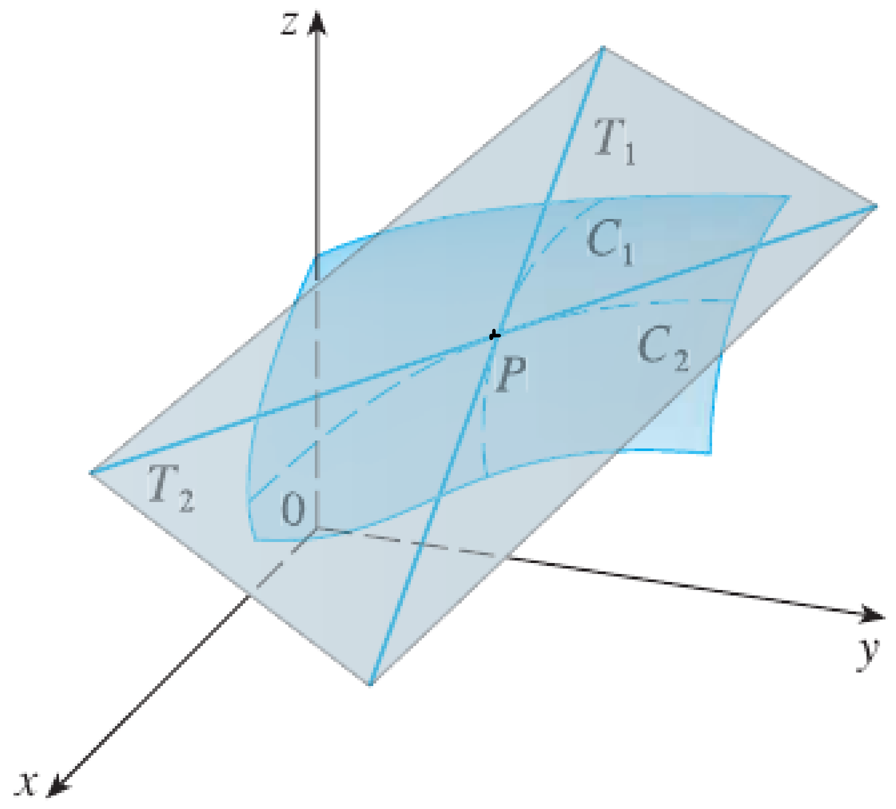
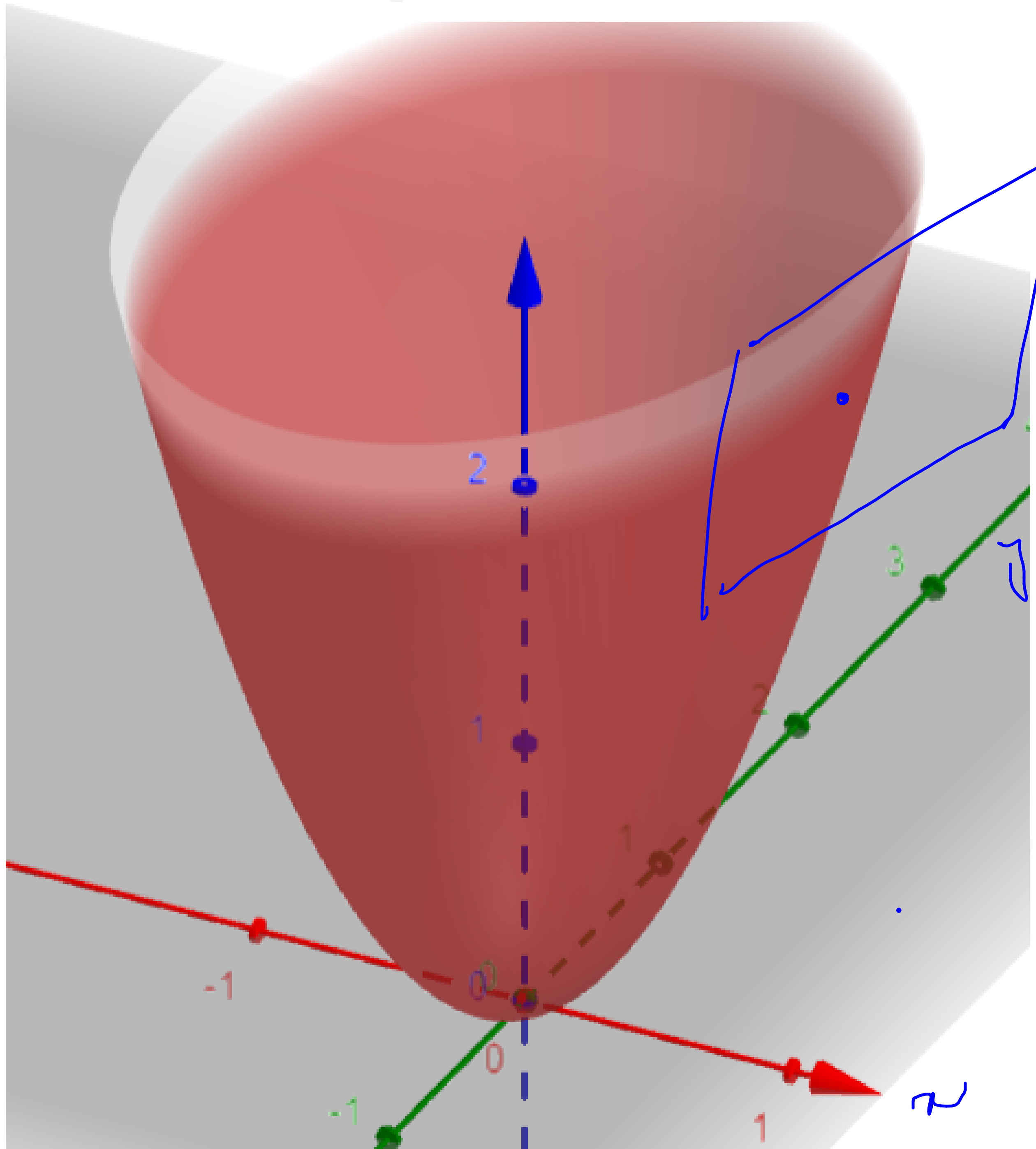
