Oil Tracking on the TX-LA Shelf

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1 Introduction

1.1 Particle Tracking

2 Tracking Algorithm Sensitivity and Details

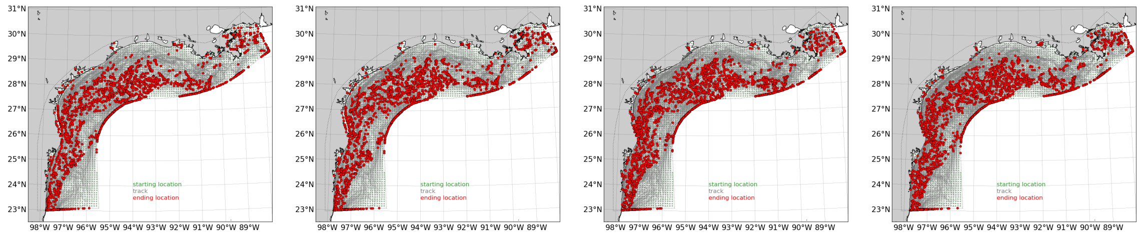
2.1 Explain Algorithm

2.1.1 2D Boundaries

Due to the basic algorithm of TRACMASS, at boundaries within the numerical domain, drifters will be stopped according to the bounding fluxes. For a given grid cell in the 2D case, there are four fluxes controlling a drifter's movement. Drifters have nonzero fluxes on active sides of the cell and zero fluxes along masked land. They can run along these walls but should not penetrate them. At open numerical boundaries, the drifters will be stopped according to a check built into tracmass itself, and will be left with their final position along the open boundary and a flag indicating that they have exited the domain so they will not be stepped forward.

The addition of subgrid turbulence parameterizations can affect this. One method is to add parameterized turbulent values to the fluxes used to calculate drifter movements. These do not affect the fact that fluxes will be zero at masked land because they are multiplied by the original ufluxes to get the fluctuation to add to the original flux values.

However, there are two methods of adding in a random walk to the particle positions directly, and these were affecting the boundary behavior of drifters near walls. The problem was that when a drifter was alongside a masked land cell, if the random new position of the drifter was just right to move the drifter from its current cell into the land cell, then an error check later in the code for the volume of the cell would catch the drifter (due to its cell having zero volume since it was on land) and the drifter would be stopped at its location near land. Since drifters in the advection-only and turbulent velocity methods do not hit land, the overall behavior was different along the coastline for the diffusion and anisodiffusion methods (in these methods, many more drifters were congregated alongshore). I changed this by adding a check in the diffusion subroutine in tracmass to not accept a new



(a) No diffusion (b) Turbulence (c) Elliptical (d) Circular

Figure 1: Comparison of types of diffusion for $A_H = 20$ m^2/s, initial spacing of 10km

displacement location for a drifter if the layer thickness (dzt) of that new location is zero. Now, I think that all of the routines will have similar coastline behavior. If, on the other hand, it is desired that drifters should be able to hit the coastline and "beach," then this behavior in the diffusion routines might be desired.

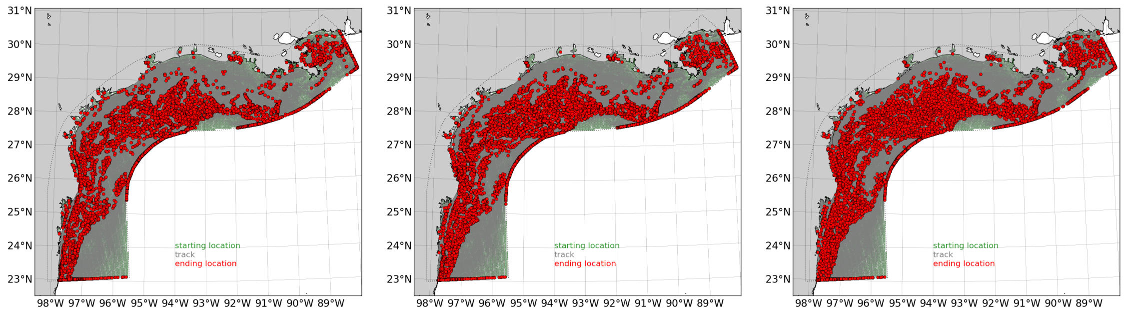
2.2 Examine Sensivity of Results to Input Parameters

A series of numerical surface drifter experiments were run for 16 days forward in time from 11/20/2009 with several changing parameters to understand their importance to the results.

