

# Introduction to Computational Neuroscience

## Lecture 5: Data analysis II

Lesson	Title
1	Introduction
2	Structure and Function of the NS
3	Windows to the Brain
4	Data analysis
5	Data analysis II
6	Single neuron models
7	Network models
8	Artificial neural networks
9	Learning and memory
10	Perception
11	Attention & decision making
12	Brain-Computer interface
13	Neuroscience and society
14	Future and outlook:AI
15	Projects presentations
16	Projects presentations

Basics

Analyses

Models

Cognitive

Applications

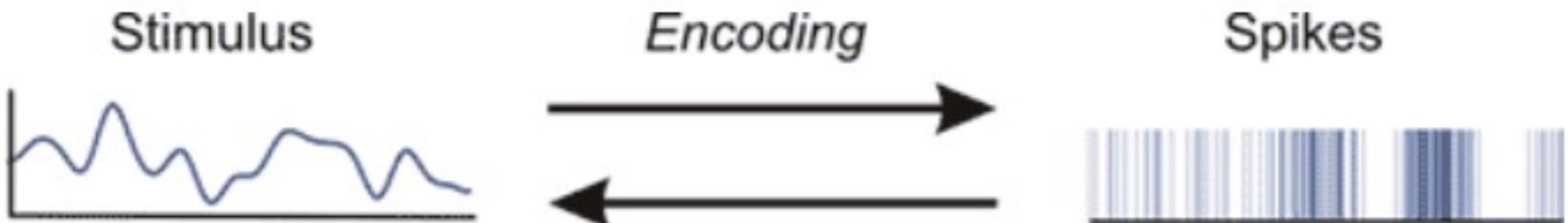
# What is real?



“What is real? How do you define real? If you’re talking about what you can hear, what you can smell, taste and feel, then real is simply electrical signals interpreted by your brain.” Morpheus to Neo

# The link between stimulus and neural response

can be studied from two different perspectives:



**External  
world**

**Activity  
inside your  
brain**

# Learning objectives

- Understand the concepts of neuronal encoding and decoding
- Describe some candidates for neural codes
- Describe some read-out strategies and their applications

# **Neuronal encoding**

Neuronal decoding

# Neuronal encoding

Neural code

Tuning curves

Some possible codes

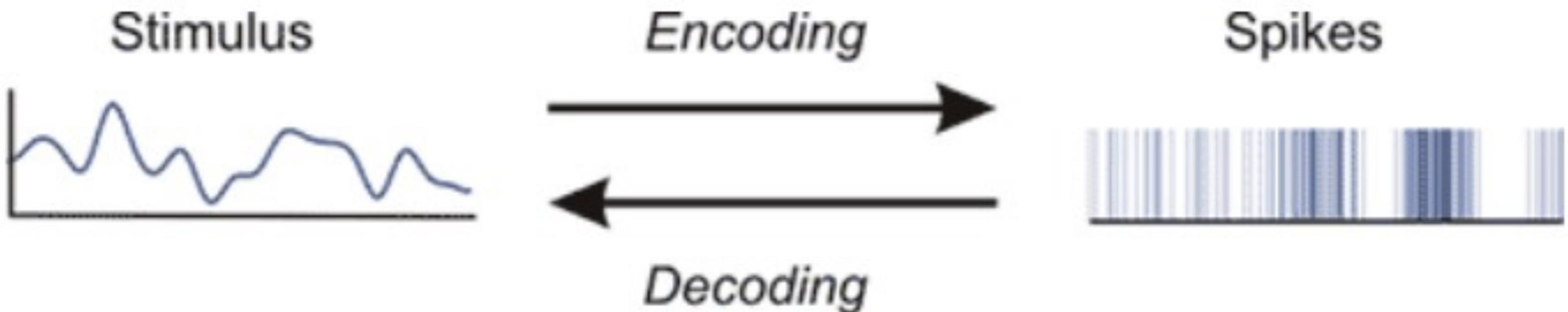
# Neural encoding

**Encoding:** discovering the map from stimulus to response

$$s_1 \rightarrow r_1 \quad s_2 \rightarrow r_2 \quad s_3 \rightarrow r_3 \quad s_{\text{NEW}} \rightarrow ?$$

How information about a stimulus is transformed into patterns of action potentials?

Given a stimulus, predict the neuronal response



# Encoding and decoding

In mathematical terms:

$P(R|S)$  **Encoding**

$P(S|R)$  **Decoding**

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**Bayes theorem** relates encoding and decoding:

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$$P(R|S) \quad \text{Encoding} \quad P(S|R) \quad \text{Decoding}$$

**Bayes theorem** relates encoding and decoding:

$$P(S, R) = P(S|R)P(R) = P(R|S)P(S)$$

# Encoding and decoding

In mathematical terms:

$$P(R|S) \quad \text{Encoding} \quad P(S|R) \quad \text{Decoding}$$

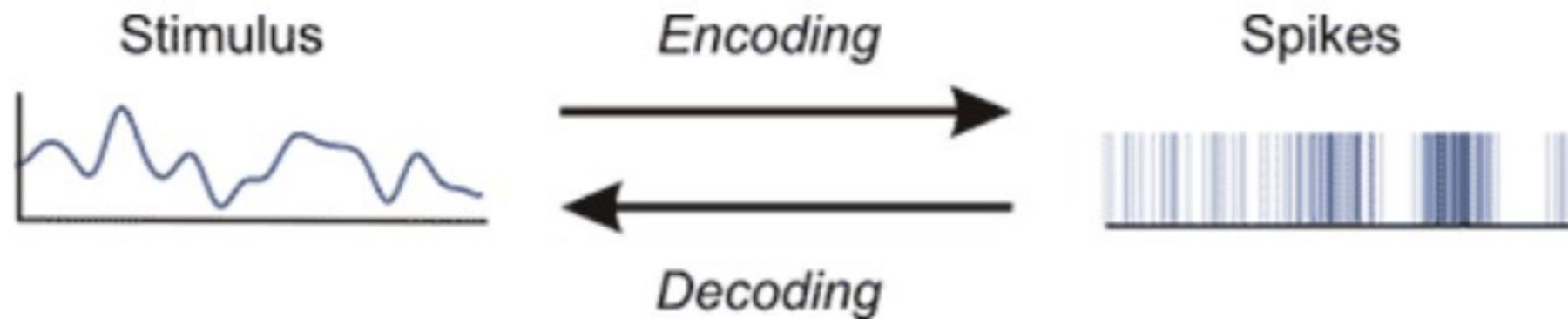
**Bayes theorem** relates encoding and decoding:

$$P(S, R) = P(S|R)P(R) = P(R|S)P(S)$$

$$P(S|R) = \frac{P(R|S)P(S)}{P(R)}$$

# Neural code

Investigating the neural code is like building a **dictionary**



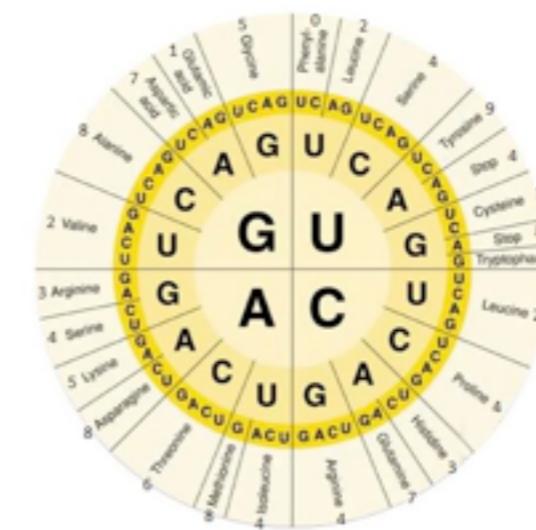
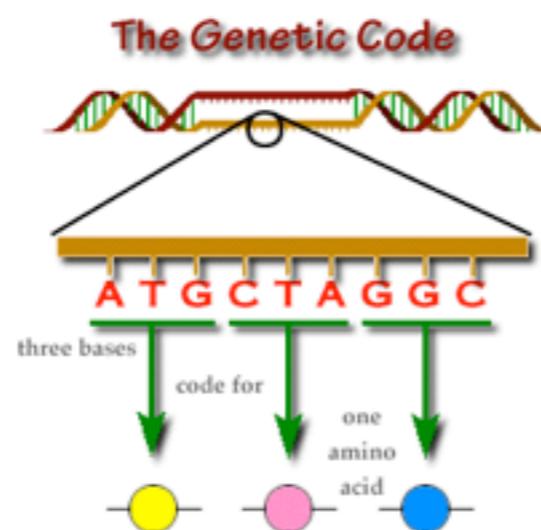
- 1 Translate **from the external world** (sensory stimuli or motor action) **to internal neural representation**
- 2 Translate **neural representations to external world**
- 3 Similar to dictionaries, one-to-many and many-to-one representations are possible

# Neural code

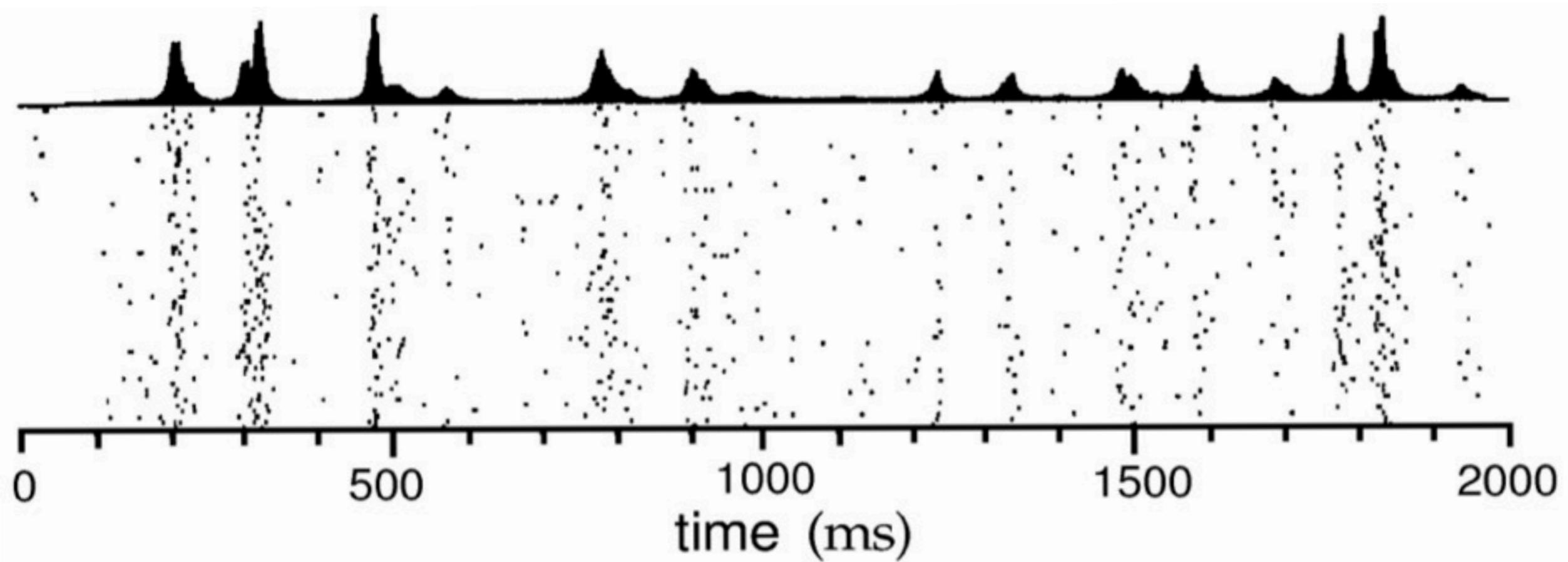
Two really hard problems for revealing the neural code:

- 1 Both stimuli and neural response are high-dimensional signals
- 2 Noise

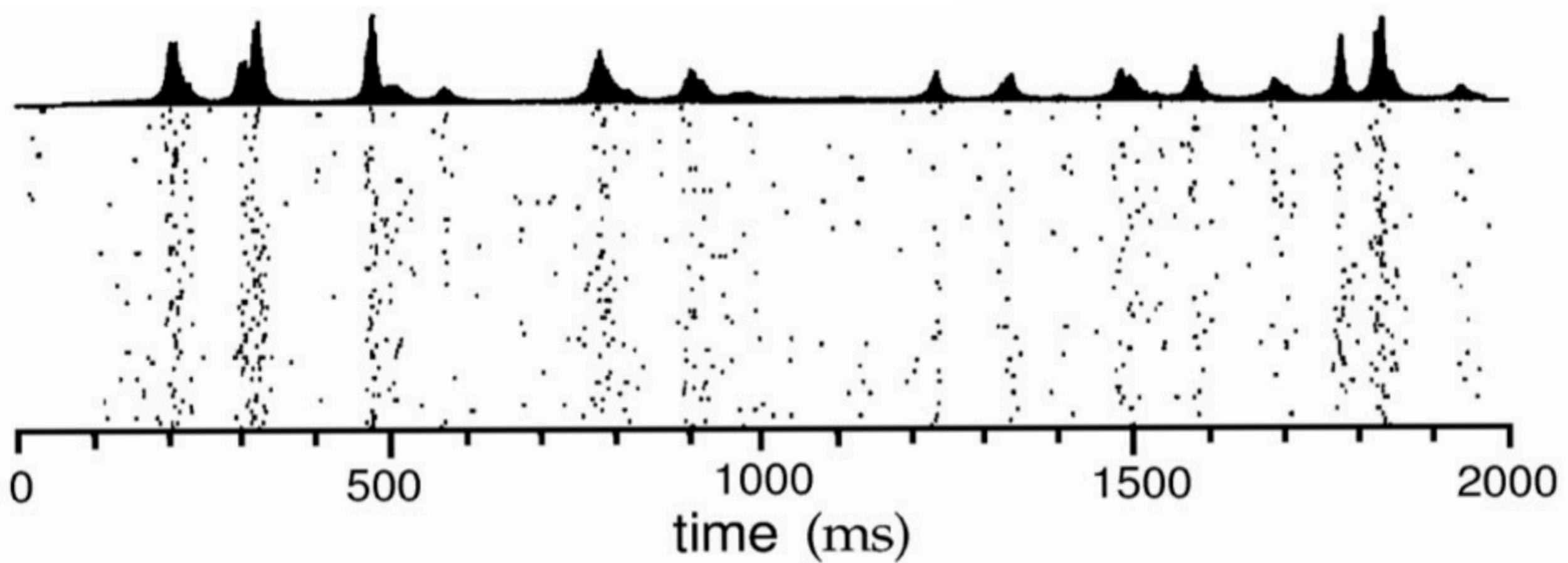
Compare to how simple is the **genetic code**: 4 letters, each ordered sequence of 3 letters determine 1 aminoacid



