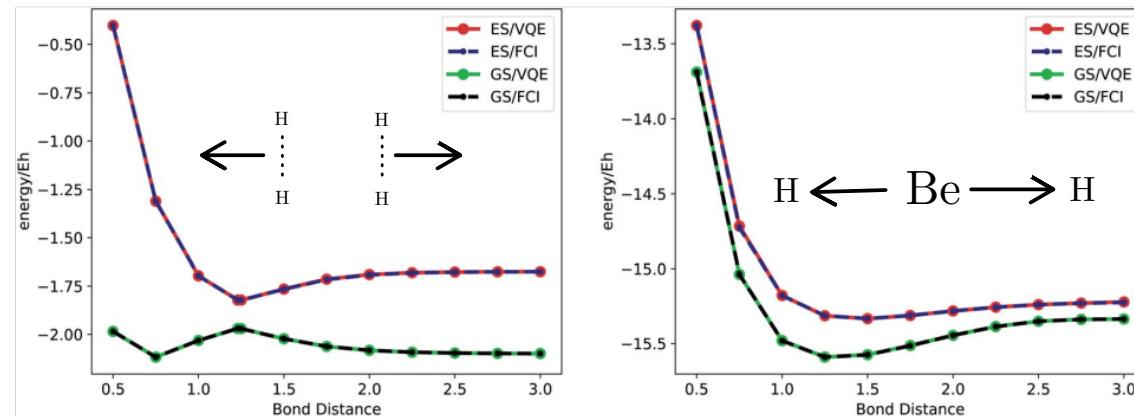


Cite this: *Chem. Sci.*, 2021, **12**, 3497

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## A feasible approach for automatically differentiable unitary coupled-cluster on quantum computers†

Jakob S. Kottmann,<sup>ab</sup> Abhinav Anand,<sup>a</sup> and Alán Aspuru-Guzik,<sup>ab,cd</sup>



**Fig. 6** Adapt-VQE for ground and excited states: adapt-VQE results using automatic differentiation for the screening and the optimization for ground and first excited state energies of  $\text{H}_4/\text{STO-3G}(4,8)$  (left) and  $\text{BeH}_2/\text{STO-3G}(6,14)$  (right). Except for the last point of the  $\text{BeH}_2$  excited state, all points agree to millihartree accuracy with the corresponding exact solution (FCI) in the given basis set. See also Fig. 4. We included the special point at distance 1.23 Å with a square configuration.

$$\langle H \rangle_U - 10 \langle P_{|00\rangle} \rangle_{U(a)U^\dagger(-1)}$$

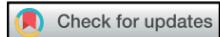
Application: Excited States in Chemistry

Chemical  
Science



EDGE ARTICLE

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Cite this: *Chem. Sci.*, 2021, 12, 3497

All publication charges for this article have been paid for by the Royal Society

## A feasible approach for automatically differentiable unitary coupled-cluster on quantum computers†

Jakob S. Kottmann, <sup>ab</sup> Abhinav Anand <sup>a</sup> and Alán Aspuru-Guzik <sup>abcd</sup>

vqe ground state : -1.9771  
vqe excited state: -1.7326  
walltime: 11s

in paper: Adaptive VQE

see “Adaptive Solver” Tutorial

[github.com/tequilahub/tequila-tutorials](https://github.com/tequilahub/tequila-tutorials)

```
geometry = "H 0.0 0.0 0.0\nH 0.0 0.0 1.23\nH {R} 0.0 0.0\nH {R} 0.0 1.23"
mol = tq.Molecule(geometry=geometry.format(R=1.0), basis_set="sto-3g")
H = mol.make_hamiltonian()
U0 = mol.make_upccgsd_ansatz(name="UpCCGSD")
# ground state opt
E0 = tq.ExpectationValue(H=H, U=U0)
result0 = tq.minimize(E0)
U0_opt = U0.map_variables(result0.variables)
# start from CIS (see ChemicalScience SI)
U1 = tq.gates.X([0,1,2,3,4])
U1 += tq.gates.H(2)
U1 + tq.gates.CNOT(2,3)
U1 + tq.gates.CNOT(2,4)
U1 + tq.gates.CNOT(2,5)
# excited state ansatz (CIS + UpCCGSD)
U1 = U1 + mol.make_upccgsd_ansatz(name="UpCCGSD", include_reference=False)
E1 = tq.ExpectationValue(H=H, U=U1)
P0 = tq.paulis.Qp(U1.qubits) #same as |0><0|
S = tq.ExpectationValue(H=P0, U=U1+U0_opt.dagger())
f_ex = E1 - result0.energy*S
result1 = tq.minimize(f_ex)
```

