ECEn 671
Midterm Exam # 1, Fall 2016
Due at the Beginning of Class, October 11, 2016

Name: _____

Instructions:

- This exam is open book, open notes, open computer, open brain, but not open neighbor.
- You will need Matlab to complete the exam.
- This is a timed exam!! You must complete the exam in **three contiguous hours**. You may not look at the problems before beginning.
- Be neat and thorough. Make your solutions clear, logical, and easy to follow.
- Print this page and attach it to your solutions.
- The due date will not be extended for any reason.

Problem 1	_____/ 25
Problem 2	_____/ 25
Problem 3	_____/ 25
Problem 4	_____/ 25
Total	_____/ 100

Start time/date: _____ Stop time/date: _____

I certify that the solutions to this exam represents my own work, obtained within the time limit, and that I did not consult with any other individual about the exam.

_____

Signature

1. (25 pts) Let S be the set of continuous real valued functions on $[0, \infty)$. The standard inner product is defined via the integral operator, i.e.,

$$\langle x(t),\ y(t) \rangle = \int_0^\infty x(t) y(t)\, dt.$$

Suppose instead that we propose to use the differential operator, i.e.,

$$\langle x(t),\ y(t) \rangle_D \stackrel{\triangle}{=} \frac{d}{dt} \left(x(t) y(t) \right) \Big|_{t=0}.$$

Is $\langle \cdot,\ \cdot \rangle_D$ a valid inner product? (Justify your answer.)

2. (25 pts) Let $V = \text{span}\{p_1(t), p_2(t)\}$ where

$$\begin{aligned} p_1(t) &= 1 - t^2, \qquad 0 \le t \le 1 \\ p_2(t) &= e^{-t} \qquad 0 \le t \le 1. \end{aligned}$$

Then V is a closed subspace of $L_2[0,1]$. By Theorem 2.10, $L_2[0,1] = V \oplus V^\perp$, and any $x \in L_2[0,1]$ can be uniquely written as $x = y + z$ where $y \in V$ and $z \in V^\perp$. Let $x = 1 + t + t^2 + t^3$. Find $y(t)$ and $z(t)$.

3. (25 pts) On the web site, you will need to download the file `mid1_p3_data.mat` which contains a Matlab "mat" file with 1001 data points $\{t_i, x_i\}_{i=1}^{1001}$. By viewing the data, you will see that it follows a trend that looks like a sinusoid of the form $A \sin(10t)$ superimposed on a quadratic function of the form $at^2 + bt + c$. Using Matlab or otherwise, find the coefficients A, a, b, and c that minimizes the mean square error between the data points and the function $\hat{x}(t) = A \sin(10t) + at^2 + bt + c$.

4. (25 pts) Suppose that we are to set up a special manufacturing company which will operate for only ten months. During the ten months the company is to produce one million copies of a single product. We assume that the manufacturing facilities have been leased for the ten-month period, but that labor has not yet been hired. Presumably, employees will be hired and fired during the ten-month period. Our problem is to determine how many employees should be hired or fired in each of the ten months. It is assumed that each employee produces one hundred items per month. The cost of labor is proportional to the number of production workers, but there is an additional cost due to hiring and firing. If $u(k)$ workers are hired in the k-th month (negative $u(k)$ corresponds to firings), the processing cost can be argued to be $u^2(k)$ because, as u increases, people must be paid to stand in line and more nonproductive employees must be paid. At the end of the ten-month period, all workers must be fired. Find $u(k)$ for $k = 1, \ldots, 10$ to minimize cost.

HINTS: Use the projection theorem and write the constraints as inner product constraints. To simplify the problem, allow $u(k)$ to be a real number that may not be an integer.