COSE312: Compilers

Lecture 3 — Syntax Analysis (1): Context-Free Grammar

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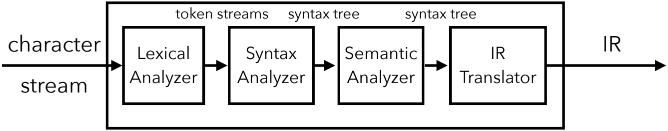
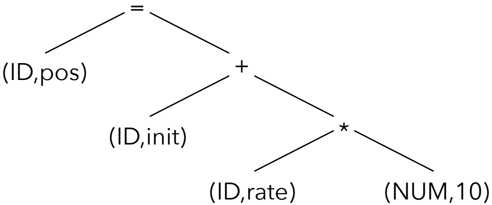
Syntax Analysis (Parsing)



Determine whether or not the input program is syntactically valid. If so, transform the stream

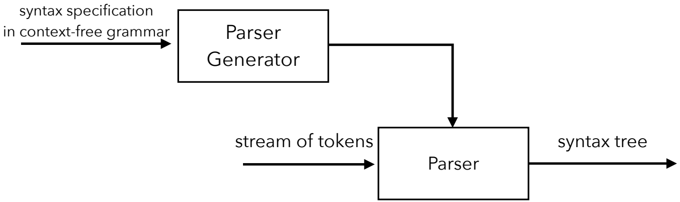
(ID, `pos`), =, (ID, `init`), +, (ID, `rate`), $*$, (NUM,`10`)

into the syntax tree (or parse tree):



Contents

- **Specification**: context-free grammars.
- **Algorithms**: top-down and bottom-up parsing algorithms
- **Tools**: automatic parser generator



Context-Free Grammar

Example: Palindrome

- A string is a palindrome if it reads the same forward and backward.
- $L = \{w \in \{0,1\}^* \mid w = w^R\}$
- L is not regular, but context-free.
- Every context-free language is defined by a recursive definition.
  - Basis: ϵ, 0, and 1 are palindromes.
  - Induction: If w is a palindrome, so are $0w0$ and $1w1$.
- The recursive definition is expressed by a context-free grammar.

$$\begin{array}{lcl} P & \to & \epsilon \\ P & \to & 0 \\ P & \to & 1 \\ P & \to & 0P0 \\ P & \to & 1P1 \end{array}$$

Context-Free Grammar

Definition (Context-Free Grammar)

A context-free grammar G is defined as a quadruple:

$$G = (V, T, S, P)$$

- V: a finite set of *variables* (*nonterminals*)
- T: a finite set of *terminal symbols* (tokens)
- $S \in V$: the start variable
- P: a finite set of *productions*. A production has the form

  $$x \rightarrow y$$

  where $x \in V$ and $y \in (V \cup T)^*$.

Example: Expressions

$$G = (\{E\}, \{(,), \mathbf{id}\}, E, P)$$

where P:

$$E \rightarrow E + E \mid E * E \mid -E \mid (E) \mid \mathbf{id}$$

The language includes $\mathbf{id} * (\mathbf{id} + \mathbf{id})$ because it is "derived" from E as follows:

$$E \Rightarrow E * E \Rightarrow \mathbf{id} * E \Rightarrow \mathbf{id} * (E) \Rightarrow \mathbf{id} * (E + E)$$
$$\Rightarrow \mathbf{id} * (\mathbf{id} + E) \Rightarrow \mathbf{id} * (\mathbf{id} + \mathbf{id})$$

Derivation

Definition (Derivation Relation, $\Rightarrow$)

Let $G = (V, T, S, P)$ be a context-free grammar. Let $\alpha A \beta$ be a string of terminals and variables, where $A \in V$ and $\alpha, \beta \in (V \cup T)^*$. Let $A \rightarrow \gamma$ is a production in G. Then, we say $\alpha A \beta$ derives $\alpha\gamma\beta$, and write

$$\alpha A \beta \Rightarrow \alpha\gamma\beta.$$

Definition ($\Rightarrow^*$, Closure of $\Rightarrow$)

$\Rightarrow^*$ is a relation that represents zero, or more steps of derivations:

- Basis: For any string α of terminals and variables, $\alpha \Rightarrow^* \alpha$.
- Induction: If $\alpha \Rightarrow^* \beta$ and $\beta \Rightarrow \gamma$, then $\alpha \Rightarrow^* \gamma$.

Language of Grammar

Definition (Sentential Forms)

If $G = (V, T, S, P)$ is a context-free grammar, then any string $\alpha \in (V \cup T)^*$ such that $S \Rightarrow^* \alpha$ is a *sentential form*.

Definition (Sentence)

A sentence of G is a sentential form with no non-terminals.

Definition (Language of Grammar)

The language of a grammar G is the set of all sentences:

$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}.$$

Derivation is not unique

At each step in a derivation, there are multiple choices to be made, e.g., a sentence $-(\mathbf{id} + \mathbf{id})$ can be derived by

$$\boldsymbol{E} \Rightarrow -\boldsymbol{E} \Rightarrow -(\boldsymbol{E}) \Rightarrow -(\boldsymbol{E} + \boldsymbol{E}) \Rightarrow -(\mathbf{id} + \boldsymbol{E}) \Rightarrow -(\mathbf{id} + \mathbf{id})$$

or alternatively by

$$\boldsymbol{E} \Rightarrow -\boldsymbol{E} \Rightarrow -(\boldsymbol{E}) \Rightarrow -(\boldsymbol{E} + \boldsymbol{E}) \Rightarrow -(\boldsymbol{E} + \mathbf{id}) \Rightarrow -(\mathbf{id} + \mathbf{id})$$

