

Chapter 11

Section 11.1

# Chapter 11

mostly about functions of two variables

$$f(x, y) = \frac{x+y}{x^2+y^2}$$

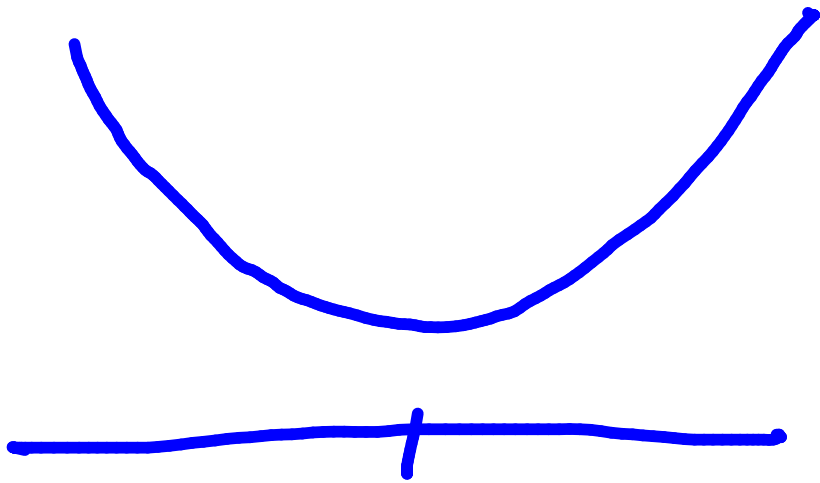
$$x^2 \sin(x)$$

→ 11.1 graphs & visualization |  $f(x, y) = \frac{x+y}{x^2+y^2}$   
 $x^2 \sin(x)$