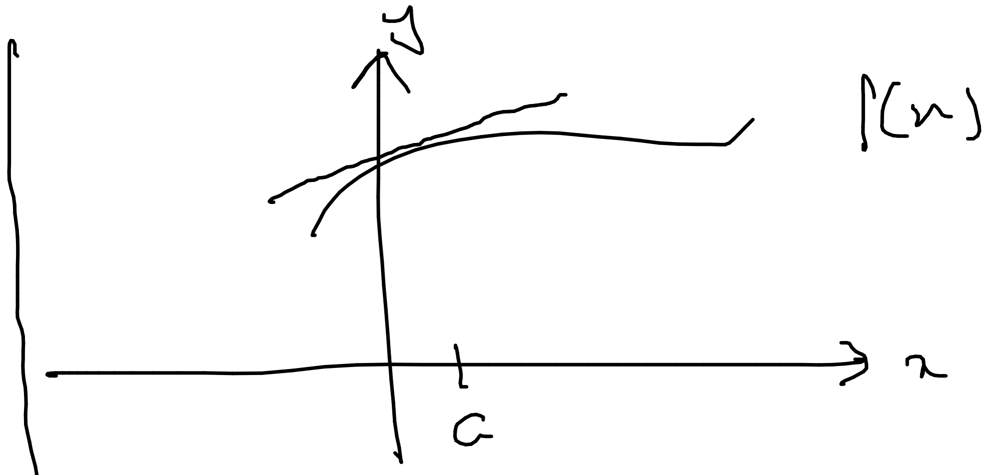
# 11.4 TANGENT PLANES AND LINEAR APPROXIMATIONS

