Asymptotic statistics of nodal domains of quantum chaotic billiards in the semiclassical limit

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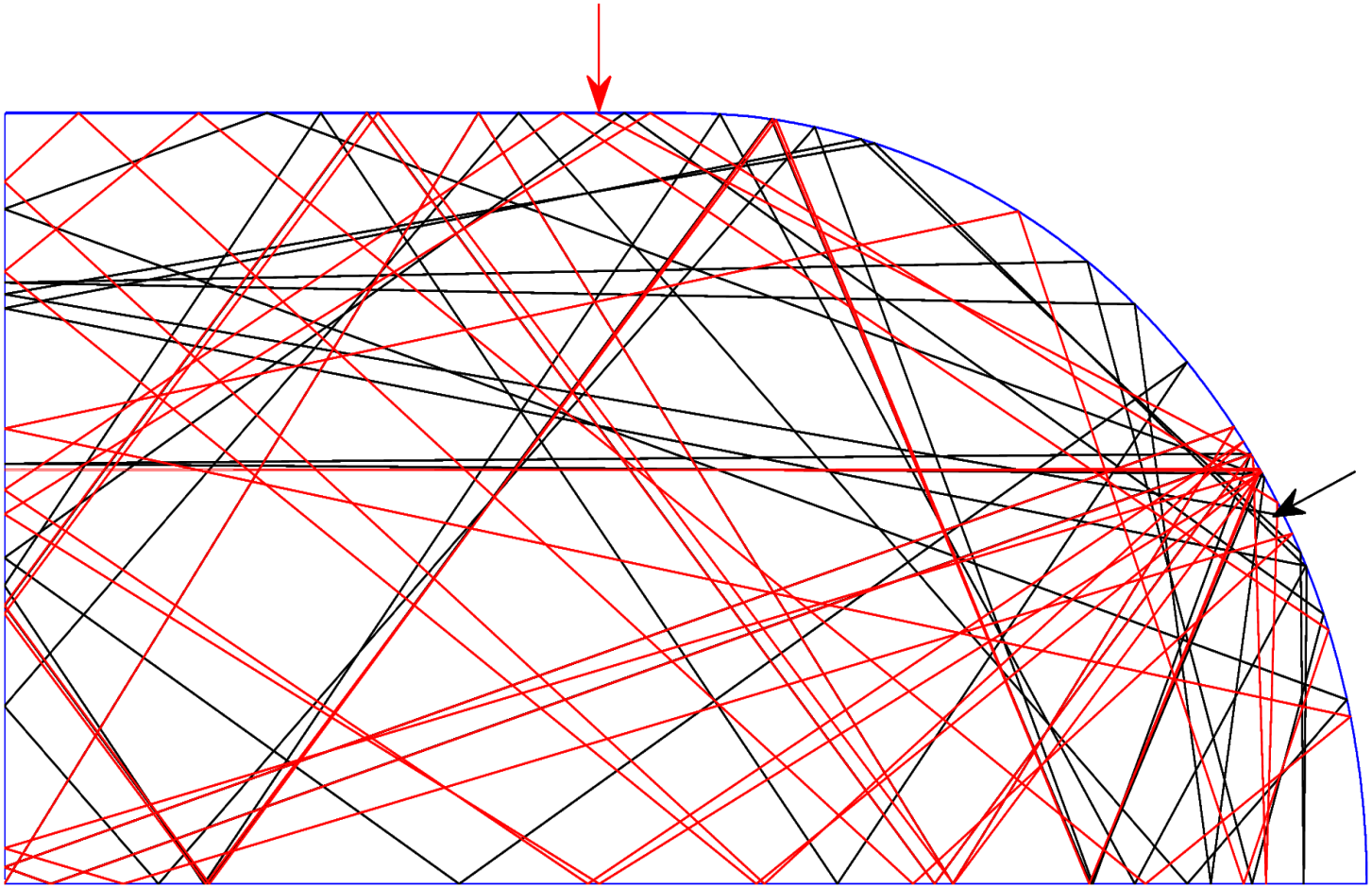
Classical Chaos



