20156 MA441-B124 QUIZ1-ANSWER (Show your work!)

Question 1 *Find the limits of functions including the infinite limits, ∞, $-\infty$. Otherwise write DNE. (Do not use L'hospital theorem.)*

$$\lim_{x\to -1}\left[\frac{2x^2+x-1}{x^2+3x+2}\right]$$

Answer.

$$\lim_{x\to -1}\left[\frac{2x^2+x-1}{x^2+3x+2}\right]=\lim_{x\to -1}\left[\frac{(2x-1)(x+1)}{(x+2)(x+1)}\right]=\lim_{x\to -1}\left[\frac{(2x-1)}{(x+2)}\right]=\frac{2(-1)-1}{(-1)+2}=\frac{-3}{1}=\mathbf{-3}$$

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Question 2 *Find the limits of functions including the infinite limits, ∞, $-\infty$. Otherwise write DNE. (Do not use L'hospital theorem.)*

$$\lim_{t\to 0}\left[\frac{\sqrt{t^2+1}-1}{t^2}\right]$$

Answer. Since $\sqrt{x}\sqrt{x}=x$ for $x\in\mathbb{R}$,

$$\lim_{t\to 0}\left[\frac{\sqrt{t^2+1}-1}{t^2}\right]=\lim_{t\to 0}\left[\frac{\sqrt{t^2+1}-1}{t^2}\cdot\frac{(\sqrt{t^2+1}+1)}{(\sqrt{t^2+1}+1)}\right]=\lim_{t\to 0}\left[\frac{t^2+1-1}{t^2(\sqrt{t^2+1}+1)}\right]=\lim_{t\to 0}\left[\frac{t^2}{t^2(\sqrt{t^2+1}+1)}\right]$$

$$=\lim_{t\to 0}\left[\frac{1}{\sqrt{t^2+1}+1}\right]=\frac{1}{\sqrt{(0)^2+1}+1}=\frac{\mathbf{1}}{\mathbf{2}}$$

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Question 3

$$\lim_{x\to -2^-}\left[\frac{4x+8}{|x+2|}\right]$$

Answer.

$$lim_{x\to -2^-}\left[\frac{4x+8}{|x+2|}\right]=lim_{x\to -2^-}\left[\frac{4(x+2)}{-(x+2)}\right]=lim_{x\to -2^-}\left[\frac{4}{-1}\right]=\mathbf{-4}$$

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Question 4 *Let*

$$f(x)=\begin{cases}-x^2 & ,x>1\\ x-1 & ,x<1\\ 2 & x=1\end{cases}$$

Show which function you used to compute limits.

a) *(3pt) Find* $\lim_{x\to 1^-} f(x)$ ***Answer.***

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(x-1)=(1)-1=\mathbf{0}$$

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b) *(3pt) Find* $\lim_{x\to 1^+} f(x)$ ***Answer.***

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(-x^2)=-(1)^2=\mathbf{-1}$$

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c) *(2pt) Does* $\lim_{x\to 1} f(x)$ *exist? Explain it.* ***Answer.*** *Since* $\lim_{x\to 1^-} f(x)\neq\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$ ***does not exist.*** ■

d) *(2pt) Is f continuous at $x=1$? Explain it.* ***Answer.*** *Since* $\lim_{x\to 1} f(x)$ *does not exist, f is **not continuous** at $x=1$* ■