why care for tangent planes?



**V EXAMPLE 1** Find the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.




tangent line

$y - f(a) = \frac{df}{dx}(a)\,(x - a)$

in 3d:

graph of $z = f(x,y)$

at point $(a,b)$

$$\boxed{z - f(a,b) = \frac{\partial f}{\partial x}(a,b)\,(x-a) + \frac{\partial f}{\partial y}(a,b)\,(y-b)}$$

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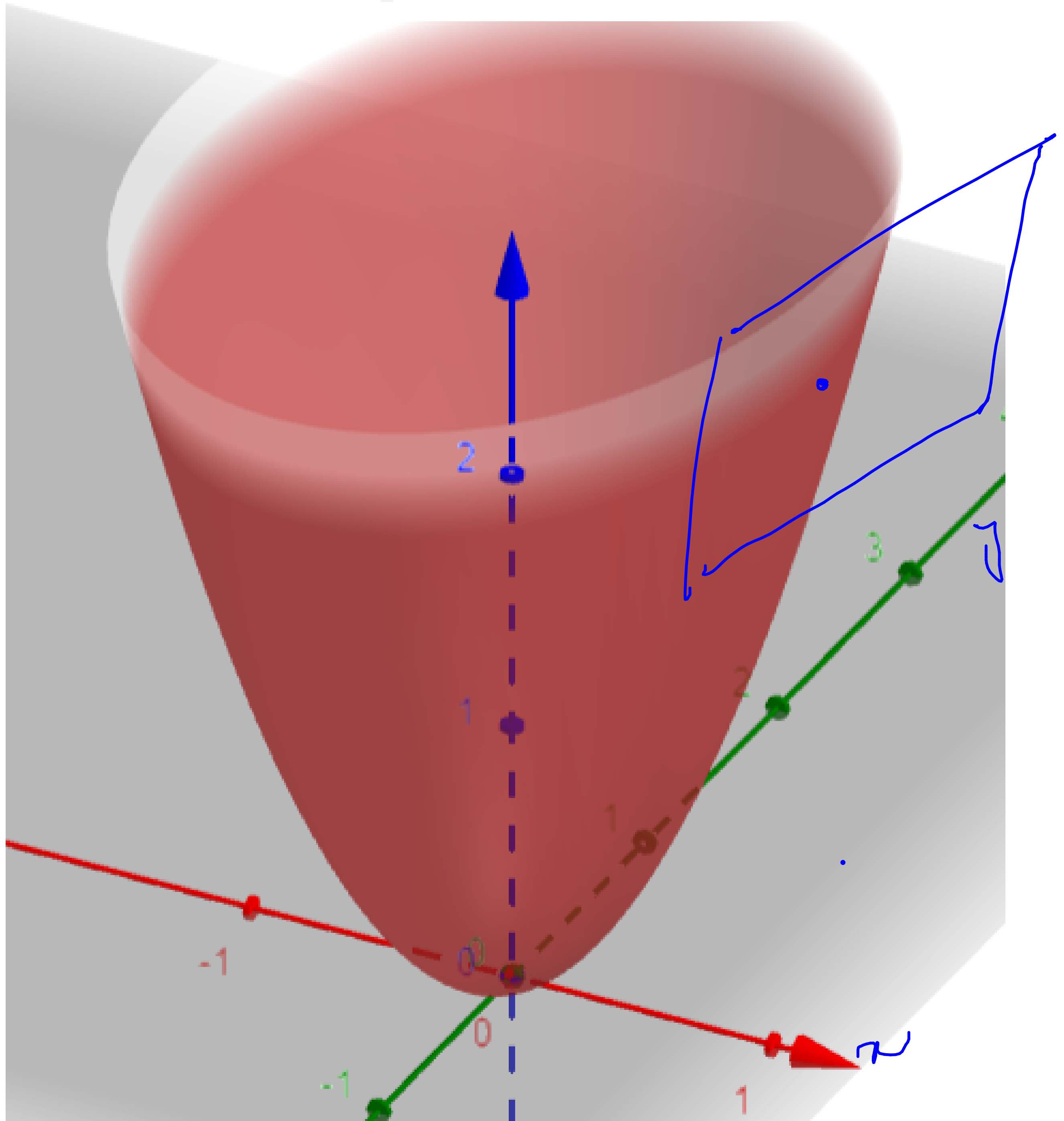



in 3d:
graph of $z = f(x,y)$
at point $(a,b)$

$$\boxed{z - f(a,b) = \frac{\partial f}{\partial x}(a,b)\,(x-a) + \frac{\partial f}{\partial y}(a,b)\,(y-b)}$$

$$z - \underbrace{f(1,1)}_{??} = \underbrace{\frac{\partial f}{\partial x}(1,1)}_{??}\,(x-1) + \underbrace{\frac{\partial f}{\partial y}(1,1)}\,(y-1)$$

$f(1,1) = 3$

$\frac{\partial f}{\partial x}(1,1) = 4x \Big|_{\substack{x=1\\y=1}} = 4$

$\frac{\partial f}{\partial y}(1,1) = 2y \Big|_{\substack{x=1\\y=1}} = 2$

the tangent plane:

$$\boxed{z-3 = 4(x-1) + 2(y-1)}$$

Q. Exercise:

find eqn of tangent plane

$$f(x,y) = xe^{y} \quad , \quad (1,0)$$

$$z - z_0 = \frac{\partial f}{\partial x}(1,0)(x-1) + \frac{\partial f}{\partial y}(1,0)(y-0)$$