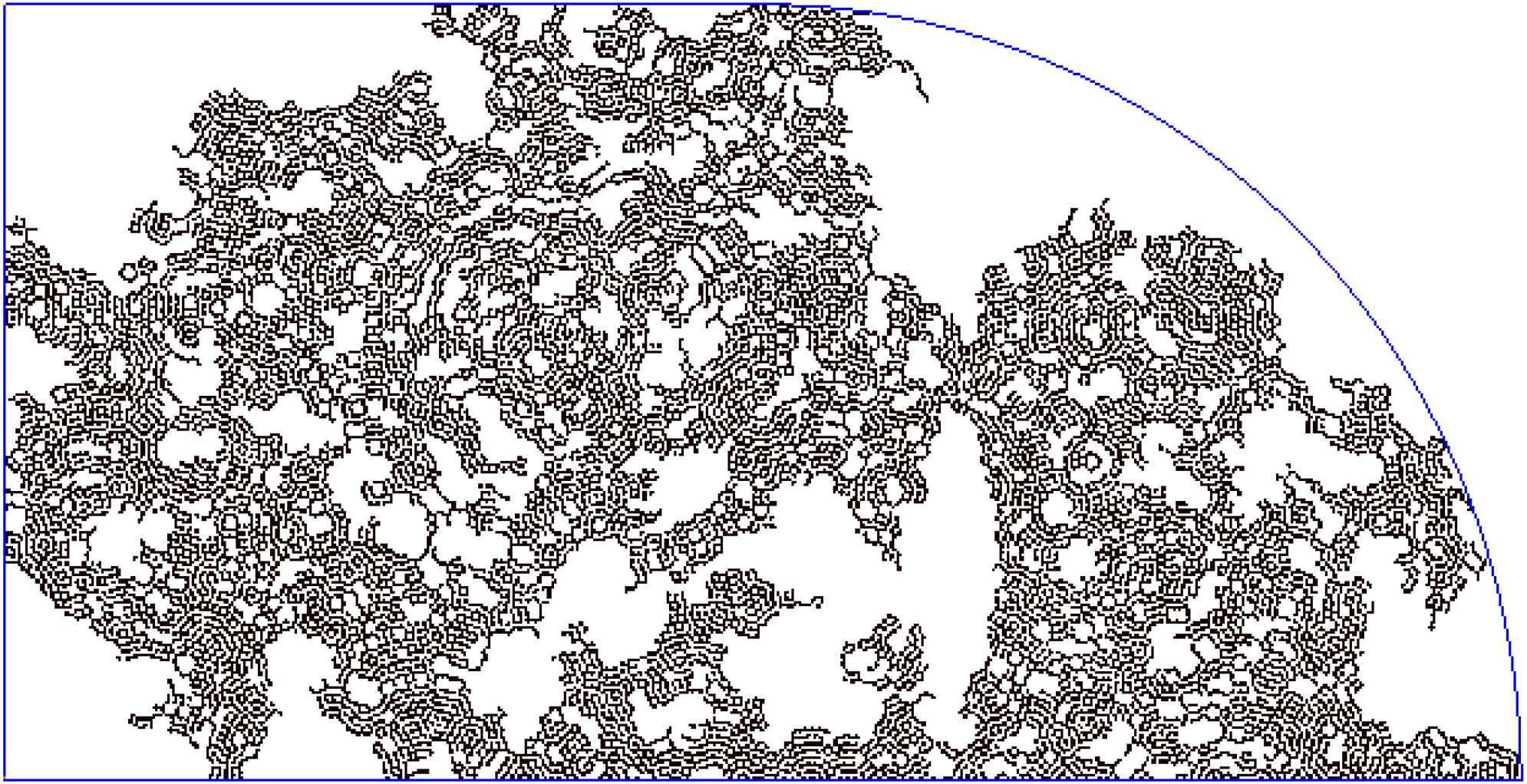
$$\lambda = \lim_{t\to\infty} \lim_{|\epsilon|\to 0} \frac{1}{t} \frac{|f(x_0,t) - f(x_0+\epsilon,t)|}{|\epsilon|}$$

Quantum Chaos

Viewer by Alex Barnett

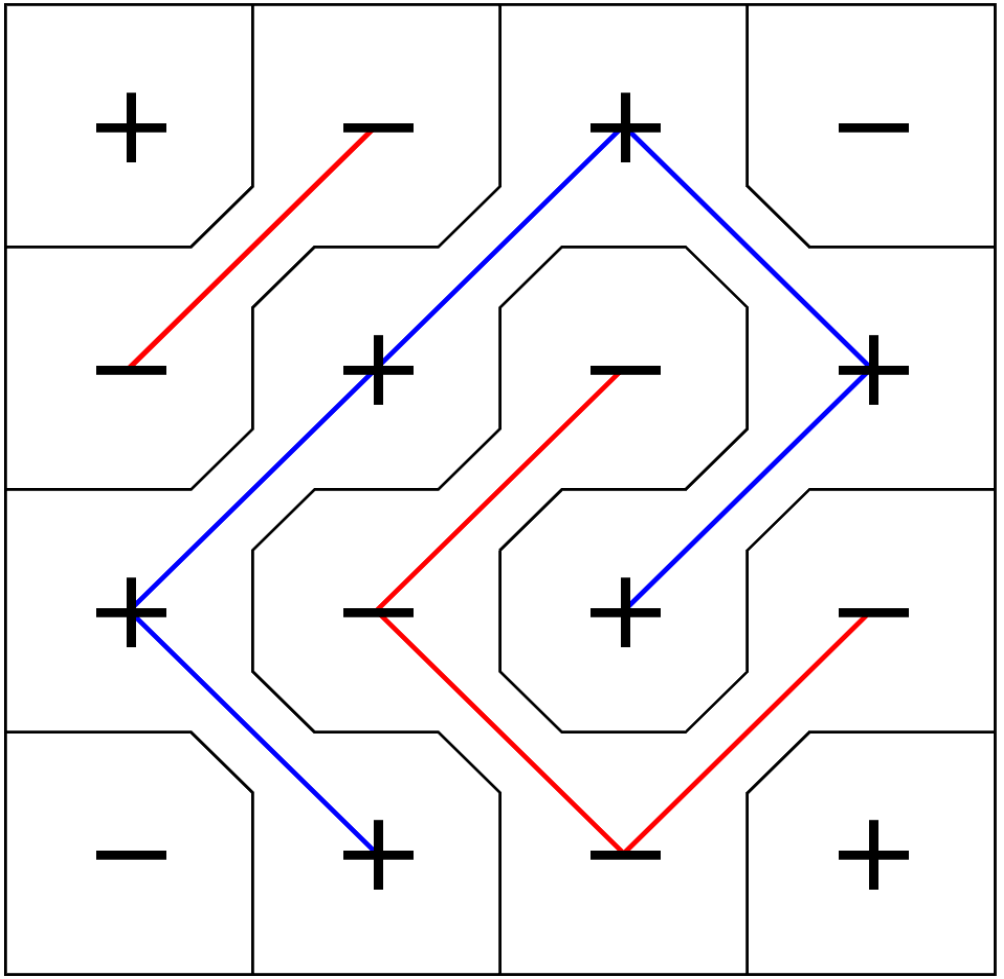
$$\begin{cases} (\Delta + k^2)u(\mathbf{r}) = 0 & \text{if } \mathbf{r} \in \Omega \\ u(\mathbf{r}) = 0 & \text{if } \mathbf{r} \in \partial\Omega \end{cases} \qquad \frac{2\pi}{\lambda} = k \in \{k_1, k_2, \ldots \to \infty\}$$

