Aided Navigation: GPS with High Rate Sensors Errata list

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February 16, 2011

Abstract. This document records and corrects errors in the book "Aided Navigation: GPS with High Rate Sensors." The most up-to-date version of this document can be obtained from the authors website "www.ee.ucr.edu/$\sim$farrell".

Thank you to the various readers who have requested clarifications or pointed out the errors corrected herein.

Chapter 1

Introduction

p.8, eqn. (1.1) – For consistency, the equation should read "$\dot{\hat{v}} = \hat{a}(t)$." The correction has no consequence in the remainder of the example since the discussion on the top of page 9 makes clear that (for this simple example) $\hat{a} = \tilde{a}$.

Chapter 2

Reference Frames

p.31, Table 2.1 – The title for the second column should be "Symbol."

p. 40, text line 3 – "realized" should be "realize"

p.45, last eqn. of Example 2.4 – The eqn. should read

$$\begin{bmatrix} n \\ e \\ d \end{bmatrix}^t = \mathbf{R}_e^t \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}^e - \begin{bmatrix} -2.430601 \\ -4.702442 \\ 3.546587 \end{bmatrix} \times 10^6 \right)$$

p.45 – The sentence at the middle of the page should read "Next, using eqns. (2.78–2.79) it is straightforward ..."

p.49, eqn. (2.42) – The eqn. should read

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}^b = \mathbf{R}_2^b \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix}$$

p.49 – The penultimate sentence should read "... rotations has singular points ...."

p.50 – The second sentence of Section 2.5.5 should read "A small angle transformation is ..."

p.51 – In the denominator of eqn. (2.45) the "$\mathbf{R}_b^t$" should be "$\mathbf{R}_b^g$."

p.52 – Eqn. (2.55) should read

$$\mathbf{\Omega}_{eg}^g = \begin{bmatrix} 0 & \dot{\lambda}\sin(\phi) & -\dot{\phi} \\ -\dot{\lambda}\sin(\phi) & 0 & -\dot{\lambda}\cos(\phi) \\ \dot{\phi} & \dot{\lambda}\cos(\phi) & 0 \end{bmatrix}.$$

The dot indicating the time derivative was misplaced on the (2,3) element.

p.55 – The last of the three equations between eqns. (2.62) and (2.63) should read

$$\frac{d}{dt}\left(e^{\int_{t_{k-1}}^{t}\mathbf{\Omega}d\tau}\mathbf{R}_b^a(t)\right) = \mathbf{0}$$

where the dt has been corrected to $d\tau$.

Chapter 3

Deterministic Systems

p.74 – In the penultimate sentence of Section 3.4, the eqn. reference should be to "(3.41)–(3.42)."

p.79 – In Example 2.12, the eigenvalues are at $-1 \pm j$. The negative sign in the real part was missing in the original text.

p.89 – Near the middle of the page, the definition should be $a_i = -\mathbf{u}_i \mathbf{\Phi}^n \mathbf{p}$. The negative sign is missing in the text.

p.92 – The role of the integer r_o is confused in the discussion. The following changes should correct the confusion.

1. The dimensions of $\mathbf{v}_1$ and $\mathbf{v}_2$ are reversed. They should be $\mathbf{v}_1 \in \mathbb{R}^{r_o}$ and $\mathbf{v}_2 \in \mathbb{R}^{n-r_o}$.
2. The column dimensions for the matrices are backwards. They should be $\mathbf{W}_1 \in \mathbb{R}^{n \times r_o}$ and $\mathbf{W}_2 \in \mathbb{R}^{n \times (n-r_o)}$.

p. 103 – In the first line after eqn. (3.105), the definition should be

$$\delta \varepsilon_a^g = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \end{bmatrix} \mathbf{u}_2^p.$$

Chapter 4

Stochastic Processes

p.111 – The second equation on the page should read:

$$P\{W_1 < w \le W_2\} = F_w(W_2) - F_w(W_1) = \int_{W_1}^{W_2} p_w(W)dW.$$

p.117 – Four lines below eqn. (4.20) "the the" should be "the".