→ 11.2, 11.3 : limits, derivatives

→ 11.7, 11.8 : solving max/min problems in  $f(x, y)$

→ 11.1 graphs & visualization



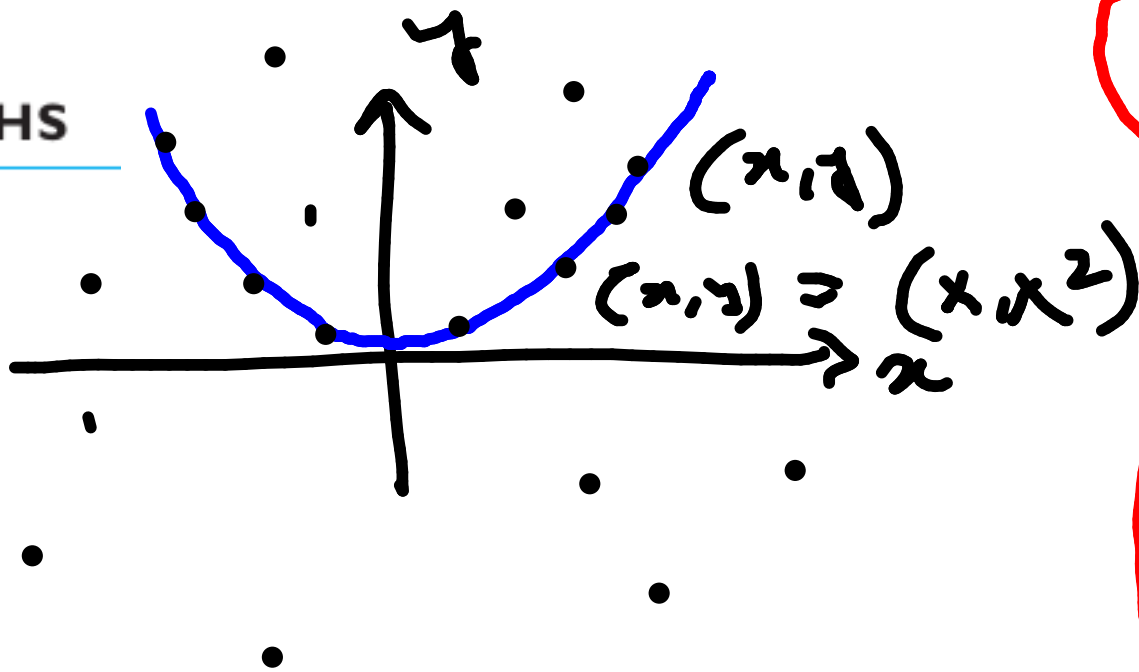
$$f(x, y) = x + y$$

$$xy$$

$$x^2 + y^2$$

**EXAMPLE 2** Find the domain and range of  $g(x, y) = \sqrt{9 - x^2 - y^2}$ .

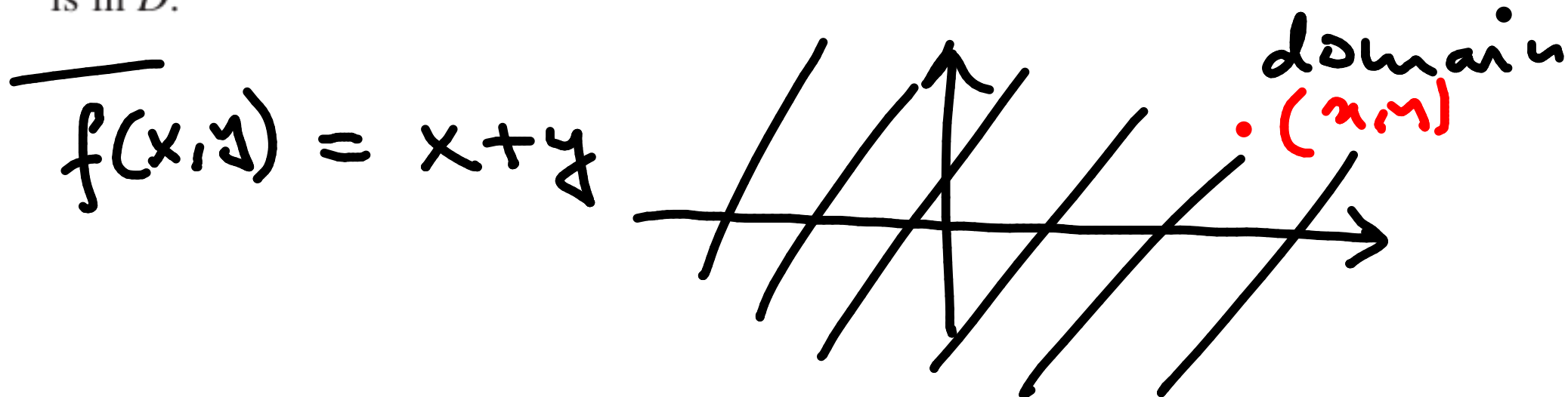
## GRAPHS



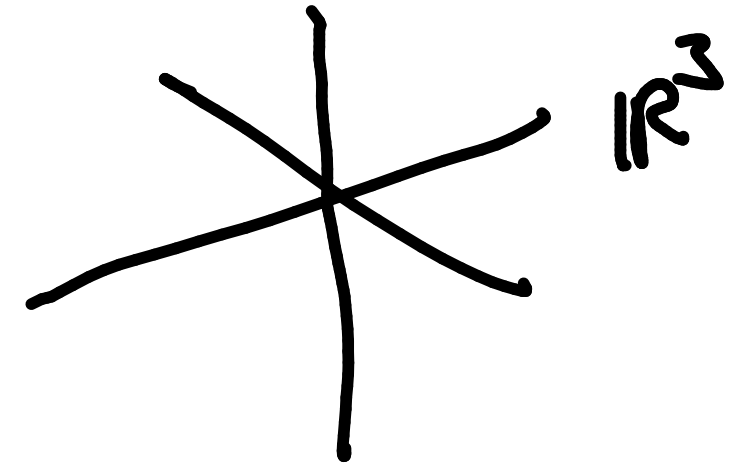
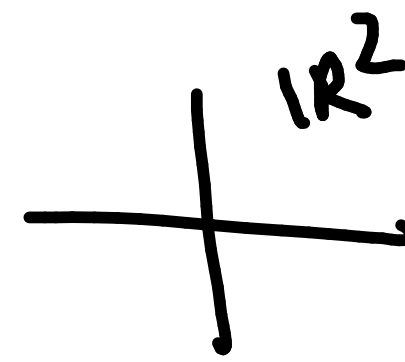
$$f(x) = x^2$$

$$\text{graph of } f(x) = x^2 \\ = \{(x, y) \mid y = x^2\}$$

**DEFINITION** If  $f$  is a function of two variables with domain  $D$ , then the **graph** of  $f$  is the set of all points  $(x, y, z)$  in  $\mathbb{R}^3$  such that  $z = f(x, y)$  and  $(x, y)$  is in  $D$ .



