Your name:

Question 1 *Find the limits of functions including the infinite limits, ∞, $-\infty$. Otherwise write DNE. (Do not use L'hospital theorem.)*

(1.6) a) $\lim_{x\to 5}(x^3 - 3x + 4)$

(1.6) b) $\lim_{x\to -2}\left[\frac{x^3 + 2x^2 - 1}{5 - 3x}\right]$

(1.6) c) $\lim_{x\to -1}\left[\frac{2x^2 + 3x + 1}{x^2 - 2x - 3}\right]$

(1.6) d) $\lim_{h\to 0}\left[\frac{(2+h)^3 - 8}{h}\right]$

(1.6) e) $\lim_{t\to 0}\left[\frac{\sqrt{t^2+4} - 2}{t^2}\right]$

(1.5) f) $\lim_{x\to 3^-}\frac{2x}{x-3}$

(1.5) g) $\lim_{x\to 3^+}\frac{2x}{x-3}$

(1.5) h) $\lim_{x\to 0}\frac{1}{x^2}$

(1.5) i) $\lim_{x\to 0}\frac{1}{x^3}$

(1.6) j) $\lim_{x\to 1}\frac{|x-1|}{x-1}$

(2.4) k) $\lim_{x\to 0}\left[\frac{\sin(5x)}{3x}\right]$

(2.4) l) $\lim_{\theta\to 0}\left[\frac{\cos\theta - 1}{\sin\theta}\right]$

(2.4) m) $\lim_{x\to 0}\left[\frac{\sin(3x)\sin(5x)}{x^2}\right]$

(2.4) n) $\lim_{\theta\to 0}\left[\frac{\cos\theta - 1}{2\theta^2}\right]$

(3.4) o) $\lim_{x\to\infty}\left[\frac{2x^2 - 7}{5x^2 + x - 3}\right]$

(3.4) p) $\lim_{x\to -\infty}\left[\frac{x-2}{x^2+1}\right]$

(3.4) q) $\lim_{x\to\infty}\left[\frac{x + 3x^2}{4x - 1}\right]$

(3.4) r) $\lim_{x\to\infty}\left[\frac{\sqrt{x + 3x^2}}{4x - 1}\right]$

(3.4) s) $\lim_{x\to -\infty}\left[\frac{\sqrt{x + 3x^2}}{4x - 1}\right]$

(3.4) t) $\lim_{x\to\infty}\left[\sqrt{9x^2 + x} - 3x\right]$

(3.4) u) $\lim_{x\to -\infty}\left[\sqrt{4x^2 + 3x} + 2x\right]$

(3.4) v) $\lim_{x\to -\infty}\left[\sqrt{4x^2 + 3x} - 2x\right]$

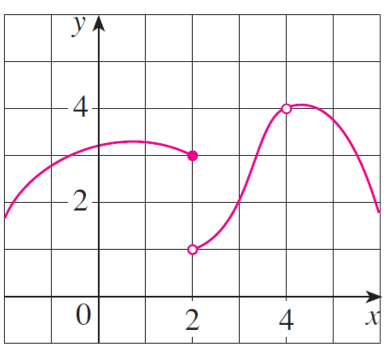
(3.4) w) $\lim_{x\to\infty} x\cdot\sin(\frac{2}{x})$

(3.4) x) $\lim_{x\to\infty} \sqrt{x}\cdot\sin(\frac{2}{x})$

(2.2, honor) y) $\lim_{x\to 1}\left[\frac{x^{2015} - 1}{x - 1}\right]$

(4.3, honor) z) $\lim_{h\to 0}\frac{1}{h}\int_2^{2+h}\sqrt{1+t^3}\,dt$

Question 2 (1.5) *Use the given graph of f to state the value of each quantity, if exists. If it does not exist, explain why.*