# Relation stimulus-response

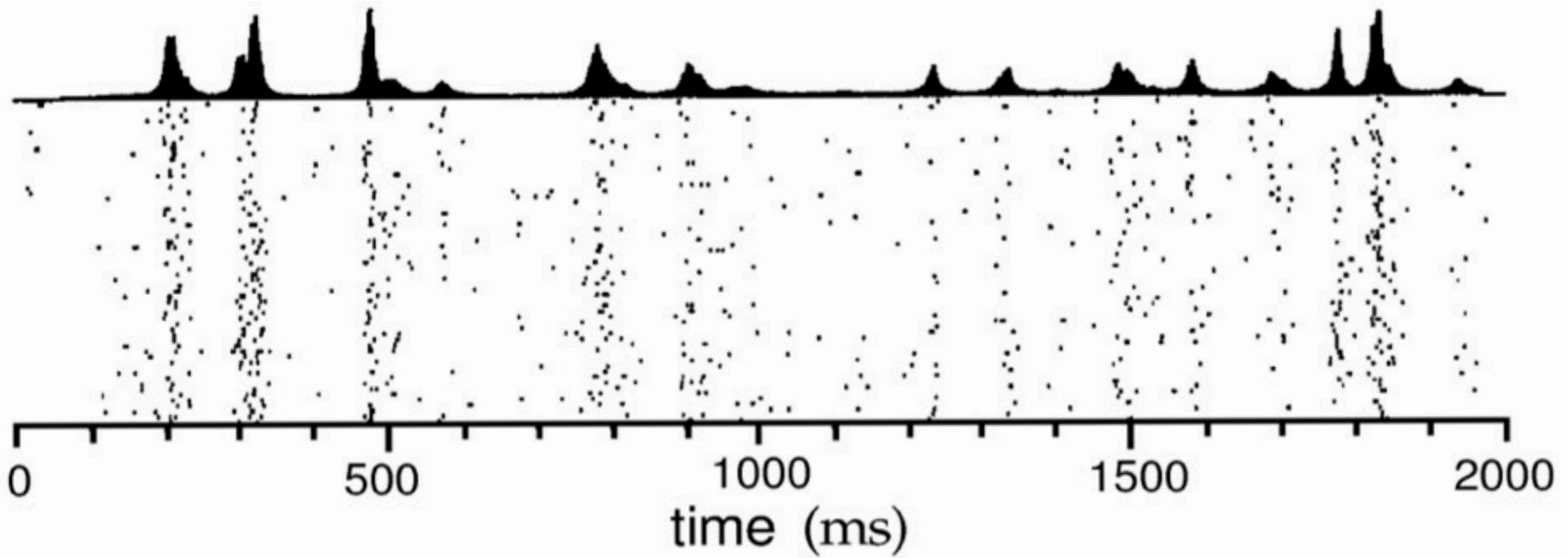


# Relation stimulus-response



Difficult to characterize! Neural responses are stochastic:

# Relation stimulus-response

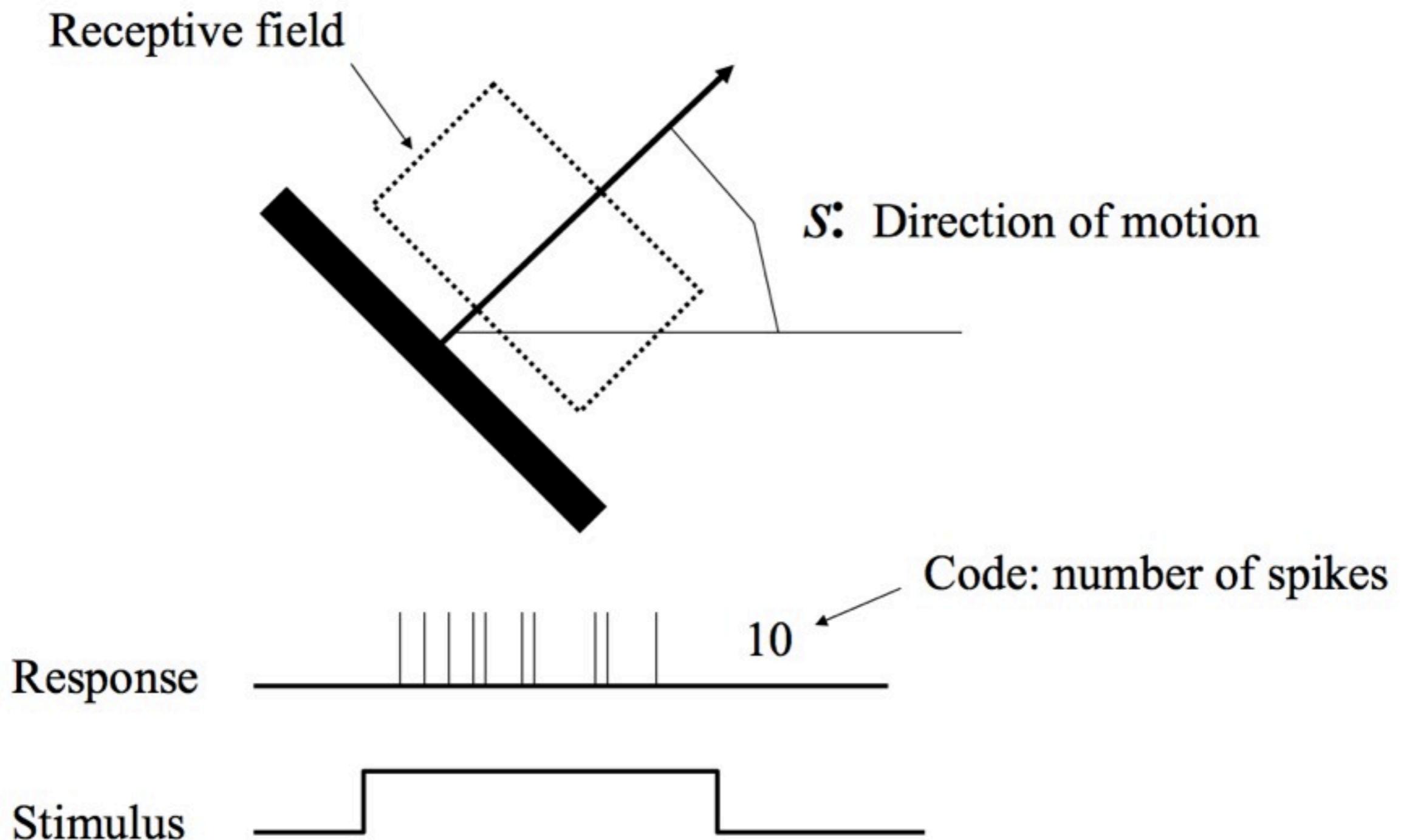


Difficult to characterize! Neural responses are stochastic:

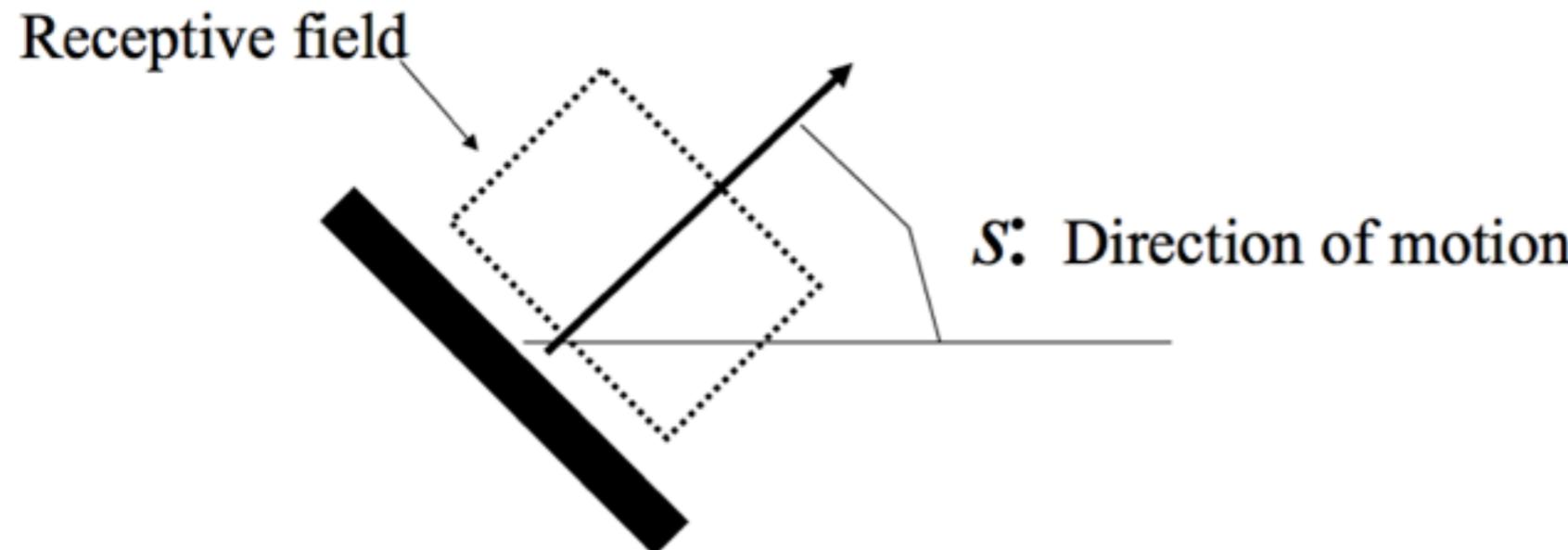
- 1 Levels of arousal and attention
- 2 Stochastic nature of biophysical processes
- 3 Other parallel cognitive processes
- 4 Large networks can be chaotic
- ...

Deterministic model unlikely, we should seek a **probabilistic** one

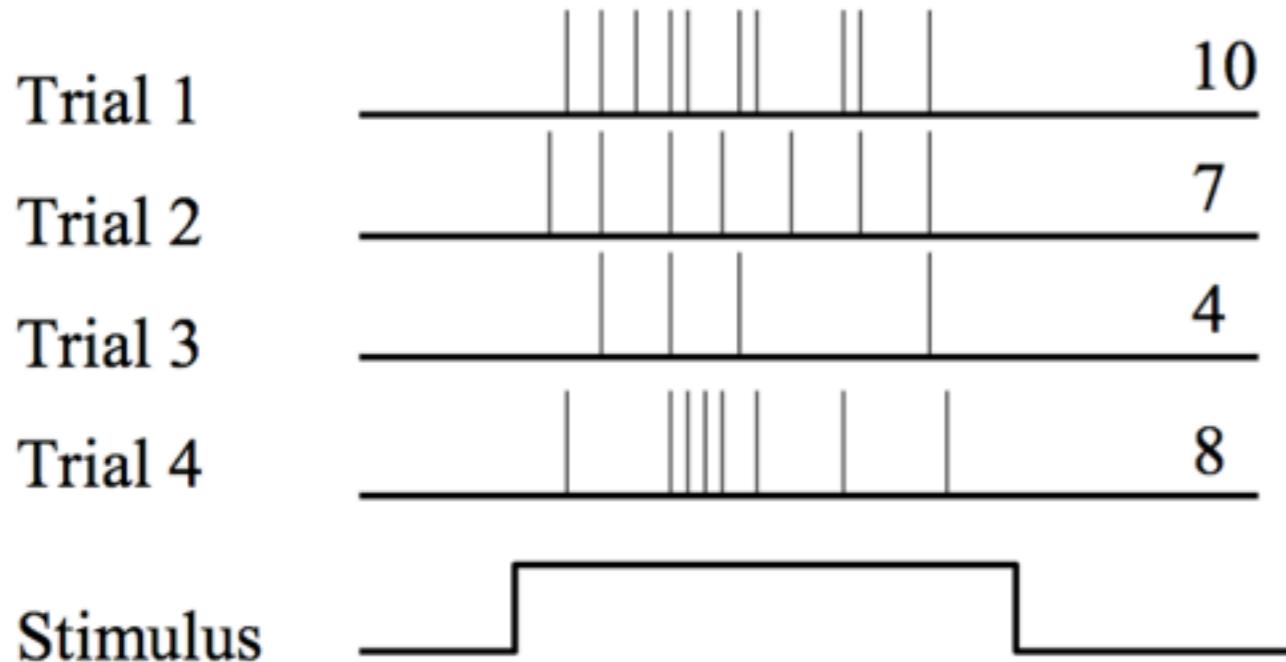
# Tuning curve (example I)



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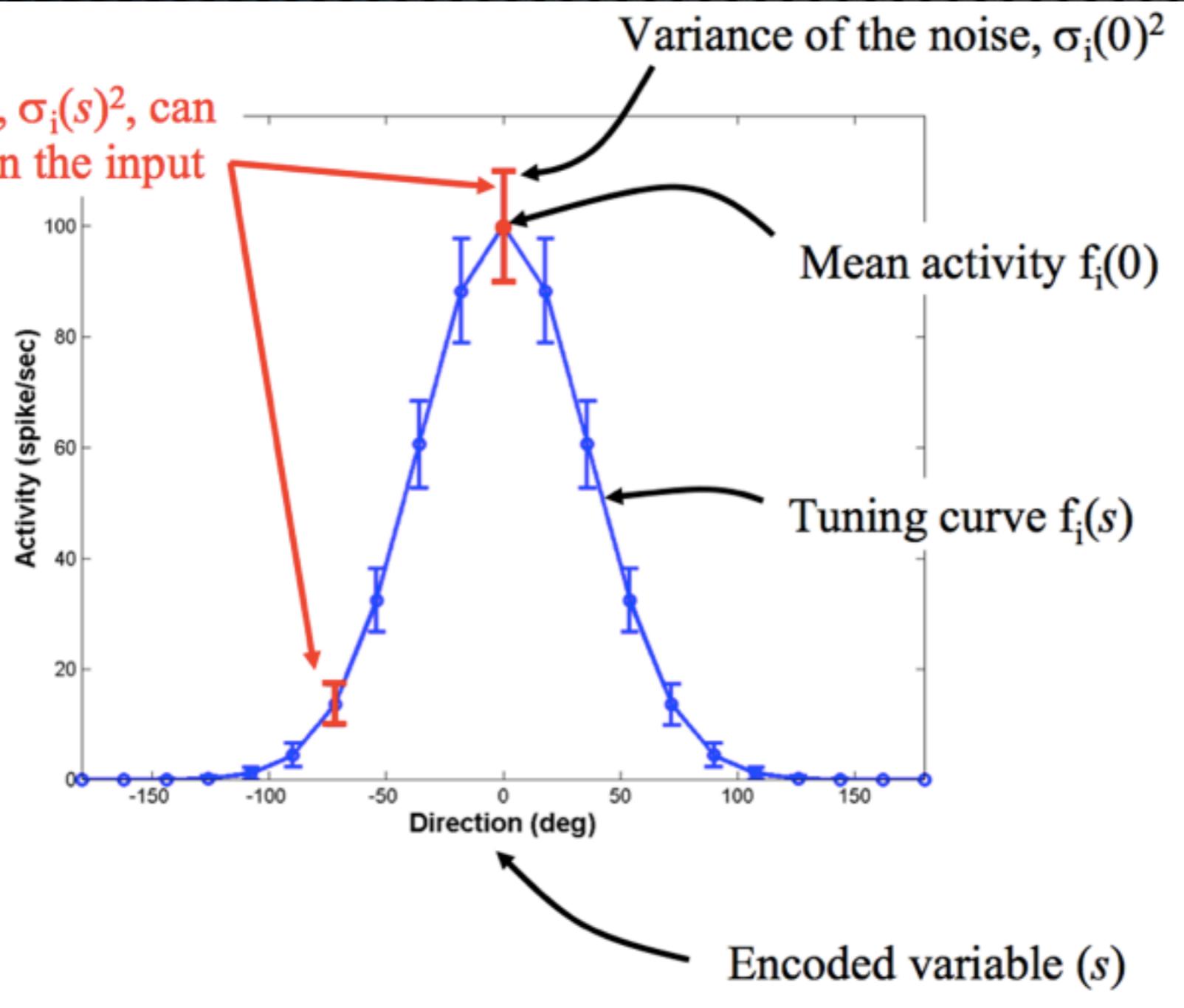
Different trials  
yield slightly  
different activity



Variability (noise?)  
typically follows a  
Poisson distrib.

