

**Question 1** Find the limits of functions including the infinite limits,  $\infty$ ,  $-\infty$ . Otherwise write DNE. (Do not use L'hospital theorem.)

$$\lim_{x \rightarrow -1} \left[ \frac{2x^2 + x - 1}{x^2 + 3x + 2} \right]$$

**Answer.**

$$\lim_{x \rightarrow -1} \left[ \frac{2x^2 + x - 1}{x^2 + 3x + 2} \right] = \lim_{x \rightarrow -1} \left[ \frac{(2x - 1)(x + 1)}{(x + 2)(x + 1)} \right] = \lim_{x \rightarrow -1} \left[ \frac{(2x - 1)}{(x + 2)} \right] = \frac{2(-1) - 1}{(-1) + 2} = \frac{-3}{1} = -3$$

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**Question 2** Find the limits of functions including the infinite limits,  $\infty$ ,  $-\infty$ . Otherwise write DNE. (Do not use L'hospital theorem.)

$$\lim_{t \rightarrow 0} \left[ \frac{\sqrt{t^2 + 1} - 1}{t^2} \right]$$

**Answer.** Since  $\sqrt{x}\sqrt{x} = x$  for  $x \in \mathbb{R}$ ,

$$\begin{aligned} \lim_{t \rightarrow 0} \left[ \frac{\sqrt{t^2 + 1} - 1}{t^2} \right] &= \lim_{t \rightarrow 0} \left[ \frac{\sqrt{t^2 + 1} - 1}{t^2} \cdot \frac{(\sqrt{t^2 + 1} + 1)}{(\sqrt{t^2 + 1} + 1)} \right] = \lim_{t \rightarrow 0} \left[ \frac{t^2 + 1 - 1}{t^2(\sqrt{t^2 + 1} + 1)} \right] = \lim_{t \rightarrow 0} \left[ \frac{t^2}{t^2(\sqrt{t^2 + 1} + 1)} \right] \\ &= \lim_{t \rightarrow 0} \left[ \frac{1}{\sqrt{t^2 + 1} + 1} \right] = \frac{1}{\sqrt{(0)^2 + 1} + 1} = \frac{1}{2} \end{aligned}$$

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**Question 3**

$$\lim_{x \rightarrow -2^-} \left[ \frac{4x + 8}{|x + 2|} \right]$$

**Answer.**

$$\lim_{x \rightarrow -2^-} \left[ \frac{4x + 8}{|x + 2|} \right] = \lim_{x \rightarrow -2^-} \left[ \frac{4(x + 2)}{-(x + 2)} \right] = \lim_{x \rightarrow -2^-} \left[ \frac{4}{-1} \right] = -4$$

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**Question 4** Let

$$f(x) = \begin{cases} -x^2 & , x > 1 \\ x - 1 & , x < 1 \\ 2 & x = 1 \end{cases}$$

Show which function you used to compute limits.

a) (3pt) Find  $\lim_{x \rightarrow 1^-} f(x)$  **Answer.**

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x - 1) = (1) - 1 = 0$$

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b) (3pt) Find  $\lim_{x \rightarrow 1^+} f(x)$  **Answer.**

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (-x^2) = -(1)^2 = -1$$

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c) (2pt) Does  $\lim_{x \rightarrow 1} f(x)$  exist? Explain it. **Answer.** Since  $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ ,  $\lim_{x \rightarrow 1} f(x)$  **does not exist.** ■

d) (2pt) Is  $f$  continuous at  $x = 1$ ? Explain it. **Answer.** Since  $\lim_{x \rightarrow 1} f(x)$  does not exist,  $f$  is **not continuous** at  $x = 1$  ■