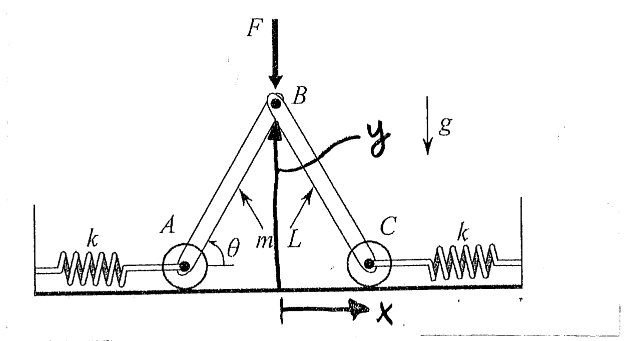
1) Problem 8.24 from Baruh



$W = 40\,lb$

$L = 2\,ft = 24\,in$

$k = 60\,lb/in$

$\theta = 15°$

Virtual work:

$$\delta W = -F\delta y - 2kx\,\delta x - 2W\left(\frac{\delta y}{2}\right)$$

The center of mass of each rod moves through $\frac{1}{2}\delta y$

$$\delta W = -(F+W)\,\delta y - 2kx\,\delta x$$

Put everything in terms of θ:

$$\cos\theta = \frac{x}{L} \Rightarrow x = L\cos\theta \Rightarrow \delta x = -L\sin\theta\,\delta\theta$$

$$\sin\theta = \frac{y}{L} \Rightarrow y = L\sin\theta \Rightarrow \delta y = L\cos\theta\,\delta\theta$$

Plug into virtual work:

$$\delta W = -(F+W)L\cos\theta\,\delta\theta - 2k(L\cos\theta)(-L\sin\theta\,\delta\theta)$$

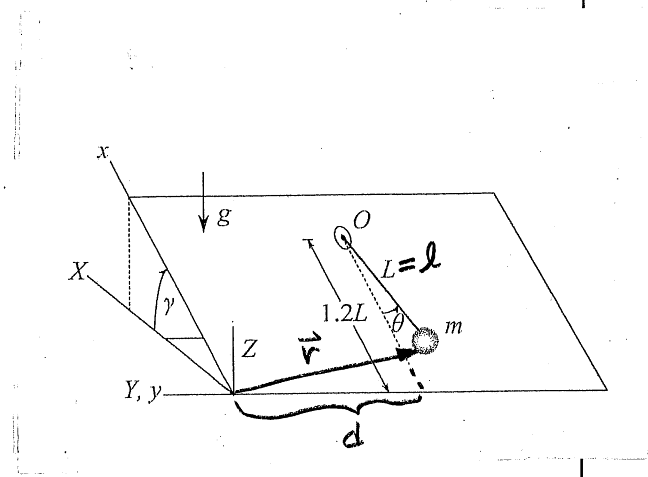
$$\delta W = \left[-F - W + 2kL\sin\theta\right]L\cos\theta\,\delta\theta = 0$$

$$-F - W + 2kL\sin\theta = 0 \Rightarrow F = 2kL\sin\theta - W$$

$$F = 2(60)(24)\sin(15°) - 40$$

$$\boxed{F = 705.4\,lb}$$

2) Problem 8.40 from Baruh



$\dot{\gamma} = \text{constant}$

$N = 1$

$n = 1$ because $\dot{\gamma} = \text{constant}$

$q = \theta$

$$\vec{r} = (1.2\ell - \ell\cos\theta)\hat{i} + (-d - \ell\sin\theta)\hat{j}$$

$$\vec{\omega}_f = -\dot{\gamma}\hat{j}$$

$$\vec{\omega}_f \times \vec{r} = -\dot{\gamma}(1.2 - \cos\theta)\ell\hat{j}\times\hat{i} = \ell\dot{\gamma}(1.2 - \cos\theta)\hat{k}$$

$$(\dot{\vec{r}})_{rel} = \ell\dot{\theta}\sin\theta\hat{i} - \ell\dot{\theta}\cos\theta\hat{j}$$

$$\dot{\vec{r}} = \ell\dot{\theta}\sin\theta\hat{i} - \ell\dot{\theta}\cos\theta\hat{j} + \ell\dot{\gamma}(1.2 - \cos\theta)\hat{k}$$

