1. 벡터공간 $M_{2\times 2}$에 대하여 다음 질문들에 답하시오.

(a) 변환 $T(A)=A^T$는 선형변환임을 증명하시오.

① $T(A+B) = T(A) + T(B)$

$(A+B)^T = A^T + B^T \qquad A^T + B^T$

② $T(c\cdot A) = c\cdot T(A)$

$(cA)^T = cA^T \qquad c\cdot A^T$

(b) Basis $S=[1,0;0,0],[0,1;0,0],[0,0;1,0],[0,0;0,1]$에 대하여 S에서 S로의 선형변환 T의 행렬표현 $T_{S\to S}$ 를 구하시오.

$$\left[T\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]_S = \left[\begin{pmatrix}1&0\\0&0\end{pmatrix}^T\right]_S = \left[\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]_S = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$\left[T\begin{pmatrix}0&1\\0&0\end{pmatrix}\right]_S = \left[\begin{pmatrix}0&1\\0&0\end{pmatrix}^T\right]_S = \left[\begin{pmatrix}0&0\\1&0\end{pmatrix}\right]_S = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\left[T\begin{pmatrix}0&0\\1&0\end{pmatrix}\right]_S = \left[\begin{pmatrix}0&0\\1&0\end{pmatrix}^T\right]_S = \left[\begin{pmatrix}0&1\\0&0\end{pmatrix}\right]_S = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$\left[T\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]_S = \left[\begin{pmatrix}0&0\\0&1\end{pmatrix}^T\right]_S = \left[\begin{pmatrix}0&0\\0&1\end{pmatrix}\right]_S = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\Rightarrow T_{S\to S} = \begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$$

2. 벡터공간 P_2와 Basis $\alpha=\langle 1,2x,x^2\rangle$, 벡터공간 P_3와 Basis $\beta=\langle 1,-x,x^2,2x^3\rangle$, 선형변환 $T(f)=\int_0 f\,dx$, $S(f)=\frac{df}{dx}$에 대하여 다음 질문들에 답하시오. (강의자료 6-1~6-11)

(a) α에서 β로의 선형변환 T의 행렬표현 $T_{\alpha\to\beta}$ 를 구하시오.

$$[T(1)]_\beta = [x]_\beta = \begin{pmatrix}0\\-1\\0\\0\end{pmatrix}$$

$$[T(2x)]_\beta = [x^2]_\beta = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$[T(x^2)]_\beta = \left[\tfrac{1}{3}x^3\right]_\beta = \begin{pmatrix}0\\0\\0\\\frac{1}{6}\end{pmatrix}$$

$$\Rightarrow T_{\alpha\to\beta} = \begin{pmatrix}0&0&0\\-1&0&0\\0&1&0\\0&0&\frac{1}{6}\end{pmatrix}$$

(b) $f=ax^2+bx+c$ 일 때, 두 좌표 $[f]_\alpha$와 $[T(f)]_\beta$를 구하시오.

$$[f]_\alpha = [ax^2+bx+c]_\alpha = \left[c\cdot 1 + \tfrac{b}{2}\cdot 2x + a\cdot x^2\right]_\alpha = \begin{pmatrix}c\\\frac{b}{2}\\a\end{pmatrix}$$

$$[T(f)]_\beta = \left[\tfrac{1}{3}ax^3 + \tfrac{1}{2}bx^2 + cx\right]_\beta = \begin{pmatrix}0\\-c\\\frac{b}{2}\\\frac{a}{6}\end{pmatrix}$$

(c) (a)(b)를 이용하여 임의의 $f=ax^2+bx+c$ 일 때, $[T(f)]_\beta = T_{\alpha\to\beta}[f]_\alpha$ 임을 확인하시오.

$$T_{\alpha\to\beta}[f]_\alpha = \begin{pmatrix}0&0&0\\-1&0&0\\0&1&0\\0&0&\frac{1}{6}\end{pmatrix}\begin{pmatrix}c\\\frac{b}{2}\\a\end{pmatrix} = \begin{pmatrix}0\\-c\\\frac{b}{2}\\\frac{a}{6}\end{pmatrix} = [T(f)]_\beta$$

2. 벡터공간 P_2와 Basis $\alpha = \langle 1, 2x, x^2 \rangle$, 벡터공간 P_3와 Basis $\beta = \langle 1, -x, x^2, 2x^3 \rangle$, 선형변환 $T(f) = \int_0 f\,dx$, $S(f) = \frac{df}{dx}$에 대하여 다음 질문들에 답하시오. (강의자료 6-1~6-11)

(d) β에서 α로의 선형변환 S의 행렬표현 $S_{\beta\to\alpha}$ 를 구하시오.

$$[S(1)]_\alpha = [0]_\alpha = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
$$[S(-x)]_\alpha = [-1]_\alpha = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$
$$[S(x^2)]_\alpha = [2x]_\alpha = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$
$$[S(2x^3)]_\alpha = [6x^2]_\alpha = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$$

$$\Rightarrow S_{\beta\to\alpha} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$

(e) α에서 α로의 선형변환 $S \circ T$ 의 행렬표현 $(S \circ T)_{\alpha\to\alpha}$ 를 구하시오. (여기서 $\circ$는 함수의 합성을 의미함)

$$(S \circ T)_{\alpha\to\alpha} = S_{\beta\to\alpha} T_{\beta\to\alpha} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{6} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(f) 임의의 $f = ax^2 + bx + c$ 일 때, $[(S \circ T)(f)]_\alpha = (S \circ T)_{\alpha\to\alpha} [f]_\alpha$ 임을 확인하시오.

$$(S \circ T)_{\alpha\to\alpha} [f]_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c \\ \frac{b}{2} \\ a \end{pmatrix} = \begin{pmatrix} c \\ \frac{b}{2} \\ a \end{pmatrix} = [(S \circ T)(f)]_\alpha$$

3. 행렬 $A = T_{S_3 \to S_2}$ = [1,1,0; 0,1,1]일 때, 행렬 A가 어떤 선형변환인지 강의자료 7-2의 그림과 같이 Singular Value Decomposion을 이용해 설명하시오.