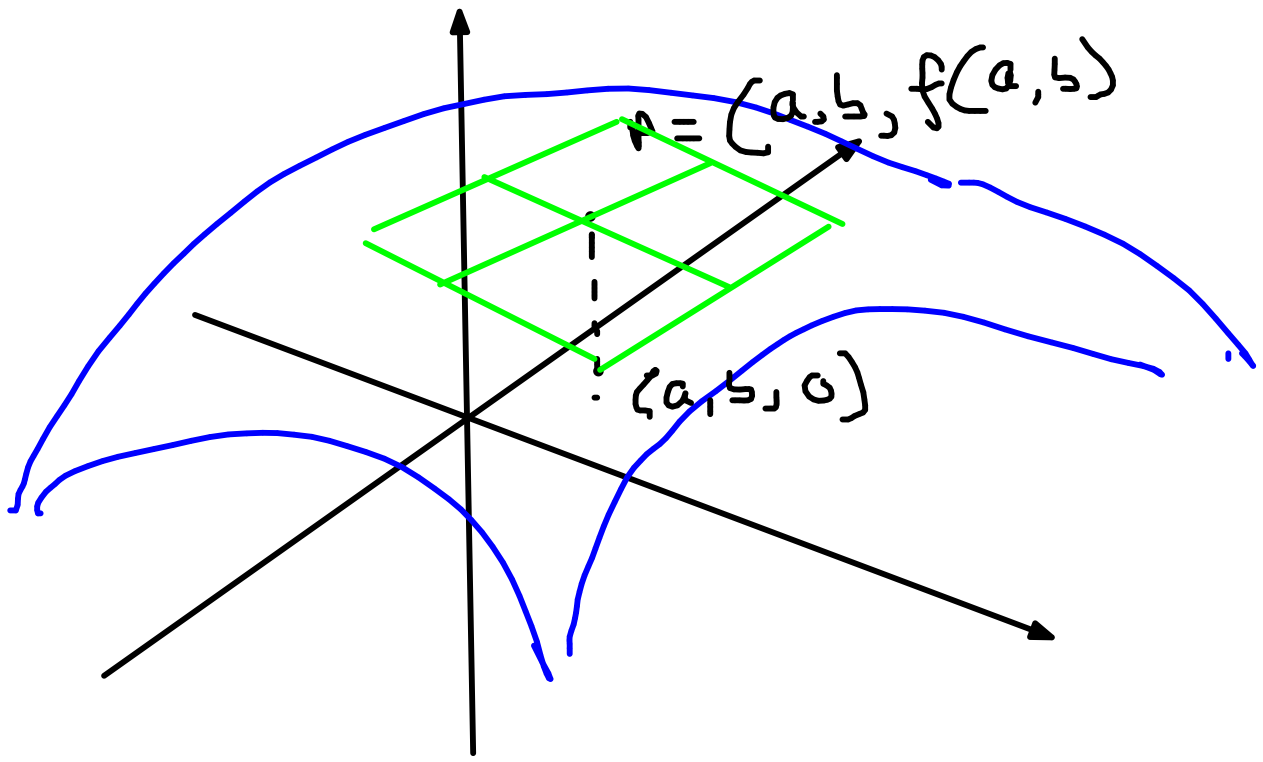
$z_0 = f(1,0) = 1$

$\frac{\partial f}{\partial x} = e^{y}\Big|_{\substack{x=1 \\ y=0}} = 1$

$\frac{\partial f}{\partial y} = xe^{y}\Big|_{x=1, y=0} = 1$

$$z - 1 = 1(x-1) + 1(y-0)$$

$$\boxed{z = x + y}$$




formula of tangent to graph of $f(x,y)$ at $(a,b)$ is

$$z - z_0 = \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b)$$

$z_0 = f(a,b)$

plane contains the point $P$ $(a, b, z_0)$

plane must look like

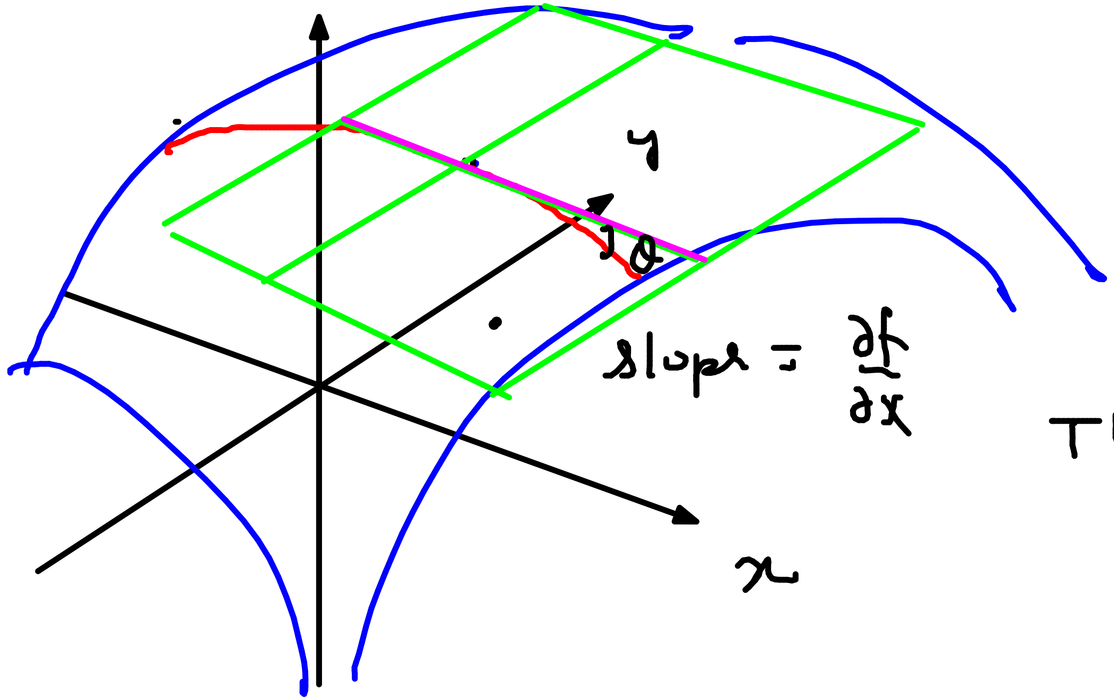
$$A(x-a) + B(y-b) + C(z-z_0) = 0$$

if $C = 0$, plane becomes vertical
not possible

→ simplified eqn eqn passing trough $(a, b, z_0)$

$z = f(x,y)$

$$\left[ z - z_0 = A(x-a) + B(y-b) \right] \text{ green plane}$$