vqe\_ex\_chem.py

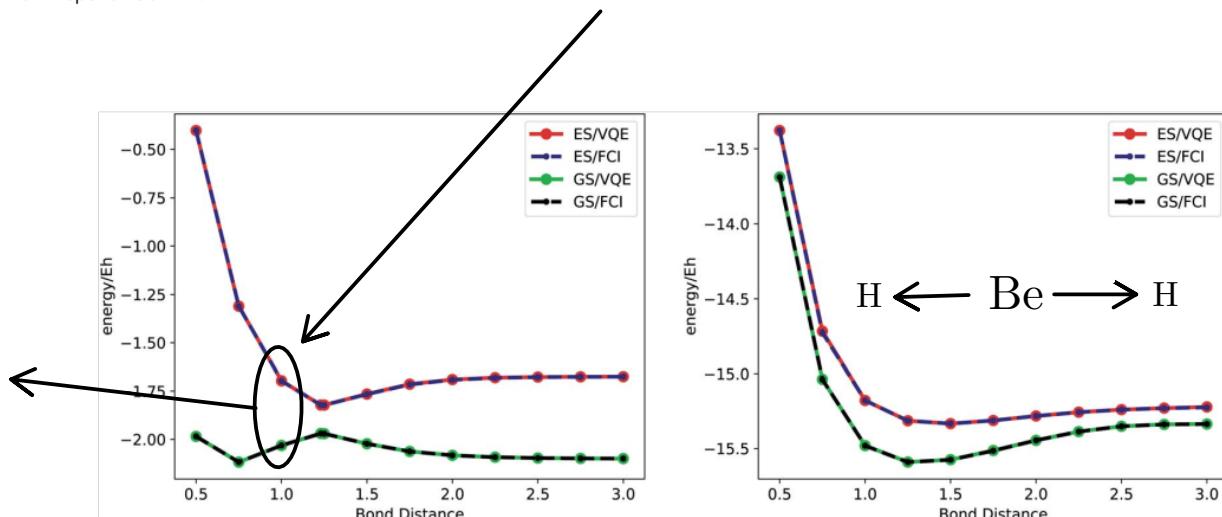
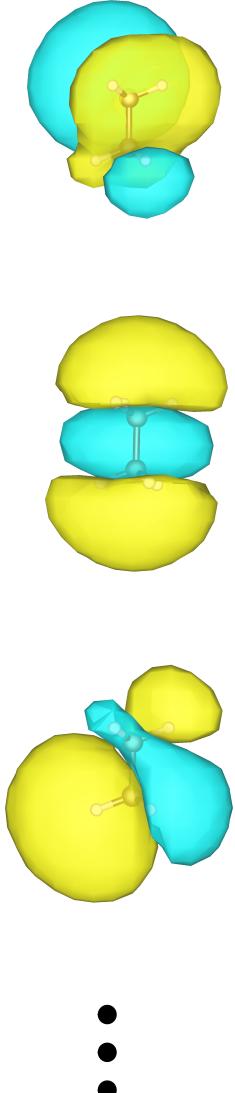
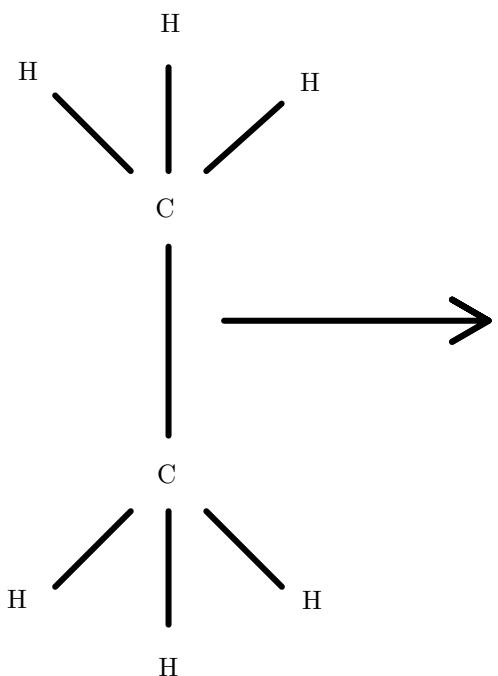


