EXTRA-NOTE- FOR definite and indefinite integrals.

Question 1 (20pt) *Find the most general anti-derivative of each of the following function.*

- $f(x) = \frac{1}{2}x^2 - 2x + 6$

$$\int f(x)dx = \frac{1}{2}\int x^2 dx - 2\int x dx + 6\int 1 dx = \frac{1}{2}\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2} + 6x + C = \frac{1}{6}x^3 - x^2 + 6x + C$$

- $f(x) = (x+1)(2x-1)$ *From* $(x+1)(2x-1) = 2x^2 + x - 1$

$$\int f(x)dx = 2\int x^2 dx + \int x dx - 1\int 1 dx = 2\cdot\frac{x^3}{3} + \frac{x^2}{2} - x + C = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$$

- $f(t) = \frac{1+t+t^2}{\sqrt{t}}$
  (Typo:$f(x)$. Everyone will get a credit.)
  From $f(t) = \frac{1}{\sqrt{t}} + \frac{t}{\sqrt{t}} + \frac{t^2}{\sqrt{t}} = t^{-\frac{1}{2}} + t^{\frac{1}{2}} + t^{\frac{3}{2}}$,

$$\int f(t)dt = \frac{2}{1}t^{\frac{1}{2}} + \frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}} + C$$

- $f(x) = \sqrt{2}$

$$\int f(x)dx = \sqrt{2}\int 1 dx = \sqrt{2}x + C$$

- $f(x) = 2\sqrt{x} + 2\sec(x)\tan(x)$
  From $f(x) = 2x^{\frac{1}{2}} + 2\sec(x)\tan(x)$,

$$\int f(x)dx = 2\int x^{\frac{1}{2}} dx + 2\int \sec(x)\tan(x)dx = 2\cdot\frac{2}{3}x^{\frac{3}{2}} + 2\sec(x) + C = \frac{4}{3}x^{\frac{3}{2}} + 2\sec(x) + C$$

Question 2 (10pt) *Find all function g such that*

$$g'(x) = x + 2\sin(x) + \sqrt{x}$$

$$f(x) = \int \left[x + 2\sin(x) + \sqrt{x}\right] dx = \frac{x^2}{2} - 2\cos(x) + \frac{2}{3}x^{\frac{3}{2}} + C$$

Question 3 (10pt) *Find f if $f'(x) = 5x^4 - 2x^5$ and $f(0) = 4$.*

$$f(x) = \int \left[5x^4 - 2x^5\right] dx = 5\cdot\frac{x^5}{5} - 2\cdot\frac{x^6}{6} + C = x^5 - \frac{x^6}{3} + C$$

From $f(0) = C = 4$,

$$f(x) = x^5 - \frac{x^6}{3} + 4$$

Question 4 (10pt) *Find f if $f''(x) = \sin(x) + \cos(x)$,$f(0) = 3$ and $f'(0) = 4$.*

$$f'(x) = -\cos(x) + \sin(x) + C$$

From $f'(0) = -\cos(0) + \sin(0) + C = 4 \Rightarrow C = 5$,

$$f'(x) = -\cos(x) + \sin(x) + 5 \Rightarrow f(x) = -sin(x) - cos(x) + 5x + D$$

From $f(0) = -0 - 1 + 5(0) + D = 3 \Rightarrow D = 4$,

$$f(x) = -sin(x) - cos(x) + 5x + 4$$

Question 5 (10pt) *Express the limit as a definite integral on the given interval.*

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1-x_i^2}{4+x_i^2}\cdot\Delta x,\ [2,6]$$

From $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$, $f(x) = \frac{1-x^2}{4+x^2}$ *and* $a = 2, b = 6$.

$$\int_2^6 \frac{1-x^2}{4+x^2}dx$$

Question 6 (10pt) *Express the limit as a definite integral.*

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5}$$

Hint: Consider $f(x) = x^4$ *and* $\int_a^b f(x)dx = \lim_{n\to\infty} f(x_i)\Delta x$ *where* $\Delta x = \frac{b-a}{n}$ *and* $x_i = a + i\cdot\Delta x$. *Let* $a = 0, b = 1$.

$$\Delta x = \frac{b-a}{n} = \frac{1}{n},\ x_i = a + i\Delta x = 0 + \frac{i}{n} = \frac{i}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^4\cdot\frac{1}{n} = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \int_0^1 x^4 dx$$

Question 7 (20pt) *Using the* ***the definition of the (definite) integral*** *to evaluate the integral.*

- $\int_2^5 (4-2x)dx$

  From $\Delta = \frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n},\ x_i = a + i\Delta x = 2 + \frac{3i}{n}$,

$$\int_2^5 (4-2x)dx = \lim_{n\to\infty}\sum_{i=1}^{n}(4-2x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}(4-2x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left(4-2(2+\frac{3i}{n})\right)\frac{3}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(-\frac{6i}{n}\right)\frac{3}{n} = \lim_{n\to\infty} -\frac{18}{n^2}\sum_{i=1}^{n} i = \lim_{n\to\infty} -\frac{18}{n^2}\frac{n(n+1)}{2} = -\frac{18}{2} = -9$$

- $\int_0^1 (x^3 - 3x^2)dx$

  From $\Delta = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n},\ x_i = a + i\Delta x = 0 + \frac{i}{n} = \frac{i}{n}$

$$\int_0^1 (x^3-3x^2)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\left(x_i^3 - 3x_i^2\right)\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left((\frac{i}{n})^3 - 3(\frac{i}{n})^2\right)\frac{1}{n} = \lim_{n\to\infty}\sum_{i=1}^{n}(\frac{i}{n})^3\frac{1}{n} - 3\sum_{i=1}^{n}(\frac{i}{n})^2\frac{1}{n}$$

$$= \lim_{n\to\infty}\left[\frac{1}{n^4}\sum_{i=1}^{n} i^3 - \frac{3}{n^3}\sum_{i=1}^{n} i^2\right] = \lim_{n\to\infty}\left[\frac{1}{n^4}\left(\frac{n(n+1)}{2}\right)^2 - \frac{3}{n^3}\frac{n(n+1)(2n+1)}{6}\right] = \lim_{n\to\infty}\frac{n^4}{4n^4} - \lim_{n\to\infty}\frac{6n^3}{6n^3} = \frac{1}{4} - 1 = -\frac{3}{4}$$

Question 8 (10pt) *If* $\int_0^9 f(x)dx = 37$ *and* $\int_0^9 g(x)dx = 16$, *find* $\int_0^9 [2f(x)+3g(x)]dx$.

$$\int_0^9 [2f(x)+3g(x)]dx = 2\int_0^9 f(x)dx + 3\int_0^9 g(x)dx = 2(37) + 3(16) = 122$$

Question 9 (Extra:10pt) *If* $\int_1^5 f(x)dx = 12$ *and* $\int_4^5 f(x)dx = 3.5$, *find* $\int_1^4 f(x)dx$.

Let $\int_1^4 f(x)dx = A$.

$$\int_1^4 f(x)dx + \int_4^5 f(x)dx = \int_1^5 f(x)dx$$

implies

$$A + 3.5 = 12 \Rightarrow A = \int_1^4 f(x)dx = 12 - 3.5 = 8.5$$