# Tuning curve (example 1)

Variance,  $\sigma_i(s)^2$ , can depend on the input

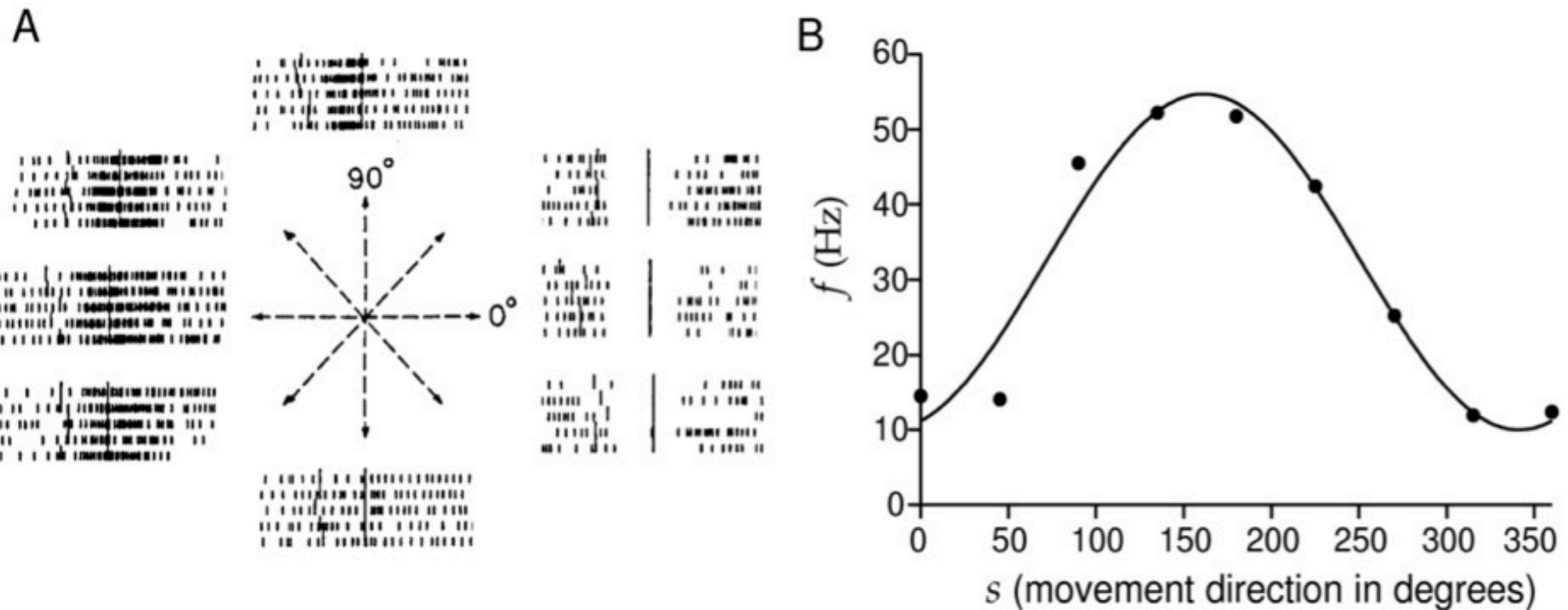


We repeat for different orientations to obtain a **tuning curve**

$$r = f(s) + n(s)$$

$$p(n|s) \rightarrow p(r|s)$$

# Tuning curve (example 2)

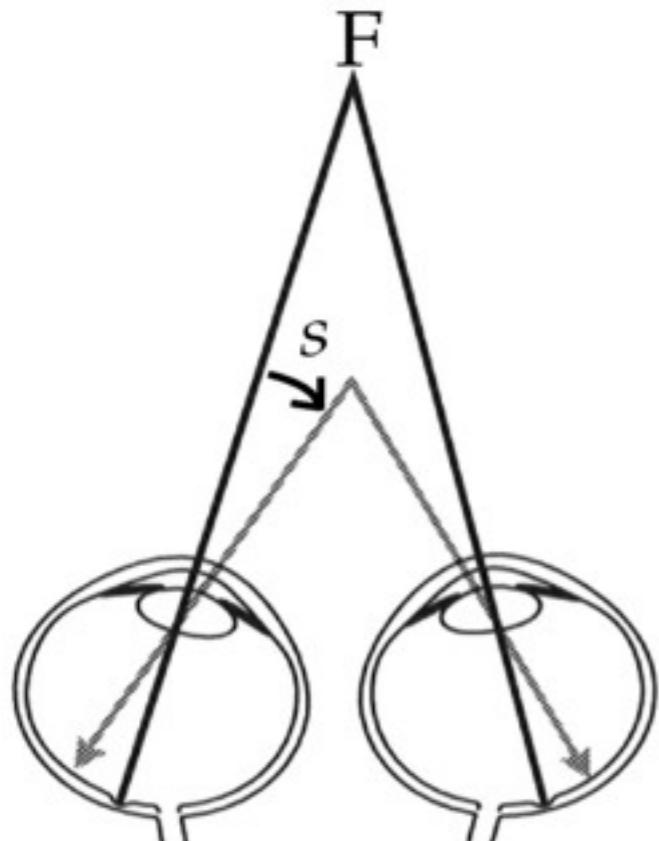


Neuron in monkey primary motor cortex (M1)

Sensitive to **reaching angle** (arm)

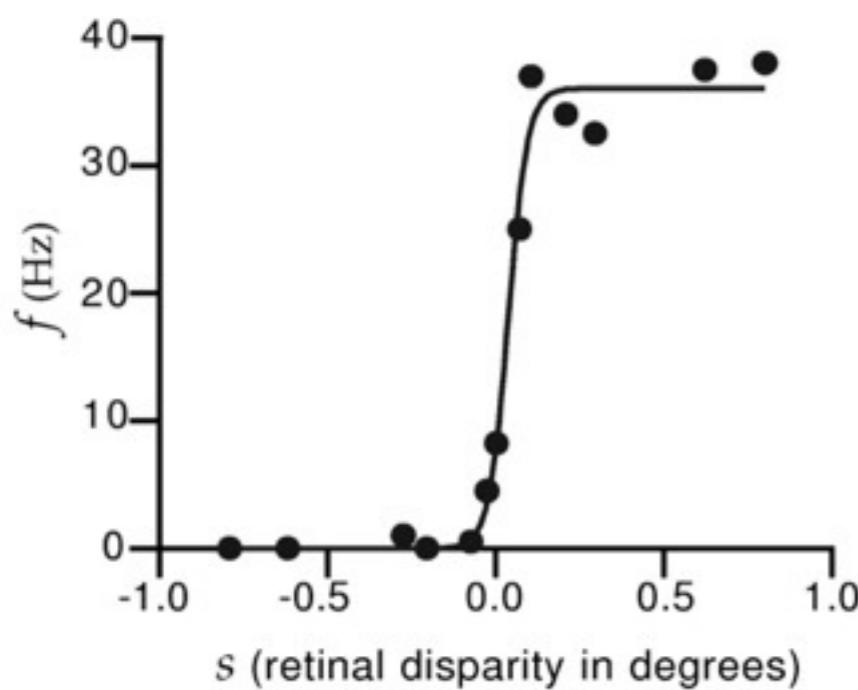
Cosine tuning curve with baseline firing rate (10 Hz)

# Tuning curve (example 3)



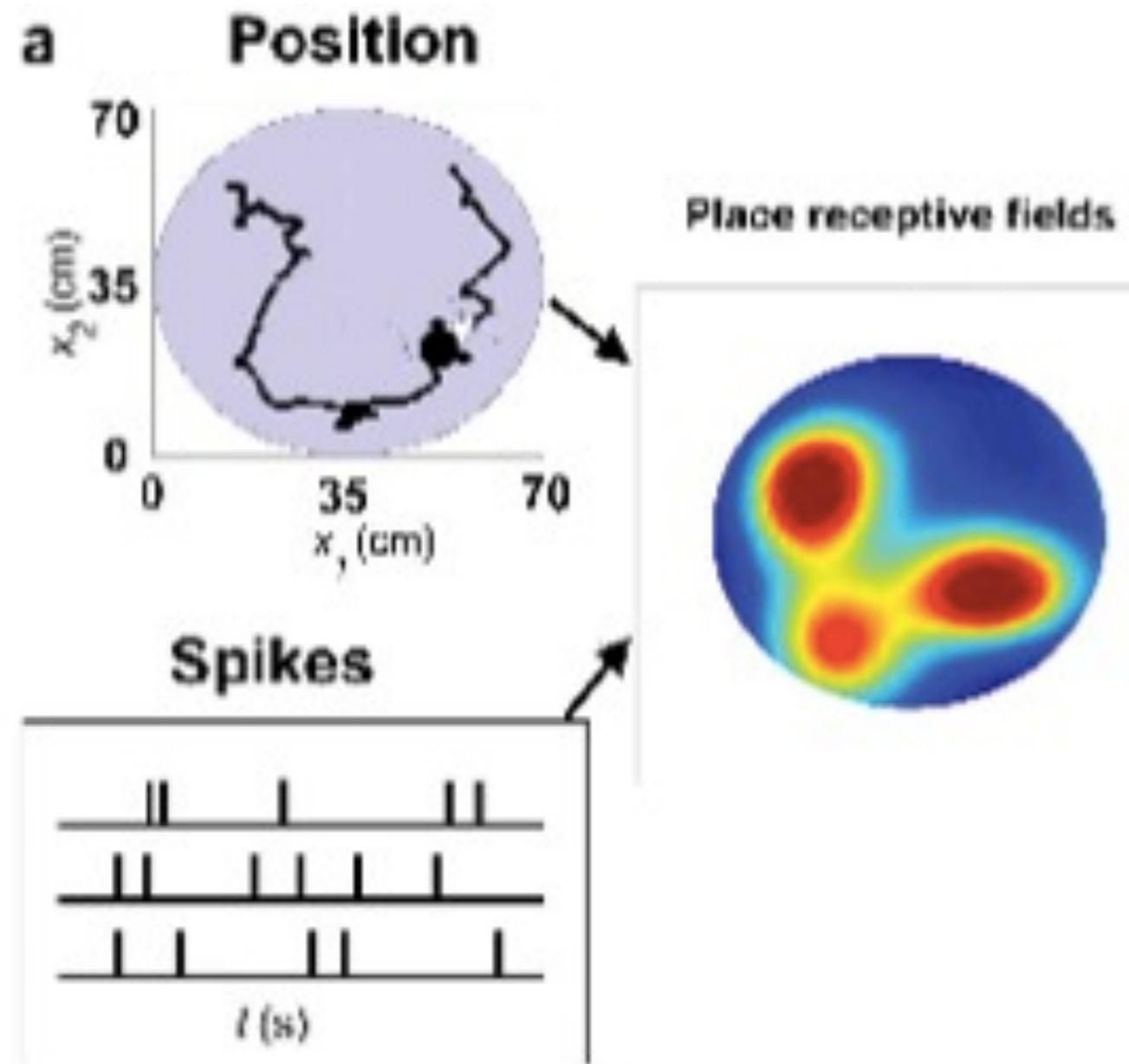
**Retinal disparity:** difference in the retinal location of an image between the two eyes

Some neurons in primary visual cortex (V1) are sensitive to retinal disparity



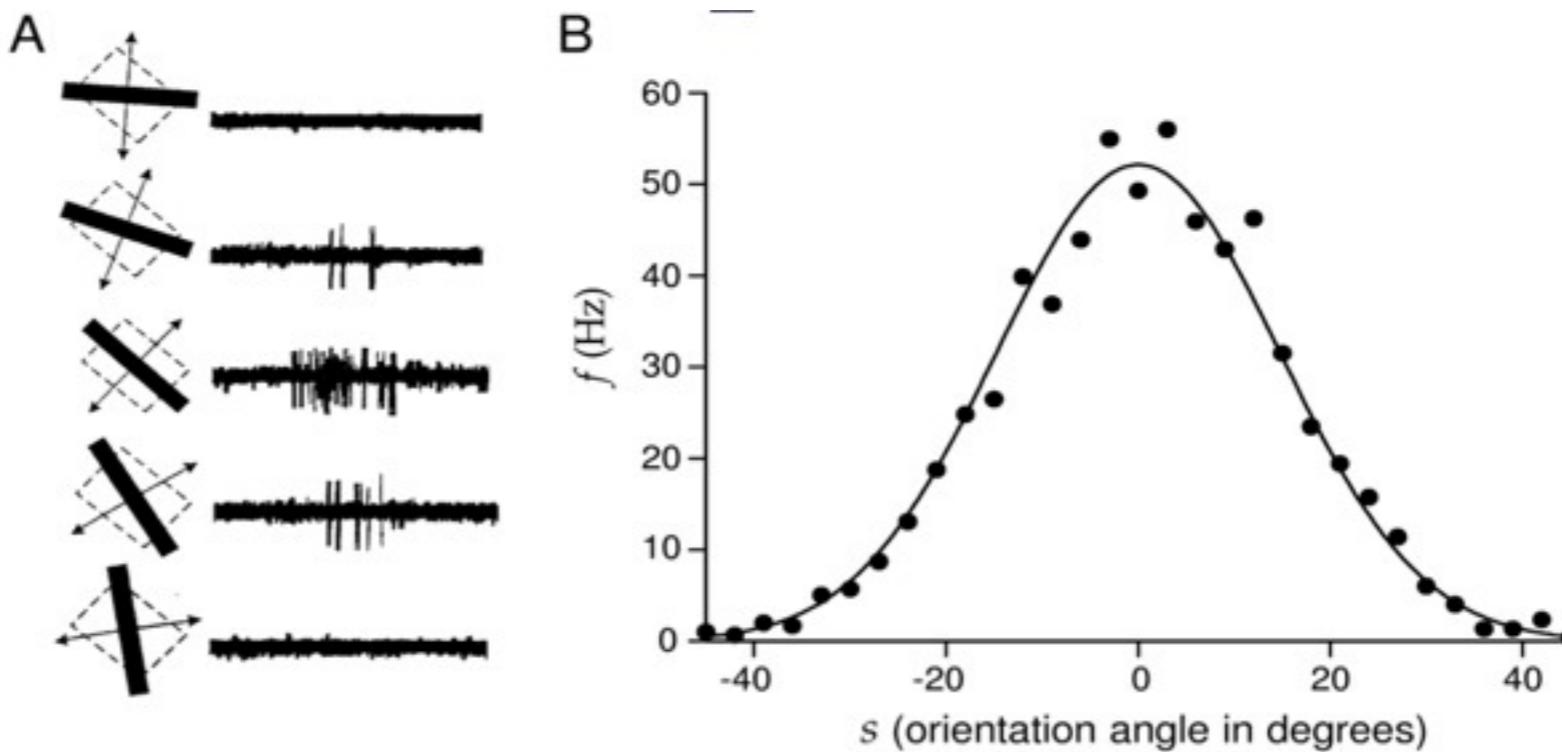
What could be the purpose  
of these neurons?

# Tuning surface (example 4)



Some neurons in the hippocampus region can be **sensitive to the location** of the animal: they are named **place cells**

# Tuning curve (summary)



Used to **characterize the sensitivity of neurons** in visual or other sensory areas **to a variety of stimulus parameters**

Measure the impact of a single stimulus attribute  $s$ , on the average neural response  $r \rightarrow r = f(s)$

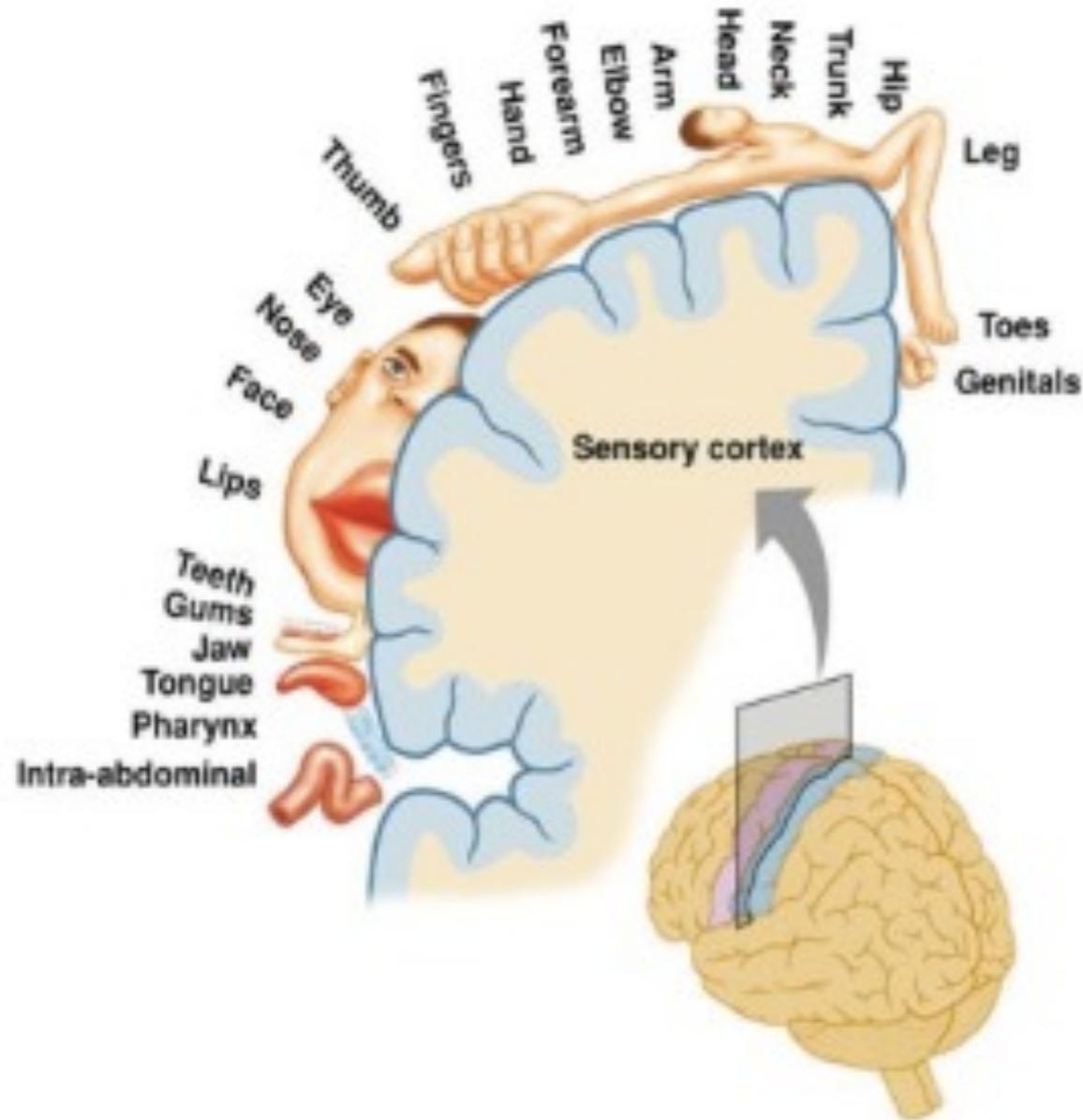
Correspond to firing rates, they are **measured** in spikes/s or **Hz**

# How do neurons encode information?



Which characteristics of spike trains can we use to code information?