Nodal domains



Percolation Model

E. Bogomolny and C. Schmit. Percolation model for nodal domains of chaotic wavefunctions. Physical Review Letters, 88(11), 2002.



Size:

$$f(s) \propto s^{-187/91}$$

Mean:

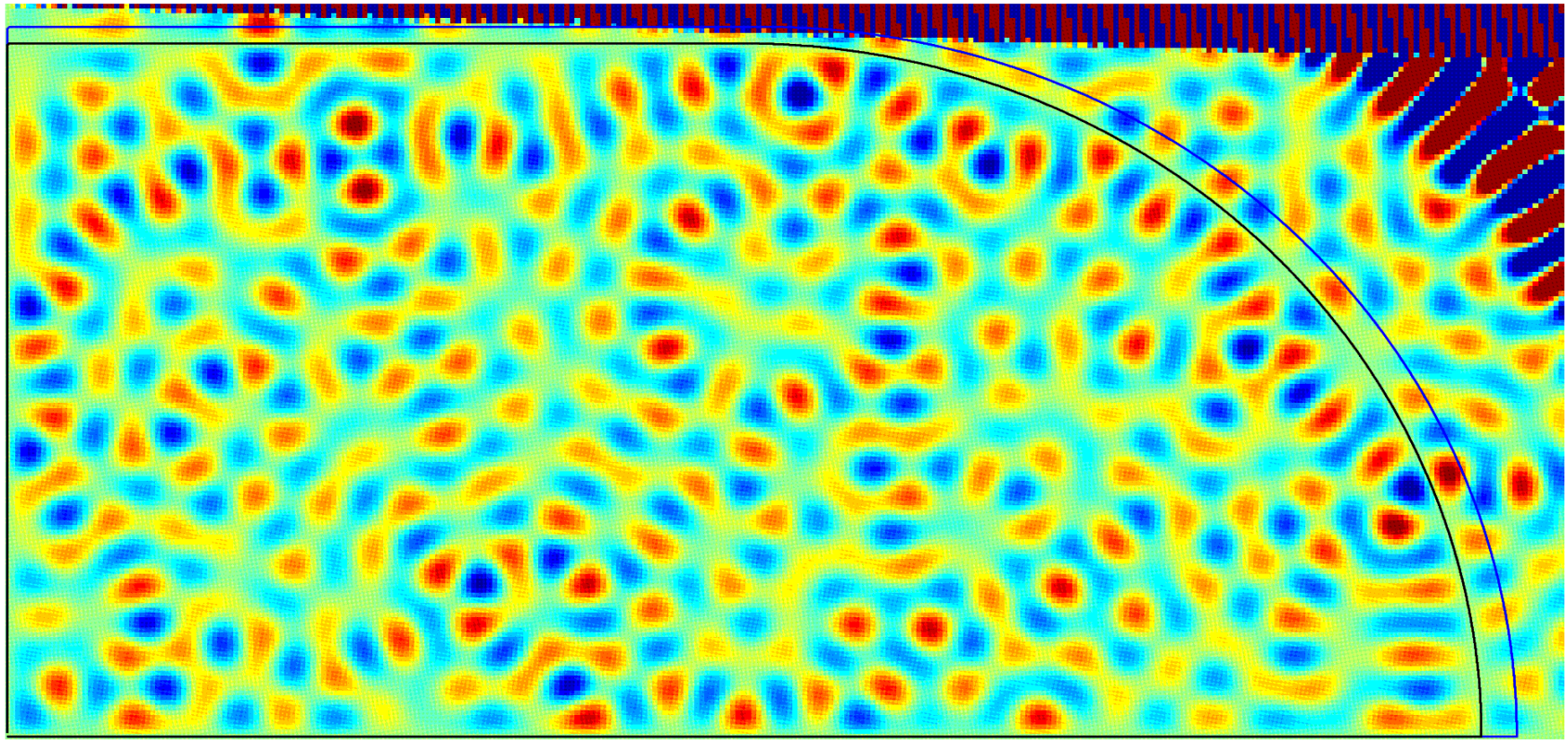
$$\frac{\bar{\nu}(E)}{\bar{N}(E)} = \frac{3\sqrt{3}-5}{\pi} \approx 0.0624$$