p.117 – Five lines below eqn. (4.20) "small random affects" should be "small random effects".

p.124 – Eqn. (4.36) should read

$$\mathrm{R}_v(\tau) = \sigma_v^2 \delta(\tau).$$

p.124 – The last equation on the page should read:

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} v^2(t)dt = R_v(0) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S_v(j\omega)d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \sigma_v^2 d\omega = \infty. \end{aligned}$$

p.124 – In the penultimate sentence of the penultimate paragraph (i.e., six lines from the bottom of the page), "Gaussion" should be "Gaussian".

p.128 – Eqn. (4.50) should read

$$S_y(j\omega) = \frac{2\beta\sigma_w^2}{\beta^2 + \omega^2}.$$

The change is the ω in the denominator.

p.135 – In the middle paragraph of the page, "given that σ_ω is the PSD" should read "given that σ_ω^2 is the PSD".

p.143 – In the last sentence of the first paragraph, "$\mathbf{Q}_d$" should be "$\mathbf{Qd}$".

p.148 – In the second line, "principle" should be "principal".

p.148 – The second line of the main equation on the page should be

$$= \frac{1}{(2\pi)^{\frac{3}{2}}} \int_{\|\mathbf{v}\| \leq c} \exp\left(-\frac{\|\mathbf{v}\|^2}{2}\right) d\mathbf{v}$$

p.148 – The phrase near the middle of the page should read "... [39], but this value was ...."

p.158 – The last sentence and a half should be:

$$\int_0^t \mathbf{\Phi}(\tau)\mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^\top\mathbf{\Phi}^\top(\tau)d\tau = \begin{bmatrix} \frac{\sigma_{v_a}^2 t^3}{3} + \frac{\sigma_{\omega_b}^2 t^5}{20} & \frac{\sigma_{v_a}^2 t^2}{2} + \frac{\sigma_{\omega_b}^2 t^4}{8} & \frac{\sigma_{\omega_b}^2 t^3}{6} \\ \frac{\sigma_{v_a}^2 t^2}{2} + \frac{\sigma_{\omega_b}^2 t^4}{8} & \sigma_{v_a}^2 t + \frac{\sigma_{\omega_b}^2 t^3}{3} & \frac{\sigma_{\omega_b}^2 t^2}{2} \\ \frac{\sigma_{\omega_b}^2 t^3}{6} & \frac{\sigma_{\omega_b}^2 t^2}{2} & \sigma_{\omega_b}^2 t \end{bmatrix}$$

where we have used the fact that $\mathbf{P}(0) = diag(P_{p_0}, P_{v_0}, P_{b_0})$. Therefore, the error variance of each of the three states is described by

$$\begin{aligned} P_p(t) &= \left(P_{p_0} + P_{v_0}t^2 + P_{b_0}\frac{t^4}{4}\right) + \left(\frac{\sigma_{v_a}^2 t^3}{3} + \frac{\sigma_{\omega_b}^2 t^5}{20}\right) \\ P_v(t) &= \left(P_{v_0} + P_{b_0}t^2\right) + \left(\sigma_{v_a}^2 t + \frac{\sigma_{\omega_b}^2 t^3}{3}\right) \\ P_b(t) &= P_{b_0} + \sigma_{\omega_b}^2 t. \end{aligned}$$

p.160 – The eigenvalues for the discrete-time system are $0.90 \pm 0.07j$, and 0.85.

Chapter 5

Optimal State Estimation

p.171, eqn. (5.3) – The 'hat' is missing from the $\mathbf{x}_{k-1}^{+}$ on the right-hand side. The equation should read

$$\hat{\mathbf{x}}_k^{-} = \mathbf{\Phi}\hat{\mathbf{x}}_{k-1}^{+} + \mathbf{G}\mathbf{u}_{k-1}.$$

p.180 – Table 5.1 shows that, for large m, the computational and memory requirements of the batch algorithm are $O(n^2 m)$ and $O(nm)$, respectively.