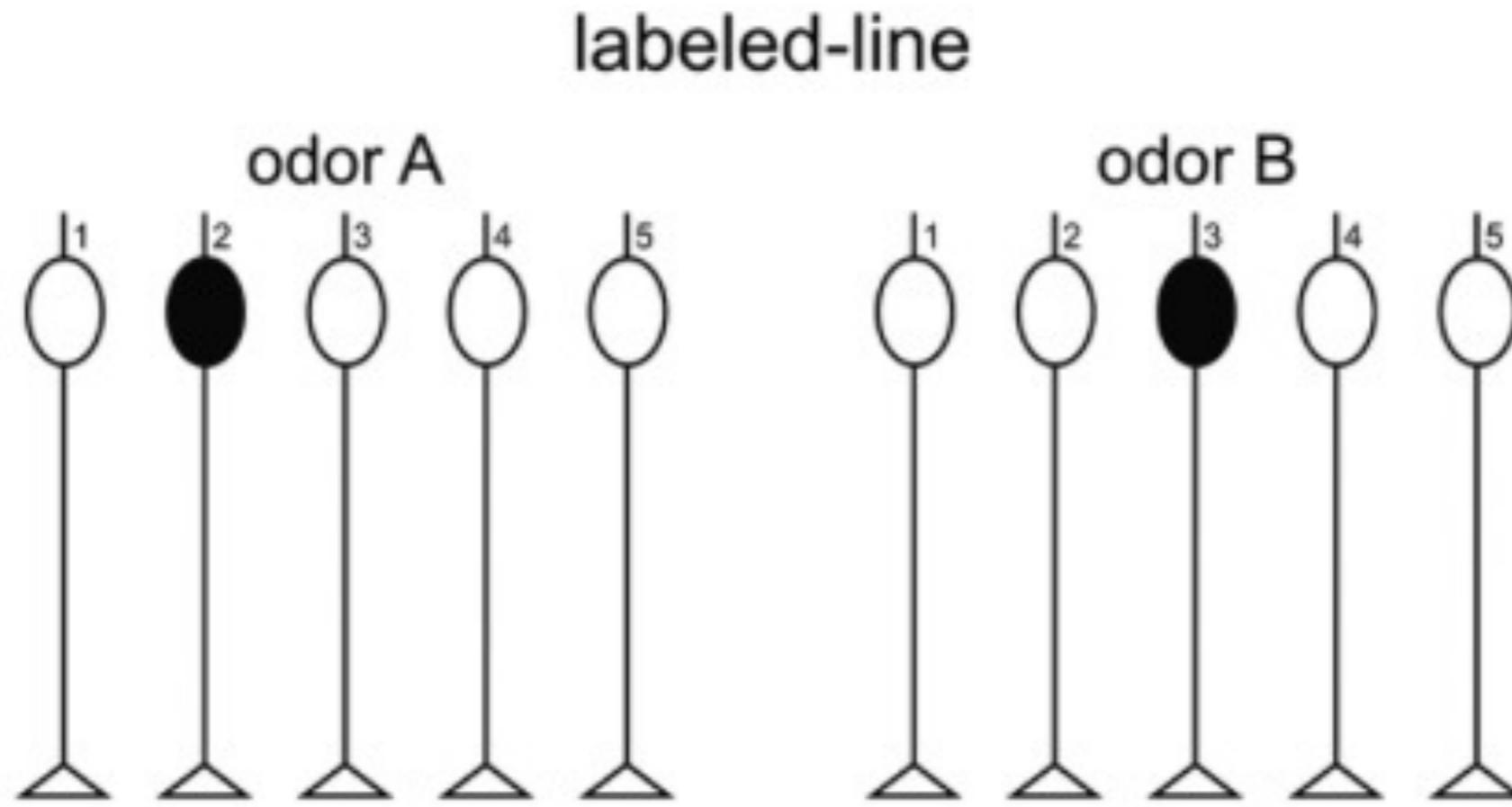
# Labeled line code



Maps maintain an orderly representation of the stimulus

Pathways carrying sensory information are specific, forming a “labeled line” regarding a particular stimulus

# Labeled line code



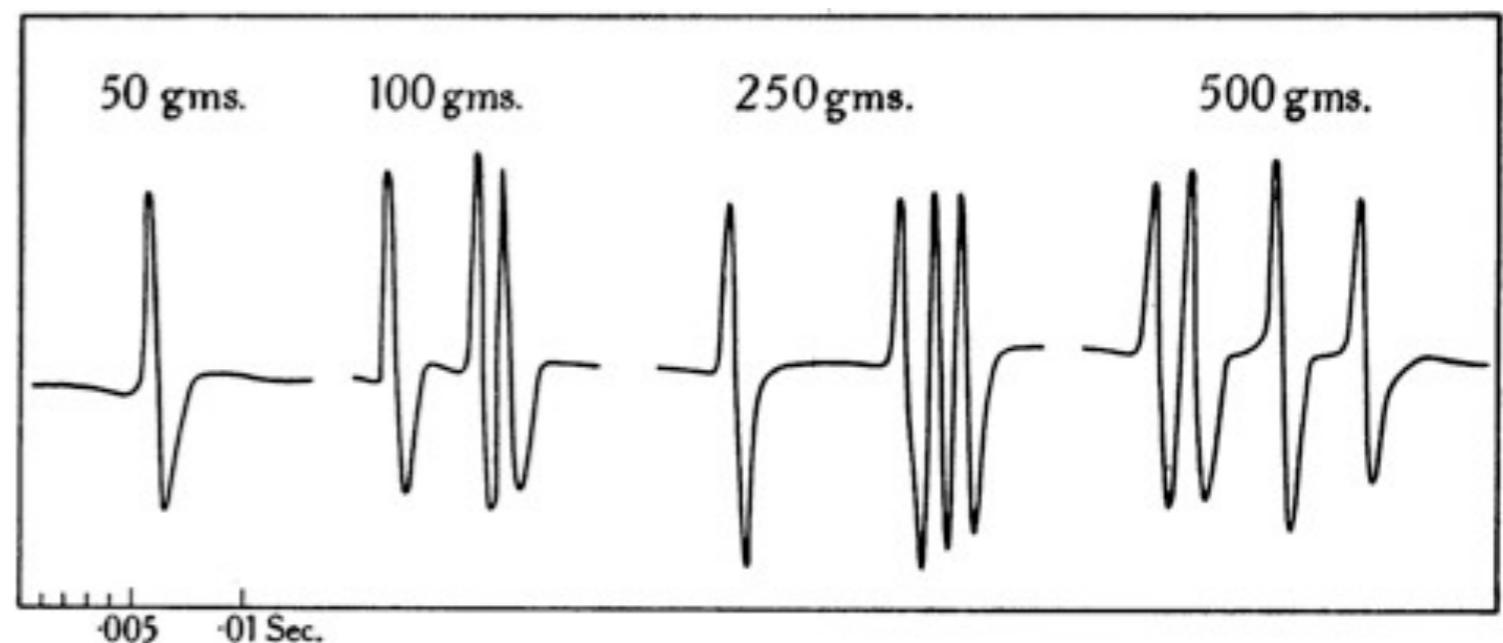
Odor is **coded in the identity of the neuron activated**

**No further processing** is needed for decoding

**Low capacity**

# Rate code

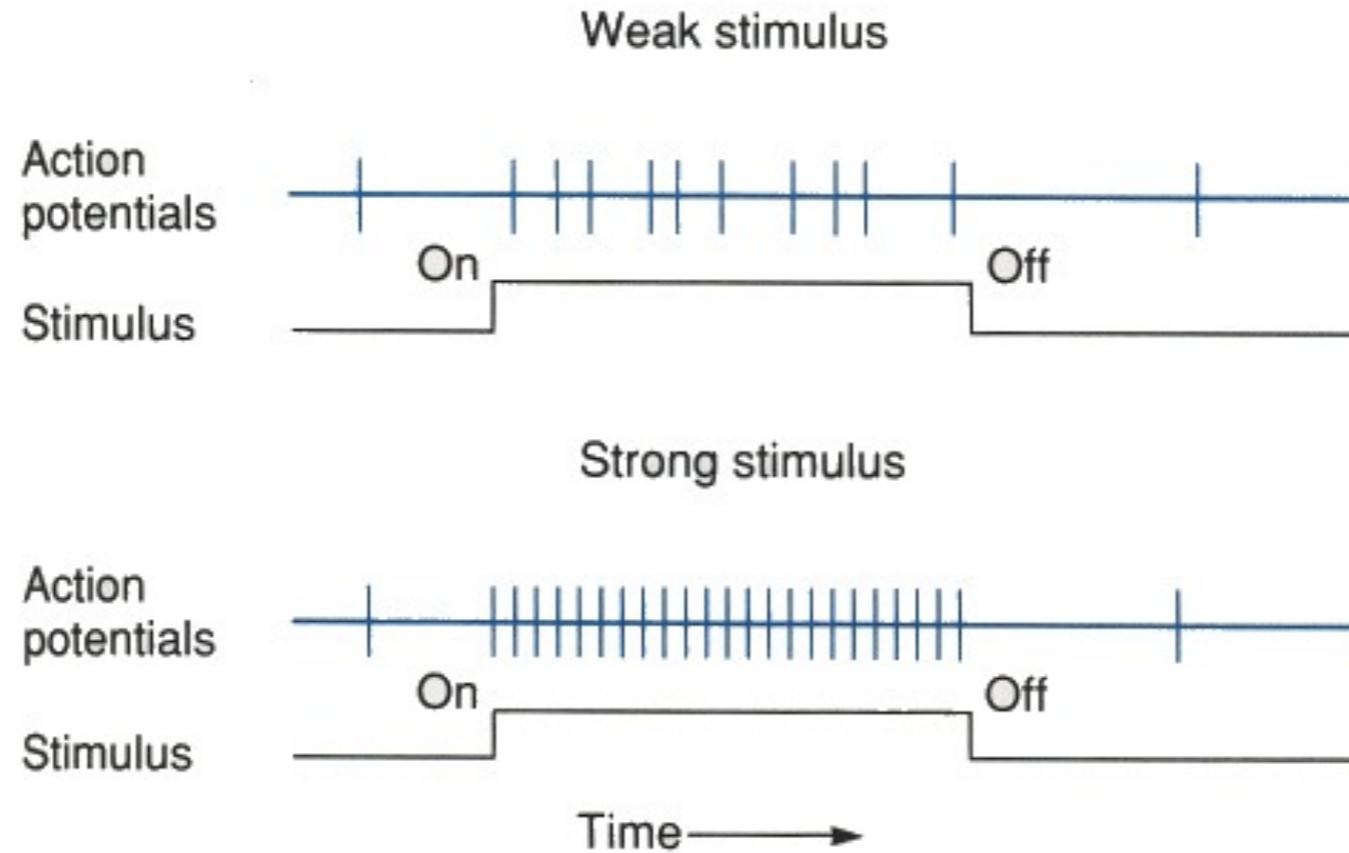
The classic → Adrian & Zotterman (1926)



Information about the stimulus is contained in the firing rate (frequency) of the neuron

In most sensory systems, the firing rate increases, generally non-linearly, with increasing stimulus intensity

# Rate code



The standard tool for describing the properties of sensory and cortical neurons, in part due to the ease of measuring

**Not efficient and slow but robust**

# Temporal code

Rate code neglects all the information possibly contained in the exact timing of the spikes (temporal code)

**String 1**

**Rate code** 000| | | 000| | | = 00| | 00| | 00| | 6

**String 2**

**Temporal code** 000| | | 000| | | ≠ 00| | 00| | 00| | 6

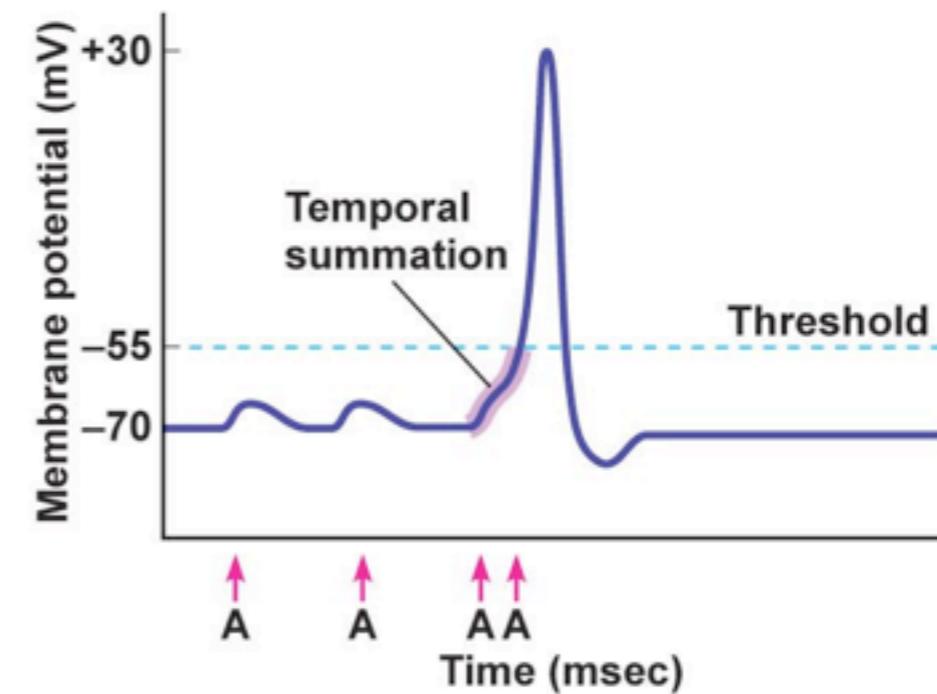
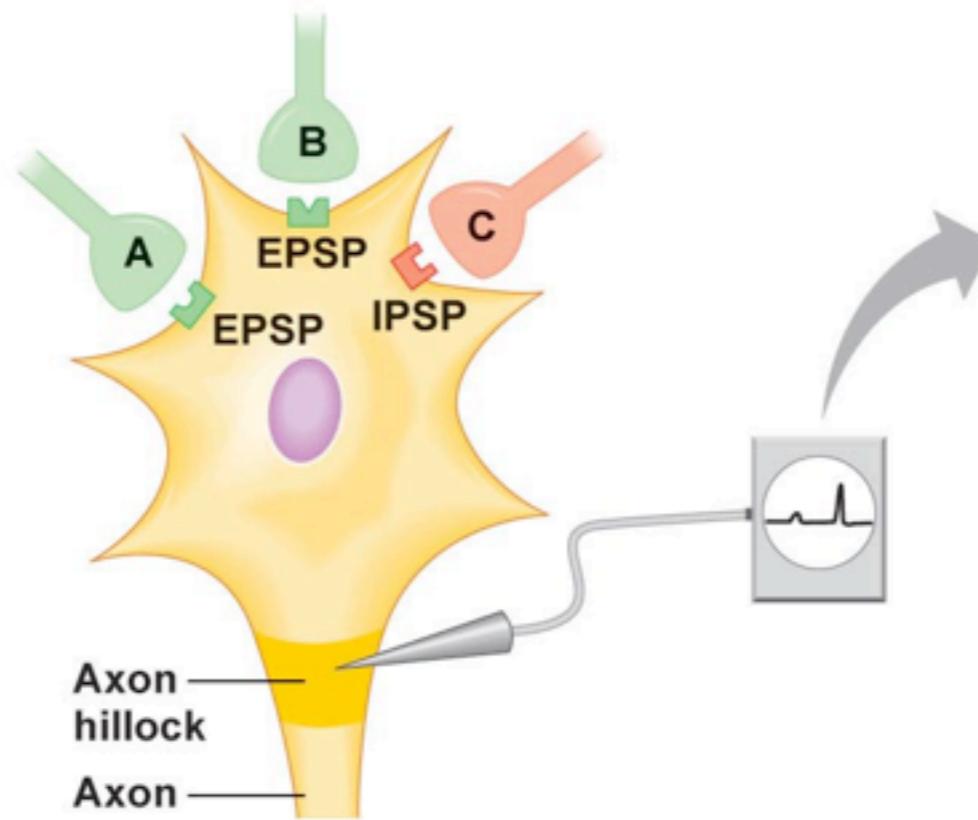
Recent studies suggest that spike timing on a millisecond time scale can be a significant element of neuronal coding

# Temporal code

Temporal code 000| | | 000| | |      00| | 00| | 00| |

How could a neuron distinguish these two inputs?

