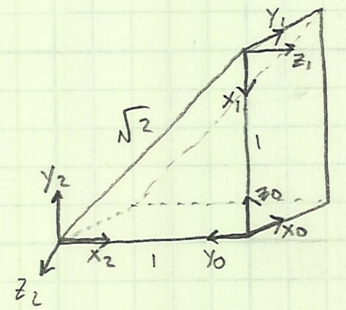
HW #2 | MEEN 537 | Dustan Kraus

2-38) Find homogeneous transformations H_1^0, H_2^0, H_2^1

Show $H_2^0 = H_1^0 H_2^1$



$$H_1^0 = \begin{array}{c} \begin{array}{cccc} x_1^0 & y_1^0 & z_1^0 & d_1^0 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad H_2^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^1 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^0 = H_1^0 H_2^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = H_2^0 \checkmark$$

2-39) Find H_1^0, H_2^0, H_3^0, H_2^3 referring to fig 2.14 on pg. 71

$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad H_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

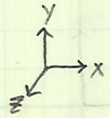
$$H_2^3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2-40) Again referring to fig 2.14 on pg 71 but camera rotated 90° about Z_3

H_1^0, H_2^0 do not change

$$H_3^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.43) Generally multiplication of H is not commutative. Consider $H = R_{x,\alpha} T_{x,b} T_{z,d} R_{z,\theta}$



- rotation & translation about the same axis are commutative because the direction of that axis does not change
  i.e $R_{x,\alpha} T_{x,b} = T_{x,b} R_{x,\alpha}$ & $T_{z,d} R_{z,\theta} = R_{z,\theta} T_{z,d}$
- translations wrt to each other are commutative because again the axes do not rotate
  i.e. $T_{x,b} T_{z,d} = T_{z,d} T_{x,b}$

Permutations: $T_{x,b} R_{x,\alpha} T_{z,d} R_{z,\theta}$; $T_{x,b} R_{x,\alpha} R_{z,\theta} T_{z,d}$; $R_{x,\alpha} T_{x,b} R_{z,\theta} T_{z,d}$

$R_{x,\alpha} T_{z,d} T_{x,b} R_{z,\theta}$

HW #2 | MEEN 537 | Dustan Krans

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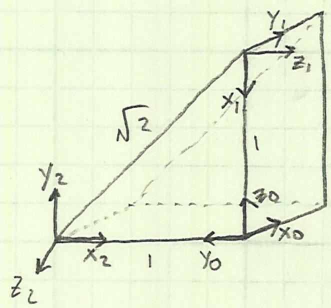
Show $H_2^0 = H_1^0 H_2^1$



$$H_1^0 = \begin{array}{c} \begin{array}{cccc} x_1^0 & y_1^0 & z_1^0 & d_1^0 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad H_2^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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2-39) Find H_1^0, H_2^0, H_3^0, H_2^3 referring to fig 2.14 on pg. 71

$$H_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad H_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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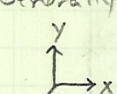
H_1^0, H_2^0 do not change

$$H_3^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & -1 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.43) Generally multiplication of H is not commutative. Consider $H = R_{x,\alpha} T_{x,b} T_{z,d} R_{z,\theta}$

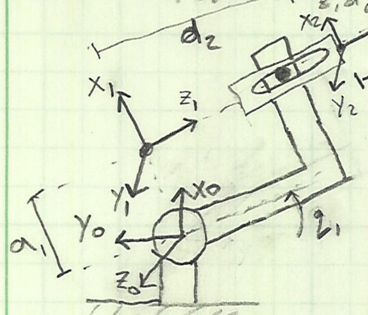


- rotation & translation about the same axis are commutative because the direction of that axis does not change
  i.e $R_{x,\alpha} T_{x,b} = T_{x,b} R_{x,\alpha}$ & $T_{z,d} R_{z,\theta} = R_{z,\theta} T_{z,d}$
- translations rex to each other are commutative because again the axes do not rotate
  i.e. $T_{x,b} T_{z,d} = T_{z,d} T_{x,b}$

Permutations: $T_{x,b} R_{x,\alpha} T_{z,d} R_{z,\theta}$; $T_{x,b} R_{x,\alpha} R_{z,\theta} T_{z,d}$; $R_{x,\alpha} T_{x,b} R_{z,\theta} T_{z,d}$