There is little overall difference for the number of time interpolation steps for these simulations (not shown).

The difference in the results from diffusion types is illustrated in Figure 1. For numerical drifter experiments with drifters initially seeded 10 km apart and using the same horizontal diffusivity, the difference in tracks and final positions is not extreme, but is noticeable. The cases with no diffusion and parameterized turbulent velocities (Figures 1(a) and 1(b)) are similar, though a larger value of A_H would presumably change this more. The cases with a random walk-type diffusion added to the particle tracks themselves (Figures 1(c) and 1(d)) show more diffused behavior and are fairly similar to each other.

Drifter tracks and final locations are shown in Figure 2 for changing the size of the horizontal diffusivity, A_H. The overall behavior is the same in all of the plots, but the drifters are somewhat noticeably more spread out as the value of the horizontal diffusivity increases. This is shown for adding diffusion using a random walk on a circle to the drifter positions, but the same type of behavior is found in the results of all of the parameterization techniques (not shown).



(a) No diffusion (b) Circular diffusion, $A_H = 5$ (c) Circular diffusion, $A_H = 20$

Figure 2: Comparison of size of A_H for initial spacing of 5km and circular trajectory diffusion

2.3 Forward/Backward

3 Performance of Model and Tracker

3.1 Mass Flux Comparison

3.2 Gyre Test

3.3 Barataria Bay

3.4 Sensitivity to Waves, Tides, and Model Output Frequency

4 Lagrangian Barotropic Stream Functions

4.1 Methodology

The Lagrangian barotropic stream function, a metric for communicating the transport for a given situation, has been used in several studies previously (*e.g.*, (Blanke et al., 1999; Döös and Engqvist, 2007)). The idea is to assign an initial volume transport property to the drifters based on their initial placement and velocity and, given a high enough density that the result does not change with more drifters, track the x and y transport as the drifters pass numerical grid cell walls. The initial volume transport is found by summing the overall flux into or out of the grid cells in which drifters are initially located and dividing by the number of initial drifters in the cells, or

$$T_0 = \frac{1}{N_0} \left(\left| u[i_0, j_0, k_0, t_0] \right| \Delta y_{i_0,j_0} \Delta z_{i_0,j_0,k_0,t_0} + \left| v[i_0, j_0, k_0, t_0] \right| \Delta x_{i_0,j_0} \Delta z_{i_0,j_0,k_0,t_0} \right),$$

where N_0 is the number of drifters initialized in a grid cell or grid cells (in which case N_0 is a vector), $u[i_0, j_0, k_0, t_0]$ and $v[i_0, j_0, k_0, t_0]$ are the zonal and meridional velocities for the initial grid cell(s) at grid index locations i_0, j_0, k_0, the drifters are seeded at time t_0, and the grid cell spacing in the zonal and meridional directions is given by $\Delta x_{i_0,j_0}$ and $\Delta y_{i_0,j_0}$ (assuming they can change horizontally but not vertically or in time), and in the vertical direction is given by $\Delta z_{i_0,j_0,k_0,t_0}$ (which can change in all dimensions) (Döös, 1995).