## Temporal summation



# Temporal code

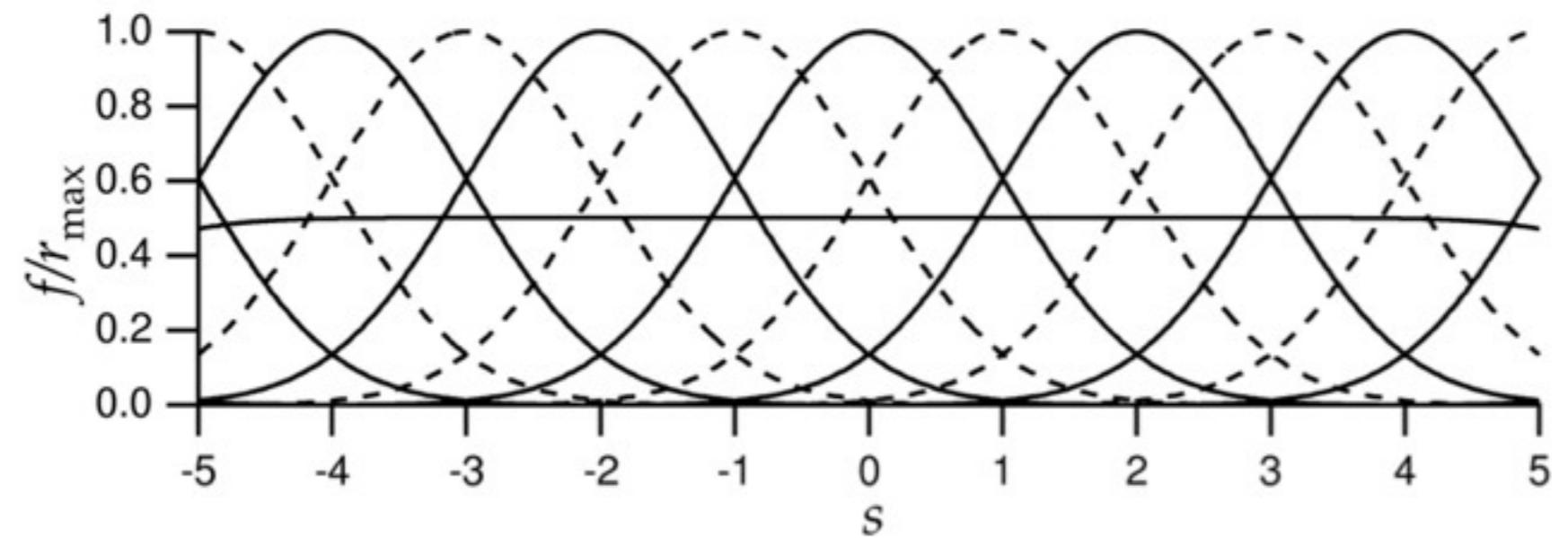
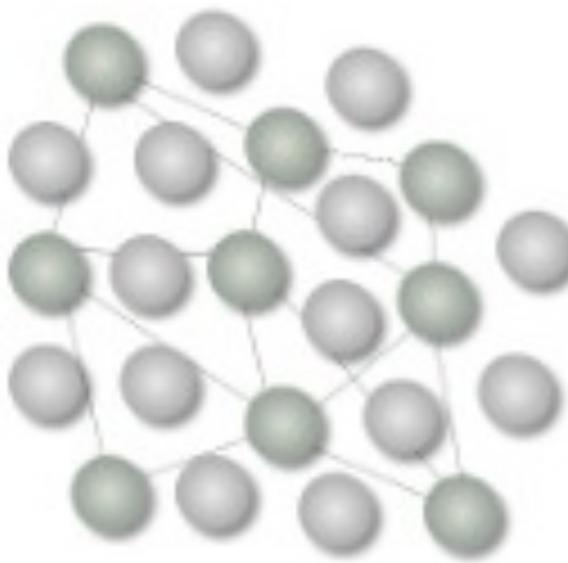
As there is no absolute clock in the brain, the information must be carried either in relative timing of spikes between neurons or with respect to ongoing brains oscillations

Examples: 1) latency of the first spike after stimulus onset,  
2) phase of an oscillation during firing, 3) sequences of firing



# Population code

Represent stimuli by using the joint activities of a number of neurons



Each neuron has a distribution of responses (is sensitive) over some set of inputs, and the responses of many neurons is combined to determine some value about the inputs

# Population code

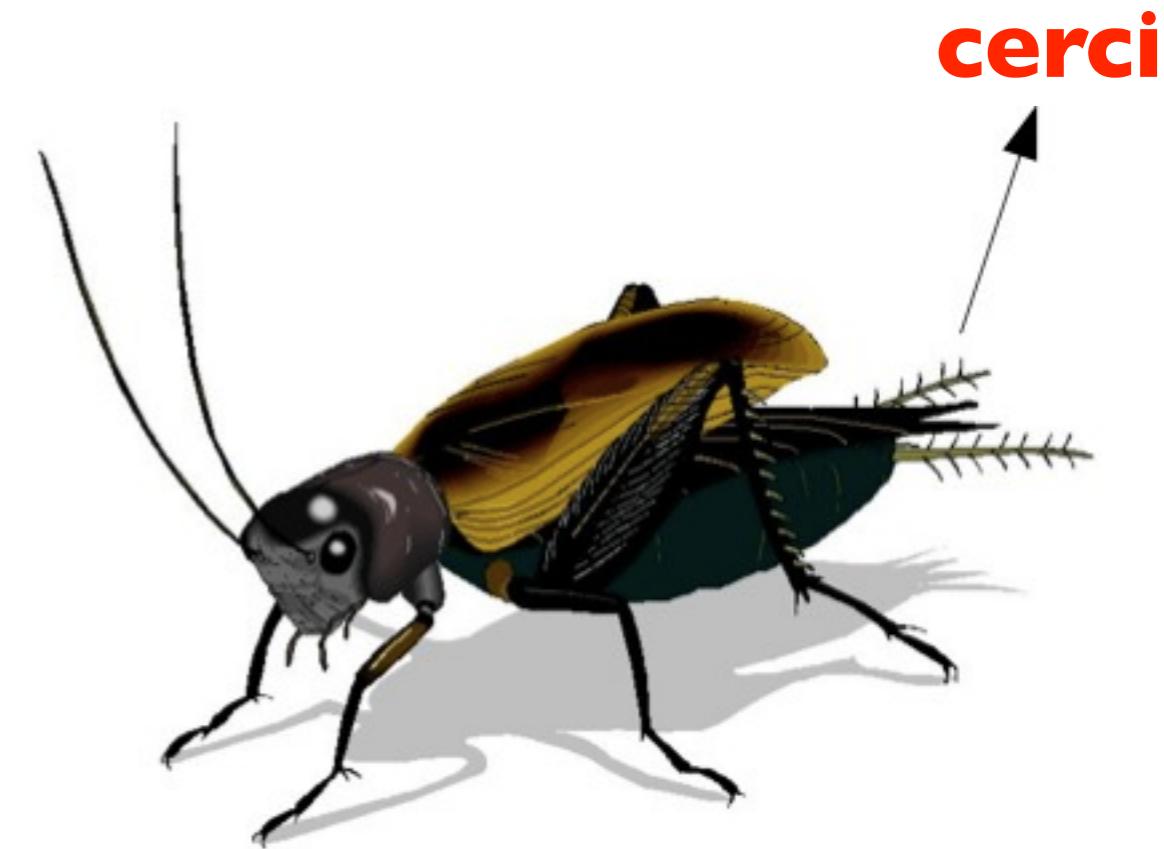
**System:** the cercal system of the cricket

**Senses air direction** as a warning  
for approaching predators

Sensory neurons project to  
interneurons

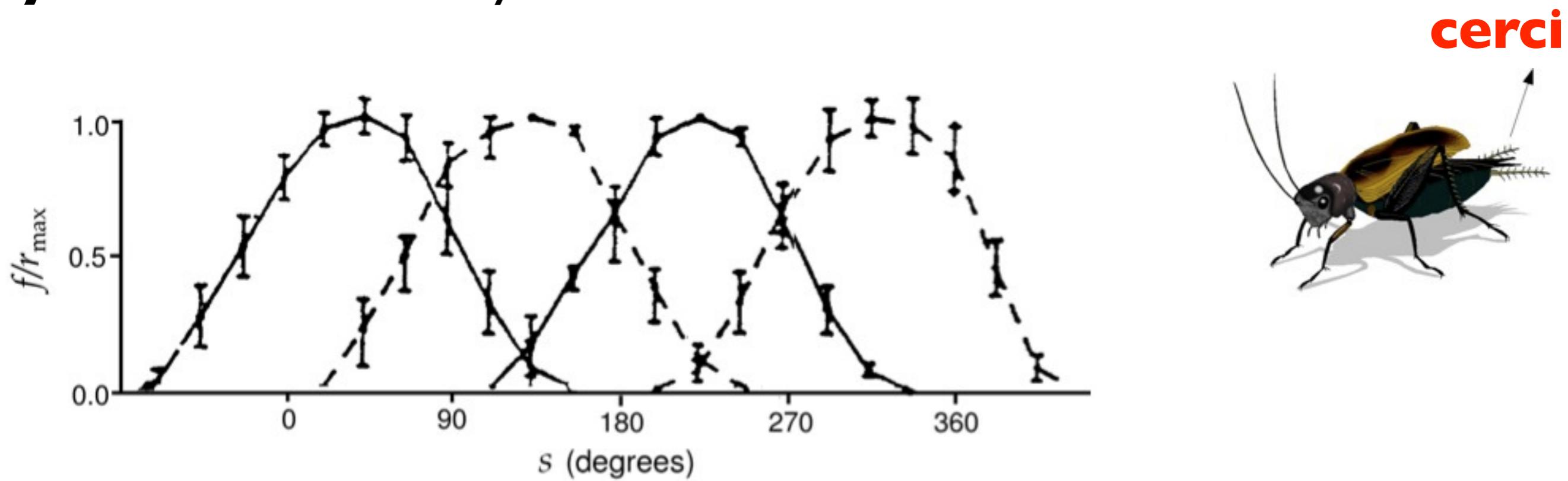
At low air-speed **information is encoded by just 4 inter-neurons**

No single neurons responds to all wind directions, **multiple neurons respond to any wind direction**



# Population code

**System:** the cercal system of the cricket



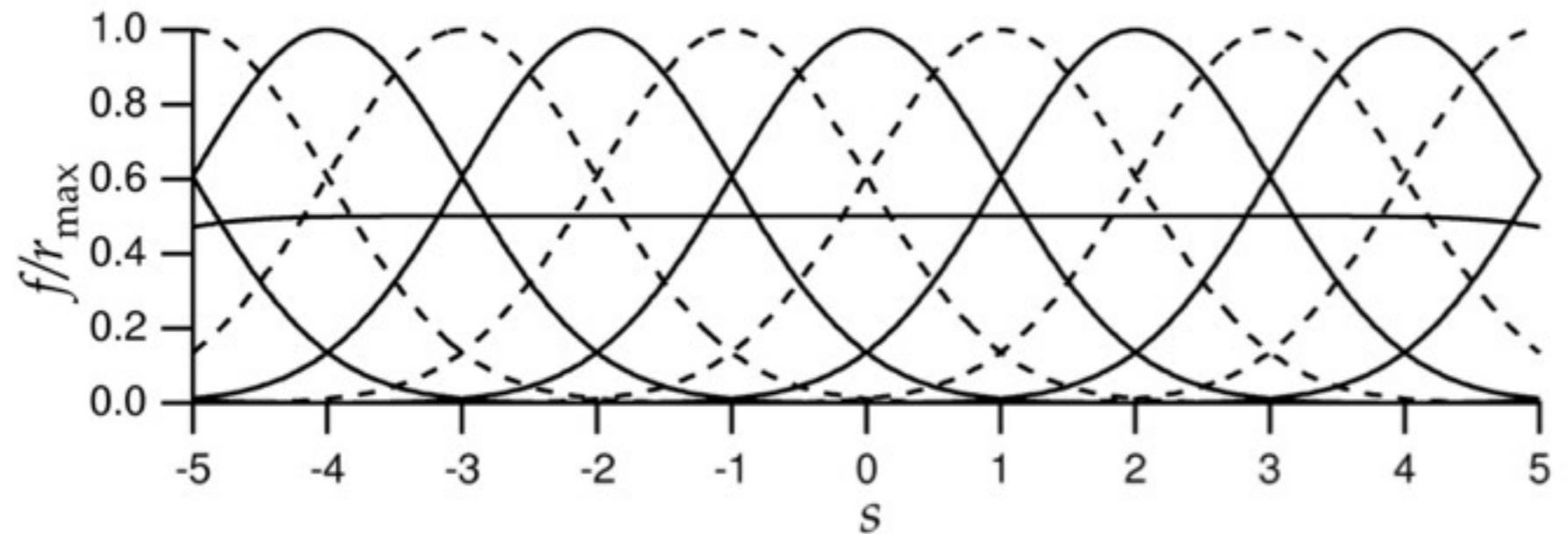
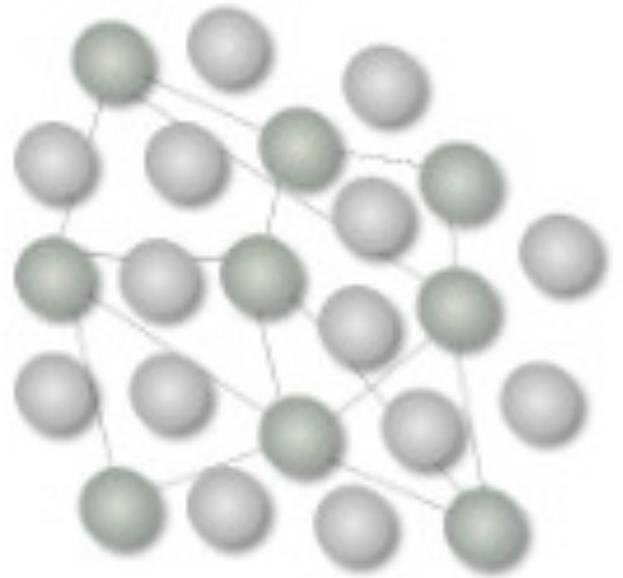
These 4 neurons are sensitive to the angle of wind

Preferred directions  $\mathbf{c}_a$ :  $-135^\circ, -45^\circ, 45^\circ, 135^\circ$

Note: rate code also assumed

**Crickets  
are  
cartesian!**

# Population code



By using a large number of neurons to represent information:

1. **Reduction of uncertainty (& fast)**
2. **Immune to fluctuations existing in single neuron's signal**
3. Ability to represent a number of **different attributes simultaneously**

# THE neural code?

**Q:**

What kind of code do neurons use to represent sensory information and communicate with each other?

