Lecture Notes

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2022-01-04

1 Lecture 1

1.1 Introduction to Dynamical Systems

Models of real-world phenomena can often be classified as either *static* or *dynamic*. Furthermore, these systems can either be *discrete* ($x_1, x_2, \ldots$ where $x_i \in \mathbb{R}$ for $i \geq 1$) or *continuous* ($x = x(t)$ where $t \geq 0$ and $x \in \mathbb{R}$, and $\dot{x} = f(x)$).

1.1.1 Where Do "Dynamical Systems" Come From?

1. Observed phenomena

2. Mathematical model

3. "Solve" the model

4. Make predictions

1.2 Autonomous ODEs

> **Definition.** *Autonomous ODEs*
> We say that an ordinary differential equation is *autonomous* if the right-hand side does not depend on t.

- The SIR (susceptible, infected, recovered) model is an example of a *first order* system of *autonomous* ODEs.
$$\begin{cases} \dot{x}_1 = f_1(x_1, \ldots, x_n) \\ \vdots \\ \dot{x}_n = f_1(x_1, \ldots, x_n) \end{cases}$$

- We will refer to n as the *dimension* of the system.