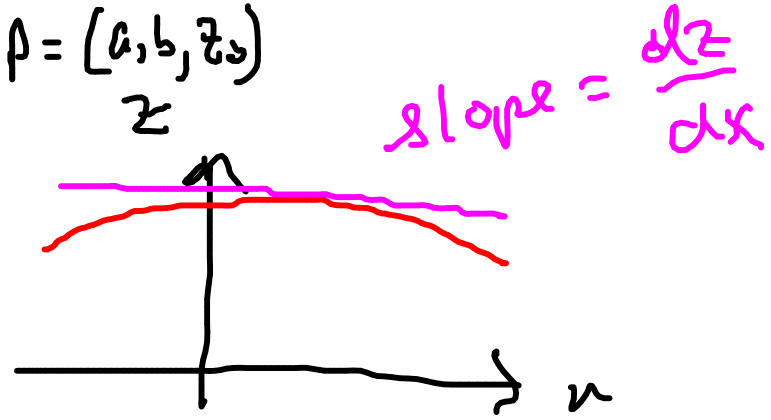



Q: $A = \frac{\partial f}{\partial x}(a,b)$, $B = \frac{\partial f}{\partial y}(a,b)$ } why??

Think: slice the graph $z = f(x,y)$ with plane $y = b$

& also slice the plane $z - z_0 = A(x-a) + B(y-b)$ with $y = b$

$$z - z_0 = A(x-a)$$




Q: what will be the slope of pink line??

$A$ = slope of the pink line

$= \frac{\partial z}{\partial x}$

$$z - z_0 = A(x-a) + B(y-b)$$

we just discussed: $A = \frac{\partial f}{\partial x}(a,b)$:

H.W. repeat the argument to convince why

$$B = \frac{\partial f}{\partial y}(a,b)$$

**8 THEOREM** If the partial derivatives $f_x$ and $f_y$ exist near $(a, b)$ and are continuous at $(a, b)$, then $f$ is differentiable at $(a, b)$.