Is precise timing important (temporal code) or are firing rates (rate code) enough?

Which variability is noise and which information?

**A:**

Probably, we should talk about the neural codes of the brain, where **different codes** (rate, temporal, population, ?) interact and are used to a **different degree in different regions** of the brain (and different animals)

# Some principles for codes

**I. Efficient coding hypothesis:** Neural codes minimize the number of spikes (cost) needed to transmit a given signal

Neurons in the visual (or auditory) system are optimized for coding images (or sounds) from their natural environment

To investigate neural codes we should test with **natural stimuli**

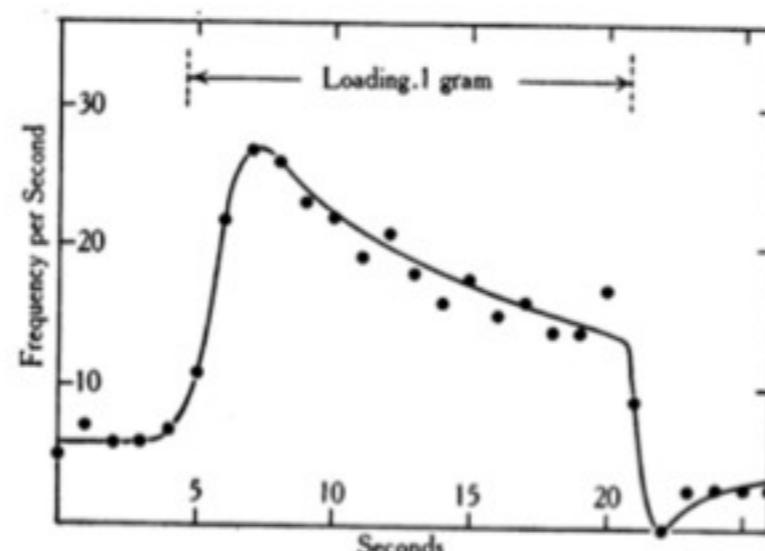
# Some principles for codes

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**2. Adaptive coding:** Neural codes adapt to the regularities of stimuli



# **Neuronal decoding**

Read-outs and information theory

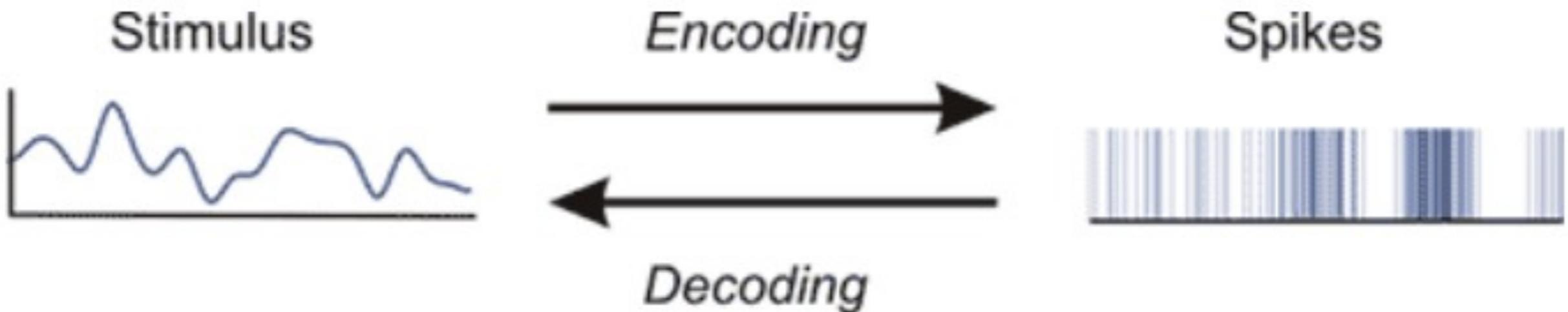
Estimators

# Neural decoding

**Decoding:** opposite map, from response to stimulus

Given a response, what was the stimulus?

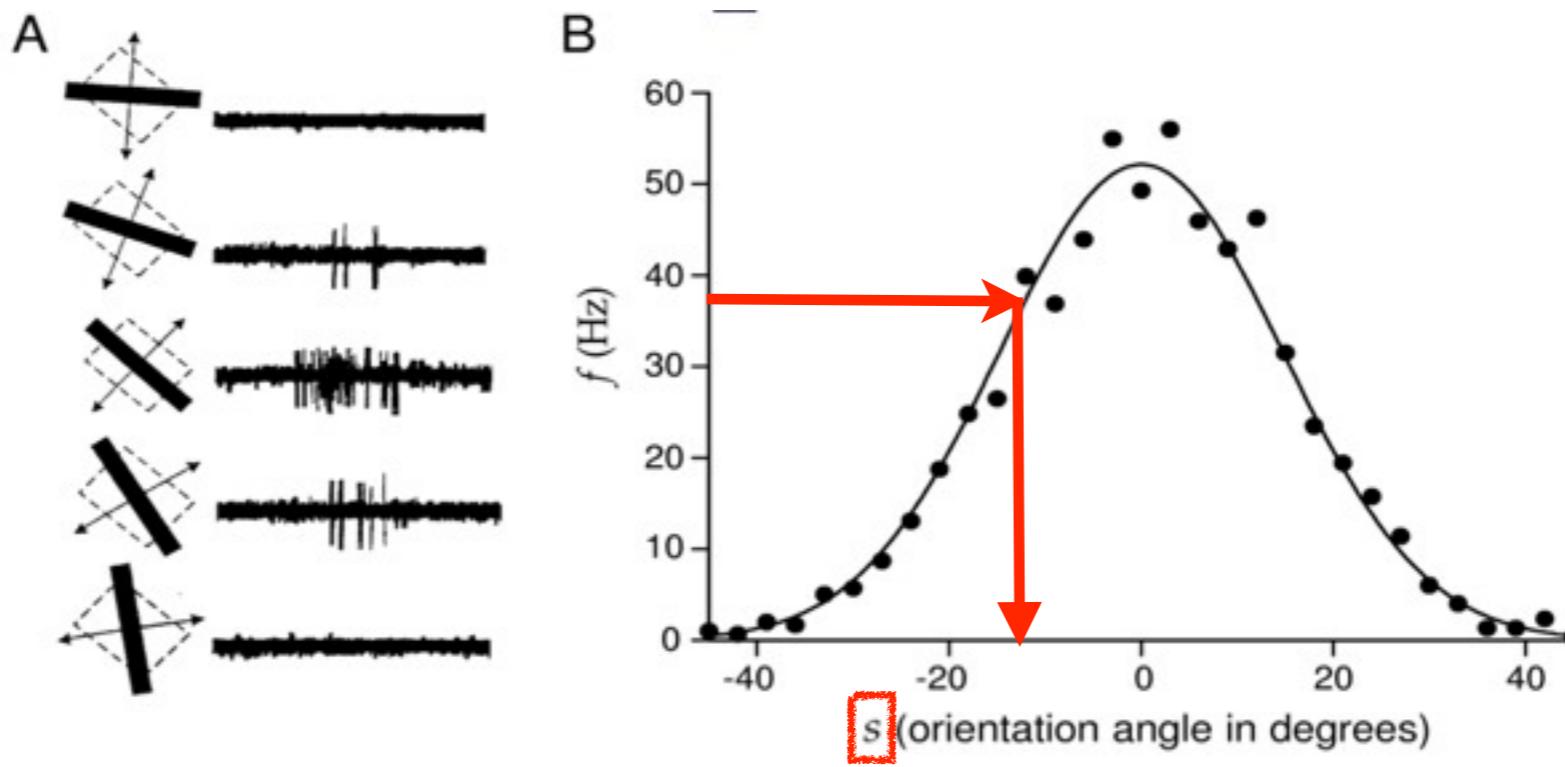
Given a firing pattern, what will be the motor behavior?



# Neural decoding

How to determine what is going on in the real world from the cortical dynamics?

Using the responses of one or more neurons to identify the stimulus



# Why?

## Fundamental

A lot of information processing in the brain consists of routing or discarding information...

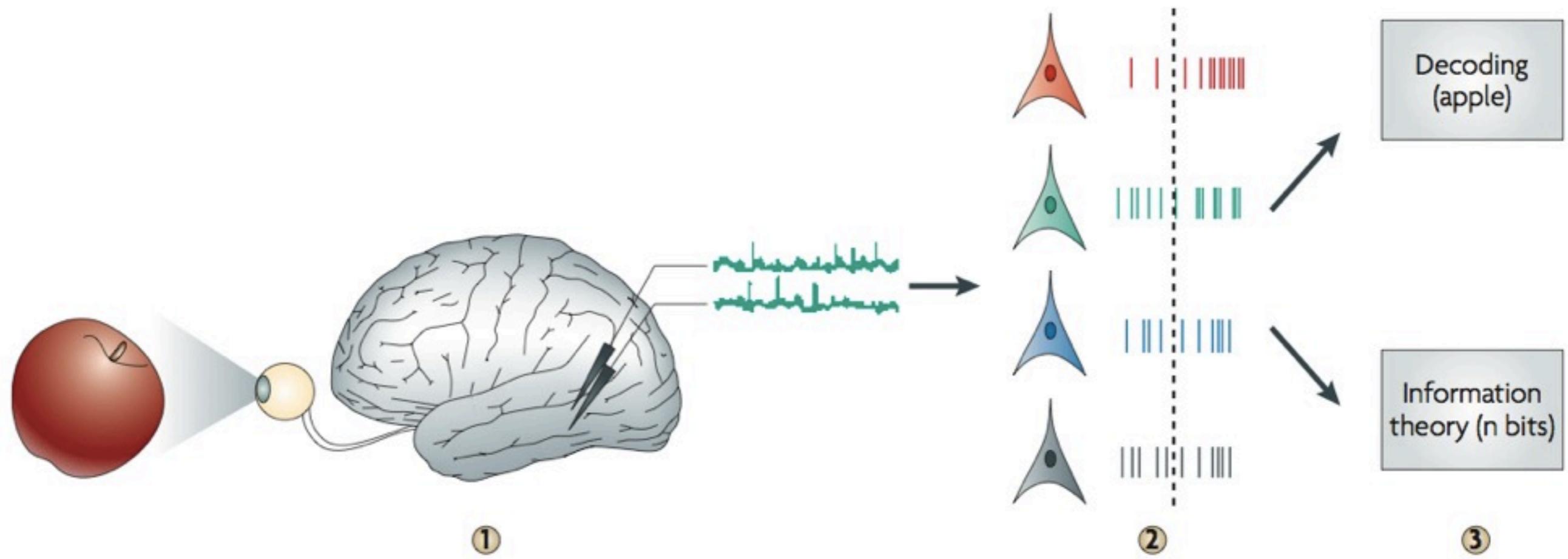
What aspects of a stimulus are important for each brain area?

## Practical

Neural prosthetics (ex. neural signals into movements)

