

Lecture Notes

Kyle Chui

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1 Lecture 1

1.1 Overview of This Course

“Ancient” history of mathematics

- Egyptian mathematics (c. 3200 BCE – 300 BCE): Number system in base 10, fractions.
- Babylonian mathematics (c. 3300 BCE – 500 BCE): Place-value sexagesimal number system, roots of algebra.
- Greek mathematics (c. 700 BCE – 400 CE): Deduction method, geometry, roots of number theory.
- Mathematics in Asia (c. 1100 BCE – 1200 CE): Roots of number theory and algebra.
- Polynomial equations.
- Development of calculus, infinite series (16th–17th centuries CE).

1.2 The Beginnings

What is “Mathematics”?

1. Logical-deductive mathematics (initiated by the Greeks). This is not the generally accepted view of “mathematics” anymore.
2. Abstract counting problems: “mathematical” exercises without a direct practical use.
3. Practical counting problems: Bookkeeping for bureaucracy, inventories of goods and harvest, calculations involving lengths and areas for farming purposes, etc.
4. Development of a number system.

Note. We will see later that 2–4 all occur almost simultaneously.

1.3 Egyptian Mathematics

Most writing was done on papyrus, which doesn’t preserve well, so we don’t have much evidence from this period.

1.3.1 Egyptian Number System

The Egyptians had a base 10 number system, with a new Hieroglyphic symbol for each power of 10. This number system is *not* place valued. One simply adds all the symbols to get the number. For instance $\cap\cap\cap||| = 34$. By comparison, our modern number system is place-valued, which means that the position of each digit matters, i.e. $254 \neq 452$.

1.3.2 Egyptian Arithmetic

Summation is extremely easy, you just write the numbers next to each other to get their sum. Multiplication was done via consecutive doubling and adding. We do this by decomposing one number into powers of 2, and then distributing the other number across this sum. Because of this method, Egyptians had tablets with powers of 2, and tablets with consecutive doublings of many numbers.

We also have sources of:

- Fractions—The Egyptians used almost exclusively fractions of the form $\frac{1}{n}$ (aside from $\frac{2}{3}$ and $\frac{3}{4}$) and used sums of these to express more general fractions.

- Notation—Since any fraction can be written as the sum of fractions of the form $\frac{1}{n}$, any general fraction could be written by just writing $\frac{1}{n}$ fractions next to each other.
- Algebra—We have evidence that they solved some linear and second order (quadratic) equations.
- Geometry—Though limited, we have evidence that the Egyptians could calculate areas and volumes of triangles, circles, pyramids, etc.

However we see no evidence for a general theory for solving these questions, and all of the problems that we know of are elementary and mostly practical. The earliest “advanced” mathematical resources from the Egyptians are from around 1900 BCE, and are predated by Babylonian mathematics.