Fig. 5.3, p. 188 – The signs for the difference between y_k and z_k are reversed.

Example 5.4, p.188 – In the example, the notation $P_x(k)$ is used, when it should have been $P_{\hat{x}}(k)$.

Example 5.4 – Following is additional information regarding the derivation of $P_{\hat{x}}(k)$.

Some preliminary statements are useful that aid the derivation.

1. The example goes through for two different sets of assumptions. First, x_k and B can be considered deterministic, but unknown. Second, x_k and B can be considered as random variables that are uncorrelated with each other and with n_k and v_k. Under either assumption, B is a constant. Those assumptions were not clearly stated, but either is reasonable.

   Under either of these assumptions:

$$\begin{aligned} var(z_k) &= E\Big\langle (z_k - E\langle z_k \rangle)^2 \Big\rangle \\ &= E\Big\langle (x_k + v_k + B - E\langle x_k + v_k + B \rangle)^2 \Big\rangle \end{aligned}$$

$$
\begin{aligned}
&= E\Big\langle (x_k - E\langle x_k\rangle + v_k + B - E\langle B\rangle)^2\Big\rangle \\
&= E\Big\langle (x_k - E\langle x_k\rangle)^2 + E\big\langle v_k^2\big\rangle + E\big\langle B - E\langle B\rangle)^2\Big\rangle \\
&= 0 + (\mu\sigma)^2 + 0 \qquad (5.1)
\end{aligned}
$$

2. The estimate $\hat{x}_k$ is unbiased. This is easily shown:

$$
\begin{aligned}
E\langle x_k - \hat{x}_k\rangle &= E\langle x_k - \Big(z_k - \hat{B}_k\Big) \\
&= E\langle x_k - \Big(x_k + v_k + B - \hat{B}_k\Big)) \\
&= E\langle \hat{B}_k - B\rangle + E\langle v_k\rangle \\
&= 0 \qquad (5.2)
\end{aligned}
$$

where the fact that $E\langle \hat{B}_k - B\rangle = 0$ is true from Example 5.3.

3. Note that z_k and $\hat{B}_k$ are cross-correlated:

$$
\begin{aligned}
E\langle z_k\hat{B}_k\rangle &= E\left\langle z_k\left(\hat{B}_{k-1} + \frac{1}{k}\left(r_k - \hat{B}_{k-1}\right)\right)\right\rangle \\
&= E\left\langle z_k\left(\hat{B}_{k-1} + \frac{1}{k}\left(z_k - y_k - \hat{B}_{k-1}\right)\right)\right\rangle \\
&= E\left\langle z_k\hat{B}_{k-1}\left(1 - \frac{1}{k}\right) + \frac{1}{k}z_k^2 - \frac{y_k z_k}{k}\right\rangle \\
&= 0 + \frac{\mu^2\sigma^2}{k} + 0.
\end{aligned}
$$

4. Similarly, v_k and $\hat{B}_k$ are cross-correlated:

$$
\begin{aligned}
E\langle v_k\hat{B}_k\rangle &= E\left\langle v_k\left(\hat{B}_{k-1} + \frac{1}{k}\left(r_k - \hat{B}_{k-1}\right)\right)\right\rangle \\
&= E\left\langle v_k\left(\hat{B}_{k-1} + \frac{1}{k}\left(z_k - y_k - \hat{B}_{k-1}\right)\right)\right\rangle \\
&= E\left\langle v_k\hat{B}_{k-1}\left(1 - \frac{1}{k}\right) + \frac{1}{k}z_k v_k - \frac{y_k v_k}{k}\right\rangle \\
&= 0 + \frac{\mu^2\sigma^2}{k} + 0.
\end{aligned}
$$