a) $\lim_{x\to 2^-} f(x)$

b) $\lim_{x\to 2^+} f(x)$

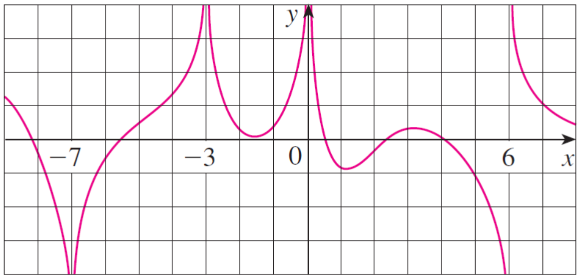
c) $\lim_{x\to 2} f(x)$

d) $f(2)$

e) $\lim_{x\to 4} f(x)$

f) $f(4)$

Question 3 (1.5) *For the function f whose graph is shown, state the following.*



a) $\lim_{x\to -7} f(x)$

b) $\lim_{x\to -3} f(x)$

c) $\lim_{x\to 0} f(x)$

d) $\lim_{x\to 6^-} f(x)$

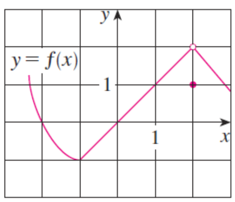
e) $\lim_{x\to 6^+} f(x)$

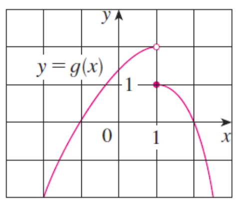
f) *The equations of the vertical asymptote(s).*

Question 4 (1.6) *Using squeeze theorem, show*

$$\lim_{x\to 0} x \sin\frac{1}{x} = 0$$

Question 5 (1.6) *The graphs of f and g are given. State the following. Use $\infty, -\infty$ for infinite limits. If the limit does not exist or is not infinite limits, explain why.*





a) $\lim_{x\to 1^-} (f(x) + g(x))$.

b) $\lim_{x\to -1} \dfrac{f(x)}{g(x)}$.

c) $\lim_{x\to 1} \sqrt{3 + f(x)}$.

Question 6 (1.6) a) *What is wrong with the following equation?*

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

b) *Explain why the following equation is correct.*

$$\lim_{x\to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x\to 2}(x + 3)$$

Question 7 (1.6,1.8) *Let*

$$f(x) = \begin{cases} x^2 + 1 & , x < 1 \\ (x-2)^2 & , x \geq 1 \end{cases}$$

a) *Find* $\lim_{x\to 1^-} f(x)$ *and* $\lim_{x\to 1^+} f(x)$

b) *Does* $\lim_{x\to 1} f(x)$ *exist?*

c) *Is f continuous at $x = 1$? Explain.*

d) *Sketch the graph f.*

Question 8 (1.6,1.8) *Let*

$$f(x) = \frac{x^2 + x - 6}{|x - 2|}$$

a) *Find* $\lim_{x\to 2^-} f(x)$ *and* $\lim_{x\to 2^+} f(x)$

b) *Does* $\lim_{x\to 2} f(x)$ *exist?*

c) *Is f continuous at $x = 2$? Explain.*

d) Sketch the graph f.

Question 9 (1.6) *If* $\lim_{x\to 1} \frac{f(x)-8}{x-1} = 10$, *find* $\lim_{x\to 1} f(x)$.

Question 10 (1.8) *Explain why the function f is discontinuous at the given number a.*

a) $a = 0$

$$f(x) = \begin{cases} \cos x & if\ x < 0 \\ 0 & if\ x = 0 \\ 1 - x^2 & if\ x > 0 \end{cases}$$

b) $a = 1$

$$f(x) = \begin{cases} \frac{x^2-x}{x^2-1} & if\ x \neq 1 \\ 1 & if\ x = 1 \end{cases}$$

Question 11 (1.8) *Find the values of a and b that make f continuous everywhere.*

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & if\ x < 2 \\ ax^2 - bx + 3 & if\ 2 \le x < 3 \\ 2x - a + b & if\ x \ge 3 \end{cases}$$