Fig. 6 Adapt-VQE for ground and excited states: adapt-VQE results using automatic differentiation for the screening and the optimization for ground and first excited state energies of  $\text{H}_4/\text{STO-3G}(4,8)$  (left) and  $\text{BeH}_2/\text{STO-3G}(6,14)$  (right). Except for the last point of the  $\text{BeH}_2$  excited state, all points agree to millihartree accuracy with the corresponding exact solution (FCI) in the given basis set. See also Fig. 4. We included the special point at distance 1.23 Å with a square configuration.

# Overview: QuantumChemistry



default: System adapted orbitals

1 spatial orbital for each electron

need more orbitals?

```
tq.Molecule(geometry="c2h6.xyz", n_qubits=28)
```

"basis sets" also possible

```
tq.Molecule(geometry="c2h6.xyz", basis_set="sto-3g")
```

map to qubits



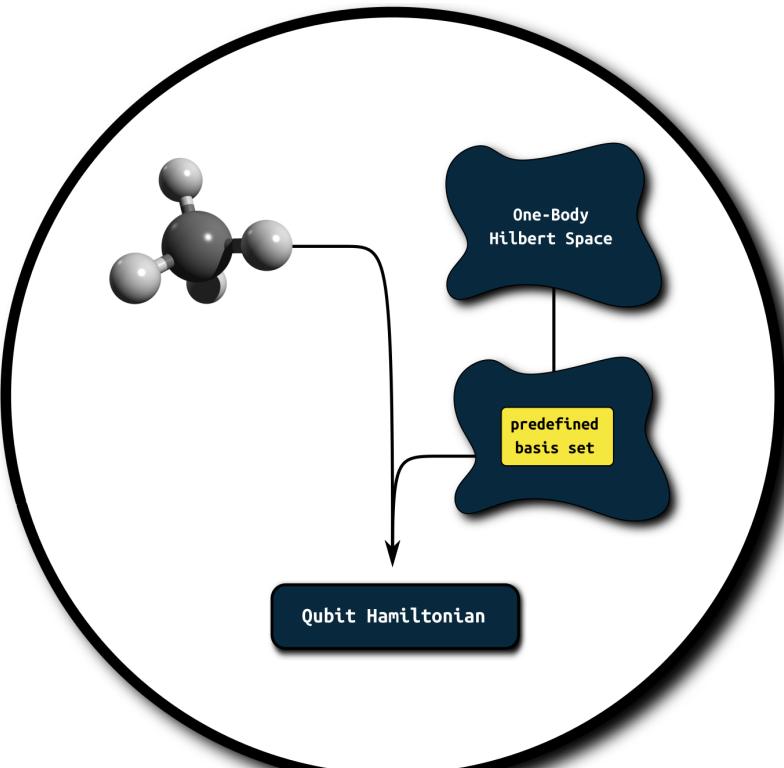
example

```
import tequila as tq
mol = tq.Molecule(geometry="c2h6.xyz")
H_HCB = mol.make_hardcore_boson_hamiltonian()
U_HCB = mol.make_ansatz("HCB-SPA")
E = tq.ExpectationValue(H=H_HCB, U=U_HCB)
result = tq.minimize(E)
```

HCB: Hard-Core-Boson approximation

requires only half the qubits (here 14 instead of 28)

c2h6.py



```

mol = tq.Molecule(geometry="ch4.xyz", basis_set="sto-3g")
H = mol.make_hamiltonian()
U = mol.make_ansatz(name="UpCCD")
E = tq.ExpectationValue(H=H, U=U)

result = tq.minimize(E)
exact = tq.compute_energy("fci")