$R_{x,\alpha} T_{z,d} T_{x,b} R_{z,\theta}$

3-4) $A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$



	θ	d	a	α
1	q_1	0	a_1	$\pi/2$
2	0	d_2+q_2	0	0

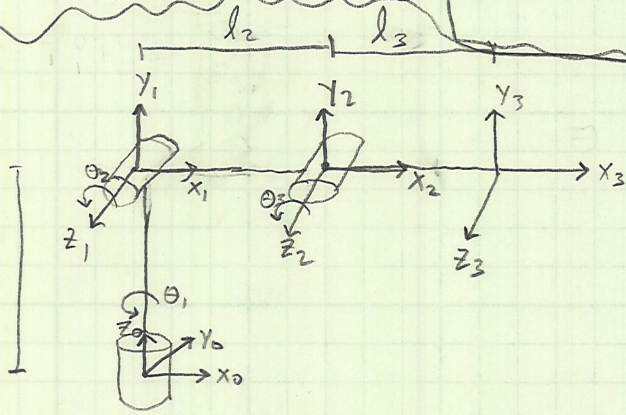
DH Parameters

$$A_1 = T_1^0 = \begin{bmatrix} C_{q_1} & 0 & S_{q_1} & a_1 C_{q_1} \\ S_{q_1} & 0 & -C_{q_1} & a_1 S_{q_1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ ⇐ eqn. 3.10 on pg 77

$$A_2 = T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2+q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_2^0 = T_1^0 T_2^1 = \begin{bmatrix} C_{q_1} & 0 & S_{q_1} & S_{q_1}(d_2+q_2)+a_1 C_{q_1} \\ S_{q_1} & 0 & -C_{q_1} & -C_{q_1}(d_2+q_2)+a_1 S_{q_1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3-6)



	θ	d	a	α
1	θ_1	l_1	0	$\pi/2$
2	θ_2	0	l_2	0
3	θ_3	0	l_3	0

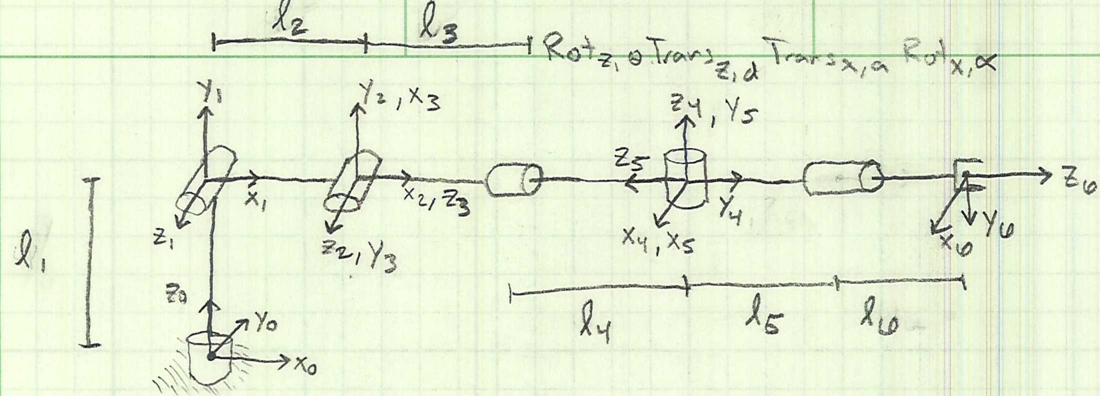
$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & l_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & l_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & l_3 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & l_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_3^0 = A_1 A_2 A_3 \Rightarrow$ matlab $\Rightarrow$

$$\boxed{T_3^0 = \begin{bmatrix} C\theta_1 C\theta_2 C\theta_3 - C\theta_1 S\theta_2 S\theta_3 & -C\theta_1 C\theta_2 S\theta_3 - C\theta_1 S\theta_2 C\theta_3 & S\theta_1 & l_2 C\theta_1 C\theta_2 + l_3 C\theta_1 C\theta_2 C\theta_3 - l_3 C\theta_1 S\theta_2 S\theta_3 \\ S\theta_1 C\theta_2 C\theta_3 - S\theta_1 S\theta_2 S\theta_3 & -S\theta_1 C\theta_2 S\theta_3 - S\theta_1 S\theta_2 C\theta_3 & -C\theta_1 & l_2 S\theta_1 C\theta_2 + l_3 S\theta_1 C\theta_2 C\theta_3 - l_3 S\theta_1 S\theta_2 S\theta_3 \\ C\theta_2 S\theta_3 + S\theta_2 C\theta_3 & C\theta_2 C\theta_3 - S\theta_2 S\theta_3 & 0 & l_1 + l_2 S\theta_2 + l_3 C\theta_2 S\theta_3 + l_3 S\theta_2 C\theta_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3-8)

