HW 1 | MEEN 537 | Dustan Kraus

1-2) forward kinematics - given joint variables (angles), determine end effector location

inverse kinematics - given end effector coordinates, determine joint variables

trajectory planning - generate reference trajectories that determine time history of the manipulator

workspace - total volume swept by end effector in all possible motions

accuracy - how close a manipulator can get to a given point in its workspace

repeatability - how close a manipulator can return to a previously taught point

resolution - smallest increment of motion the controller can sense. Total distance travelled divided by 2^n n = bits of encoder accuracy

joint variable - either θ or d, relative displacement b/w adjacent links

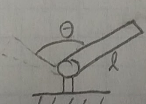
spherical wrist - 3 joint axes intersect @ a common point (before end effector)

end effector - end of a kinematic chain, does gripping, etc.

1-8)
1. Picking up an egg
2. Manipulation near fragile objects like plants
3. Movement near humans
4. lifting a box (keeping enough friction) pushing on sides
5. Keeping a button pushed

1-12)
- holding an object while moving the elbow around something
- exerting additional force @ the same position by getting better angle

1-14) $l = 50$ cm
$\theta = 180°$



$$\text{cotrol resolution} = \frac{(180/360)\,2\pi(50)}{2^8}$$

$$\boxed{\text{control resolution} = 0.6136 \text{ cm}}$$

1-16) Repeatability is only affected really be controller resolution, while accuracy is affected by forward kinematic math error, machining errors, assumed geometry, rigidity etc.

1-20) This would only be possible if one of the joints was spherical.

1-21) lower mass on distal links would make for a lower moment of inertia about the base link which could result in quicker movements. this would likely decrease rigidity though

2-10) $R = R_{y,\psi} R_{x,\phi} R_{z,\theta} = \boxed{\begin{bmatrix} C_\psi & 0 & S_\psi \\ 0 & 1 & 0 \\ -S_\psi & 0 & C_\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix} \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = R}$ ⟵

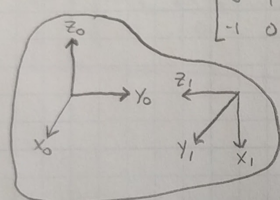
☆ for the rest of this hw: $R_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi & C_\phi \end{bmatrix}$ $R_{y,\phi} = \begin{bmatrix} C_\phi & 0 & S_\phi \\ 0 & 1 & 0 \\ -S_\phi & 0 & C_\phi \end{bmatrix}$ $R_{z,\phi} = \begin{bmatrix} C_\phi & -S_\phi & 0 \\ S_\phi & C_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ☆

2-11) $\boxed{R = R_{z,\theta} R_{x,\phi} R_{x,\psi}}$

2-12) $\boxed{R = R_{z,\alpha} R_{x,\phi} R_{z,\theta} R_{x,\psi}}$

2-13) $\boxed{R = R_{z,\alpha} R_{z,\theta} R_{x,\phi} R_{x,\psi}}$

2-14) $R = R_{y,\pi/2} R_{x,\pi/2} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \boxed{\begin{array}{c} \begin{matrix} x_1^0 & y_1^0 & z_1^0 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \end{array} = R}$ ⟵



2-15) $R_3^1 = R_2^1 R_3^2 \Rightarrow (R_2^1)^T R_3^1 = R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & 1/2 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$R_3^2 = \begin{bmatrix} 0 & 0 & -1 \\ \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & -\sqrt{3}/2 & 0 \end{bmatrix}$ ⟵

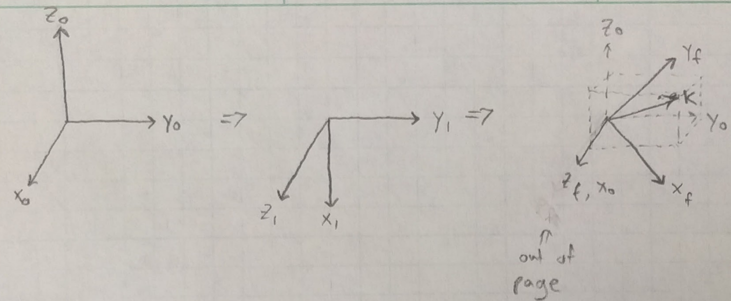
2-22) using matlab syms: $R = \begin{bmatrix} S_\phi^2 - C_\phi^2 & -2C_\phi S_\phi S_\theta & 2C_\phi C_\theta S_\phi \\ -2C_\phi S_\phi S_\theta & S_\theta(C_\phi^2 S_\theta - S_\phi^2 S_\theta) - C_\theta^2 & -C_\theta S_\theta - C_\theta(C_\phi^2 S_\theta - S_\phi^2 S_\theta) \\ 2C_\phi C_\theta S_\phi & -C_\theta S_\theta - S_\theta(C_\phi^2 C_\theta - C_\theta S_\phi^2) & C_\theta(C_\phi^2 C_\theta - C_\theta S_\phi^2) - S_\theta^2 \end{bmatrix}$

2-23) $R = R_{y,90} R_{z,45} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix} = R$

eq. 2.45; $k = \dfrac{1}{2\sin\theta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$ ①

$\theta = \cos^{-1}\left(\dfrac{r_{11} + r_{22} + r_{33} - 1}{2}\right) = \cos^{-1}\left(\dfrac{0 + \sqrt{2}/2 + 0 - 1}{2}\right) = 1.7178$ ②

sub ② → ① ⇒ $k = \dfrac{1}{2\sin(1.7178)} \begin{bmatrix} \sqrt{2}/2 - 0 \\ 1 + \sqrt{2}/2 \\ \sqrt{2}/2 - 0 \end{bmatrix} \Rightarrow \boxed{k = \begin{bmatrix} 0.3574 \\ 0.8629 \\ 0.3574 \end{bmatrix} \quad \theta = 98.4°}$ ⟵



X_f & Y_f are in plane of page

2-24) $\phi = \pi/2 \quad \theta = 0 \quad \psi = \pi/4$ assume ZYZ

$$R = R_{z,\pi/2} R_{y,0} R_{z,\pi/4} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{R_1^0 = \begin{bmatrix} -\sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \qquad \boxed{x_1^0 = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}}$$