1) This is a central force problem, so angular momentum is conserved.

Energy is also conserved

$$\underline{E_0 = E_f}$$

$$T_0 + V_0 = T_f + V_f$$

$$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v_f^2 + \frac{1}{2} k \left(\frac{4 l_0}{3} - l_0\right)^2$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_f^2 + \frac{1}{18} k l_0^2$$

$$m v_0^2 = m v_f^2 + \frac{1}{9} k l_0^2 \quad ①$$

We don't know v_f, so use conservation of angular momentum.

$$\underline{H_0 = H_f}$$

$$H_0 = m l_0 v_0 \sin 45$$

$H_f = m r_f v_f$ because all of the final velocity is tangential because the spring has reached its maximum length, which means that the radial velocity is zero

$$H_f = m\left(\frac{4 l_0}{3}\right) v_f = \frac{4 m l_0}{3} v_f$$

$$\frac{4 \cancel{m l_0}}{3} v_f = \cancel{m l_0} v_0 \sin 45$$

$$v_f = \frac{3}{4} v_0 \sin 45$$

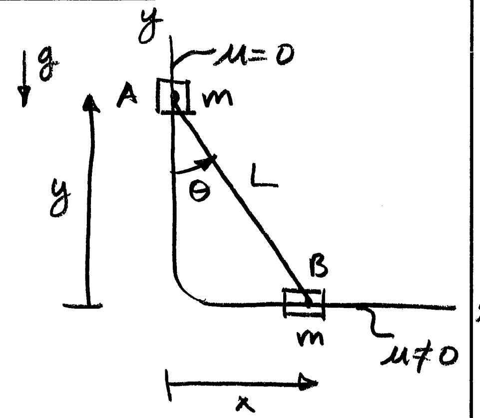
Plug into ①:

$$m v_0^2 = \frac{9}{16} m v_0^2 \sin^2 45 + \frac{1}{9} k l_0^2$$

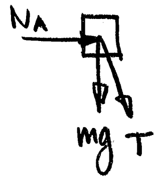
$$\Rightarrow \boxed{k = \frac{207 m v_0^2}{32 l_0^2} = 6.46875 \frac{m v_0^2}{l_0^2}}$$

2) Find the equation of motion

Look at each mass individually:



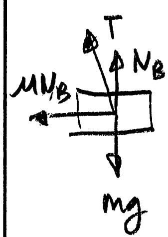
Mass A



$$[F_y = m\ddot{y}]$$

$$-mg - T\cos\theta = m\ddot{y}$$

Mass B



$$[F_x = m\ddot{x}]$$

$$-T\sin\theta - \mu N_B = m\ddot{x}$$

$$\Downarrow$$

$$[F_y = 0]$$

$$N_B + T\cos\theta - mg = 0$$

$$N_B = mg - T\cos\theta$$

$$-\mu(mg - T\cos\theta) - T\sin\theta = m\ddot{x}$$

$$m\ddot{x} = T(\mu\cos\theta - \sin\theta) - \mu mg$$

and (from above)...

$$m\ddot{y} = -T\cos\theta - mg \Rightarrow T = \frac{-m\ddot{y} - mg}{\cos\theta}$$

$$m\ddot{x} = \frac{-m\ddot{y} - mg}{\cos\theta}(\mu\cos\theta - \sin\theta) - \mu mg$$

$$m\ddot{x} = m\ddot{y}(\tan\theta - \mu) + mg(\tan\theta - 2\mu)$$

But $\cos\theta = \frac{y}{L} \Rightarrow y = L\cos\theta \Rightarrow \dot{y} = -L\dot{\theta}\sin\theta$

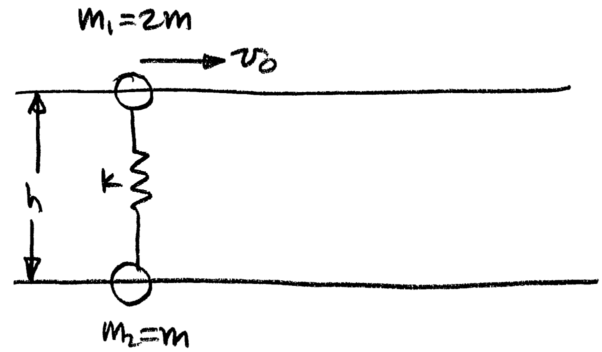
$$\Rightarrow \ddot{y} = -L\ddot{\theta}\sin\theta - L\dot{\theta}^2\cos\theta$$

$\sin\theta = \frac{x}{L} \Rightarrow x = L\sin\theta \Rightarrow \dot{x} = L\dot{\theta}\cos\theta$

$$\Rightarrow \ddot{x} = L\ddot{\theta}\cos\theta - L\dot{\theta}^2\sin\theta$$

$$\boxed{\ddot{\theta}\left[\cos\theta(1+\tan^2\theta) - \mu\sin\theta\right] - \dot{\theta}^2\mu\cos\theta - \frac{g}{L}\tan\theta = -\frac{2\mu g}{L}}$$

3)



Cons. of lin. mom:

Initial:

$P_0 = 2mv_0$

Final:

$P_f = 2mv_1 + mv_2$

$P_f = P_0 \Rightarrow 2v_0 = 2v_1 + v_2$ ①

Cons. of energy:

Initial:

$E_0 = \frac{1}{2}(2m)v_0^2 = mv_0^2$

Final:

$E_f = \frac{1}{2}(2m)v_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}k\delta^2 = mv_1^2 + \frac{m}{2}v_2^2 + \frac{k}{2}\delta^2$

$E_f = E_0 \Rightarrow mv_1^2 + \frac{m}{2}v_2^2 + \frac{k}{2}\delta^2 = mv_0^2$ ②

Solve ① and ② simultaneously for v_2:

$$v_2 = \frac{\frac{4v_0}{3} \pm \sqrt{\left(\frac{4v_0}{3}\right)^2 - 4\left(\frac{2k}{3m}\delta^2\right)}}{2}$$

v_{2max} occurs with the positive sign and when $\delta = 0$:

$$\boxed{v_{2max} = \frac{4v_0}{3}}$$

δ_{max} occurs when $\dot{\delta} = 0$, which occurs when $v_1 = v_2 = v$. From ①, $2v_0 = 2v + v \Rightarrow v = \frac{2v_0}{3}$

From ②,

$$m\left(\frac{2v_0}{3}\right)^2 + \frac{m}{2}\left(\frac{2v_0}{3}\right)^2 + \frac{k}{2}\delta_{max}^2 = mv_0^2$$

$$\boxed{\delta_{max} = \sqrt{\frac{2m}{3k}}\, v_0}$$