Variance:

$$\frac{\sigma^2(\nu(E))}{\bar{N}(E)} = \frac{18}{\pi^2} + \frac{4\sqrt{3}}{\pi} - \frac{25}{2\pi} \approx 0.0502$$

Scaling method

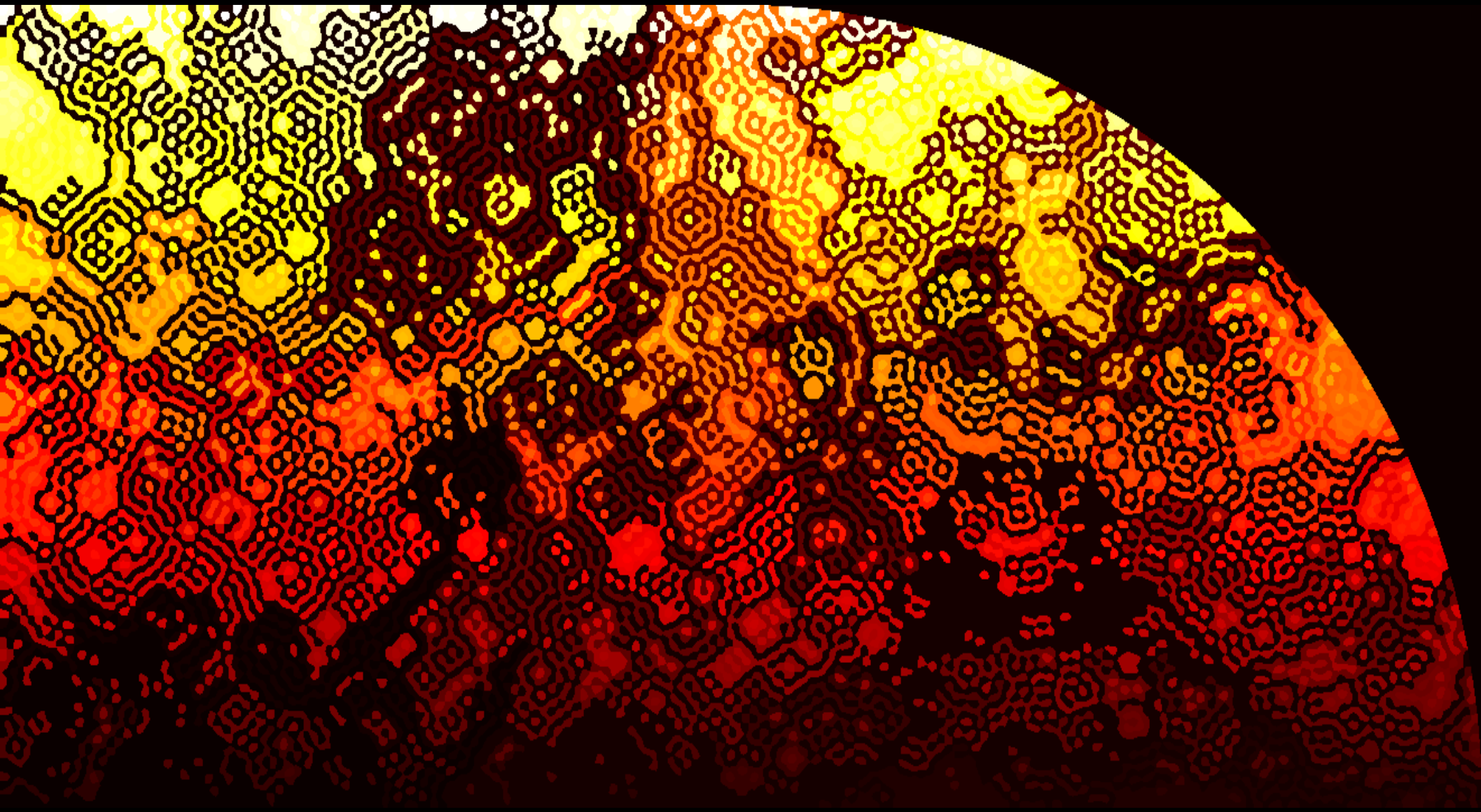
$$\chi_i(k,\mathbf{r}) = u_i\left(\frac{k}{k_i}\mathbf{r}\right) = \sum_{l=1}^{B} X_{li}\xi_l(k,\mathbf{r}) + \epsilon_i(\mathbf{r})$$