$$T = \frac{1}{2}m\dot{\vec{r}}\cdot\dot{\vec{r}} = \frac{m\ell^2}{2}\dot{\theta}^2\sin^2\theta + \frac{m\ell^2}{2}\dot{\theta}^2\cos^2\theta + \frac{m\ell^2\dot{\gamma}^2}{2}(1.2 - \cos\theta)^2$$

$$T = \frac{m\ell^2}{2}\dot{\theta}^2 + \frac{m\ell^2\dot{\gamma}^2}{2}(1.2 - \cos\theta)^2$$

$$V = mgh = mg\left(1.2\ell\sin\gamma - \ell\cos\theta\sin\gamma\right)$$

$$V = mg\ell(1.2 - \cos\theta)\sin\gamma$$

$$L = T - V = \frac{m\ell^2}{2}\dot{\theta}^2 + \frac{m\ell^2\dot{\gamma}^2}{2}(1.2 - \cos\theta)^2 + mg\ell(\cos\theta - 1.2)\sin\gamma$$

$$\frac{\partial L}{\partial \dot{\theta}} = m\ell^2 \dot{\theta}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = m\ell^2 \ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = \frac{m\ell^2 \dot{\gamma}^2}{\cancel{2}}(\cancel{2})(1.2-\cos\theta)(\sin\theta) - mg\ell \sin\theta \sin\gamma$$

$$\frac{\partial L}{\partial \theta} = 1.2 m\ell^2 \dot{\gamma}^2 \sin\theta - m\ell^2 \dot{\gamma}^2 \cos\theta \sin\theta - mg\ell \sin\theta \sin\gamma$$

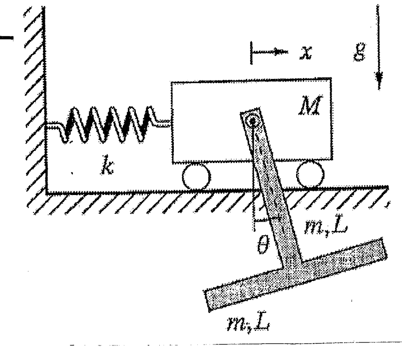
$$Q = 0$$

$$m\ell^2 \ddot{\theta} - 1.2 m\ell^2 \dot{\gamma}^2 \sin\theta + m\ell^2 \dot{\gamma}^2 \cos\theta \sin\theta + mg\ell \sin\theta \sin\gamma = 0$$

$$\ddot{\theta} - 1.2 \dot{\gamma}^2 \sin\theta + \dot{\gamma}^2 \cos\theta \sin\theta + \frac{g}{\ell} \sin\theta \sin\gamma = 0$$

$$\boxed{\ddot{\theta} + \left[\dot{\gamma}^2(\cos\theta - 1.2) + \frac{g}{\ell}\sin\gamma\right]\sin\theta = 0}$$

3)



$N=2,\ n=2$

$$T = \tfrac{1}{2}M\dot{x}^2 + \tfrac{1}{2}(2m)\dot{\vec{r}}_G \cdot \dot{\vec{r}}_G + \tfrac{1}{2}I_G\dot{\theta}^2$$

$$\vec{r}_G = \frac{m}{2m}\left[(x+\tfrac{L}{2}\sin\theta)\hat{\imath} - \tfrac{L}{2}\cos\theta\,\hat{\jmath} + (x+L\sin\theta)\hat{\imath} - L\cos\theta\,\hat{\jmath}\right]$$

$$= \tfrac{1}{2}\left[(2x+\tfrac{3L}{2}\sin\theta)\hat{\imath} - \tfrac{3L}{2}\cos\theta\,\hat{\jmath}\right]$$

$$\vec{r}_G = (x+\tfrac{3L}{4}\sin\theta)\hat{\imath} - \tfrac{3L}{4}\cos\theta\,\hat{\jmath}$$

$$\dot{\vec{r}}_G = (\dot{x}+\tfrac{3L}{4}\dot{\theta}\cos\theta)\hat{\imath} + \tfrac{3L}{4}\dot{\theta}\sin\theta\,\hat{\jmath}$$