**DEFINITION** If  $f$  is a function of two variables with domain  $D$ , then the **graph** of  $f$  is the set of all points  $(x, y, z)$  in  $\mathbb{R}^3$  such that  $z = f(x, y)$  and  $(x, y)$  is in  $D$ .



$$f(x, y) = x + y$$

$\bullet (x, y)$

the entire plane  
is the domain of  
 $f(x, y) = x + y$

Q: Express the graph of  $f(x, y) = x + y$  as a set

$$= \left\{ (x, y, z) \in \mathbb{R}^3 \mid \underbrace{z = x + y}_{\text{plane}} \right\}$$

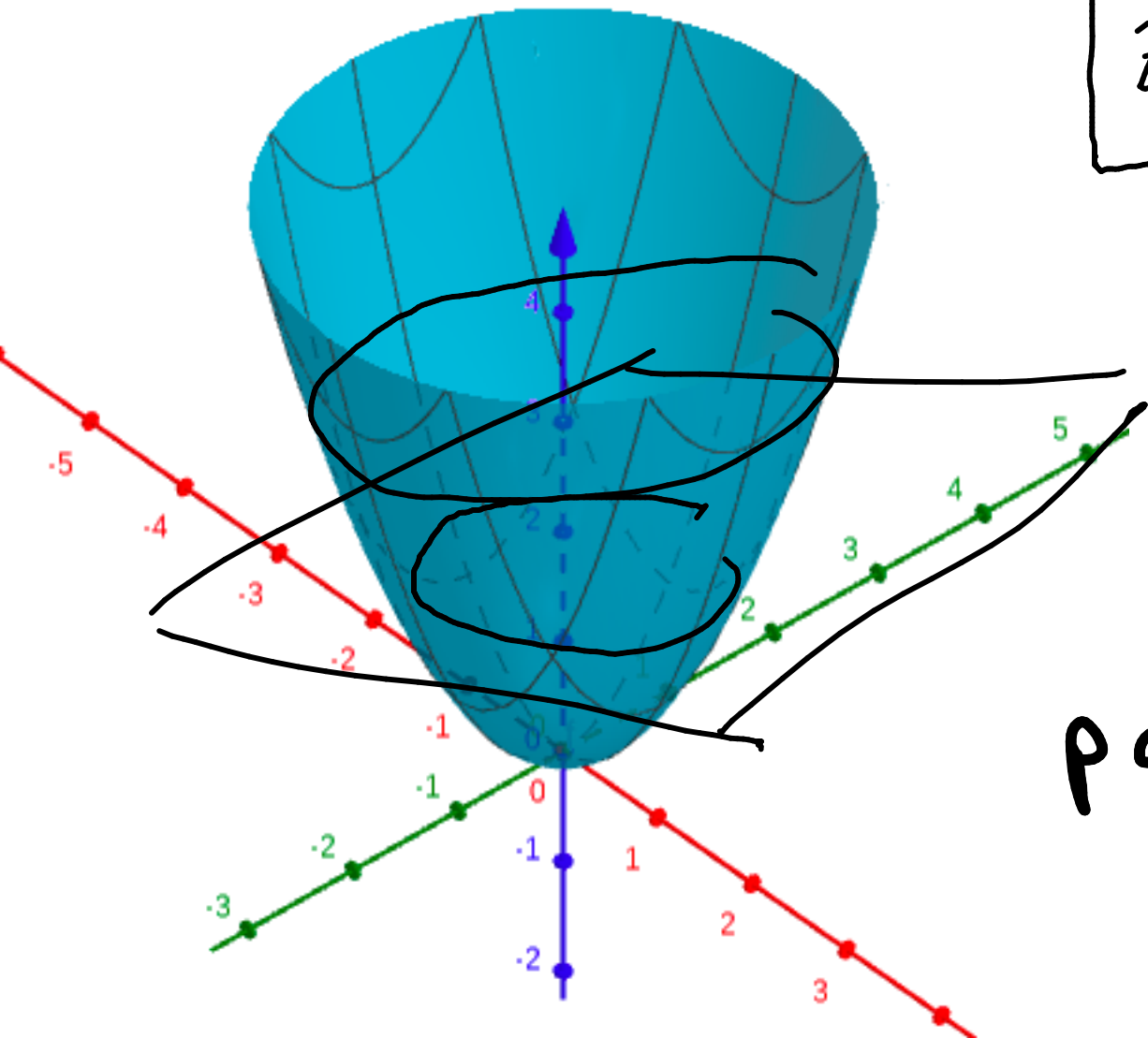
plane



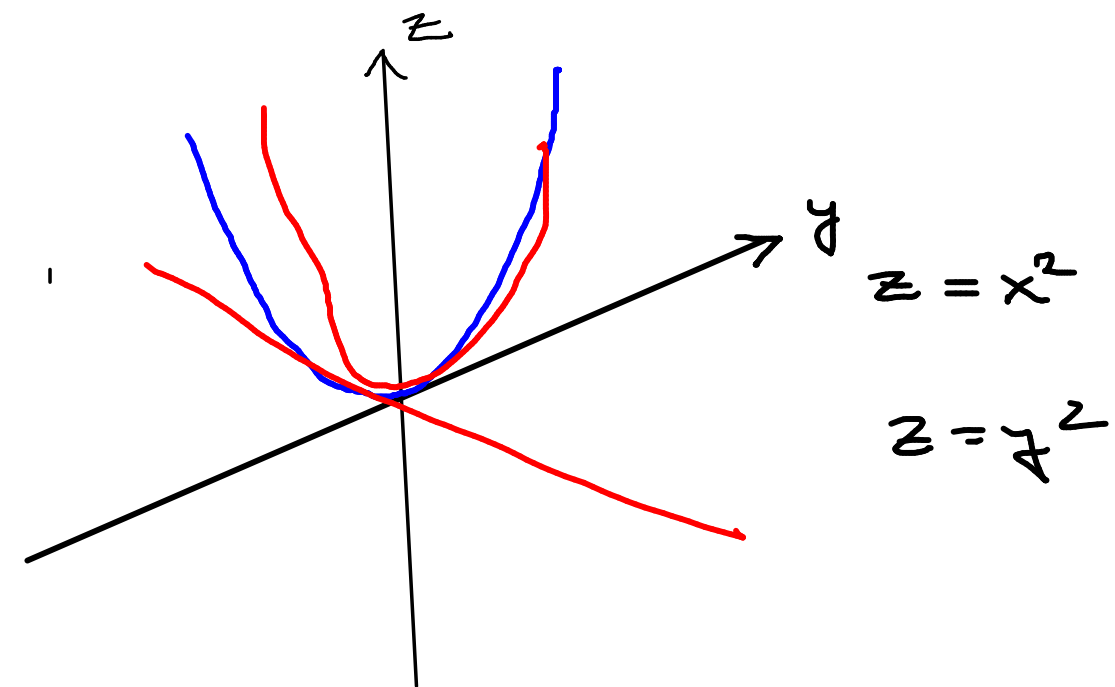
Q. Draw the graph of

$$f(x,y) = x^2 + y^2$$