E. Vergini. Calculation by scaling of highly excited states of billiards. Physical Review E, (3):2204–2207, 1995.

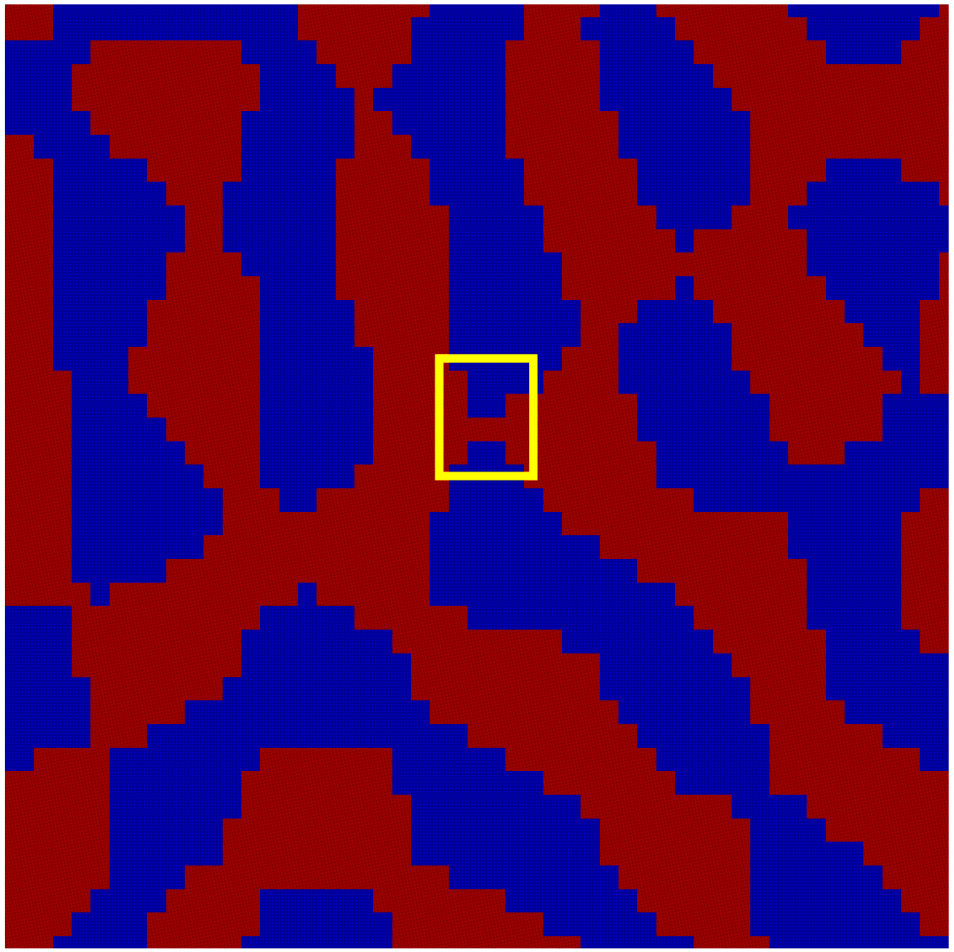
Counting

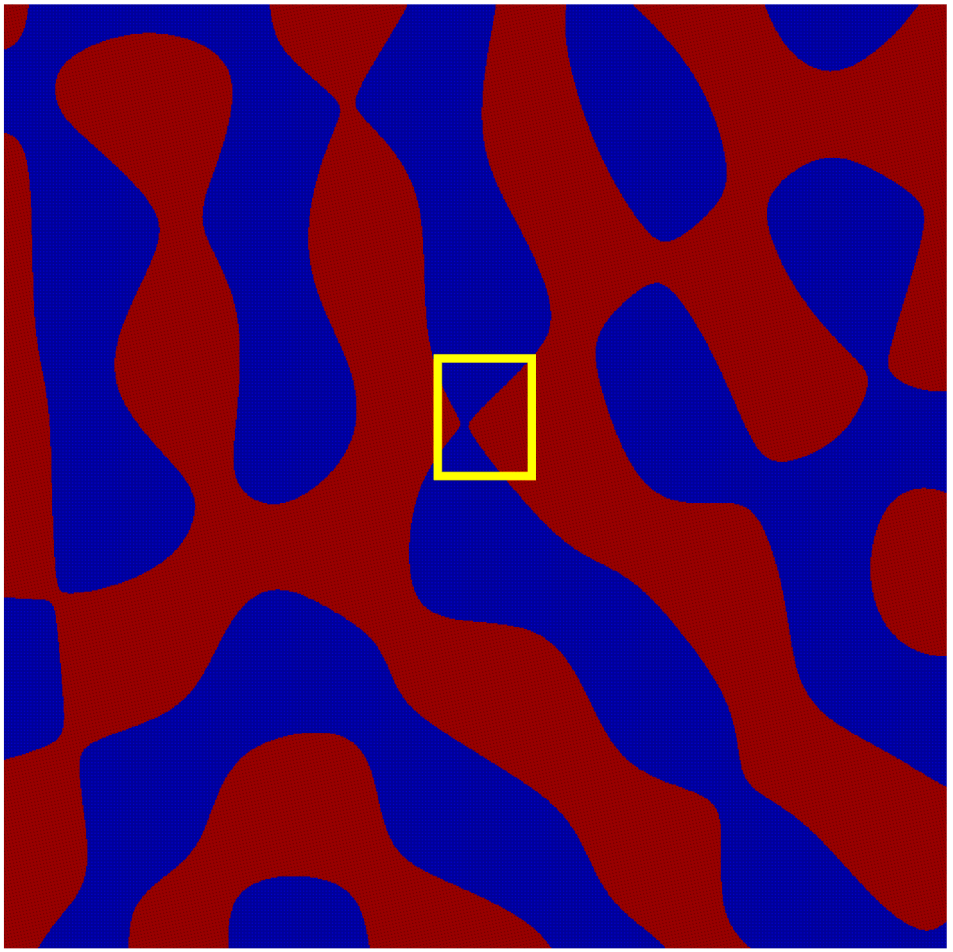
- DFS/BFS hybrid nodal domain traversal
  - Maintain bit array of what is already in stack
  - Smaller constant that DFS or BFS
- O(N) time and space
  - N is number of evaluation points
  - Visits each point once
  - Uses 2N bits to store sign and “already seen”