Question 12 (1.8) *Use IVT to show there is a root of the given equation in the specified interval.*

a)

$$x^4 + x - 3 = 0, \quad (1,2)$$

b) Use the calculator.

$$\sin x = x^2 - x, \quad (1,2)$$

(honor) c) Use the calculator.

$$\sqrt{x-5} = \frac{1}{x+3}, \quad (-\infty, \infty)$$

Question 13 (1.6,1.8, honor) *For what values of x is f continuous?*

a)

$$f(x) = \begin{cases} 0 & \textit{if } x \textit{ is rational} \\ 1 & \textit{if } x \textit{ is irrational} \end{cases}$$

b)

$$f(x) = \begin{cases} 0 & \textit{if } x \textit{ is rational} \\ x & \textit{if } x \textit{ is irrational} \end{cases}$$

Question 14 *Find derivatives.*

(2.3) a) $f(x) = 2^{12}$

(2.3) b) $f(x) = 3x^2 - 4x + 1$

(2.3) c) $f(x) = \dfrac{1}{2}x - \dfrac{1}{3}$

(2.3) d) $f(x) = (x-2)(2x+3)$

(2.3) e) $f(x) = (1 + x + x^2)(2 - x^4)$

(2.3) f) $f(x) = \dfrac{2x+1}{x+4}$

(2.3) g) $f(x) = \dfrac{x^2-1}{2x-3}$

(2.3) h) $f(x) = \dfrac{x}{(x-1)^2}$

(2.3) i) $f(x) = \dfrac{x}{x^2-1}$

(2.3) j) $f(x) = x^{-2}$

(2.3) k) $f(x) = \sqrt{x} - x$

(2.3) l) $f(x) = \dfrac{x^2+4x+3}{\sqrt{x}}$

(2.3) m) $f(x) = \dfrac{\sqrt{x}+x}{x^2}$

(2.3) n) $f(x) = \dfrac{4}{1-x}$

(2.4) o) $f(x) = 3x^2 - 2\cos x$

(2.4) p) $f(x) = \sec x \cdot \tan x$

(2.4) q) $f(x) = \dfrac{\tan x - 1}{\sec x}$

(2.4) r) $f(x) = \sqrt{x}\sin x$

(2.5) s) $f(x) = \sin(\sqrt{x})$

(2.5) t) $f(x) = \sqrt{\sin x}$

(2.5) u) $f(x) = (2x^3 + 5)^4$

(2.5) v) $f(x) = \sqrt{1-2x}$

(2.5) w) $f(x) = \sqrt{x^2+4}$

(2.5) x) $f(x) = \sqrt{x + \sqrt{x}}$

(2.5) y) $f(x) = \sec^2 x + \tan^2 x$

(2.5) z) $f(x) = \sin(x^2) + \cos x$

(2.5) aa) $f(x) = \sin(\tan(2x))$

Question 15 *Find $f'(x)$ using the **definition of the derivative**.*

(2.1) a) $f(x) = 3x^2 - 4x + 1$

(2.1) b) $f(x) = \dfrac{1}{x}$

(2.1) c) $f(x) = 2\sqrt{x}$

(2.4) d) $f(x) = \sin x$

Question 16 (2.3) *Suppose that $f(4) = 2, g(4) = 5, f'(4) = 6, g'(4) = -3$. Find $h'(4)$.*

a) $h(x) = 3f(x) + 8g(x)$

b) $h(x) = f(x)g(x)$

c) $h(x) = \frac{f(x)}{g(x)}$

d) $h(x) = \frac{g(x)}{f(x)+g(x)}$

Question 17 (2.3) *If $h(2) = 4$ and $h'(2) = -3$, find*

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

Question 18 (2.5) *If $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$, find $h'(1)$.*

Question 19 *Find an equation of the tangent line to the curve at the given point.*