Leftmost and Rightmost Derivations

- Leftmost derivation: the leftmost non-terminal in each sentential is always chosen. If $\alpha \Rightarrow \beta$ is a step in which the leftmost non-terminal in α is replaced, we write $\alpha \Rightarrow_l \beta$.

$$E \Rightarrow_l -E \Rightarrow_l -(E) \Rightarrow_l -(E+E) \Rightarrow_l -(\mathbf{id}+E) \Rightarrow_l -(\mathbf{id}+\mathbf{id})$$

- Rightmost derivation (canonical derivation): the rightmost non-terminal in each sentential is always chosen. If $\alpha \Rightarrow \beta$ is a step in which the rightmost non-terminal in α is replaced, we write $\alpha \Rightarrow_r \beta$.

$$E \Rightarrow_r -E \Rightarrow_r -(E) \Rightarrow_r -(E+E) \Rightarrow_r -(E+\mathbf{id}) \Rightarrow_r -(\mathbf{id}+\mathbf{id})$$

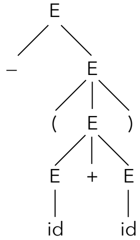
- If $S \Rightarrow_l^* \alpha$, α is a *left sentential form*.
- If $S \Rightarrow_r^* \alpha$, α is a *right sentential form*.

Parse Tree

A graphical tree-like representation of a derivation. E.g., the derivation

$$\boldsymbol{E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(\mathbf{id}+E) \Rightarrow -(\mathbf{id}+\mathbf{id})}$$

is represented by the parse tree:



- Each interior node represents the application of a production.
- The interior node is labeled by the head of the production.
- Children are labeled by the symbols in the body of the production.

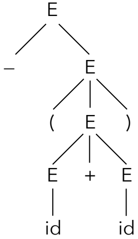
Parse Tree

A parse tree ignores variations in the order in which symbols are replaced. Two derivations

$$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(\mathbf{id}+E) \Rightarrow -(\mathbf{id}+\mathbf{id})$$

$$E \Rightarrow -E \Rightarrow -(E) \Rightarrow -(E+E) \Rightarrow -(E+\mathbf{id}) \Rightarrow -(\mathbf{id}+\mathbf{id})$$

produce the same parse tree:



The parse trees for two derivations are identical if the derivations use the same set of rules (they apply those rules only in a different order).

Ambiguity

A grammar is *ambiguous* if

- it produces more than one parse tree for some sentence,
- it has multiple leftmost derivations, or
- it has multiple rightmost derivations.

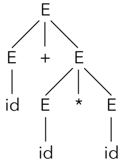
Example

The grammar

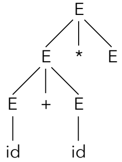
$$E \to E + E \mid E * E \mid -E \mid (E) \mid \mathbf{id}$$

is ambiguous, because it permits two different leftmost derivations for $\mathbf{id} + \mathbf{id} * \mathbf{id}$:

1. $E \Rightarrow E{+}E \Rightarrow \mathbf{id}{+}E \Rightarrow \mathbf{id}{+}E{*}E \Rightarrow \mathbf{id}{+}\mathbf{id}{*}E \Rightarrow \mathbf{id}{+}\mathbf{id}{*}\mathbf{id}$



2. $E \Rightarrow E{*}E \Rightarrow E{+}E{*}E \Rightarrow \mathbf{id}{+}E{*}E \Rightarrow \mathbf{id}{+}\mathbf{id}{*}E \Rightarrow \mathbf{id}{+}\mathbf{id}{*}\mathbf{id}$



Writing a Grammar

Transformations to make a grammar more suitable for parsing:

- eliminating ambiguity
- eliminating left-recursion
- left factoring

Eliminating Ambiguity

We can usually eliminate ambiguity by transforming the grammar. E.g., an ambiguous grammar:

$$E \rightarrow E + E \mid E * E \mid (E) \mid \mathbf{id}$$

To eliminate the ambiguity, we express in grammar

- (precedence) bind $*$ tighter than $+$
  - $1 + 2 * 3$ is always parsed by $1 + (2 * 3)$
- (associativity) $*$ and $+$ associate to the left
  - $1 + 2 + 3$ is always parsed by $(1 + 2) + 3$

An unambiguous grammar:

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow \mathbf{id} \mid (E) \end{aligned}$$

- parse tree for $\mathbf{id} + \mathbf{id} + \mathbf{id}$
- parse tree for $\mathbf{id} + \mathbf{id} * \mathbf{id}$

Exercise

Transform the grammar

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow \mathbf{id} \mid (E) \end{aligned}$$

so that $*$ associate to the right.

Eliminating Left-Recursion

A grammar is left-recursive if it has a non-terminal A such that there A appears as the first right-hand-side symbol in an A-production, e.g.,

$$E \to E + T \mid T$$

To eliminate left-recursion, rewrite the grammar using right recursion:

$$\begin{aligned} &E \to T\ E' \\ &E' \to +\ T\ E' \\ &E' \to \epsilon \end{aligned}$$

Left Factoring

The grammar

$$S \to \textbf{if } E \textbf{ then } S \textbf{ else } S$$
$$S \to \textbf{if } E \textbf{ then } S$$

has rules with the same prefix. We can *left factor* the grammar as follows:

$$S \to \textbf{if } E \textbf{ then } S\ X$$
$$X \to \epsilon$$
$$X \to \textbf{else } S$$

Summary

- The syntax of a programming language is usually specified by a context-free grammar.
  - derivation, left/rightmost derivations
  - parse trees
  - ambiguous/unambiguous grammars
  - grammar transformation (eliminating ambiguity, eliminating left-recursion, left factoring)