Derivation:

$$
\begin{aligned}
P_{\hat{x}}(k) &= E\left\langle (x_k - \hat{x}_k)^2\right\rangle \\
&= E\left\langle \left(x_k - (z_k - \hat{B}_k)\right)^2\right\rangle \\
&= E\left\langle \left(x_k - (x_k + v_k + B - \hat{B}_k)\right)^2\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= E\left\langle \left((\hat{B}_k - B) - v_k \right)^2 \right\rangle \\
&= E\left\langle (\hat{B}_k - B)^2 \right\rangle - 2E\left\langle (\hat{B}_k - B)v_k \right\rangle + E\left\langle v_k^2 \right\rangle \\
&= \quad \left(1+\mu^2\right)\frac{\sigma^2}{k} \quad - \quad 2\frac{\mu^2\sigma^2}{k} \quad + \quad \mu^2\sigma^2 \\
&= \left(\frac{1-\mu^2}{k} + \mu^2 \right)\sigma^2
\end{aligned}
$$

Checks: To investigate the validity of $P_{\hat{x}_k}$ consider the following two checks on the result.

1. For $k = 1$, it is true that $\hat{x}_1 = y_1$. Therefore, $P_{\hat{x}_k} = \sigma^2$.
2. As $k \to \infty$, $P_{\hat{x}_k} \to (\mu\sigma)^2$.

The result derived above satisfies both of these checks.

Chapter 6

Performance Analysis

p.218, eqn. (6.4) – The matrix $\hat{\mathbf{\Phi}}$ should be $\hat{\mathbf{\Phi}}_k$.

p.220, Step 3 – The $\mathbf{Q}d_k$ should be $\hat{\mathbf{Q}}d_k$.

p.225 — In line 9, "principal" should be "principle."

p.233 — "Exercise" should be "Exercises."

Chapter 7

Navigation System Design

p.241 − The following paragraphs are meant to clarify the ideas of Section 7.2.5.1. The discussion assumes that $\lambda > 0$. The text should state that the system is only observable over time intervals during which the acceleration $u(t)$ is not constant. The following discussion supports this statement.

Assuming that the acceleration measurement $u(t)$ is constant, the observability matrix is

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -\lambda_y & 0 \\ 0 & 0 & 1 & \lambda_y^2 & u \\ 0 & 0 & 0 & -\lambda_y^3 & 0 \\ 0 & 0 & 0 & \lambda_y^4 & 0 \end{bmatrix}$$

as stated on page 241. At any time, given that $u(t)$ is constant, $rank(\mathcal{O}) = 4$, because the last two rows are linearly dependent. Given this observability matrix, using the methods of Section 3.6.3, the subspace spanned by the vectors:

$$\mathbf{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{q}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{q}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ u \end{bmatrix}, \mathbf{q}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

is observable. Vectors in the subspace spanned by the vector:

$$\mathbf{q}_5 = \begin{bmatrix} 0 \\ 0 \\ -u \\ 0 \\ 1 \end{bmatrix}$$

are not observable. This mathematically shows the fact that for a constant acceleration, a constant bias of magnitude b_u is indistinguishable from a scale factor error of magnitude $\alpha = \frac{-b_u}{u}$.

For a constant acceleration, the estimation error converges to the subspace spanned by $\mathbf{q}_5$. Note however that this subspace is different for different values of the acceleration (i.e., the vector $\mathbf{q}_5(u)$ is a function of u). The only vector in the intersection of the subspace spanned by $\mathbf{q}_5(u_1)$ and subspace spanned by $\mathbf{q}_5(u_2)$ for $u_1 \neq u_2$ is the zero vector. Therefore, over intervals of time where the acceleration u is changed, the system should be observable; however, the results in the Aided Navigation book only discuss observability for time invariant systems.

