$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_{k,nc} + \sum_{j=1}^{p} \lambda_j a_{jk}, \quad k = 1 \ldots m$$

$$\sum_{k=1}^{m} a_{jk}\dot{q}_k + a_{j0} = 0, \; j = 1 \ldots p$$

$$Q_k = \sum_{i=1}^{N} \mathbf{F}_i \cdot \frac{\partial \mathbf{r}_i}{\partial q_k} = \sum_{i=1}^{N} \mathbf{F}_i \cdot \frac{\partial \dot{\mathbf{r}}_i}{\partial \dot{q}_k}$$

$$[I'] = [R][I][R]^T$$

$$T_{rot} = \frac{1}{2}\{\omega\}^T [I]\{\omega\}$$

$$[I_B] = [I_G] + m[\{d\}^T\{d\}[1] - \{d\}\{d\}^T]$$

$$I_{B_{xx}} = I_{G_{xx}} + m(d_y^2 + d_z^2)$$

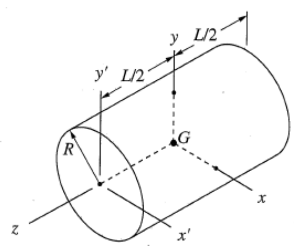
$$I_{B_{xy}} = I_{G_{xy}} + m d_x d_y$$

$$\{H\} = [I]\{\omega\}$$

$$\{M\} = \{\dot{H}\}$$

$$I_{xx}\alpha_x - I_{xy}(\alpha_y - \omega_x\omega_z) - I_{xz}(\alpha_z + \omega_x\omega_y) - (I_{yy} - I_{zz})\omega_y\omega_z - I_{yz}(\omega_y^2 - \omega_z^2) = M_x$$
$$I_{yy}\alpha_y - I_{yz}(\alpha_z - \omega_x\omega_y) - I_{xy}(\alpha_x + \omega_y\omega_z) - (I_{zz} - I_{xx})\omega_x\omega_z - I_{xz}(\omega_z^2 - \omega_x^2) = M_y$$
$$I_{zz}\alpha_z - I_{xz}(\alpha_x - \omega_y\omega_z) - I_{yz}(\alpha_y + \omega_x\omega_z) - (I_{xx} - I_{yy})\omega_x\omega_y - I_{xy}(\omega_x^2 - \omega_y^2) = M_z$$

$$I_{xx}\alpha_x - (I_{yy} - I_{zz})\omega_y\omega_z = M_x$$
$$I_{yy}\alpha_y - (I_{zz} - I_{xx})\omega_x\omega_z = M_y$$
$$I_{zz}\alpha_z - (I_{xx} - I_{yy})\omega_x\omega_y = M_z$$



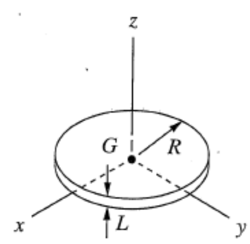
Circular Cylinder

Volume $= \pi R^2 L$

$I_{zz} = \frac{1}{2}mR^2$

$I_{xx} = I_{yy} = \frac{1}{4}mR^2 + \frac{1}{12}mL^2$

$I_{x'x'} = I_{y'y'} = \frac{1}{4}mR^2 + \frac{1}{3}mL^2$

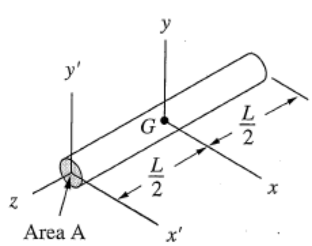


Thin Disk $\left(\frac{L}{R} \text{ very small}\right)$

Volume $= \pi R^2 L$

$I_{zz} = \frac{1}{2}mR^2$

$I_{xx} = I_{yy} = \frac{1}{4}mR^2$



Slender Rod $\left(\frac{R}{L} \text{ very small}\right)$

Volume $= AL$

$I_{zz} \approx 0$

$I_{xx} = I_{yy} = \frac{1}{12}mL^2$

$I_{x'x'} = I_{y'y'} = \frac{1}{3}mL^2$