Infer subjective human experience directly from brain activation

# Read-Outs & Information Theory

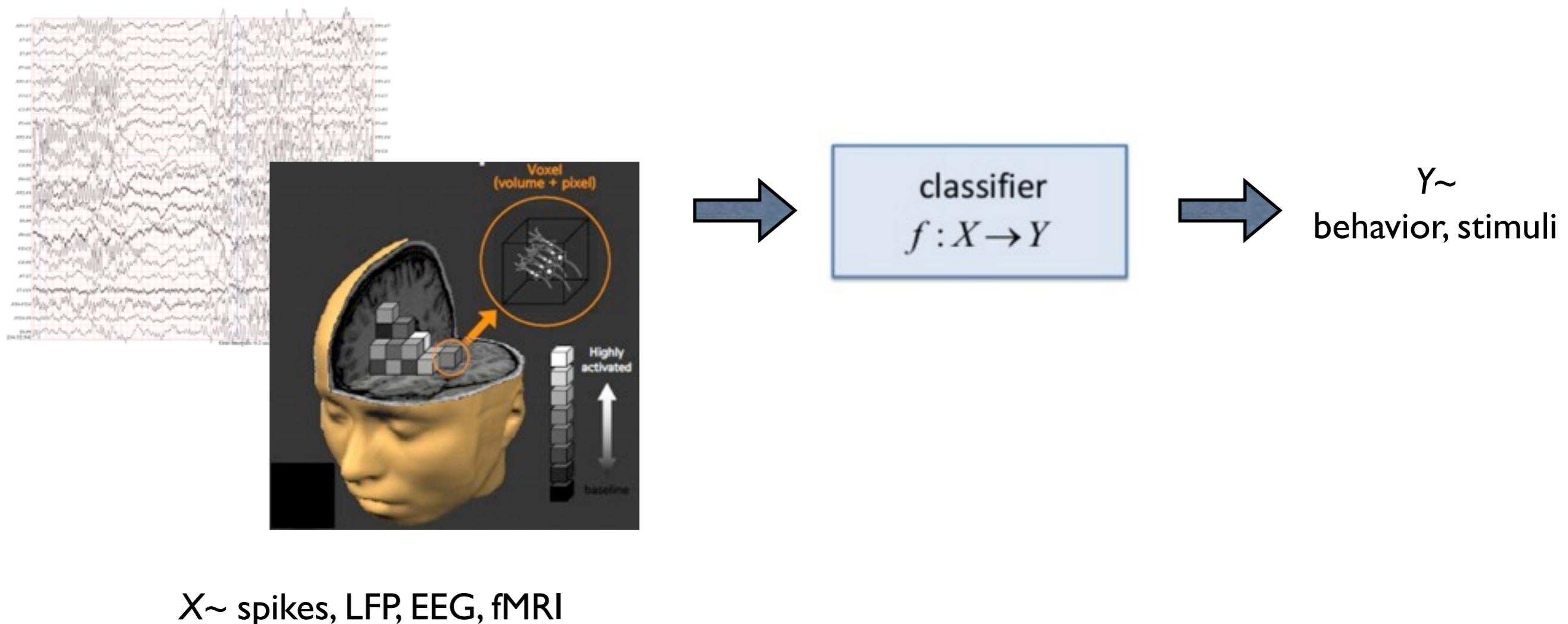


**RO**: predict which stimulus or behavior elicits an observed neural response (ex., classifiers in % accuracy)

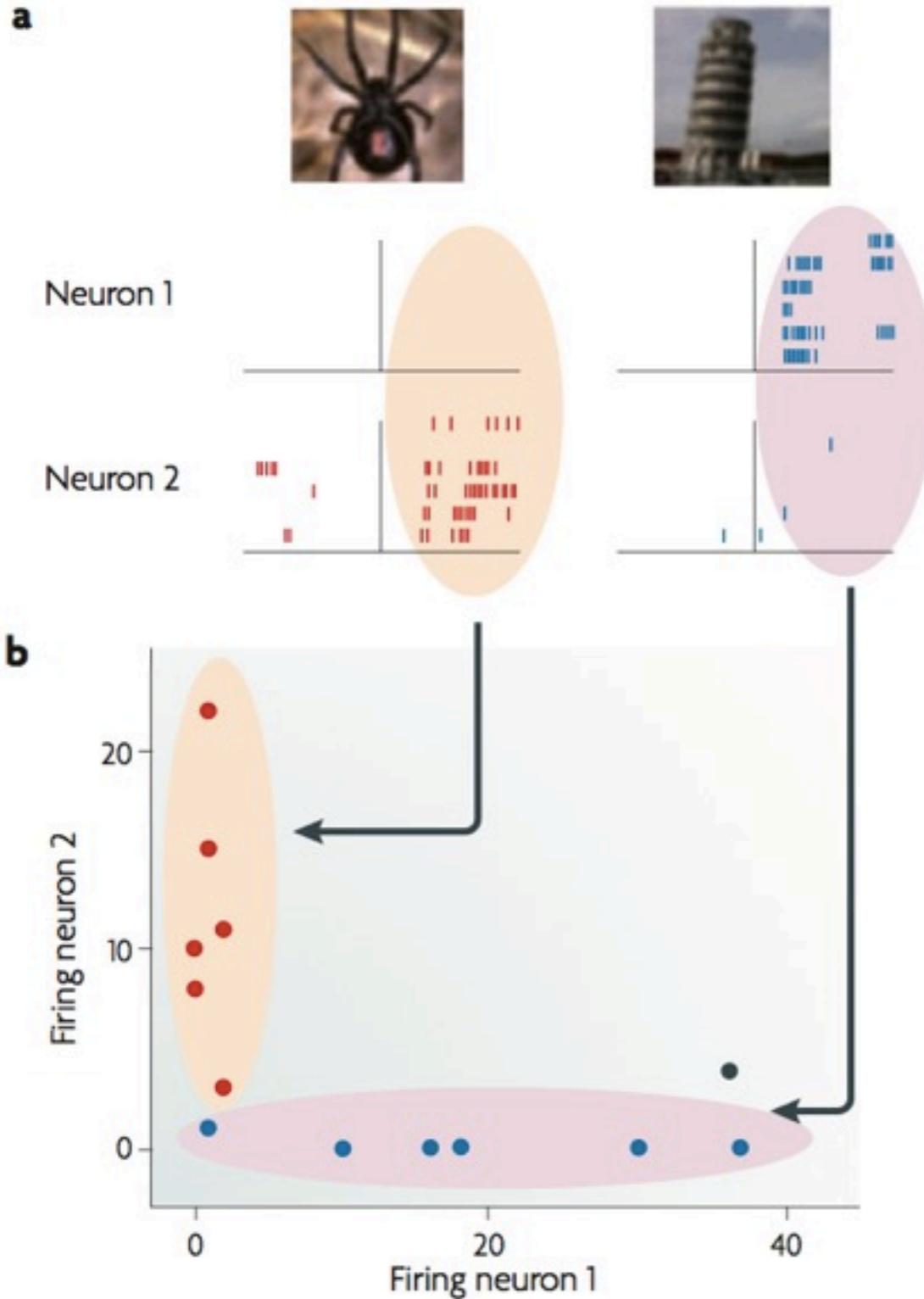
**IT**: reduction of uncertainty about the stimulus obtained by knowing the neural response (ex., mutual information in bits)

# Machine Learning: algorithms that learn from data

- “Teaches” the algorithm with correct examples to learn the relation between neuronal signals  $X$  and behavior  $Y$  (or stimulus)
- What? Where? When?



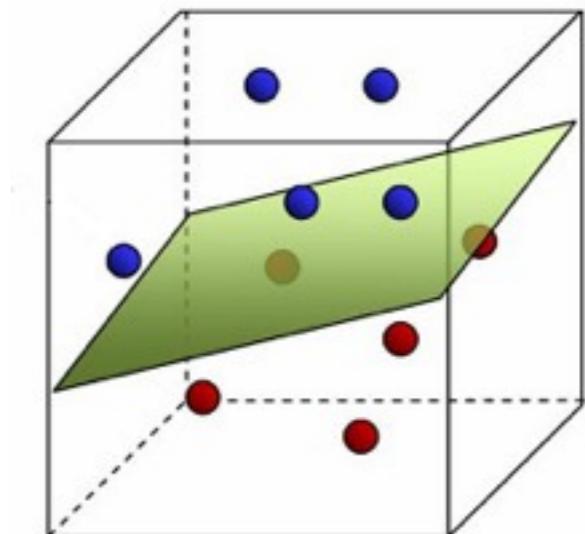
# Read-Outs: the problem



**Prediction:** given the firing rate of the two neurons...which class was the stimulus?

**Features:**  $\{r_1, r_2\}$

**Labels:**  $\{0, 1\}$  (**animal, building**)



Machine learning

# Read-Outs: formulation

**Idea:** train a classifier to discriminate between different classes of stimuli (or decisions) and used to predict novel examples

**Features:** can be firing rates in intracranial recordings, power of oscillations in EEG, voxel activity in fMRI,...

$$\mathbf{x} = (x_1, x_2, \dots, x_v)$$

**Classifier:** a function  $f(\cdot)$  that takes the values of the observed features (ex., voxels) and predicts to which class  $y$  the observation belongs

$$y = f(\mathbf{x})$$

# Read-Outs: training & test

## Training data

A classifier has a number of parameters that have to be learned

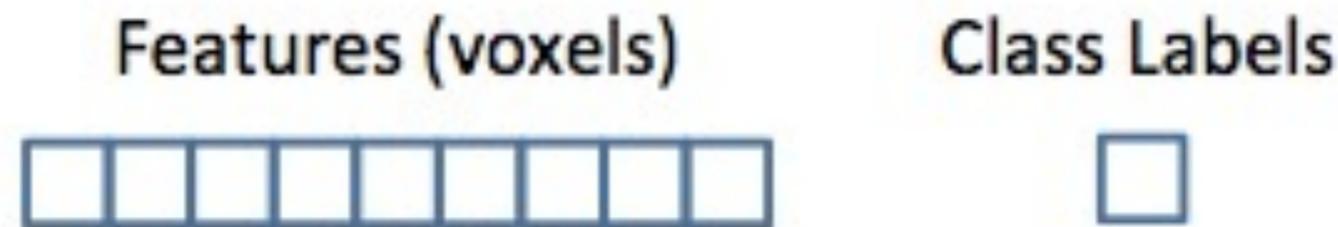
A learned classifiers models the relation between features and class labels in the **training data set**

## Test data

If the classifiers truly captures the relation between features and labels, it should predict the class label for data it has not seen before

Once trained the classifier is evaluated using an independent set of observations (**test data**)

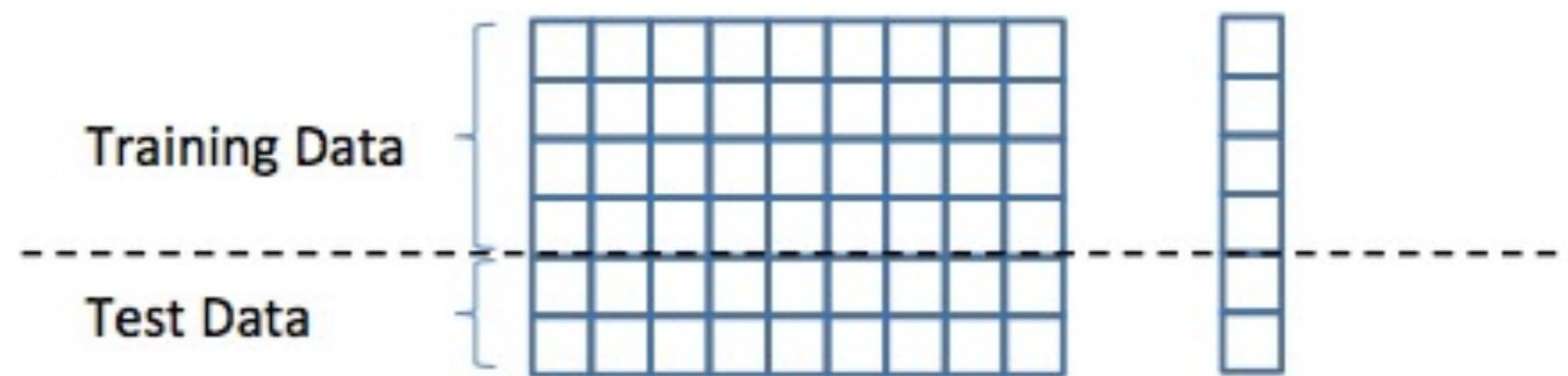
# Read-Outs: illustration



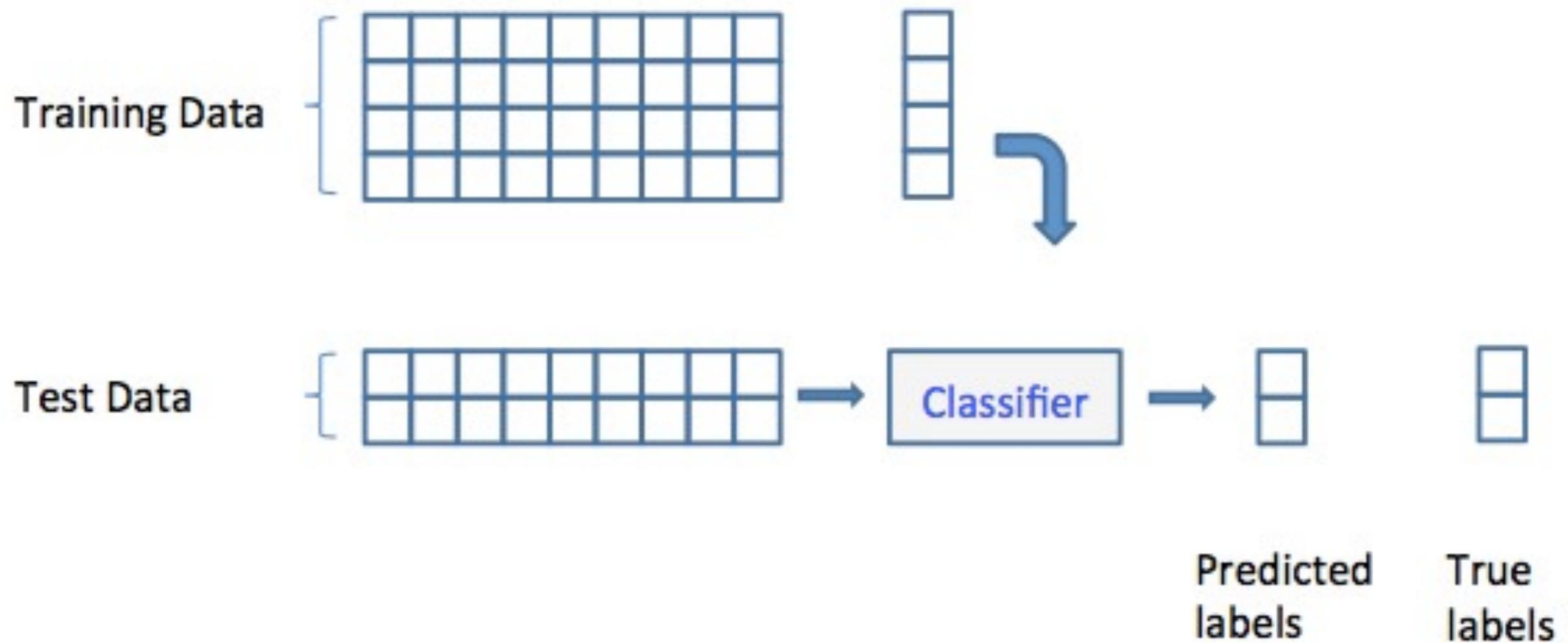
$$\underline{x} = (x_1, \dots, x_v) \quad y$$



# Read-Outs: illustration



# Read-Outs: illustration



# Read-Outs: summary

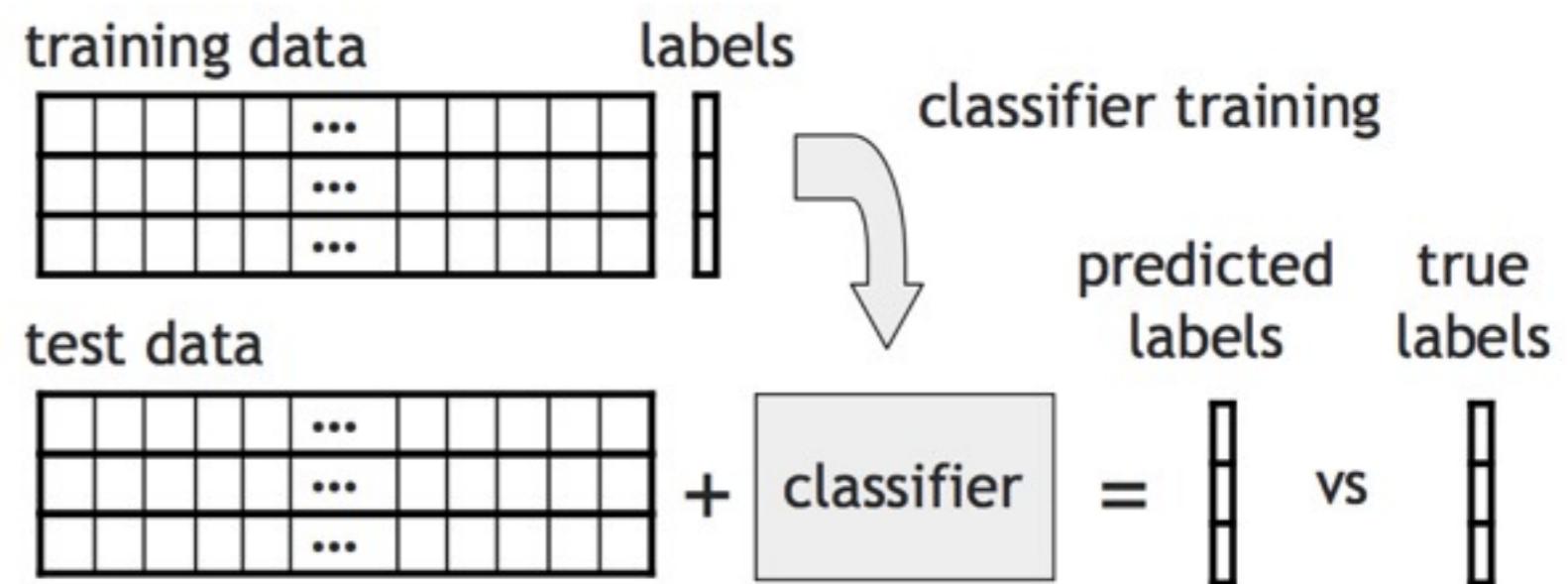
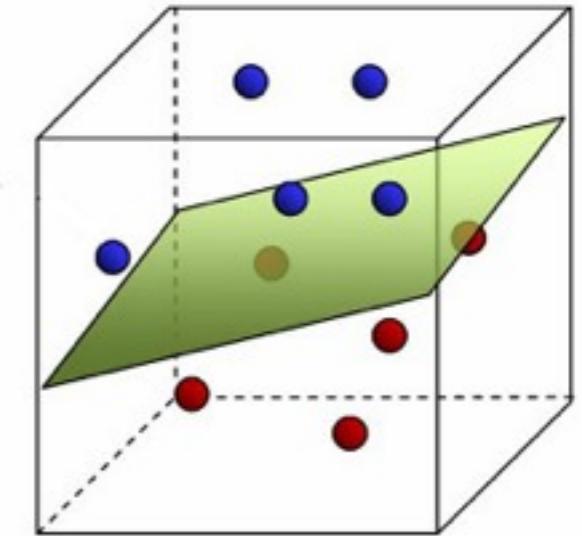
1. Defining features and classes