Interpolation

- Problem: Coarse sampling creates errors





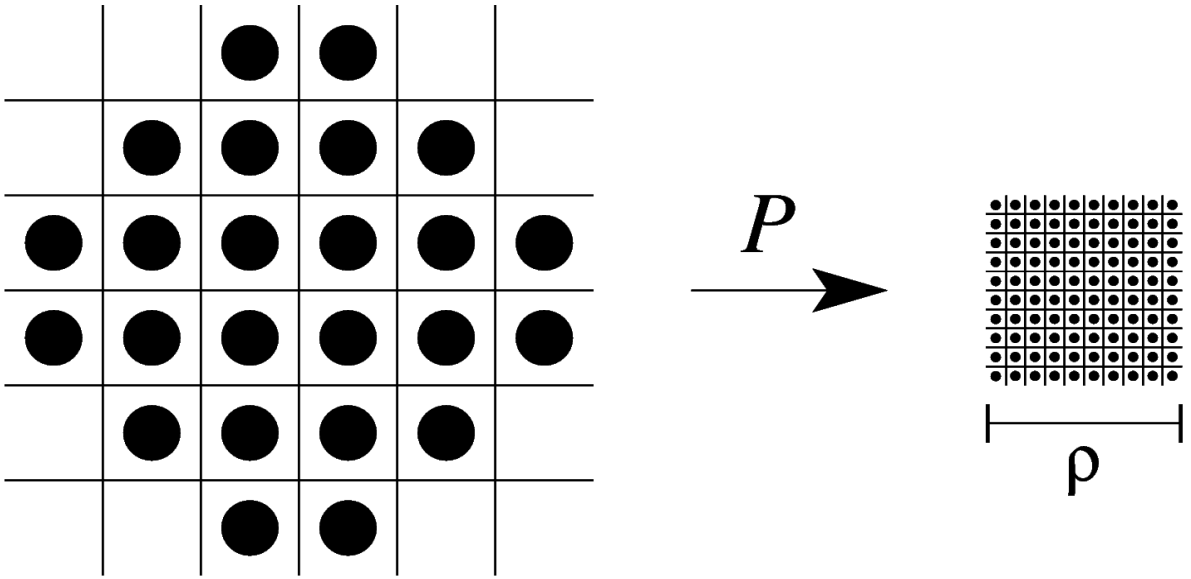
- Solution: use the Helmholtz equation to interpolate