```

backends: PySCF or Psi4

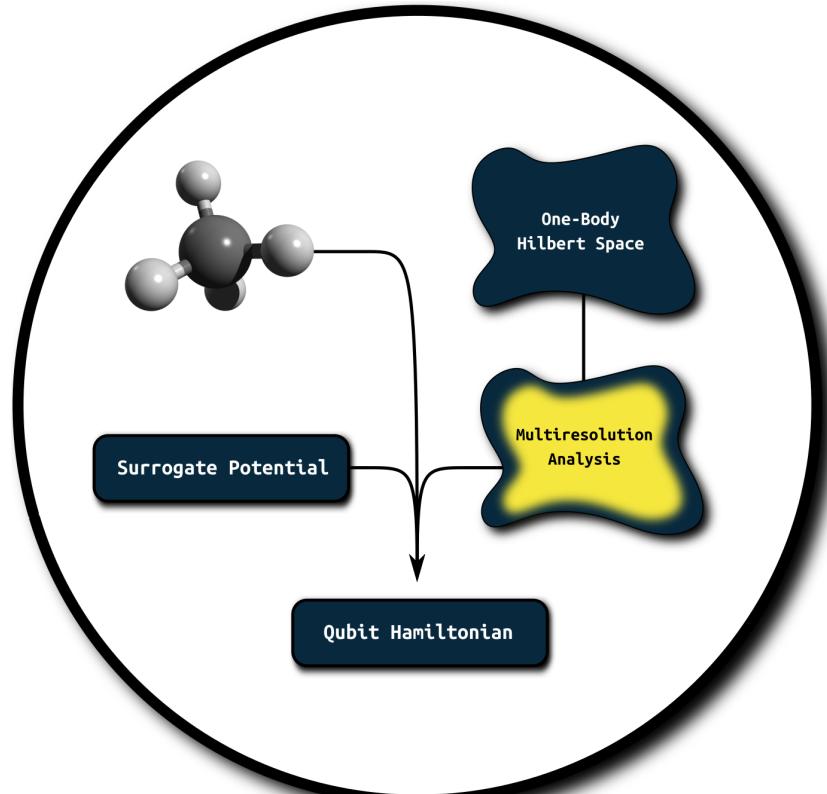
**System Adapted:**

8 orbitals	VQE/MRA-PNO : -40.2761
	FCI/MRA-PNO : -40.2926

**Basis Sets:**

9 orbitals	VQE/STO-3G : -39.7580
	FCI/STO-3G : -39.8060

17 orbitals	FCI/6-31G : -40.3013
-------------	----------------------



```

mol = tq.Molecule(geometry="ch4.xyz")
H = mol.make_hamiltonian()
U = mol.make_ansatz(name="UpCCD")
E = tq.ExpectationValue(H=H, U=U)

result = tq.minimize(E)
exact = tq.compute_energy("fci")