$$\dot{\vec{r}}_G \cdot \dot{\vec{r}}_G = \dot{x}^2 + \frac{3L}{2}\dot{x}\dot{\theta}\cos\theta + \frac{9L^2}{16}\dot{\theta}^2\cos^2\theta + \frac{9L^2}{16}\dot{\theta}^2\sin^2\theta$$

$$\dot{\vec{r}}_G \cdot \dot{\vec{r}}_G = \dot{x}^2 + \frac{3L}{2}\dot{x}\dot{\theta}\cos\theta + \frac{9L^2}{16}\dot{\theta}^2$$

$$I_G = 2\left[\tfrac{1}{12}mL^2 + m\left(\tfrac{L}{4}\right)^2\right] = 2\left[\frac{mL^2}{12} + \frac{mL^2}{16}\right] = 2\left[\frac{4+3}{48}\right]mL^2$$

$$I_G = \frac{7}{24}mL^2$$

$$T = \tfrac{1}{2}M\dot{x}^2 + m\dot{x}^2 + \frac{3mL}{2}\dot{x}\dot{\theta}\cos\theta + \frac{9mL^2}{16}\dot{\theta}^2 + \frac{7}{48}mL^2\dot{\theta}^2$$

$$T = \left(\frac{M}{2}+m\right)\dot{x}^2 + \frac{3mL}{2}\dot{x}\dot{\theta}\cos\theta + \frac{17}{24}mL^2\dot{\theta}^2$$

$$V_g = -(2m)g\left(\tfrac{3L}{4}\cos\theta\right) = -\frac{3mgL}{2}\cos\theta$$

$$V_k = \tfrac{1}{2}kx^2$$

$$V = \tfrac{1}{2}kx^2 - \frac{3mgL}{2}\cos\theta$$

$$L = \left(\frac{M}{2}+m\right)\dot{x}^2 + \frac{3mL}{2}\dot{x}\dot{\theta}\cos\theta + \frac{17}{24}mL^2\dot{\theta}^2 - \tfrac{1}{2}kx^2 + \frac{3mgL}{2}\cos\theta$$

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER
COMET

$\underline{k=1}$ $(q_1 = x)$

$$\frac{\partial L}{\partial \dot{x}} = (M+2m)\dot{x} + \frac{3mL}{2}\dot{\theta}\cos\theta$$

$$\frac{d}{dt}(\ ) = (M+2m)\ddot{x} + \frac{3mL}{2}\ddot{\theta}\cos\theta - \frac{3mL}{2}\dot{\theta}^2\sin\theta$$

$$\frac{\partial L}{\partial x} = -Kx$$

$$\boxed{(M+2m)\ddot{x} + \frac{3mL}{2}\ddot{\theta}\cos\theta - \frac{3mL}{2}\dot{\theta}^2\sin\theta + Kx = 0}$$

$\underline{k=2}$ $(q_2 = \theta)$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{3mL}{2}\dot{x}\cos\theta + \frac{17}{12}mL^2\dot{\theta}$$

$$\frac{d}{dt}(\ ) = \frac{3mL}{2}\ddot{x}\cos\theta - \frac{3mL}{2}\dot{x}\dot{\theta}\sin\theta + \frac{17}{12}mL^2\ddot{\theta}$$

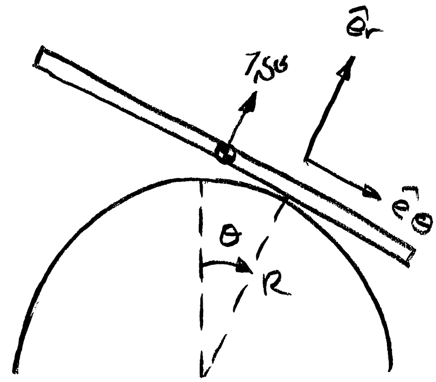
$$\frac{\partial L}{\partial \theta} = -\frac{3mL}{2}\dot{x}\dot{\theta}\sin\theta - \frac{3mgL}{2}\sin\theta$$

$$\frac{3mL}{2}\ddot{x}\cos\theta - \cancel{\frac{3mL}{2}\dot{x}\dot{\theta}\sin\theta} + \frac{17}{12}mL^2\ddot{\theta} + \cancel{\frac{3mL}{2}\dot{x}\dot{\theta}\sin\theta} + \frac{3mgL}{2}\sin\theta = 0$$

$$\boxed{\frac{3mL}{2}\ddot{x}\cos\theta + \frac{17}{12}mL^2\ddot{\theta} + \frac{3mgL}{2}\sin\theta = 0}$$