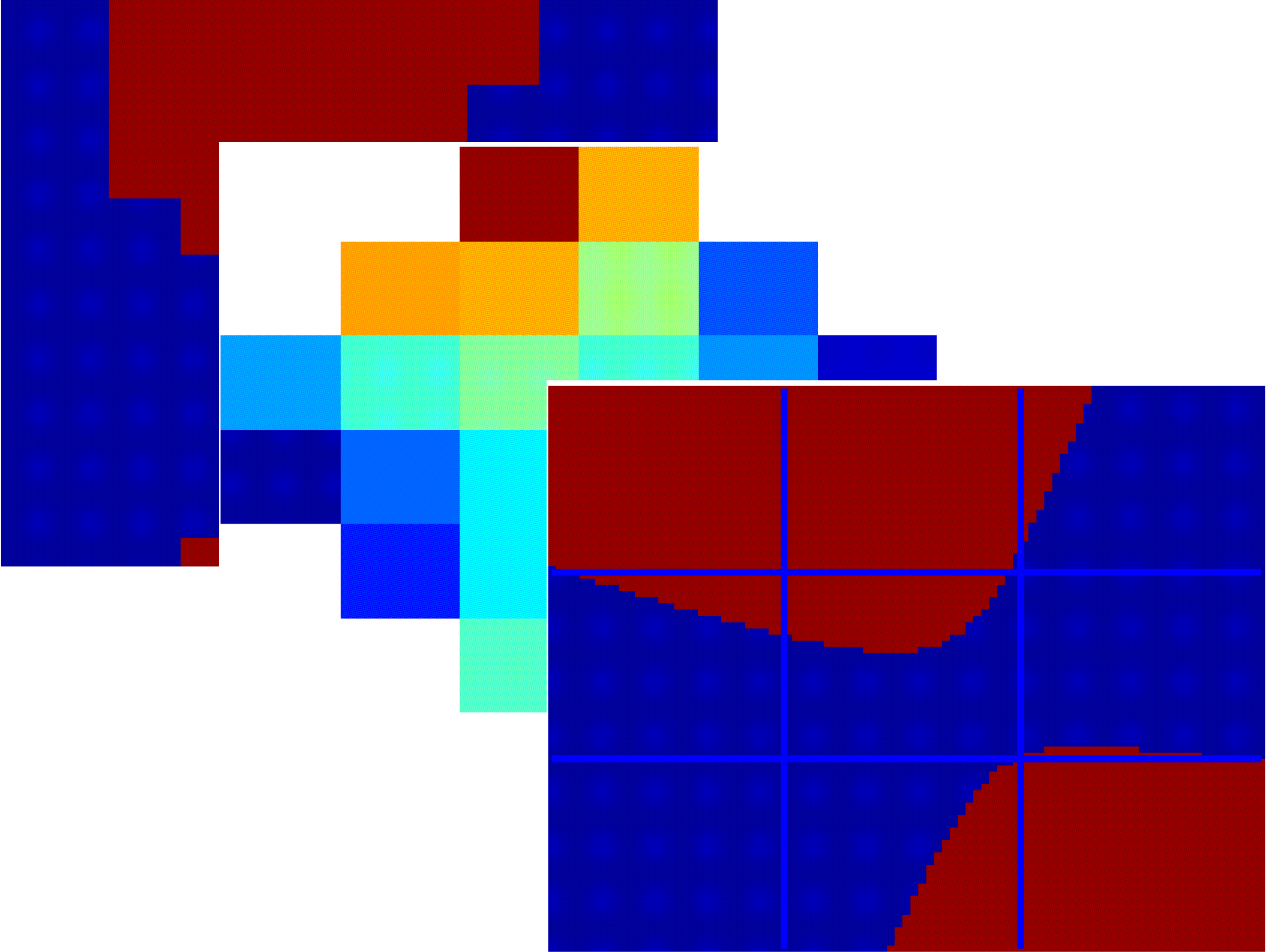
Interpolation

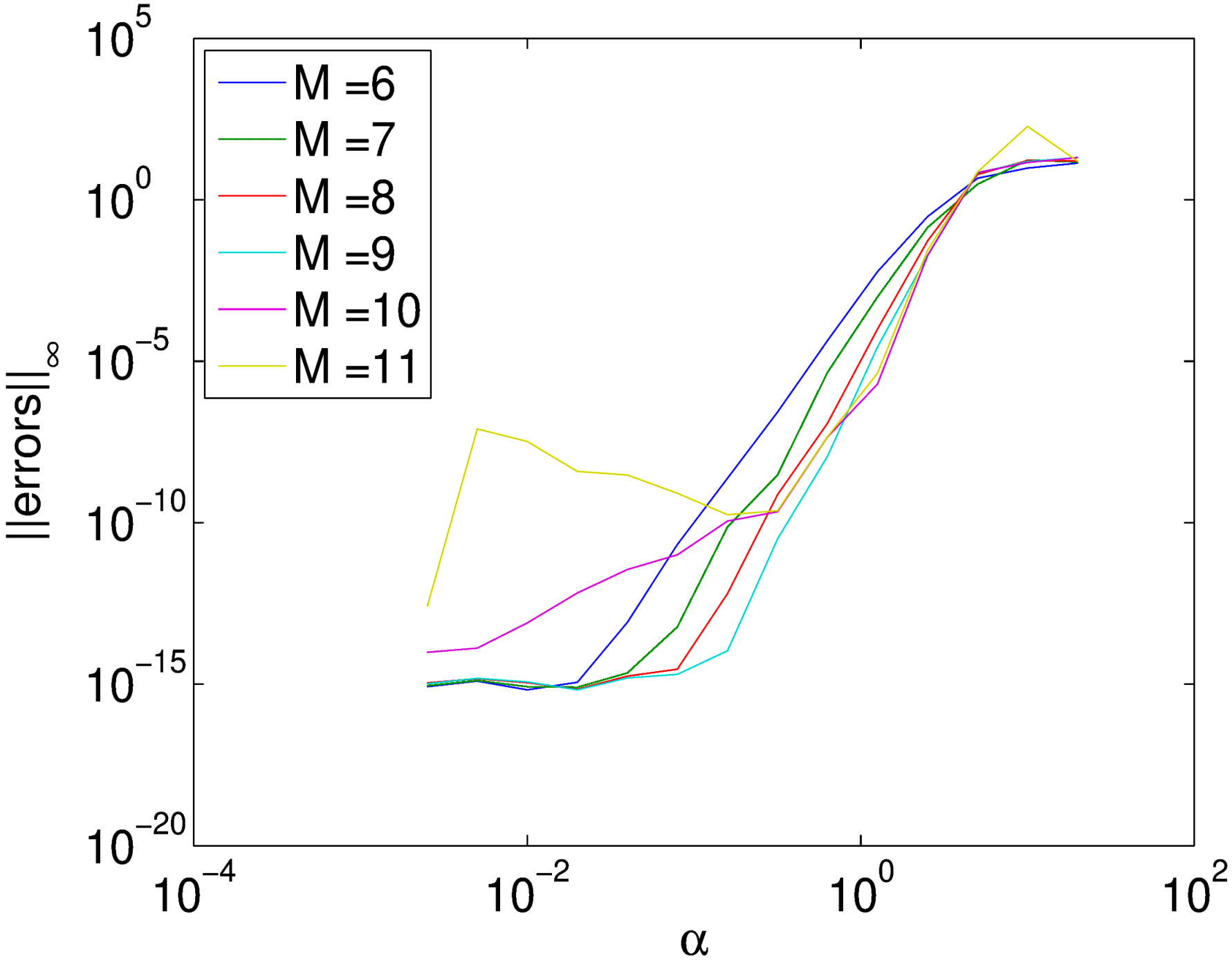
$$\tilde{u}(\mathbf{r}) = \sum_{i=0}^{2M} c_i \zeta_i(\mathbf{r})$$