$$z = x^2 + y^2$$



paraboloid



$$z = x^2 + y^2$$



**V EXAMPLE 4** Sketch the graph of  $g(x, y) = \sqrt{9 - x^2 - y^2}$ .

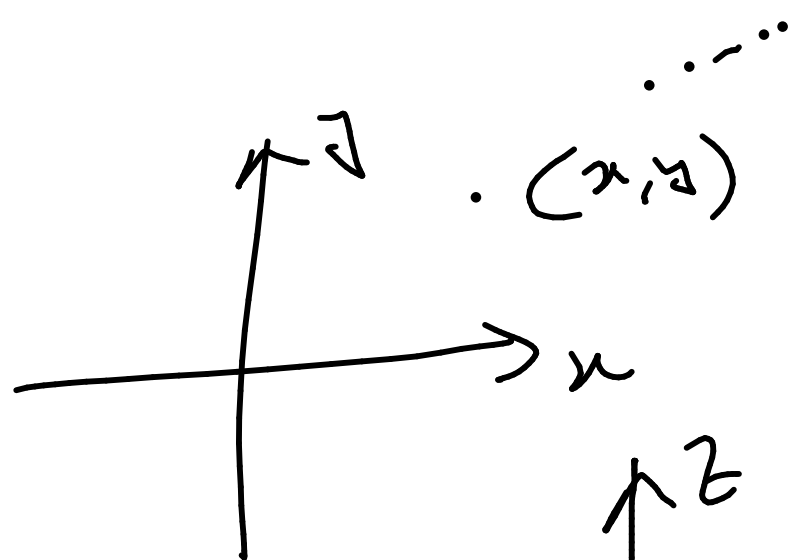
$$z = \sqrt{9 - x^2 - y^2} \quad ??$$

<https://www.geogebra.org/3d?lang=en>

(a)  $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$

