$$\int_1^2 \frac{1}{x} dx = \ln(2)$$

Since

$$\frac{1}{1-x} = 1 + x + x^2 + \dots$$

with $|x| < 1$

$$\int \frac{1}{1-x} dx = C + x + \frac{x^2}{2} + \dots$$

Since $\ln(1-0) = 0$ and $[\ln(1-x)]' = \frac{-1}{1-x}$,

$$-\ln(1-x) = x + \frac{x^2}{2} + \dots$$

By abel's theorem, we can still evaluate at $x = 1$.

$$\ln(2) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{(-1)^{i+1}}{i}$$

Then

$$\int_1^2 \frac{1}{x} dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{1+\frac{i}{n}} \cdot \frac{1}{n} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n+i} = \lim_{n\to\infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right]$$

$$= \lim_{n\to\infty} \left[\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{2n} - (\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n}) \right] = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{(-1)^{i+1}}{i} = \ln(2)$$

Exercise 73. Let $x_i = 1 + \frac{i}{n}$ and $x_{i-1} \le x_i^* = \sqrt{x_{i-1} \cdot x_i} \le x_i$.

$$\int_1^2 \frac{1}{x^2} dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} n \sum_{i=1}^{n} \left[\frac{1}{(n+i-1)(n+i)} \right]$$

$$= \lim_{n\to\infty} n \sum_{i=1}^{n} \left[\frac{1}{n+i-1} - \frac{1}{n+i} \right] = \lim_{n\to\infty} n \left[\frac{1}{n} - \frac{1}{2n} \right] = \lim_{n\to\infty} \frac{n}{2n} = \frac{1}{2}$$