```

backend: MADNESS

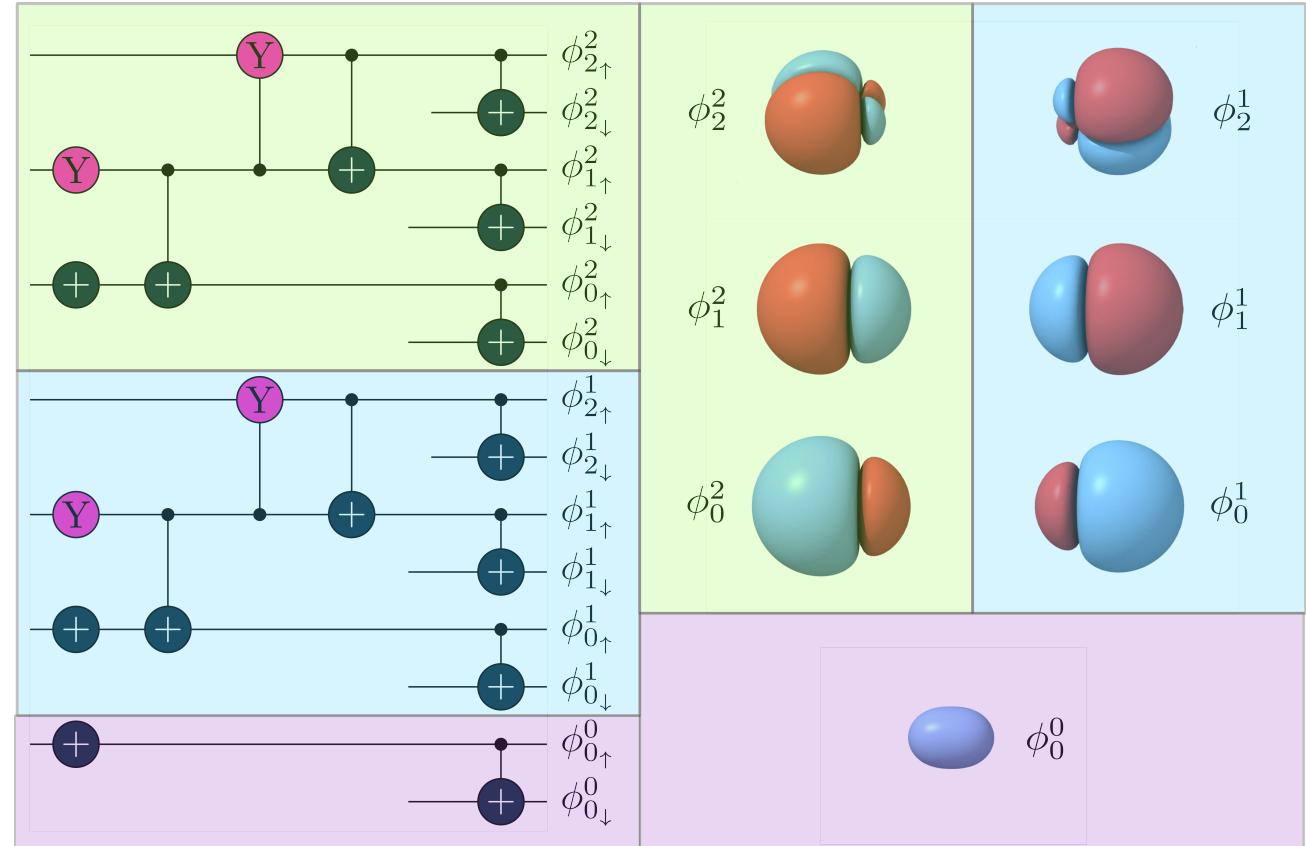
ch4.py

Application: Dequantized VQE via Separable Pair Approximations (SPA)

## Optimized Low-Depth Quantum Circuits for Molecular Electronic Structure using a Separable Pair Approximation

Jakob S. Kottmann<sup>1,2,\*</sup> and Alán Aspuru-Guzik<sup>1,2,3,4,†</sup>

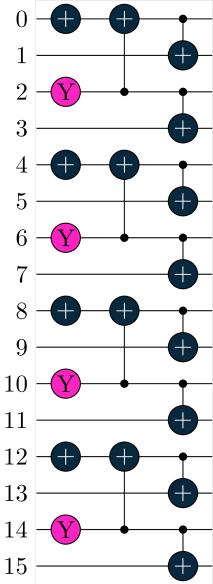
Molecule( $N_e, N_q$ )	$N_{\text{param}}$	$N_{\text{cnot}}$	Depth
$\text{H}_2(2,4)$	1	3	3
$\text{LiH}(2,10)$	4	15	18
$\text{BeH}_2(4,8)$	2	6	3
$\text{BeH}_2(6,14)$	4	15	7
$\text{BH}_3(6,12)$	3	9	3
$\text{N}_2(6,12)$	3	9	3
$\text{C}_2\text{H}_4(12,24)$	6	18	3
$\text{H}_2\text{O}_2(14,28)$	7	21	3
$\text{C}_2\text{H}_6(14,28)$	7	21	3
$\text{C}_2\text{H}_6(2,12)$	5	19	23
$\text{C}_2\text{H}_6(14,84)$	35	133	23