Assuming that all drifters that enter a grid cell will exit via another grid cell wall, the 3D transport field is non-divergent, that is,

$$\partial_i U + \partial_j V + \partial_k W = 0, \tag{1}$$

for zonal, meridional, and vertical volume transports U, V, W and directions i, j, k. Alternatively, this can be written in terms of the numerical discretization as:

$$U_{i,j,k,n} - U_{i-1,j,k,n} + V_{i,j,k,n} - V_{i,j-1,k,n} + W_{i,j,k,n} + W_{i,j,k-1,n} = 0, \tag{2}$$

where $U_{i,j,k,n}, V_{i,j,k,n}, W_{i,j,k,n}$ are the (x, y, z) volume transports registered for drifter instance n for grid cell located at indices (i, j, k). Equations 1 or 2 can be integrated along an axis to obtain a 2D non-divergent field which can be presented using a stream function. For the vertical case, this gives

$$\frac{\partial \psi}{\partial i} = \sum_k V \tag{3a}$$

$$\frac{\partial \psi}{\partial j} = -\sum_k U, \tag{3b}$$

where ψ is the stream function.

In this particular surface-limited drifter case, we assume that there is little vertical motion at the surface such that $\partial_k W \approx 0$, so that $\sum_k V \approx V$ and $\sum_k U \approx U$. All following analysis applies without this assumption but would just include a step of first summing the transports in the vertical direction. This analysis was originally presented by Blanke et al. (1999) for numerical drifters stepped forward for a given time. Döös and Engqvist (2007) applied the method to model output that changes in time and for a physical regime that changes over larger time scales as well. This work also presents paths that are averaged over many drifter simulations starting at different times in order to find the overall transport. To do so, transports calculated over time for a given simulation are then combined with transports from subsequent simulations before calculating the stream function.

To apply this to a numerical simulation of drifters, $U_{i,j}$ and $V_{i,j}$ have to be calculated for a given simulation (or $U_{i,j,k}$ and $V_{i,j,k}$ if changes in z are important). This is accomplished by registering each time a drifter exits one grid cell and enters another by passing a cell wall, and subtracting or adding that drifter's initial volume transport from the cumulative grid cell drifter transport value. Once $U_{i,j}$ and $V_{i,j}$ are found for all drifter simulations, they can be added together.

We can use the equations for the stream function, Equation 3 with the vertical direction excluded or already integrated away, to find the explicit formulation for the stream function, using also the continuity equation assuming that the vertical direction is unimportant or has been integrated away:

$$U_x + V_y = 0. \tag{4}$$

Equation 3b gives:

$$\psi(x, y) = \int_{y_0}^{y} U \, \mathrm{d}y' + a(x), \tag{5}$$

where y_0 is a specific y value and y' is a dummy variable. Differentiating Equation 5, using Equation 4, and comparing with Equation 3a gives

$$\begin{aligned}
\psi_x &= \int_{y_0}^{y} U_x \, \mathrm{d}y' + a_x(x) \\
&= \int_{y_0}^{y} -V_y \, \mathrm{d}y' + a_x(x) && \text{from Equation 4} \\
&= -V(x, y) + V(x, y_0) + a_x(x) \\
&= -V(x, y) && \text{from Equation 3a} \\
\Rightarrow a_x(x) &= -V(x, y_0) \\
a(x) &= -\int_{x_0}^{x} V(x, y_0) \, \mathrm{d}x' && (6)
\end{aligned}$$

Using Equation 5 with Equation 6 gives

$$\psi(x, y) = \int_{y_0}^{y} U(x, y) \, \mathrm{d}y' - \int_{x_0}^{x} V(x, y_0) \, \mathrm{d}x'. \tag{7}$$

Equation 7 can be solved numerically by simply cumulatively summing in each direction according to the integrals and combining the terms, that is (in Python),

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psi_i = np.cumsum(V, axis=1)
psi_j = np.cumsum(U, axis=0)
psi = psi_j - psi_i.
```

4.2 Results

5 Results for Different Conditions

5.1 Dependence of Circulation on Weatherband

5.2 Seasonal Variability

5.3 Cross-Shelf Behavior

6 Analysis

References

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