(2.3) a)

$$y = x^4 + 2x^2 - x, \ (1,2)$$

(2.3) b)

$$y = \frac{2x}{x+1}, \ (1,1)$$

(2.3) c)

$$y = \frac{\sqrt{x}}{1+x^2}, \ (1, \frac{1}{2})$$

(2.4) d)

$$y = \sec x, \ (\frac{\pi}{3}, 2)$$

(2.5) e)

$$y = \sqrt{1+x^3},\ (2,3)$$

(2.6) f)

$$x^2 + xy + y^2 = 3,\ (1,1)$$

Question 20 (2.6) *Find* $\frac{dy}{dx}$.

a)

$$x^2 + xy - y^2 = 4$$

b)

$$x^4 + x^2y^2 + y^3 = 5$$

c)

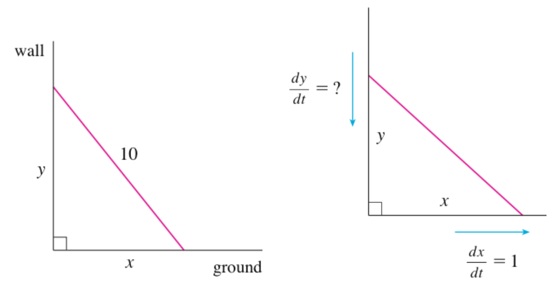
$$xy = \sqrt{x^2+y^2}$$

(honor) d)

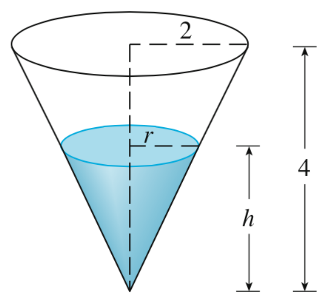
$$\tan\left(\frac{x}{y}\right) = x + y$$

Question 21 (2.8) *Air is being pumped into a spherical balloon so that its volume increase at a rate of* $100cm^3/s$*. How fast is the radius of the balloon increasing when the diameter is* $50cm$*. Hint:* $V = \frac{4\pi r^3}{3}$.

Question 22 (2.8) *A ladder 10ft long rest against a vertical wall. If the bottom of the ladder slide away from the wall at a rate* $1ft/s$*, how fast is the top of the latter slding down the wall when the botton of the ladder is 6ft from the wall?*



Question 23 (2.8) *A water tank has the shape of an inverted circular cone with base radius* $2m$ *and height* $4m$*. If water is being pumped into the tank at a rate of* $2m^3/min$*, find* ***the rate at which the water level is rising*** *when the water is 3m deep. Hint:* $V = \frac{1}{3}\pi r^2 h$.



Question 24 (2.1, honor) *Determine whether* $f'(0)$ *exists.*

a)

$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

b)

$$f(x) = \begin{cases} x^2\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Question 25 (2.2, honor) *Let*

$$f(x) = \begin{cases} x^2+1, & x < 1 \\ x+1, & x \geq 1 \end{cases}$$

Is f differentiable at 1?
Hint: Use

$$f_-'(a) := \lim_{h\to a^-}\frac{f(h+a)-f(a)}{h} \quad \text{, Left-hand derivative of } f$$

$$f_+'(a) := \lim_{h\to a^+}\frac{f(h+a)-f(a)}{h} \quad \text{, Right-hand derivative of } f$$

and

$$f'(a) \text{ exists } \Leftrightarrow f_+'(a) = f_-'(a)$$

Question 26 (2.3) *Let*

$$f(x) = \begin{cases} x^2, & x \le 2 \\ mx + b, & x > 2 \end{cases}$$

Find the value of m and b that make f differentiable everywhere.