$Rot_{z,\theta}\, Trans_{z,d}\, Trans_{x,a}\, Rot_{x,\alpha}$



* note, I defined the axes different in the previous problem, so I'm using those same ones in this problem

DH Parameters

	θ	d	a	α
1	θ_1	l_1	0	$\pi/2$
2	θ_2	0	l_2	0
3	$\theta_3 + \pi/2$	0	0	$\pi/2$
4	$\theta_4 + \pi/2$	$l_3 + l_4$	0	$\pi/2$
5	θ_5	0	0	$\pi/2$
6	θ_6	$-l_5 - l_6$	0	π

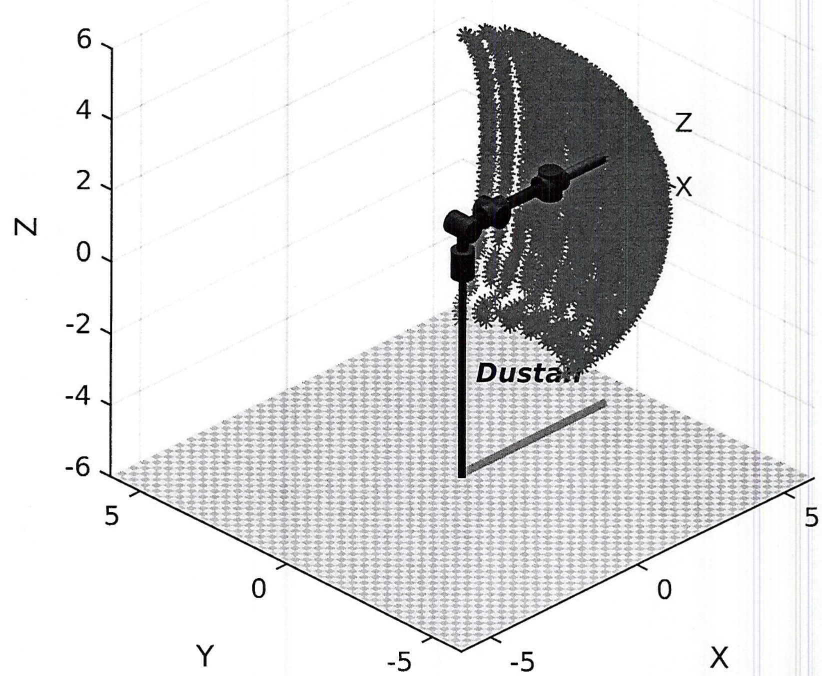
I solved for the T_6^0 in matlab, but it's too long to write here.