4)

$$T = \frac{1}{2} m \dot{\vec{r}}_G \cdot \dot{\vec{r}}_G + \frac{1}{2} I_G \dot{\theta}^2$$



$$\vec{r}_G = R\hat{e}_r - R\theta\hat{e}_\theta$$

$$\dot{\vec{r}}_G = R\dot{\hat{e}}_r - R\dot{\theta}\hat{e}_\theta - R\theta\dot{\hat{e}}_\theta$$

$$\dot{\vec{r}}_G = R\dot{\theta}\hat{e}_\theta - R\dot{\theta}\hat{e}_\theta + R\theta\dot{\theta}\hat{e}_r$$

$$\dot{\vec{r}}_G = R\theta\dot{\theta}\hat{e}_r$$

$$\dot{\vec{r}}_G \cdot \dot{\vec{r}}_G = R^2\theta^2\dot{\theta}^2$$

$$I_G = \frac{1}{12} m L^2$$

$$T = \frac{1}{2} m R^2\theta^2\dot{\theta}^2 + \frac{1}{2}\left(\frac{1}{12} m L^2\right)\dot{\theta}^2 = \frac{1}{2} m R^2\theta^2\dot{\theta}^2 + \frac{1}{24} m L^2\dot{\theta}^2$$

$$V = mg\left(R\cos\theta + R\theta\sin\theta\right) = mgR(\cos\theta + \theta\sin\theta)$$

$$L = T - V = \frac{1}{2} m R^2\theta^2\dot{\theta}^2 + \frac{1}{24} m L^2\dot{\theta}^2 - mgR(\cos\theta + \theta\sin\theta)$$

$$\frac{\partial L}{\partial\dot{\theta}} = mR^2\theta^2\dot{\theta} + \frac{1}{12} m L^2\dot{\theta}$$

$$\frac{d}{dt}(\;) = 2mR^2\theta\dot{\theta}^2 + mR^2\theta^2\ddot{\theta} + \frac{1}{12} m L^2\ddot{\theta}$$

$$\frac{\partial L}{\partial\theta} = mR^2\theta\dot{\theta}^2 - mgR(-\sin\theta + \sin\theta + \theta\cos\theta)$$

$$\frac{\partial L}{\partial\theta} = mR^2\theta\dot{\theta}^2 - mgR\theta\cos\theta$$

$$2mR^2\theta\dot{\theta}^2 + mR^2\theta^2\ddot{\theta} + \frac{1}{12} m L^2\ddot{\theta} - mR^2\theta\dot{\theta}^2 + mgR\theta\cos\theta = 0$$

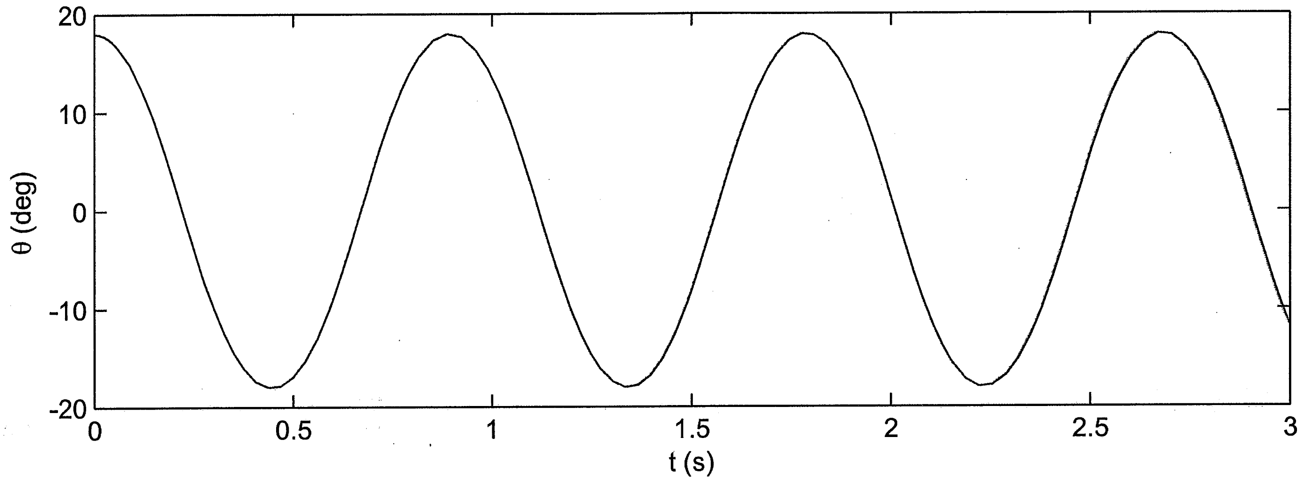
$$\boxed{m\left(R^2\theta^2 + \frac{L^2}{12}\right)\ddot{\theta} + mR^2\theta\dot{\theta}^2 + mgR\theta\cos\theta = 0}$$

