A. 다음 언어들이 Regular language가 아님을 증명하시오.

1.29(b) $\{www \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$

$A = \{www \mid w \in (a,b)^*\}$

Let $p \geq 1$

Let $s = 0^p 1 0^p 1 0^p 1 \in A$

Let $x = 0^u$, $y = 0^v$ $(1 \leq v \leq p)$, $z = 0^{p-(u+v)} 1 0^p 1 0^p 1$

$xy^0z = 0^{u+p-(u+v)} 1 0^p 1 0^p 1.$

$= 0^{p-v} 1 0^p 1 0^p 1.$ $\quad 1 \leq v \leq p$ 이므로 $p-v \neq p$

$\therefore xy^0z \notin A$ $\quad \therefore$ Regular Language 아님

1.46(a) $\{0^n 1^m 0^n \mid m, n \geq 0\}$

$\{0^* 1^* 0^*\} \cap \{0^n 1^m 0^n\}^c = \{0^n 1^n 0^n\}$

$B = \{w \mid w \in (0^n 1^n 0^n),\ m, n \geq 0\}$

Let $p \geq 1$

Let $s = 0^p 1^p 0^p \in B$

Let $x = 0^u$, $y = 0^v$, $z = 0^{p-(u+v)} 1^p 0^p$

$xy^0z = 0^{p-v} 1^p 0^p.$ $\quad p-v \neq p$ 이므로 $xy^0z \notin B$

$\therefore$ Regular Language 아님

1.29(c) $\{a^{2^n} \mid n \geq 0\}$

$\quad \overset{\shortparallel}{A}$

A: reg $\Rightarrow \exists p := p_A \geq 1$

$s = a^{2^p} \in A$

$= \exists xyz$

Let $x = a^u,\ y = a^v\ (1 \leq v \leq p),\ z = a^{2^p-(u+v)}$

$xy^2z = a^{2^p+v}$

$2^p < 2^p + v < 2^p + p < 2^p + 2^p = 2^{p+1}$

즉 $2^p < 2^{p+v} < 2^{p+1}$ 이므로

$xy^2z = a^{2^p+v} \notin A$

1.46(c') $\{w \in \{0,1\}^* \mid w \text{ is a palindrome, that is, } w = w^R.\}$

$\quad \overset{\shortparallel}{B}$

B: reg $\Rightarrow \exists p := p_B \geq 1$

$s = 0^p 1 0^p \in B$

$= \exists xyz$

Let $x = 0^u,\ y = 0^v\ (1 \leq v \leq p),\ z = 0^{p-(u+v)} 1 0^p$

$xy^2z = 0^{p+v} 1 0^p = w$

$w^R = 0^p 1 0^{p+v}$) $w \neq w^R$

$xy^0z = 0^{p-v} 1 0^p$ 로도 증명가능

1.46(d) $\{wtw \mid w, t \in \{0,1\}^+\}$

C

$s = 0^p 1 0^p$

$xy^2z = \underbrace{0^{p+v}}_{w}\underbrace{1}_{t}\underbrace{0^p}_{w}$ 라고 하면 됨 ∴ 이 예시로는 증명 불가능

$s = \underbrace{0^p 1}_{w}\underbrace{0}_{t}\underbrace{0^p 1}_{w} \in C$

Let $x = 0^u$, $y = 0^v$ $(1 \le v \le p)$, $z = 0^{p-(u+v)} 1 0 0^p 1$

$xy^2z = 0^{p+v} 1 0 0^p 1 = wtw$ 꼴?

$w = 0^{p+v} 1$ 이어야 하는데 $s \in C$ 려면 $v = 1$ 이어야 함

근데 이러면 $t = \varepsilon$ 이 되고, 문제 조건과 맞지 않음 $(\therefore xy^2z \notin C)$

$v \ge 2$ 여도 $0^{p+v}1 \ne 0^{p+1}1$ 이므로 $xy^2z \notin C$

cf) $xy^0z = \underbrace{0^{p-v} 1}_{w}\underbrace{0}_{t}\underbrace{0^p 1}_{w}$ 는 C에 들어감

cf) $s = \underbrace{0^p 1}_{w} 1 \underbrace{0^p 1}_{w}$ 로 두면 더 편하게 증명가능

1.47 $\{w \in \{1, \sharp\}^* \mid w = x_1 \sharp x_2 \sharp \cdots \sharp x_k$ for $k \ge 0$, $x_i \in 1^*$, $x_i \ne x_j$ for $i \ne j\}$

D

$k = 0$ 이면 $w = \varepsilon$

$D^c = \{1, \#\}^* - D$. (D^c이 regular가 아니면 D도 regular가 아님)

$D^c = \{w \in \{1, \#\}^* \mid w = x_1 \# x_2 \# \cdots \# x_k,\ k \ge 2,\ \exists x_i = x_j \text{ for } i \ne j\}$

$s = 1^p \# 1^p = \exists xyz$

Let $x = 1^u$, $y = 1^v$ $(1 \le v \le p)$, $z = 1^{p-(u+v)} \# 1^p$

$xy^2z = \underbrace{1^{p+v}}_{x_1} \# \underbrace{1^p}_{x_2} \in D^c$

$x_1 \ne x_2$