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3. 행렬 $A = T_{S_3 \to S_2}$ = [1,1,0; 0,1,1]일 때, 행렬 A가 어떤 선형변환인지 강의자료 7-2의 그림과 같이 Singular Value Decomposion을 이용해 설명하시오.

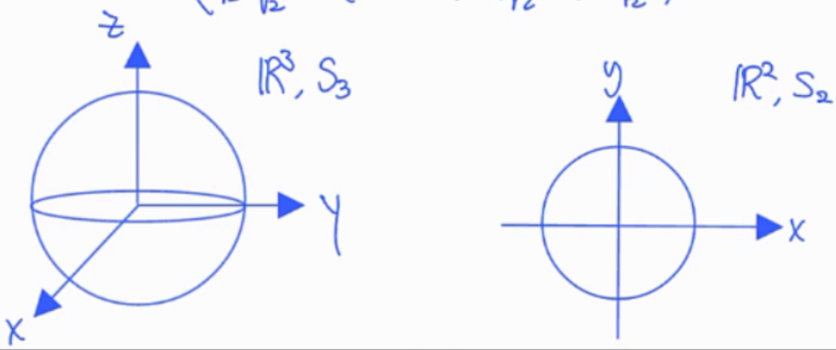
$S_3 = \mathbb{R}^3$, $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$

$S_2 = \left\{\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right\}$

$\sigma_1 = \sqrt{3}, \quad v_1 = \frac{1}{\sqrt{6}}\begin{pmatrix}1\\2\\1\end{pmatrix}, \quad u_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\sigma_2 = 1, \quad v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\1\end{pmatrix}, \quad u_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$

$$A = U\Sigma V^T = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\sqrt{3} & \\ & 1\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}}\\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\end{pmatrix}$$



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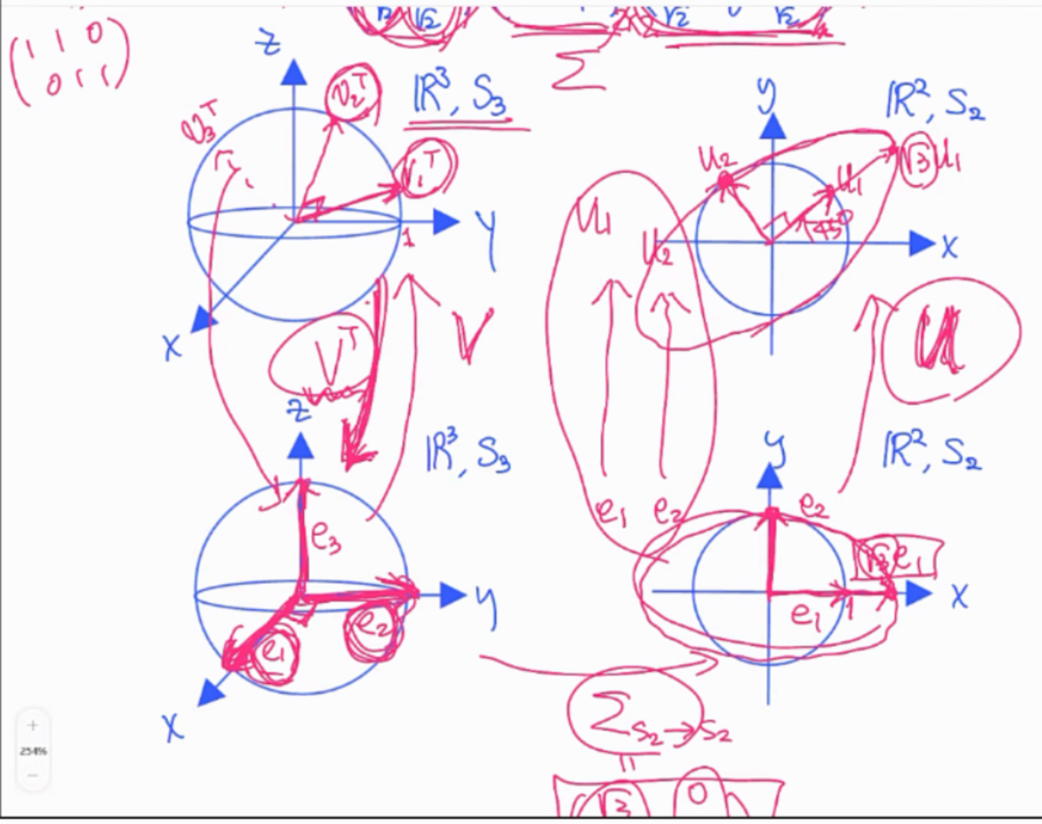

$V^{-1} = V^T$

$(v_1 \; v_2)$

$(u_1 \; u_2)_{2\times 2}$