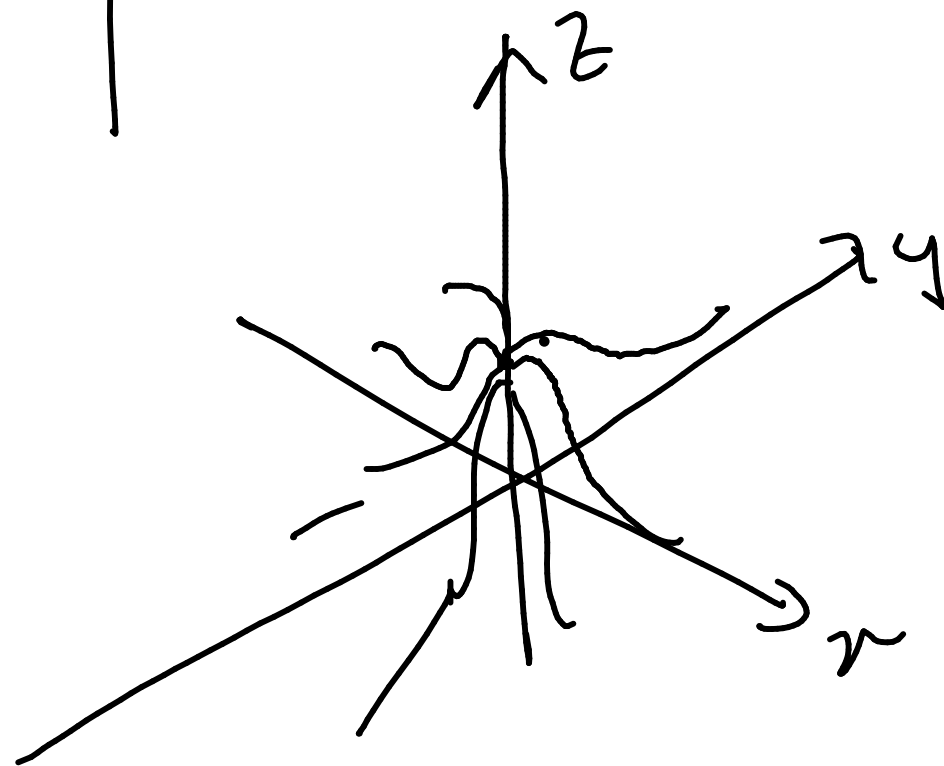
          

(d)  $f(x, y) = \frac{\sin x \sin y}{xy}$



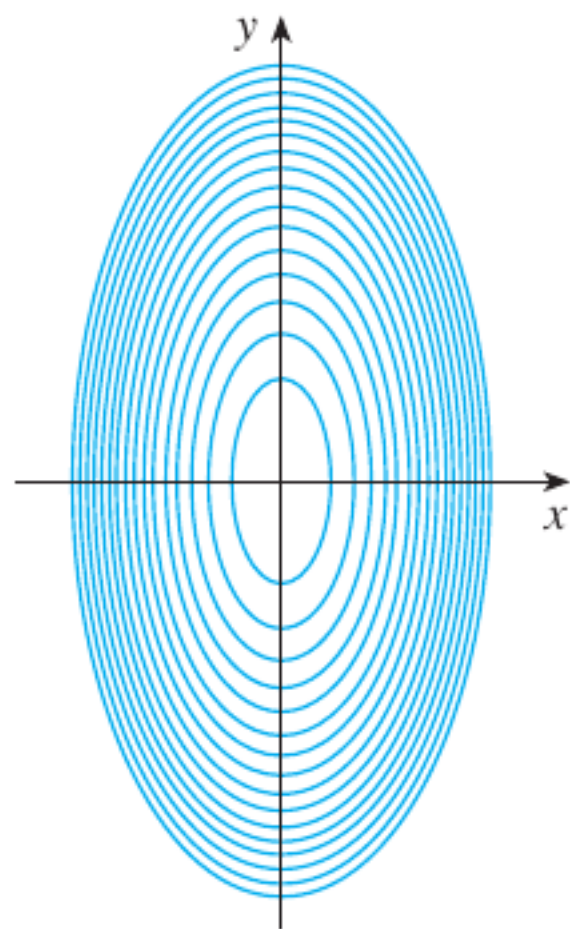
$$f(x, y) = e^{-x^2 - y^2}$$

$$= e^{-(\text{sq of distance of point from origin})}$$

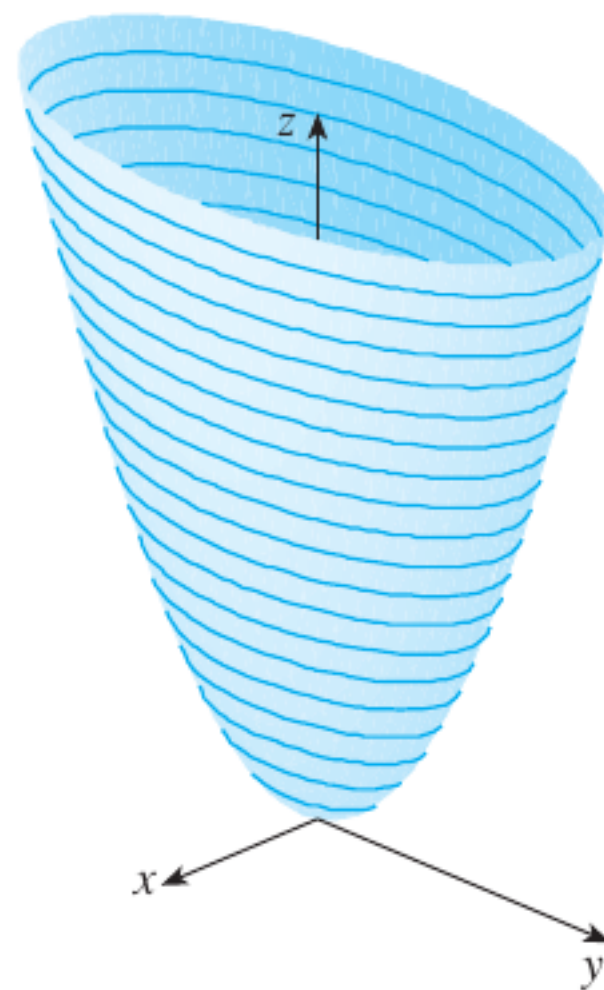


bell shaped