$$\dot{\theta} = \omega$$

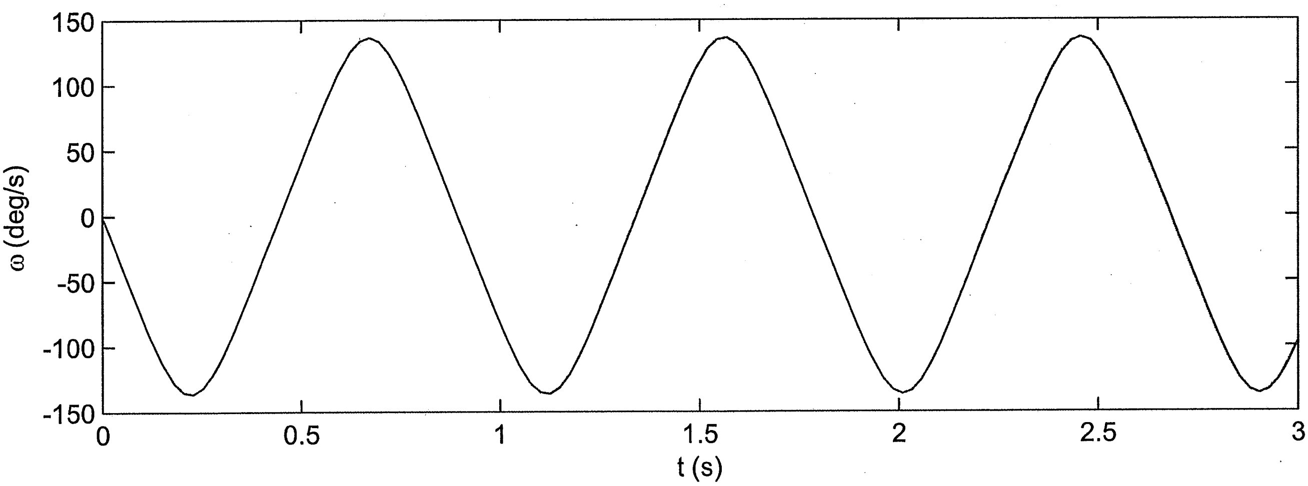
$$m\left(R^2\theta^2 + \frac{L^2}{12}\right)\dot{\omega} + mR^2\theta\omega^2 + mgR\theta\cos\theta = 0$$

$$\dot{\omega} = \frac{1}{m\left(R^2\theta^2 + \frac{L^2}{12}\right)}\left[-mR^2\theta\omega^2 - mgR\theta\cos\theta\right]$$

See attached m-file and plot



θ (deg)
20
10
0
-10
-20
0
0.5
1
1.5
2
2.5
3
t (s)



ω (deg/s)
150
100
50
0
-50
-100
-150
0
0.5
1
1.5
2
2.5
3
t (s)

```
clear;
clc;

% Model parameters
m = 2.1;     % Mass (kg)
L = 1;       % Rod Length (m)
g = 9.81;    % Gravity (m/s^2)
R = 0.5;     % Cylinder Radius (m)

% Solve for theta and omega using ode45
x0 = [18*pi/180 0];
tspan = [0 3];
[t,x] = ode45(@rod_on_cylinder, tspan, x0, [], m, L, g, R);
theta = x(:,1);
omega = x(:,2);
clear x;

% Plot results
figure(1);
subplot(2,1,1);
plot(t,theta*180/pi);
xlabel('t (s)');
ylabel('\theta (deg)');
subplot(2,1,2);
plot(t,omega*180/pi);
xlabel('t (s)');
ylabel('\omega (deg/s)');
```