$$\zeta_i \in \{J_j(kr)\sin(j\theta), J_j(kr)\cos(j\theta)\}_{j=0}^{M}$$

- Performed with a single matrix multiply (at each point)



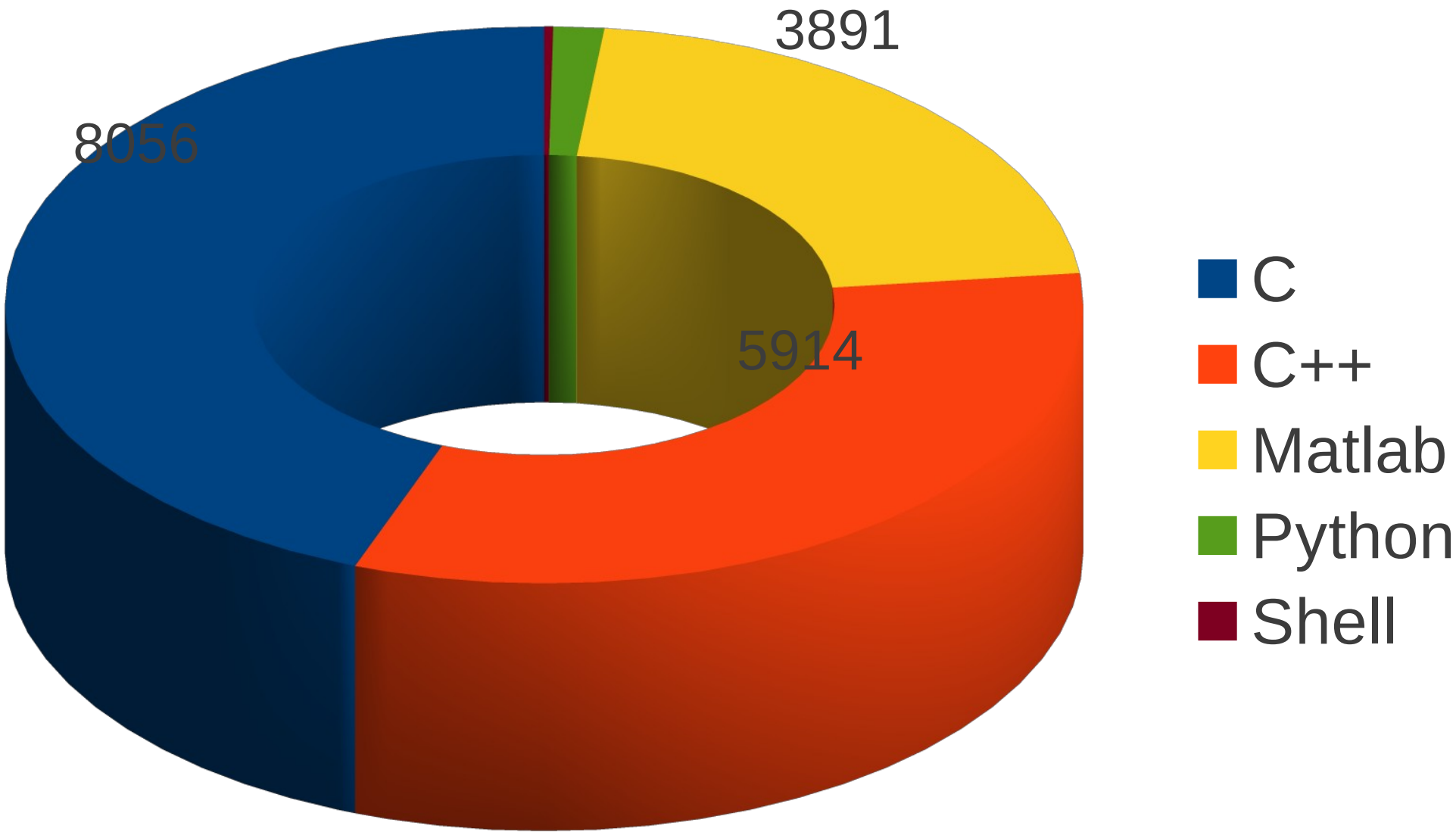



4x4 +2/side
M =6
M =7
M =8
M =9
M =10
M =11
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
$||errors||_{\infty}$
10^{-4}
10^{-2}
10^{0}
10^{2}
α

Results

Conjecture	Sinai	Stadium
Size	8.65e-4	
Mean		
Variance		

- Result figures...


3891
8056
5914
C
C++
Matlab
Python
Shell

- Runtime and data size...

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Spring 2012

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Montréal, Canada

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