*contour curves, or level curves.*

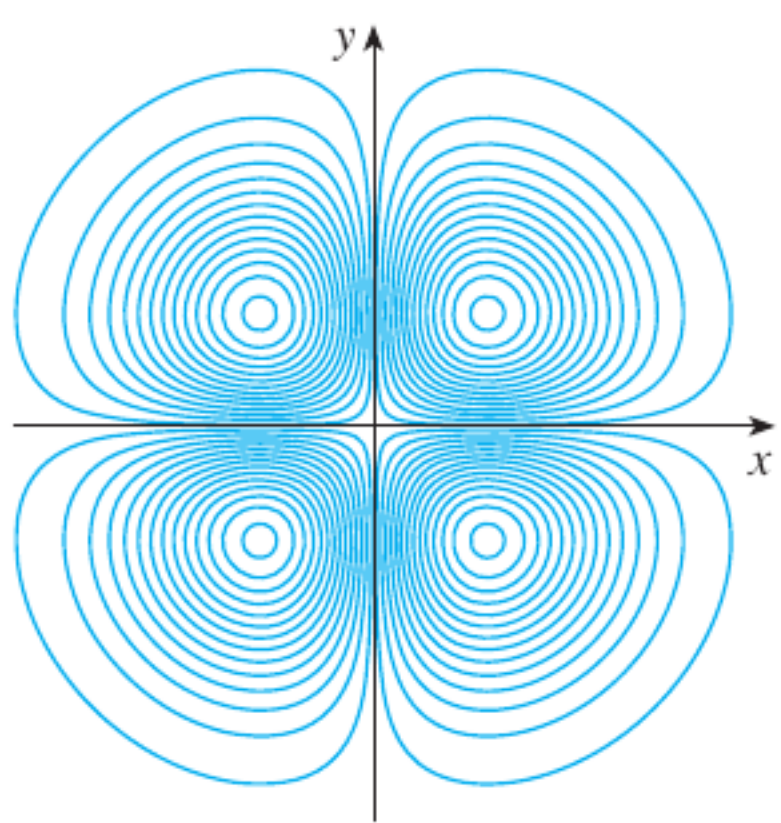


(a) Contour map



(b) Horizontal traces are raised level curves

*next time.*

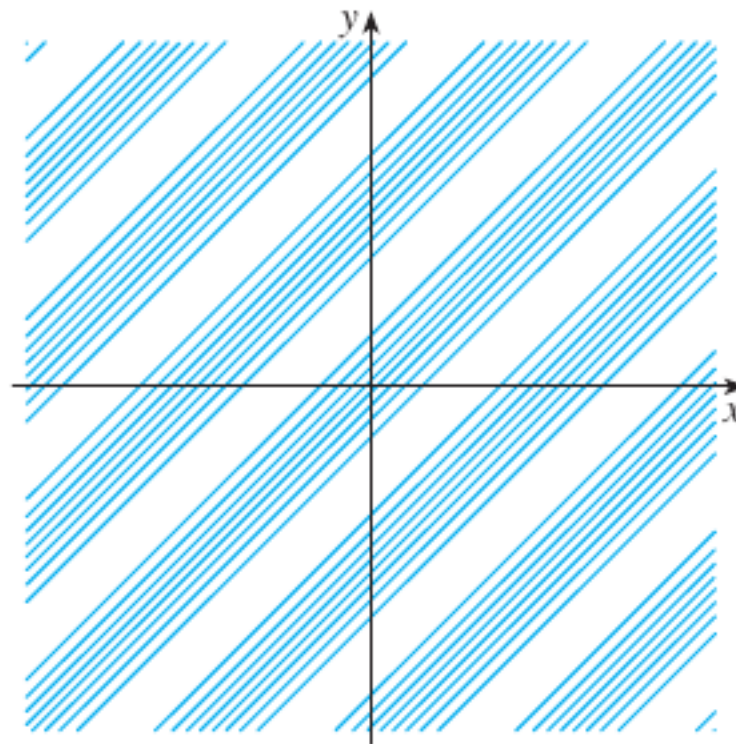
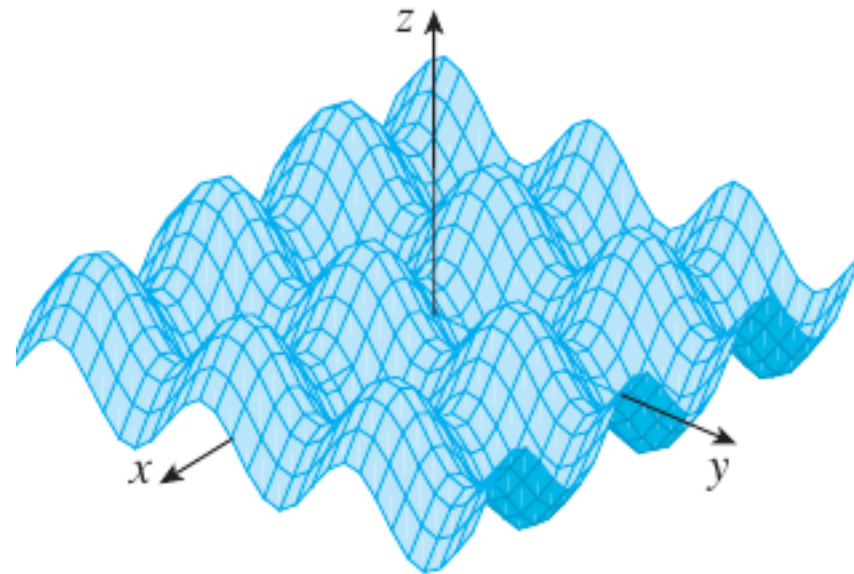
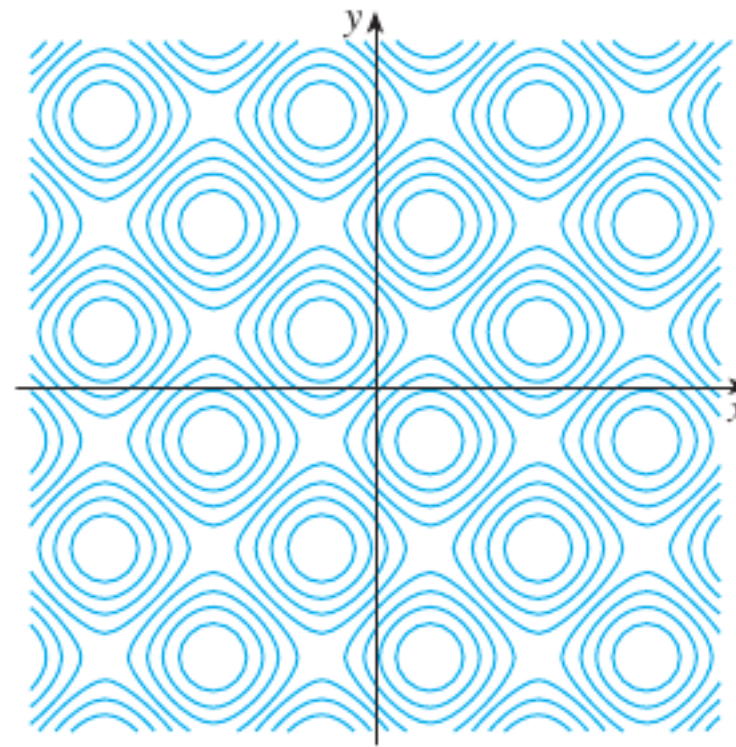
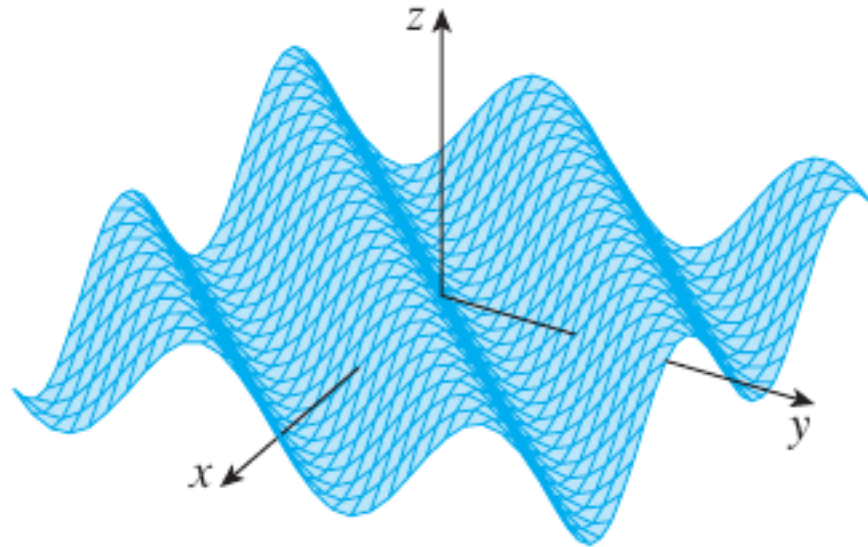


(a) Level curves of  $f(x, y) = -xye^{-x^2-y^2}$

**EXAMPLE 10** Find the domain of  $f$  if  $f(x, y, z) = \ln(z - y) + xy \sin z$ .

**EXAMPLE II** Find the level surfaces of the function  $f(x, y, z) = x^2 + y^2 + z^2$ .

Match the function



Contour plots  
or level curves