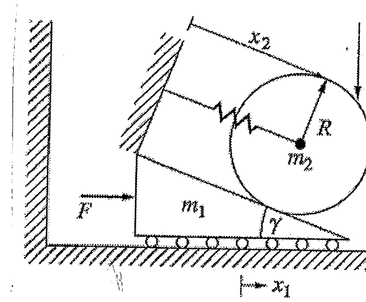
```
function xdot = rod_on_cylinder_function(t,x,m,L,g,R)

theta = x(1);
omega = x(2);

thetadot = omega;
omegadot = 1/m/(R^2*theta^2 + L^2/12)*(-m*R^2*theta*omega^2 -
m*g*R*theta*cos(theta));

xdot = [thetadot;omegadot];
```

5)



$$T = \tfrac{1}{2} m_1 \dot{x}_1^2 + \tfrac{1}{2} m_2 \dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2 + \tfrac{1}{2} I_2 \dot{\theta}^2$$

$$\vec{r}_2 = (x_1 + x_2 \cos\gamma)\hat{\imath} - x_2 \sin\gamma\, \hat{\jmath}$$

$$\dot{\vec{r}}_2 = (\dot{x}_1 + \dot{x}_2 \cos\gamma)\hat{\imath} - \dot{x}_2 \sin\gamma\, \hat{\jmath}$$

$$\dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2 = \dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 \cos\gamma + \dot{x}_2^2 \cos^2\gamma + \dot{x}_2^2 \sin^2\gamma$$

$$\dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2 = \dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 \cos\gamma + \dot{x}_2^2$$

$$\dot{\theta} = \frac{\dot{x}_2}{R}$$

$$T = \tfrac{1}{2} m_1 \dot{x}_1^2 + \tfrac{1}{2} m_2 \dot{x}_1^2 + m_2 \dot{x}_1 \dot{x}_2 \cos\gamma + \tfrac{1}{2} m_2 \dot{x}_2^2 + \tfrac{1}{2}\left(\tfrac{1}{2} m_2 \cancel{R^2}\right)\frac{\dot{x}_2^2}{\cancel{R^2}}$$

$$T = \tfrac{1}{2}(m_1 + m_2)\dot{x}_1^2 + \tfrac{3}{4} m_2 \dot{x}_2^2 + m_2 \dot{x}_1 \dot{x}_2 \cos\gamma$$

$$V = \tfrac{1}{2} k x_2^2 - m_2 g x_2 \sin\gamma$$

$$L = T - V = \tfrac{1}{2}(m_1 + m_2)\dot{x}_1^2 + \tfrac{3}{4} m_2 \dot{x}_2^2 + m_2 \dot{x}_1 \dot{x}_2 \cos\gamma$$
$$- \tfrac{1}{2} k x_2^2 + m_2 g x_2 \sin\gamma$$

$\underline{k=1}$ $(q_1 = x_1)$

$$\frac{\partial L}{\partial \dot{x}_1} = (m_1 + m_2)\dot{x}_1 + m_2 \dot{x}_2 \cos\gamma \qquad \boxed{(m_1 + m_2)\ddot{x}_1 + m_2 \ddot{x}_2 \cos\gamma = F}$$

$$\frac{d}{dt}(\;) = (m_1 + m_2)\ddot{x}_1 + m_2 \ddot{x}_2 \cos\gamma$$

$$\frac{\partial L}{\partial x_1} = 0$$

$\underline{k=2}$ $(q_2 = x_2)$

$$\frac{\partial L}{\partial \dot{x}_2} = \tfrac{3}{2} m_2 \dot{x}_2 + m_2 \dot{x}_1 \cos\gamma \qquad \boxed{\tfrac{3}{2} m_2 \ddot{x}_2 + m_2 \ddot{x}_1 \cos\gamma + k x_2 - m_2 g \sin\gamma = 0}$$