Question 27 (2.4) *Find the given derivative by finding the first few derivatives and observing the pattern that occurs.*

a) $\dfrac{d^{99}}{dx^{99}}(\sin x)$

(honor) b) $\dfrac{d^{30}}{dx^{30}}(x \cdot \sin x)$

Question 28 (2.8) *Suppose $y = \sqrt{2x+1}$ where x and y is functions of t.*

- *If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = 4$.*
- *If $\frac{dy}{dt} = 5$, find $\frac{dx}{dt}$ when $x = 12$.*

Question 29 (2.8) *Suppose $4x^2 + 9y^2 = 36$ where x and y is functions of t.*

- *If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = -2$ and $y = \frac{2\sqrt{5}}{3}$.*
- *If $\frac{dy}{dt} = \frac{1}{3}$, find $\frac{dx}{dt}$ when $x = 2$ and $y = \frac{2\sqrt{5}}{3}$.*

Question 30 (2.9) *Find the linearization $L(x)$ of the function f at a where*

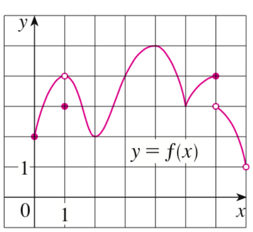
$$L(x) = f(a) + f'(a)(x - a)$$

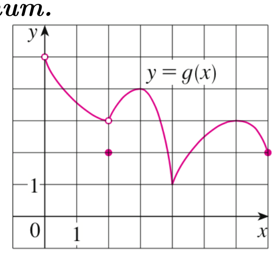
- $f(x) = x^4 + 3x^2, \ a = -1$
- $f(x) = \sqrt{x}, \ a = 4$

Question 31 (2.9) *Find the differential dy of each function.*

- $y = x^2 \cdot \sin 2x$
- $y = \frac{s}{1+2s}$

Question 32 (3.1) *Use the graph to state the absolute and local maximum and minimum **values** and corresponding **points** of the function f and g.**Note:** By Stewart's definition, **endpoints cannot be a local maximum or minimum.***





Question 33 (3.1) *Find the the critical number(s) of the function f.*

a) $f(x) = 2x^3 - 3x^2 - 36x$.

b) $f(x) = |3x - 4|$.

c) $f(x) = \dfrac{x-1}{x^2+4}$.

d) $f(x) = 2\cos x + \sin^2 x$.

Question 34 (3.1) *Find the the absolute maximum and minimum **values** of the function f.*

a) $f(x) = 2x^3 - 3x^2 - 12x + 1, \ [-2, 3]$.

b) $f(x) = 3x^4 - 16x^3 + 18x^2,\ [-1, 4]$.

c) $f(x) = x\sqrt{4 - x^2},\ [-1, 2]$.

Question 35 (3.2) *Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval, Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.*

$$f(x) = x^3 - 3x + 2,\ [-2, 2]$$

Question 36 (3.2) *Show that the equation* $x^3 - 12x + 1 = 0$ *has exact one root in the interval* $[-1, 1]$.
HINT: *For the existence, use IVT. For the uniqueness, use Rolle's Theorem.*

Question 37 (3.3,3.4) *Given* f,

a) $f(x) = 2x^3 + 3x^2 - 36x$ *b)* $f(x) = 3x^4 - 8x^3 - 24x^2 + 96x$ *c)* $f(x) = \frac{x}{x^2+1}$

- *Find the intervals on which* f *is increasing. (Exclude endpoint(s) if exist(s).)*
- *Find the intervals on which* f *is decreasing. (Exclude endpoint(s) if exist(s).)*
- *Find the local minimum value(s).*
- *Find the local maximum value(s).*
- *Find the intervals on which* f *is concave upward.*
- *Find the intervals on which* f *is concave downward.*
- *Find (an) inflection point(s) if exists.*