**EXAMPLE 2** Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use it to approximate $f(1.1, -0.1)$.

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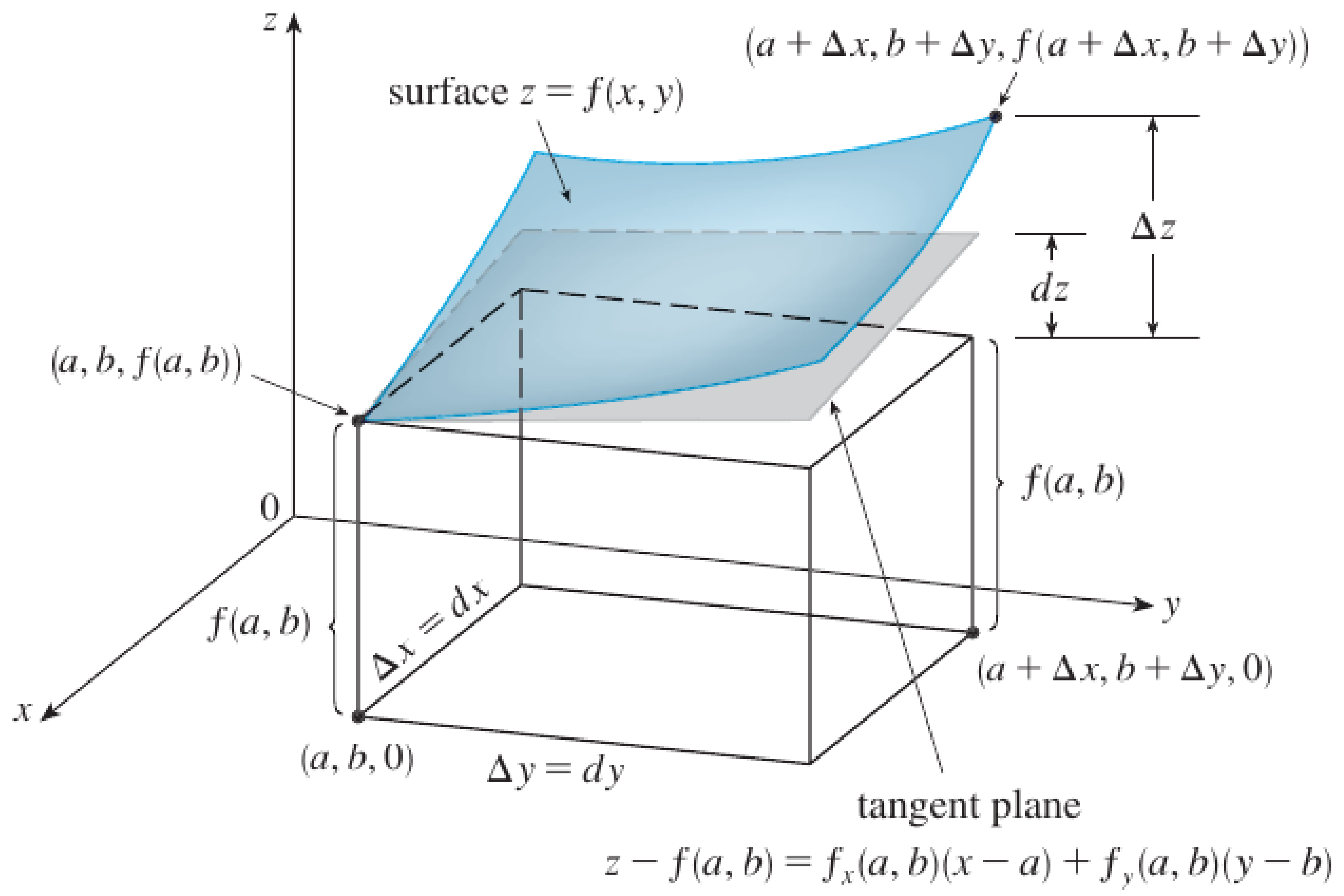
## DIFFERENTIALS

$$dz = f_x(x, y)\,dx + f_y(x, y)\,dy = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$$



**EXAMPLE 3**

(a) If $z = f(x, y) = x^2 + 3xy - y^2$, find the differential $dz$.

(b) If $x$ changes from 2 to 2.05 and $y$ changes from 3 to 2.96, compare the values of $\Delta z$ and $dz$.

**EXAMPLE 4** The base radius and height of a right circular cone are measured as 10 cm and 25 cm, respectively, with a possible error in measurement of as much as 0.1 cm in each. Use differentials to estimate the maximum error in the calculated volume of the cone.