For the following discussion of time varying systems, consider without loss of generality the time interval $t \in [0, t]$. For a time varying system, we are interested in the rank of the observability grammian defined as

$$\mathbf{M}(t, 0) = \int_0^t \Phi(\tau, 0)^\top H^\top H \Phi(\tau, 0) d\tau$$

where $\Phi(t, 0)$ is the state transition matrix (see Section 3.5.3) for the time varying matrix $\mathbf{F}$. For this example, with $\tau = 0$ and $\mathbf{F}$ as defined in eqn. (7.15), the state transition matrix is

$$\mathbf{\Phi}(t, 0) = \begin{bmatrix} 1 & t & \frac{t^2}{2} & 0 & \int_0^t \int_0^s u(\tau) d\tau ds \\ 0 & 1 & t & 0 & \int_0^t u(\tau) d\tau \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & e^{-\lambda t} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Therefore, the observability grammian is

$$\mathbf{M}(t, 0) = \int_0^t \begin{bmatrix} 1 & \tau & \frac{\tau^2}{2} & e^{-\lambda\tau} & g(\tau) \\ \tau & \tau^2 & \frac{\tau^3}{2} & \tau e^{-\lambda\tau} & \tau g(\tau) \\ \frac{\tau^2}{2} & \frac{\tau^3}{2} & \frac{\tau^4}{4} & \frac{\tau^2}{2} e^{-\lambda\tau} & \frac{\tau^2}{2} g(\tau) \\ e^{-\lambda\tau} & \tau e^{-\lambda\tau} & \frac{\tau^2}{2} e^{-\lambda\tau} & e^{-2\lambda\tau} & e^{-\lambda\tau} g(\tau) \\ g(\tau) & \tau g(\tau) & \frac{\tau^2}{2} g(\tau) & g(\tau) & (g(\tau))^2 \end{bmatrix} d\tau.$$

where $g(t) = \int_0^t \int_0^s u(\tau) d\tau ds$. This matrix has rank 5 unless $u(t)$ is a constant vector, in which case the third and fifth columns are identical and the grammian has rank 4. Therefore, the system is observable over time intervals where the acceleration is not constant.

p.246 – In Table 7.1, the heading of the fourth column should be σ_{b_y}.

p.252 – In eqn. (7.29), the left-hand side should be $\tilde{\mathbf{f}}(\hat{\mathbf{x}}, \mathbf{0})$.

Chapter 8

Global Positioning System

p.265 – On line 14, “course” should be “coarse.”

p.268 – The second sentence in the last paragraph should read “Therefore, the estimated value of $\mathbf{x}$ will be affected by the error $\left(\mathbf{p}^i - \hat{\mathbf{p}}^i\right)$.”

p.274 – Two lines after eqn. (8.18) the left-hand side of the equation should be $\bar{\mathbf{R}}_a^e$.

p. 306 – Following eqn. (8.84) the phrase should be “where E_{cm} is ... than ...”

Chapter 9

GPS Aided Encoder-Based Dead-Reckoning

No known errors.

Chapter 10

AHRS

No known errors.

Chapter 11

Aided Inertial Navigation

p.381 – The last phrase on the page should read "In an inertial reference frame ..."

p.406 – In the penultimate paragraph in the last sentence, the phrase "Specific definitions for F_{vg} and ..." should be "Specific definitions for $F_{\rho g}$ and ..."

p.412– In eqn. (11.134), ν_a should be ν_g

Chapter 12

LBL and Doppler Aided INS

p.447 – In the second paragraph from the bottom, the end of the first sentence should read "... it will always be the case that $\delta\hat{\mathbf{x}}^{+}(t_0)$ will be zero."

Appendix A

Notation

No known errors.

Appendix B

Linear Algebra Review

p.467 – Lemma B.5.2 eqn. (B.32) is missing an inverse. The equation should read

$$(\mathbf{A} + \mathbf{BCD})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{B}\left(\mathbf{DA}^{-1}\mathbf{B} + \mathbf{C}^{-1}\right)^{-1}\mathbf{DA}^{-1}.$$

p.472 – In Section B.11, in the final result for $\mathbf{P}$, the first column of the first row that reads

$$d_{11} + d_{22}u_{a2}^2 + d_{33}u_{13}^2$$

should read

$$d_{11} + d_{22}u_{12}^2 + d_{33}u_{13}^2.$$

Appendix C

Calculation of GPS Satellite Position & Velocity

No known errors.

Appendix D

Quaternions

No known errors.