## Optimized Low-Depth Quantum Circuits for Molecular Electronic Structure using a Separable Pair Approximation

Jakob S. Kottmann<sup>1,2,\*</sup> and Alán Aspuru-Guzik<sup>1,2,3,4,†</sup>

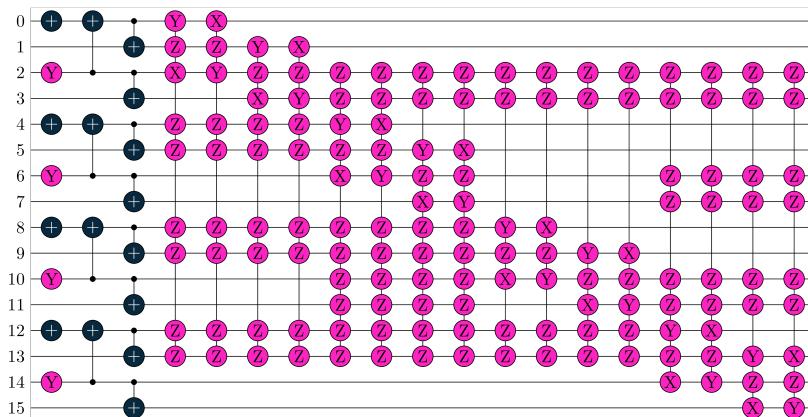
```
U = mol.make_ansatz("SPA")
U += tq.gates.QubitExcitation("a", [2,3,4,5])
U += tq.gates.Ry(angle="b", target=0)
U+= tq.gates.CNOT(0,1)
```



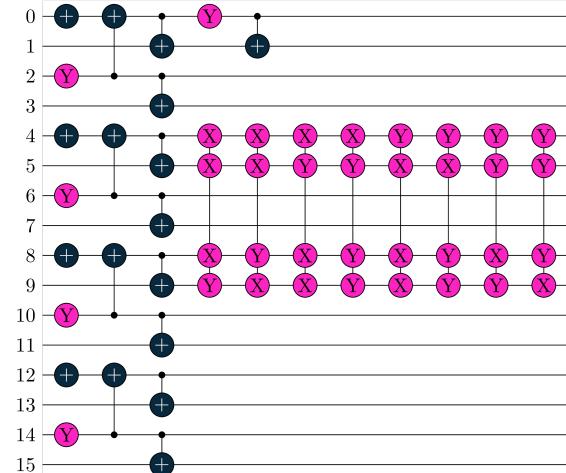
`mol.make_ansatz("SPA")`

make your own ...

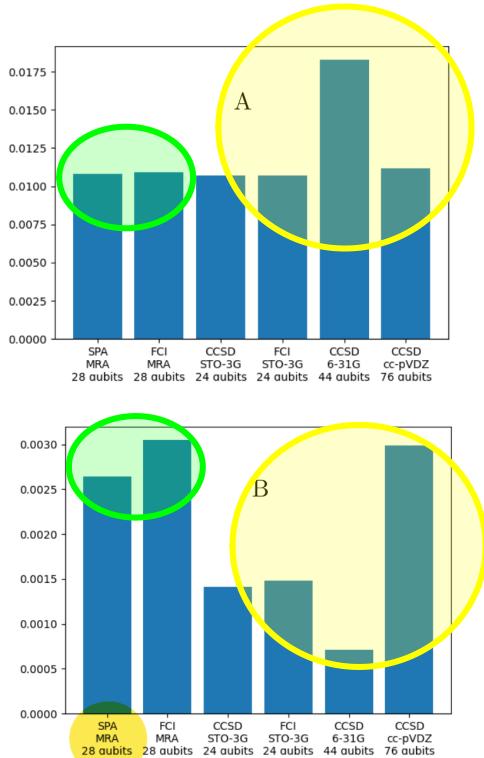
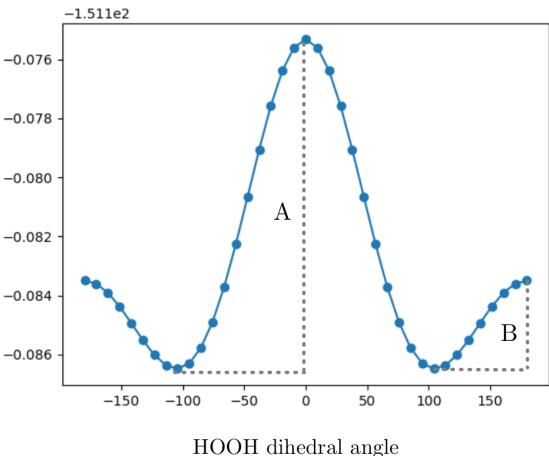
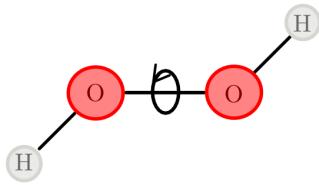
`mol.make_ansatz("SPA+S")`