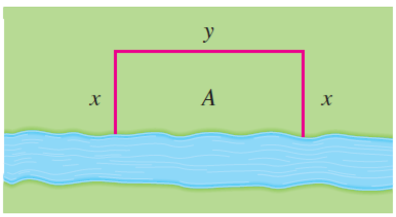
Question 38 (3.4) *Sketch the curve.*

a) $y = \frac{2x^2}{x^2-1}$ *c)* $y = \frac{x^2}{\sqrt{x+1}}$

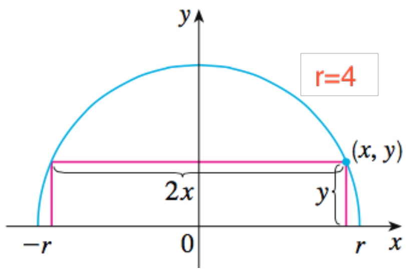
b) $y = \frac{x}{x^2-9}$ *d)* $y = \frac{x}{\sqrt{x^2+1}}$

- *Find the domain.*
- *Find* x*-and* y*-intercepts.*
- *Find the symmetry if exists.(even, odd, periodic).*
- *Find the asymptote(s).*
- *Find the interval(s) of Increase or Decrease.*
- *Find the local maximum and minimum value(s).*
- *Find concavity and points of inflection.*
- *Sketch the Curve.*

Question 39 (3.7) *A farmer has 2400ft of fencing and wants to fence off a rectangular file that border a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?*



Question 40 (3.8) *Find the area of the largest rectangle that can be inscribed in a semicircle of radius* $r = 4$.
Hint: Describe y *in terms of* x. $0 \le x \le 4,\ 0 \le y \le 4$.



Question 41 (3.9) *Find the most general anti-derivative of each of the following functions.*

(3.9) a) $f(x) = \frac{1}{2}x^2 - 2x + 6$

(3.9) b) $f(x) = (x-1)^2$

(3.9) c) $f(x) = \frac{10}{x^9}$

(3.9) d) $f(x) = \sqrt{2}$

(3.9) e) $f(x) = \frac{3}{\sqrt{x}}$

(3.9) f) $f(x) = (x+1)(2x-1)$

(3.9) g) $f(t) = \frac{1+t+t^2}{\sqrt{t}}$

(3.9) h) $f(x) = \sqrt{\frac{3}{x}}$

(4.4) i) $f(x) = 2\sqrt{x} + 2\sec(x)\tan(x)$

(4.4) j) $f(x) = \sqrt{x} + 2\sin(x) + x$

(4.5) k) $f(x) = x^2\sqrt{x^3+1}$

(4.5) l) $f(x) = (1-2x)^9$

(4.5) m) $f(x) = (x^2+1)(x^3+3x)^4$

(4.5) n) $f(x) = \sec^3(x)\tan(x)$

(4.5) o) $f(x) = x\sqrt{1+x^2}$

(4.5) p) $f(x) = x^3\sqrt{1+x^2}$

Question 42 (3.9) *Find the function* f *if*

a) $f'(x) = 5x^4 - 2x^5$ *and* $f(0) = 4$.

b) $f''(x) = \sin(x) + \cos(x)$, $f(0) = 3$ *and* $f'(0) = 4$.

Question 43 (4.2) *Express the limit as a definite integral on the given interval.*

a)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1-x_i^2}{4+x_i^2} \cdot \Delta x,\ [2,6]$$

b)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5},\ [0,1]$$

Question 44 (4.2) *Using the* ***the definition of the (definite) integral*** *to evaluate the integral.*

a) $\int_2^5 (4-2x)dx$

b) $\int_0^1 x^3 dx$

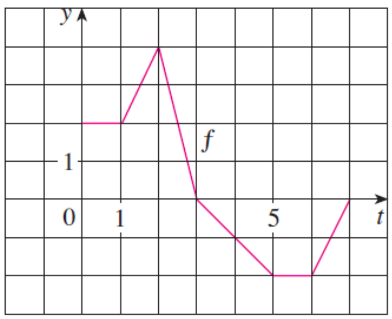
Question 45 (4.2) *If* $\int_0^9 f(x)dx = 37$ *and* $\int_0^9 g(x)dx = 16$, *find* $\int_0^9 [2f(x) + 3g(x)]dx$.