$$\frac{d}{dt}(\;) = \tfrac{3}{2} m_2 \ddot{x}_2 + m_2 \ddot{x}_1 \cos\gamma$$

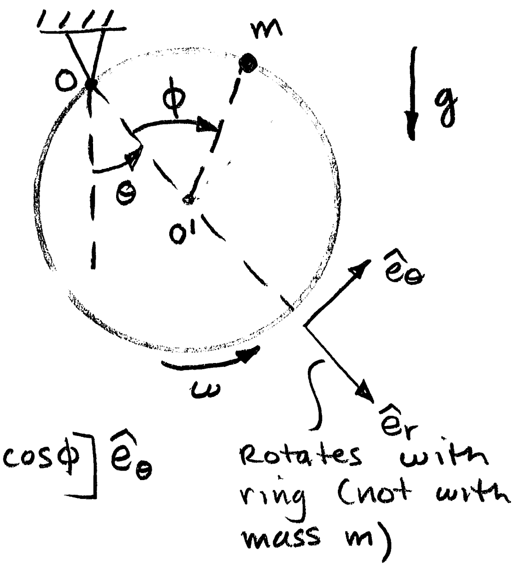
$$\frac{\partial L}{\partial x_2} = -k x_2 + m_2 g \sin\gamma$$

6)

$N=1$

$n=1$

$q=\phi$



$$\vec{r} = (r - r\cos\phi)\,\hat{e}_r + r\sin\phi\,\hat{e}_\theta$$

$$\vec{r} = r(1-\cos\phi)\,\hat{e}_r + r\sin\phi\,\hat{e}_\theta$$

$$\dot{\vec{r}} = r\dot{\phi}\sin\phi\,\hat{e}_r + r(1-\cos\phi)\,\omega\,\hat{e}_\theta + r\dot{\phi}\cos\phi\,\hat{e}_\theta - r\omega\sin\phi\,\hat{e}_r$$

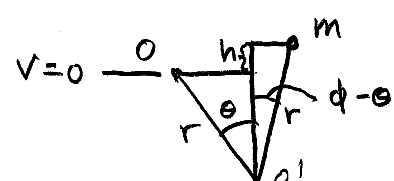
$$\dot{\vec{r}} = r(\dot{\phi}-\omega)\sin\phi\,\hat{e}_r + r\left[\omega + (\dot{\phi}-\omega)\cos\phi\right]\hat{e}_\theta$$

$$T = \frac{1}{2} m \dot{\vec{r}} \cdot \dot{\vec{r}}$$

$$T = \frac{mr^2}{2}\left[(\dot{\phi}-\omega)^2\sin^2\phi + \omega^2 + 2\omega(\dot{\phi}-\omega)\cos\phi + (\dot{\phi}-\omega)^2\cos^2\phi\right]$$

$$T = \frac{mr^2}{2}\left[(\dot{\phi}-\omega)^2 + \omega^2 + 2\omega(\dot{\phi}-\omega)\cos\phi\right]$$

$$V = mgh = mgr\left(\cos(\phi-\theta) - \cos\theta\right)$$



$$L = T - V$$

$$L = \frac{mr^2}{2}\left[(\dot{\phi}-\omega)^2 + \omega^2 + 2\omega(\dot{\phi}-\omega)\cos\phi\right] - mgr\left[\cos(\phi-\theta) - \cos\theta\right]$$

$$\frac{\partial L}{\partial \dot{\phi}} = \frac{mr^2}{2}\left[2(\dot{\phi}-\omega) + 2\omega\cos\phi\right] = mr^2\left[\dot{\phi} - \omega + \omega\cos\phi\right]$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = mr^2\left[\ddot{\phi} - \omega\dot{\phi}\sin\phi\right]$$

$$\frac{\partial L}{\partial \phi} = -mr^2\omega(\dot{\phi}-\omega)\sin\phi + mgr\sin(\phi-\theta)$$

$$\cancel{m}r^{\cancel{2}}\left[\ddot{\phi} - \cancel{\omega\dot{\phi}\sin\phi}\right] + \cancel{m}r^{\cancel{2}}\omega(\cancel{\dot{\phi}}-\omega)\sin\phi - \cancel{m}g\cancel{r}\sin(\phi-\theta) = 0$$

But $\theta = \omega t$

$$\boxed{r\ddot{\phi} - r\omega^2\sin\phi - g\sin(\phi-\omega t) = 0}$$