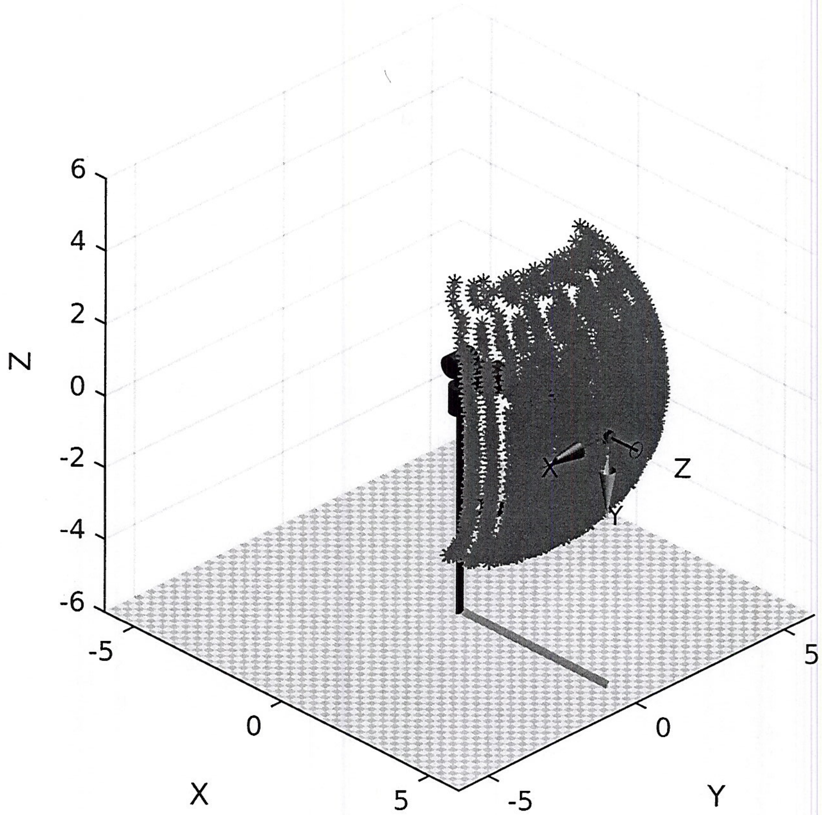
See code

```
%% Problem 3-6
%I did this part before I wrote the function calc_A
syms th1 th2 th3 l1 l2 l3
A1 = [cos(th1)  0  sin(th1)    0;...
   sin(th1)    0  -cos(th1)  0;...
   0           1    0         l1;...
   0           0    0         1];
A2 = [cos(th2)  -sin(th2)  0  l2*cos(th2);...
   sin(th2)    cos(th2)    0  l2*sin(th2);...
   0           0           1      0;...
   0           0           0      1];
A3 = [cos(th3)  -sin(th3)  0  l3*cos(th3);...
   sin(th3)    cos(th3)    0  l3*sin(th3);...
   0           0           1      0;...
   0           0           0      1];
T_3_0 = A1*A2*A3;

%% Problem 3-8 or 1(g)
syms th1 th2 th3 th4 th5 th6 l1 l2 l3 l4 l5 l6
A1 = calc_A(th1, l1, 0, 90);
A2 = calc_A(th2, 0, l2, 0);
A3 = calc_A(th3+(pi/2),0,0,90);
A4 = calc_A(th4 + (pi/2), (l3+l4),0,90);
A5 = calc_A(th5,0,0,90);
A6 = calc_A(th6,(-l5-l6),0,180);

T6_0 = A1*A2*A3*A4*A5*A6;

%% 1(h)
%Select link lengths for the robot in 3-8. Find a way to determine
%   the reachable workspace using forward kinematics
clear all
close all

% Create Robot
L1 = Link('d',1,'a',0,'alpha',pi/2);
L2 = Link('d',0,'a',1,'alpha',0);
L3 = Link('d',0,'a',0,'alpha',pi/2,'offset',pi/2);
L4 = Link('d',2,'a',0,'alpha',pi/2,'offset',pi/2);
L5 = Link('d',0,'a',0,'alpha',pi/2);
L6 = Link('d',-2,'a',0,'alpha',pi);

bot = SerialLink([L1 L2 L3 L4 L5 L6], 'name', 'Dustan');
%position of the origin of the end effector in the end effector frame
end_eff_pos = [0;0;0;1];
inc = 0.25;
pos = [];
%joint 1
for i = -pi/6:inc:pi/6,
   %joint 2
   for j = -pi/6:inc:pi/6,
      %joint 3
      for k = -pi/6:inc:pi/6,
         %joint 4
         for l = -pi/6:inc:pi/6,
            %joint 5
            for m = -pi/6:inc:pi/6,
               H = bot.fkine([i,j,k,l,m,0]);
```

```
            pos(:,end+1) = H*end_eff_pos;
          end
        end
      end
    end
end
figure()
plot3(pos(1,:),pos(2,:),pos(3,:),'*');
hold on
bot.plot([0 0 0 0 0 0])
view(-45,30)
xlim([-6, 6])
ylim([-6,6])

figure()
plot3(pos(1,:),pos(2,:),pos(3,:),'*');
hold on
bot.plot([0 0 0 0 0 0])
view(45,30)
xlim([-6, 6])
ylim([-6,6])
```

I constrained every joint to plus or minus 30 degrees. The reachable workspace is shown below.





6
4
2
0
-2
-4
-6
Z
-5
0
5
X
-5
0
5
Y
X
Y
Z