1. t=-1일 때 y=7, t=1일 때 y=7, t=2일 때 y=21의 값을 가질 때, 직선 $y = C + Dt$의 Least Square Approximation을 구하시오.

$$\underset{A}{\begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}} \underset{x}{\begin{pmatrix} C \\ D \end{pmatrix}} = \underset{b}{\begin{pmatrix} 7 \\ 7 \\ 21 \end{pmatrix}}$$

$$\hat{x} = (A^TA)^{-1}A^Tb = \left[\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}\right]^{-1}\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \end{pmatrix}\begin{pmatrix} 7 \\ 7 \\ 21 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}^{-1}\begin{pmatrix} 35 \\ 42 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 35 \\ 42 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 6 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 18 \\ 8 \end{pmatrix} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$$

$$\therefore \boxed{y = 9 + 4t}$$

2. t=-2일 때 y=4, t=-1일 때 y=2, t=0일 때 y=-1, t=1일 때 y=0, t=2일 때 y=0의 값을 가질 때, 포물선 $y = C + Dt + Et^2$의 Least Square Approximation을 구하시오.

$$\begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}\begin{pmatrix} C \\ D \\ E \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

$$\hat{x} = \left[\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \\ 4 & 1 & 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 & 4 \\ 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}\right]^{-1}\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \\ 4 & 1 & 0 & 1 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 10 & 0 \\ 10 & 0 & 34 \end{pmatrix}^{-1}\begin{pmatrix} 5 \\ -10 \\ 8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{17}{35} & 0 & -\frac{1}{7} \\ 0 & \frac{1}{10} & 0 \\ -\frac{1}{7} & 0 & \frac{1}{14} \end{pmatrix}\begin{pmatrix} 5 \\ -10 \\ 8 \end{pmatrix} = \begin{pmatrix} \frac{1}{7} \\ -1 \\ \frac{4}{7} \end{pmatrix}$$

$$\therefore \boxed{y = \frac{1}{7} - t + \frac{4}{7}t^2}$$

3. a=(1;2;-2), b=(1;-1;4) c=(2,0,-2)일 때, Gram-Schmidt 방법으로 Orthonormal Basis를 구하시오.

$$A := a = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$$

$$B := b - \frac{A^Tb}{A^TA}A = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} - \frac{-9}{9}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

검산 : $AB = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0 \quad (\therefore A \perp B)$

$$C := c - \frac{A^Tc}{A^TA}A - \frac{B^Tc}{B^TB}B = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} - \frac{6}{9}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} - \frac{0}{9}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{3} \\ -\frac{4}{3} \\ -\frac{2}{3} \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

상수가 없어도 $A \perp B \perp C$ 임은 변함없으므로

$$C = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

Orthogonal Basis

$$A = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$

Orthonormal Basis

$$A' = \frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad B' = \frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \quad C' = \frac{1}{3}\begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$$