Question 46 (4.2) *If* $\int_1^5 f(x)dx = 12$ *and* $\int_4^5 f(x)dx = 3.5$, *find* $\int_1^4 2f(x)dx$.

Question 47 (4.2) *Use the properties of integrals to verify the inequality.*

$$2 \le \int_{-1}^{1} \sqrt{1+x^2}dx \le 2\sqrt{2}$$

Question 48 (4.3) *Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.*



- *Evaluate $g(0), g(1), g(2), g(3), g(6)$.*
- *On what interval is g increasing?*
- *Where g have a maximum value?*
- *Sketch a rough graph of g.*

Question 49 (4.3) *Use the fundamental theorem of Calculus to find the* ***derivative*** *of the function.*

a) $f(x) = \int_1^x \frac{1}{t^3+1}dt$

b) $f(x) = \int_x^\pi \sqrt{1+\sec(t)}dt$

c) $f(x) = \int_0^{x^4} \cos^2(\theta)d\theta$

d) $f(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4+1}dt$

Question 50 *Evaluate.*

(4.3) a) $\displaystyle\int_{-1}^{2}(x^3-2x)dx$

(4.3) b) $\displaystyle\int_{\frac{\pi}{6}}^{\pi}\sin\theta d\theta$

(4.3) c) $\displaystyle\int_{1}^{2}(1+2y)^2 dy$

(4.4) d) $\displaystyle\int_{0}^{2}|2x-1|dx$

(4.4) e) $\displaystyle\int_{-1}^{2}(x-2|x|)\,dx$

(4.5) f) $\displaystyle\int_{-1}^{1}x(1-x)^2dx$

(4.5) g) $\displaystyle\int_{1}^{4}\sqrt{\frac{5}{x}}dx$

(4.5) h) $\displaystyle\int_{0}^{\frac{\pi}{2}}\cos(x)\sin(\sin(x))dx$

(4.5) i) $\displaystyle\int_{1}^{2}x\sqrt{x-1}dx$

(4.5) j) $\displaystyle\int_{0}^{1}x^3\sqrt{1+x^2}dx$

(4.4) k) $\displaystyle\int_{1}^{9}\frac{x-1}{\sqrt{x}}dx$

(4.4) l) $\displaystyle\int_{-2}^{2}f(x)dx$ *where* $f(x) = \begin{cases} 2 & if\ -2 \le x \le 0 \\ 4-x^2 & if\ 0 < x \le 2 \end{cases}$

Question 51 (4.4) *What is wrong with the equation? Explain.*

$$\int_{-2}^{1}x^{-4}dx = \left.\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$$

Question 52 (4.3) *Evaluate the limit by recognizing the sum as a Riemman sum.*

$$\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^3+\left(\frac{2}{n}\right)^3+\ldots+\left(\frac{n}{n}\right)^3\right]$$

Question 53 (4.3) *Evaluate the limit by recognizing the sum as a Riemman sum.*

$$\lim_{n\to\infty}\frac{1}{n}\left(\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+\ldots+\sqrt{\frac{n}{n}}\right)$$

Question 54 (4.4) *The velocity function (in meters per second) is given for a particle moving along a line.*

$$v(t) = 3t-5, 0 \le t \le 3$$

- *Find the displacement*

- *Find the distance traveled.*

Question 55 (4.5) *If f is continuous and $\int_0^4 f(x)dx = 10$, find $\int_0^2 f(2x)dx$.*

Question 56 (4.5) *If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 xf(x^2)dx$.*

Question 57 (5.1) *Find the area of the region enclosed by the parabola $y = 5x - x^2$ and the line $y = x$.*

- *Find the points of intersection.*
- *Find the area.*

Question 58 (5.1) *Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.*

- *Find the points of intersection.*
- *Find the area.*