1. 다음 행렬방정식 $AX = B$을 만족하는 해 X를 구하는 과정을 쓰시오.(강의자료 p11-12의 내용-가우스 소거법과 Backward 대입법- 이용)

(a) $A = (1,1,1;2,0,3;0,4,0)$, $X = (x,y,z)^T$, $B = (1,3,-4)^T$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 0 & 3 & 3 \\ 0 & 4 & 0 & -4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 4 & 0 & -4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 2 & -2 \end{array}\right)$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \leftarrow B'$$

$-2y-1=1$ $\quad x-1-1=1$

$\therefore z=-1,\ y=-1,\ x=3 \quad \therefore X=(3,-1,-1)^T$

(b) $A = (1,-1,2,0;1,1,0,2;2,-3,3,3;-1,1,-3,4)$, $X = (x,y,z,w)^T$, $B = (-1,9,3,10)^T$

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & -1 \\ 1 & 1 & 0 & 2 & 9 \\ 2 & -3 & 3 & 3 & 3 \\ -1 & 1 & -3 & 4 & 10 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & -1 \\ 0 & 2 & -2 & 2 & 10 \\ 0 & -1 & -1 & 3 & 5 \\ 0 & 0 & -1 & 4 & 9 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & -1 \\ 0 & 2 & -2 & 2 & 10 \\ 0 & 0 & -2 & 4 & 10 \\ 0 & 0 & -1 & 4 & 9 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & -1 \\ 0 & 2 & -2 & 2 & 10 \\ 0 & 0 & -2 & 4 & 10 \\ 0 & 0 & 0 & 2 & 4 \end{array}\right)$$

$$\begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 2 & -2 & 2 \\ 0 & 0 & -2 & 4 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -1 \\ 10 \\ 10 \\ 4 \end{pmatrix}$$

$-2z+8=10 \quad 2y+2+4=10 \quad x-2-2=-1$

$\therefore w=2,\ z=-1,\ y=2,\ x=3 \quad \therefore X=(3,2,-1,2)^T$

2. 행렬 $A_1 = (-1,2,2;2,-1,0;3,0,-1)$에 대하여 다음에 답하시오. (강의자료 p18-23)

(a) 행렬 A_1에 가우스 소거법을 적용하면서 E_1, E_2, E_3를 구하고, 그들의 역행렬 $E_1^{-1}, E_2^{-1}, E_3^{-1}$도 구하시오.

$$\underbrace{\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}}_{E_3}\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}}_{E_2}\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{E_1}}_{K}\underbrace{\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 0 \\ 3 & 0 & -1 \end{pmatrix}}_{A_1}$$

$$E_1A_1 = \begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 4 \\ 3 & 0 & 1 \end{pmatrix}$$

$$E_2E_1A_1 = \begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 6 & 5 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & -3 \end{pmatrix}}_{=U}$$

2. 행렬 $A_1 = (-1, 2, 2; 2, -1, 0; 3, 0, -1)$에 대하여 다음에 답하시오. (강의자료 p18-23)

(b) (a)에서 구한 행렬들을 이용해 $K = E_3 E_2 E_1$ 와 $L = E_1^{-1} E_2^{-1} E_3^{-1}$ 를 계산하고, $KL = I$임을 확인하시오.

$$L = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{E_1^{-1}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}}_{E_2^{-1}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}}_{E_3^{-1}} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 2 & 1 \end{pmatrix}$$

$$K = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \qquad K = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{pmatrix}$$

(c) (b)에서 구한 L과 (a)에서 구한 U, Forward/Backward 대입법을 이용해 $A_1 X_1 = (-1, 3, 2)^T$, $A_1 X_2 = (7, -4, -4)^T$ 를 만족하는 해 X_1, X_2 를 구하시오.

$$\underset{L}{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 2 & 1 \end{pmatrix}} \underbrace{\underset{U}{\begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & -3 \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}} = \underset{B}{\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}, \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \therefore X_1 = (1, -1, 1)^T$$

$$\underset{L}{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 2 & 1 \end{pmatrix}} \underbrace{\underset{U}{\begin{pmatrix} -1 & 2 & 2 \\ 0 & 3 & 4 \\ 0 & 0 & -3 \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}} = \underset{B}{\begin{pmatrix} -7 \\ -4 \\ -4 \end{pmatrix}}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -3 \end{pmatrix}, \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \quad \therefore X_2 = (-1, 2, 1)^T$$

3. 행렬 $A_2 = (1,0,3;3,4,3;2,2,5)$에 대하여 다음에 답하시오. (강의자료 p18-23)

(a) 행렬 A_1에 가우스 소거법을 적용하면서 E_1, E_2, E_3를 구하고, 그들의 역행렬 $E_1^{-1}, E_2^{-1}, E_3^{-1}$도 구하시오.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 3 \\ 3 & 4 & 3 \\ 2 & 2 & 5 \end{pmatrix}$$

(b) (a)에서 구한 행렬들을 이용해 $K = E_3E_2E_1$ 와 $L = E_1^{-1}E_2^{-1}E_3^{-1}$ 를 계산하고, $KL = I$임을 확인하시오.

(c) (b)에서 구한 L과 (a)에서 구한 U, Forward/Backward 대입법을 이용해 $A_2X_1 = (-2,8,1)^T$, $A_2X_2 = (-1,7,1)^T$ 를 만족하는 해 X_1, X_2 를 구하시오.

$$\overset{L}{\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{1}{2} & 1 \end{pmatrix}}\underbrace{\overset{U}{\begin{pmatrix} 1 & 0 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 2 \end{pmatrix}}\overset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}}_{=\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \begin{pmatrix} -2 \\ 8 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -2 \\ 14 \\ -2 \end{pmatrix}. \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \quad \therefore X_1 = (1,2,-1)^T$$

$$\overset{L}{\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{1}{2} & 1 \end{pmatrix}}\underbrace{\overset{U}{\begin{pmatrix} 1 & 0 & 3 \\ 0 & 4 & -6 \\ 0 & 0 & 2 \end{pmatrix}}\overset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}}_{=\begin{pmatrix} u \\ v \\ w \end{pmatrix}} = \begin{pmatrix} -1 \\ 7 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -1 \\ 10 \\ -2 \end{pmatrix}. \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad \therefore X_2 = (2,1,-1)^T$$

[illegible] ✓

$$A = (E_3E_2E_1)^{-1}U$$

$$= \underbrace{(E_1^{-1}E_2^{-1}E_3^{-1})}_{=L}U$$

$$AX_1 = B_1$$

$$LUX_1 = B_1$$

$$\underbrace{L\begin{pmatrix} u \\ v \\ w \end{pmatrix}}_{} = B_1$$

$$= L^{-1}B_1$$

$$= E_3E_2E_1B_1$$

$$= B'$$

① 1번 방법

$$LX_1 = B_1'$$

↓

Backward 대입법

② 2번 방법

이용