- Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval, Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = x^3 - 3x + 2,\ [-2, 2]$$

$$f'(x) = 3x^2 - 3$$

From the Mean Value Theorem, there is $c \in (-2, 2)$

$$f(2) - f(-2) = f'(c)((2) - (-2))$$

$$4 - 0 = (3c^2 - 3)(2 + 2) \Rightarrow 1 = 3c^2 - 3 \Rightarrow c^2 = \frac{4}{3} \Rightarrow c = \pm\frac{2}{\sqrt{3}}$$

Since $\pm\frac{2}{\sqrt{3}}$ are within the domain $[-2, 2]$, The answer is

$$\pm\frac{2}{\sqrt{3}}$$

- $f(x) = x^4 - 2x^2 + 3$

$$f'(x) = 4x^3 - 4x = 4x(x - 1)(x + 1)$$

x	$-\infty$		-1		0		1		∞
$f'(x)$		-	0	$+$	0	-	0	$+$	
$f(x)$		$\searrow$	2	$\nearrow$	3	$\searrow$	2	$\nearrow$	

$$f''(x) = 12x^2 - 4 = 4(3x^2 - 1)$$

x	$-\infty$		$-\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$		∞
$f''(x)$		$+$		-		$+$	
$f(x)$		$\smile$	$\frac{22}{9}$	$\frown$	$\frac{22}{9}$	$\smile$	

  - Find the intervals on which f is increasing. (Exclude endpoint(s) if exist(s).)

$$(-1, 0) \cup (1, \infty)$$

  - Find the intervals on which f is decreasing. (Exclude endpoint(s) if exist(s).)

$$(-\infty, -1) \cup (0, 1)$$

  - Find the local minimum value(s).

$$f(-1) = f(1) = 2$$

  - Find the local maximum value(s).

$$f(0) = 3$$

  - Find the intervals on which f is concave upward.

$$(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$$

  - Find the intervals on which f is concave downward.

$$(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}})$$

- Find (an) inflection point(s) if exists.

$$(-\frac{1}{\sqrt{3}}, \frac{22}{9}),\ (\frac{1}{\sqrt{3}}, \frac{22}{9})$$

- $f(x) = \frac{x}{x^2+1}$

$$f'(x) = \frac{(1)(x^2+1)-(x)(2x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2} = \frac{-(x+1)(x-1)}{(x^2+1)^2}$$

Critical numbers $c = -1, 1$.
From $[(x^2+1)^2]' = 2(x^2+1)(2x) = 4x(x^2+1)$

x	$-\infty$		-1		1		∞
$f'(x)$		-		+		-	
$f(x)$		$\searrow$	$-\frac{1}{2}$	$\nearrow$	$\frac{1}{2}$	$\searrow$	

$$f''(x) = \frac{(-2x)(x^2+1)^2-(1-x^2)(4x)(x^2+1)}{(x^2+1)^4} = \frac{(x^2+1)[(-2x)(x^2+1)-(4x)(1-x^2)]}{(x^2+1)^4}$$

$$= \frac{2x[-(x^2+1)-2(1-x^2)]}{(x^2+1)^3} = \frac{2x[-x^2-1-2+2x^2]}{(x^2+1)^3} = \frac{2x(x^2-3)}{(x^2+1)^3}$$

x	$-\infty$		$-\sqrt{3}$		0		$\sqrt{3}$		∞
$f''(x)$		-	0	+	0	-	0	+	
$f(x)$		$\frown$	$-\frac{\sqrt{3}}{4}$	$\smile$	0	$\frown$	$\frac{\sqrt{3}}{4}$	$\smile$	

- Find the intervals on which f is increasing. (Exclude endpoint(s) if exist(s).)

$$(-1, 1)$$

- Find the intervals on which f is decreasing. (Exclude endpoint(s) if exist(s).)

$$(-\infty, -1) \cup (1, \infty)$$

- Find the local minimum value(s).

$$f(-1) = -\frac{1}{2}$$

- Find the local maximum value(s).

$$f(1) = \frac{1}{2}$$

- Find the intervals on which f is concave upward.

$$(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$$

- Find the intervals on which f is concave downward.

$$(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$$

- Find (an) inflection point(s) if exists.

$$(-\sqrt{3}, -\frac{\sqrt{3}}{4}),\ (0, 0),\ (\sqrt{3}, \frac{\sqrt{3}}{4})$$

- $f(x) = \frac{x^2}{x^2+1} = \frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{x^2+1}$

$$f'(x) = -\frac{(0)(x^2+1)-(1)(2x)}{(x^2+1)^2} = -\frac{-2x}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

Critical numbers $c = 0$.
From $[(x^2+1)^2]' = 2(x^2+1)(2x) = 4x(x^2+1)$

x	$-\infty$		0		∞
$f'(x)$		-		+	
$f(x)$		$\searrow$	0	$\nearrow$	

$$f''(x) = \frac{(2)(x^2+1)^2 - (2x)(4x)(x^2+1)}{(x^2+1)^4} = \frac{(2)(x^2+1)[(x^2+1)-(4x)(x)]}{(x^2+1)^4}$$

$$= \frac{2[x^2+1-4x^2]}{(x^2+1)^3} = \frac{-2[3x^2-1]}{(x^2+1)^3}$$

Therefore

$$f''(x) = 0 \ or \ f''(x) : DNE \Rightarrow 3x^2 - 1 = 0 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

x	$-\infty$		$-\frac{1}{\sqrt{3}}$		$\frac{1}{\sqrt{3}}$		∞
$f''(x)$		-	0	+	0	-	
$f(x)$		$\frown$	$\frac{1}{4}$	$\smile$	$\frac{1}{4}$	$\frown$	

- Find the intervals on which f is increasing. (Exclude endpoint(s) if exist(s).)

$$(0, \infty)$$

- Find the intervals on which f is decreasing. (Exclude endpoint(s) if exist(s).)

$$(-\infty, 0)$$

- Find the local minimum value(s).

$$f(0) = 0$$

- Find the local maximum value(s). No.
- Find the intervals on which f is concave upward.

$$(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$

- Find the intervals on which f is concave downward.

$$(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$$

- Find (an) inflection point(s) if exists.

$$(-\frac{1}{\sqrt{3}}, \frac{1}{4}), (\frac{1}{\sqrt{3}}, \frac{1}{4})$$