2. Feature selection

3. Choosing a classifier

4. Training and testing a classifier

5. Examining results



# Read-outs: choose a classifier

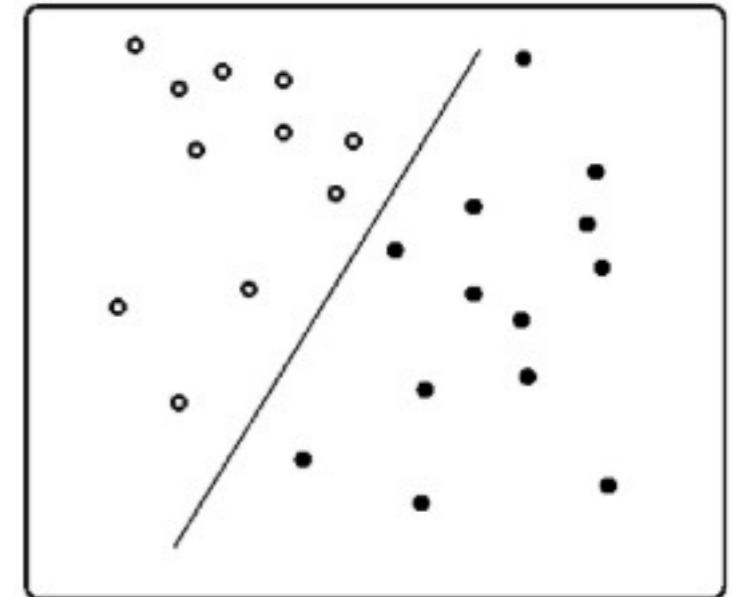
## Linear:

Naive Bayes (NB)

Support Vector Machines (SVM)

Logistic Regression (LR)

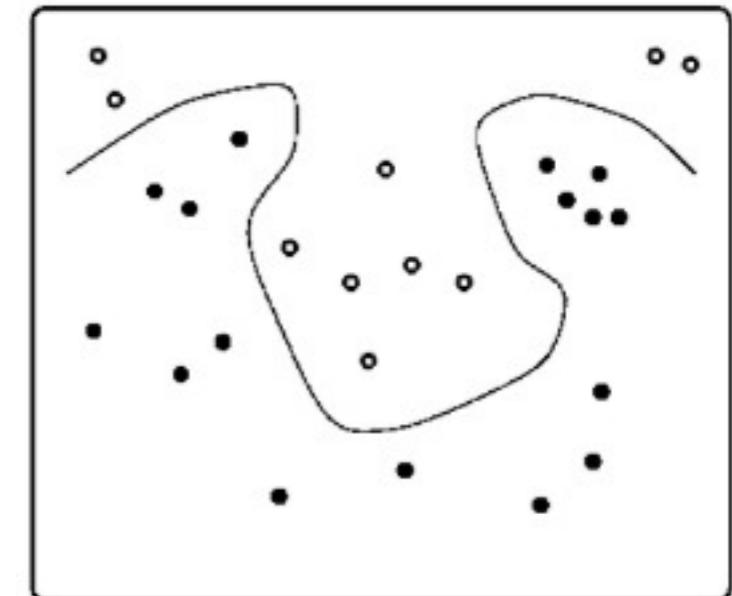
Linear Discrimination Analysis (LDA)



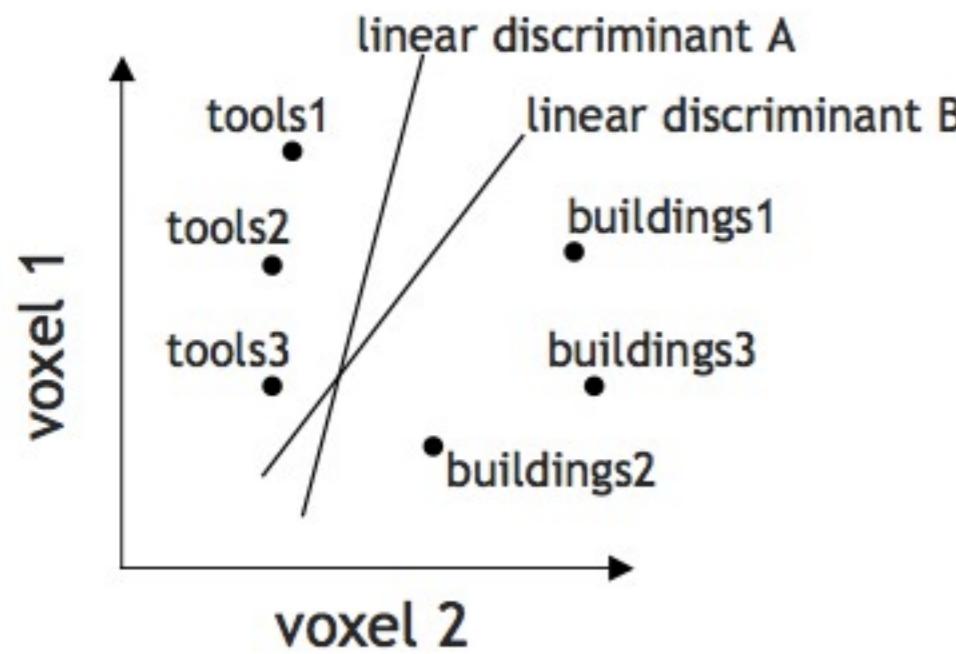
## Non-linear:

Kernel SVM

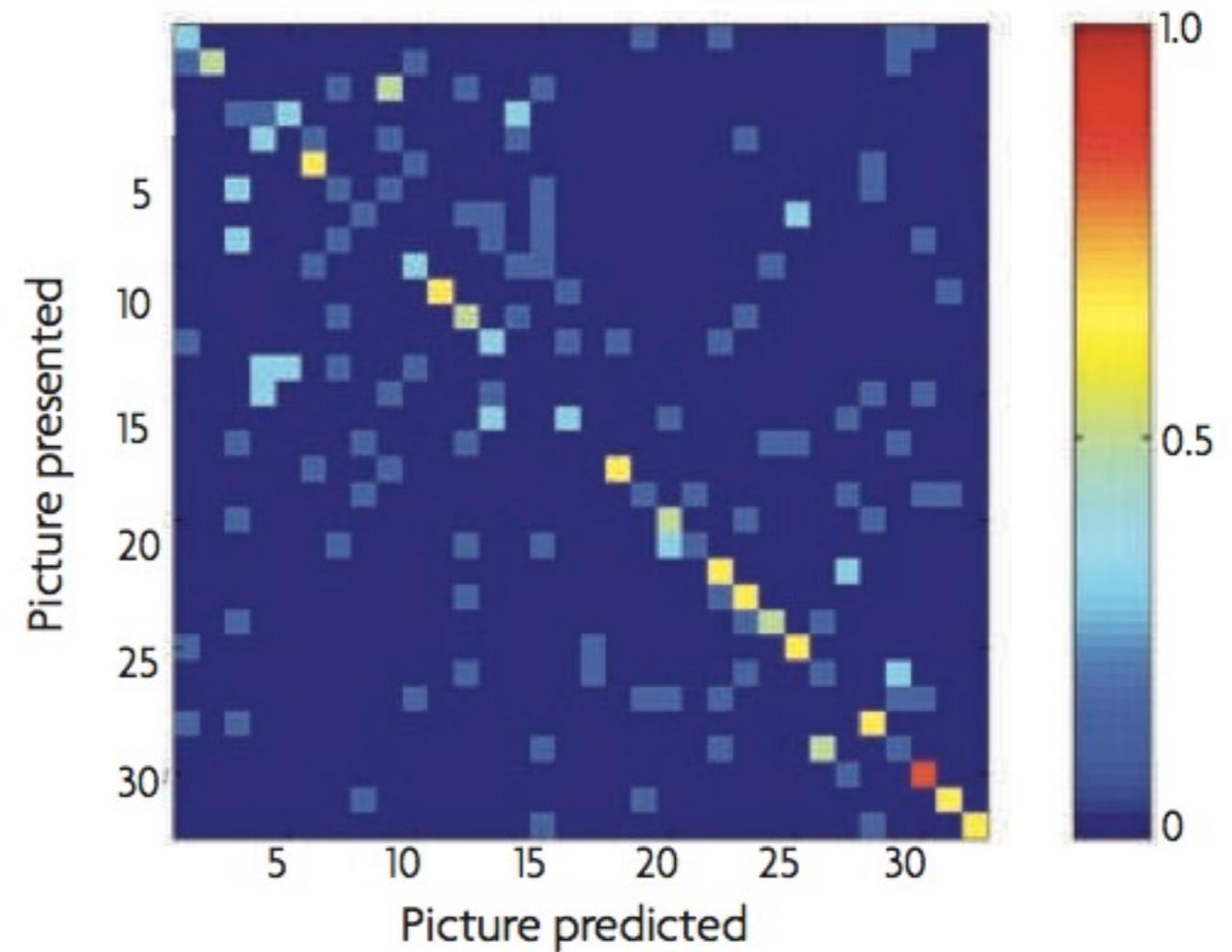
Artificial Neural Networks (ANN)



# Read-outs: presenting results

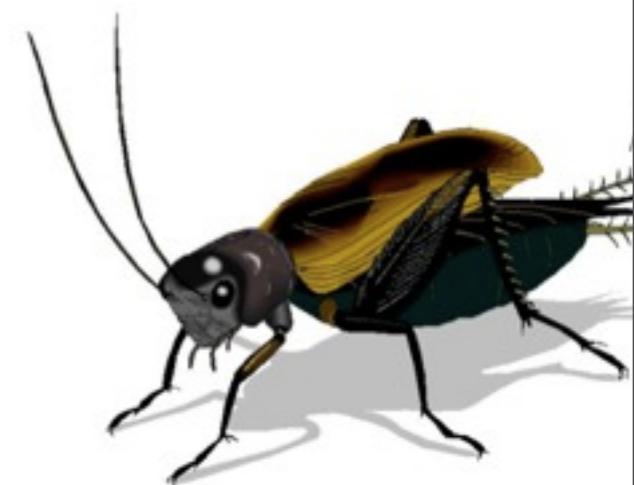
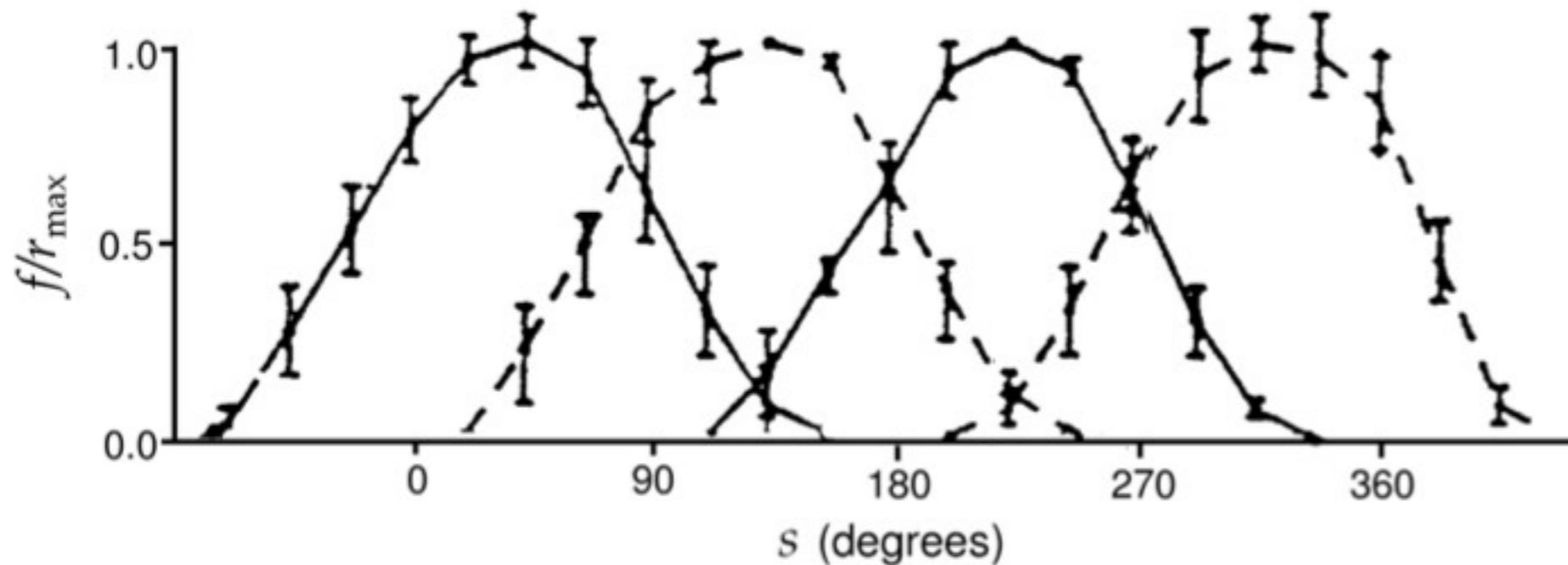


**Confusion matrix**



# Estimators: back to the cricket

**System:** the cercal system of the cricket



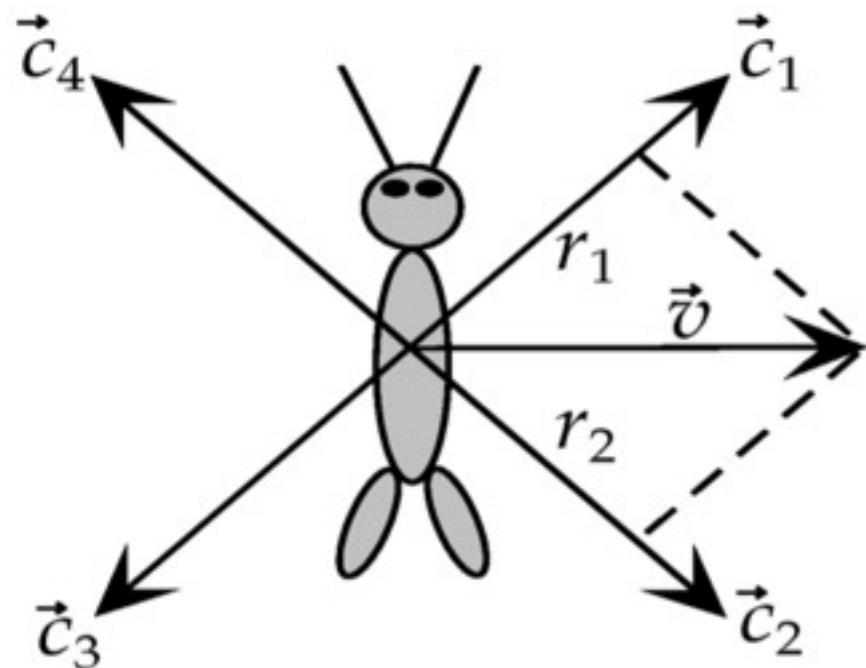
These 4 neurons are sensitive to the angle of wind

Preferred directions  $\mathbf{c}_a$ :  $-135^\circ, -45^\circ, 45^\circ, 135^\circ$

**Crickets  
are  
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# Estimators: population vector

An heuristic method to decode orientation (or any periodic variable) is to say that neuron **a** “votes” for a vector,  $\mathbf{c}_a$ , with a strength determined by its activity  $r_a$



$$\vec{v}_{\text{pop}} = \sum_{a=1}^4 \left( \frac{r}{r_{\max}} \right)_a \vec{c}_a$$

Similar results have been observed for arm reaching movements in monkeys

# Estimation: optimal decoding

The vector method is not general nor optimal

Other methods than can be called “optimal” take into account the full probabilistic model of encoding:

**Maximum likelihood estimator**

$$P(r|s)$$

**Bayesian estimates**

$$P(s|r)$$

# Estimation: ML estimator

Values of  $s$  for which  $p(r|s)$  is high are the stimuli which are likely to have produced the observed activities  $r$ ; values of  $s$  for which  $p(r|s)$  is low are unlikely

## Maximum likelihood estimator

$$\hat{s}_{ML}(r) = \arg \max_s p(r|s)$$

# Estimation: Bayesian estimator

Bayesian estimators combine the likelihood  $p(r|s)$  with any prior information about the stimulus  $s$  to produce a posterior distribution  $p(s|r)$

## Bayesian estimator

$$p(s|r) = \frac{p(r|s)p(s)}{p(r)} \quad p(r) = \sum_s p(r|s)p(s)$$

$$\hat{s}_{MAP}(r) = \arg \max_s p(s|r)$$

# Summary

- Encoding deals with the way stimulus is mapped to neural activity
- Cortex faces the opposite problem (decoding): to infer real world stimulus from neuronal dynamics
- There are several plausible and non-mutually exclusive neural codes: rate, temporal, population...
- Decoding neural activity with read-outs is a useful tool for investigating neural processing and BCI

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