• • •



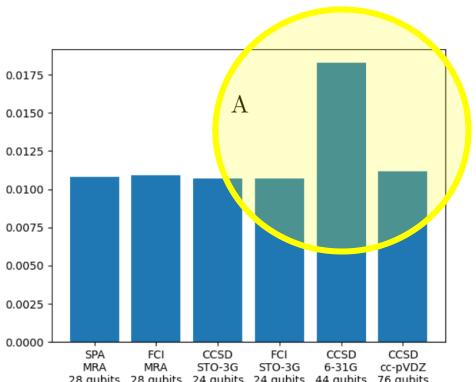
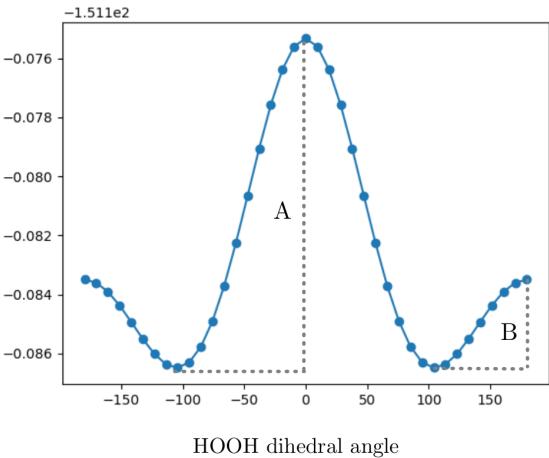
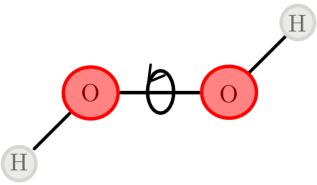
`ch4_circuits.py`



```
import tequila as tq

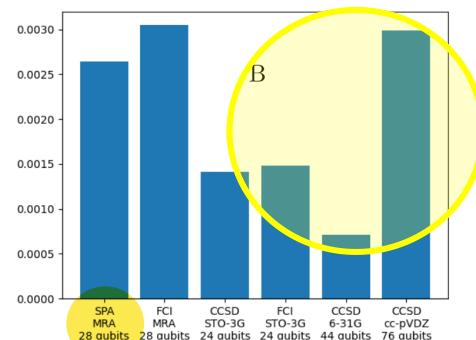
mol = tq.Molecule(geometry="h2o2.xyz", n_pno=7, pno={"maxrank":1})
H = mol.make_hardcore_boson_hamiltonian()
U = mol.make_upccgsd_ansatz(name="HCB-SPA")
E = tq.ExpectationValue(H=H,U=U)

result=tq.minimize(E)
energy=result.energy
```



In this example:

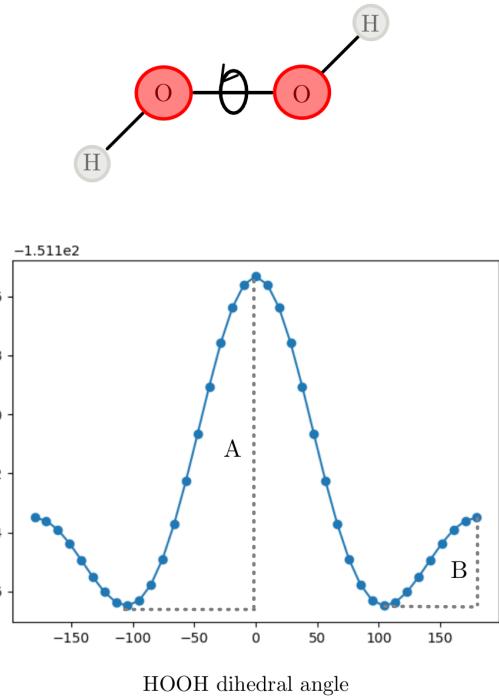
larger basis set is not always better



```
import tequila as tq

mol = tq.Molecule(geometry="h2o2.xyz", n_pno=7, pno={"maxrank":1})
H = mol.make_hardcore_boson_hamiltonian()
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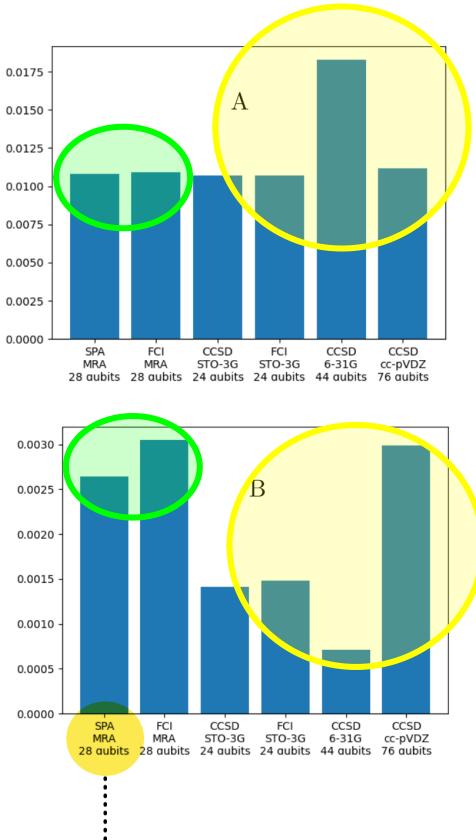
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In this example:

larger basis set is not always better

SPA is a good approximation

MRA-PNOs are accurate and compact  
"more accuracy with less qubits"



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Letter

## Reducing Qubit Requirements while Maintaining Numerical Precision for the Variational Quantum Eigensolver: A Basis-Set-Free Approach

Jakob S. Kottmann,\* Philipp Schleich, Teresa Tamayo-Mendoza, and Alán Aspuru-Guzik\*



high level blog entry:

<https://aspuru.substack.com/p/bits-are-cheap-and-qubits-expensive>



Alán  
Aspuru-Guzik  
UofT



Philipp  
Schleich  
UofT/CS



Abhinav  
Anand  
UofT/Chem



Sumner  
Alperin-Lea  
UofT/Chem



Alba  
Cervera-Lierta  
Barcelona Supercomputing Center



Phillip  
Jensen  
UofT/Chem



Thi Ha  
Kyaw  
UofT/CS



Teresa  
Tamayo-Mendoza  
Harvard



Mario  
Krenn  
MPI Erlangen



Zi-Jian  
Zhang  
UofT/CS



UNIVERSITY OF  
TORONTO

the  
matter lab

qosf

### Initial Team: Sumner Alperin-Lea, Alba Cervera-Lierta, Teresa Tamayo-Mendoza, Cyrille Lavigne

AAG-Group (UofT): Philipp Schleich, Abhinav Anand, Matthias Degroote, Skylar Chaney, Maha Kesibi, Naomi Grace Curnow, Arkaprava Choudhury

Izmaylov-Group (UofT): Tzu-Ching "Thomson" Yen, Vladyslav Verteletskyi, Zachary Bansingh

QOSF: Brandon Solo, Giorgios Tsilimigkounakis, Claudia Zendeja-Morales, Tanya Garg

Github: Alejandro de la Serna, Leo Becker, David Wierichs, Arianne van den Griend ..... You?

DS3Lab (ETH): Maurice Weber

tequila

github/tequilahub