Introduction to Structural Equation Modeling using lavaan

Multigroup Models & Measurement Invariance

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Outline of this lecture

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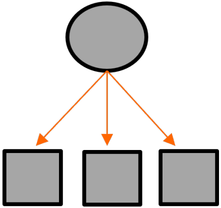
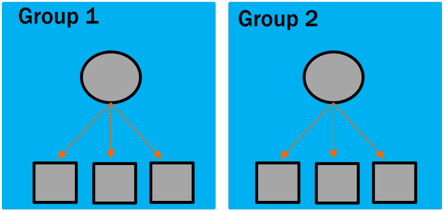
From one group to multiple groups

- EFA or CFA for single group:



- EFA or CFA for multiple groups:



General setup of multigroup analysis

1. Each group treated as **separate dataset**.
   - Like “select data” in SPSS: select people who meet a specific condition (e.g., female, Dutch, etc)
   - lavaan does it behind the scenes, after denoting your groups.
2. Model is fitted in each group separately.
3. Apply constraints to test whether parameters are the same / significantly different across different groups.

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Example: Multigroup regression

Regression example from lab meeting ‘Intro and regression’:

- Outcome: sw
- Predictors: overt and covert
- Group: gender (males and females)

Multigroup regression - lavaan model

```
# Data
data_regr <- read.table("popular_regr.txt", header = T)
data_regr[sapply(data_regr, function(x)
        as.character(x) %in% c("-99", "-999") )] <- NA
data_regr$gender <- factor(data_regr$gender,
                           labels = c("male", "female"))

# Model
model.MGregr <- '
  sw ~ overt + covert # regression
  sw ~~ sw            # residual variance
  sw ~ 1              # intercept
'
```

Multigroup regression - fit lavaan model

Use the ‘group’ argument in lavaan:

```
# Fit model
fit.MGregr <- lavaan(model.MGregr, data = data_regr,
                group = "gender") # multigroup specification

## Warning:  lavaan->lav_data_full():
##    group variable 'gender' contains missing values

# Results
#summary(fit.MGregr) # or:
#summary(fit.MGregr, standardized = TRUE)
```

Multigroup regression - lavaan output

```
                   Estimate  Std.Err  z-value  P(>|z|)
Group 1 [male]:

Regressions:
  sw ~
    overt            -0.278    0.059   -4.719    0.000
    covert           -0.497    0.039  -12.818    0.000
Intercepts:
   .sw                4.930    0.085   57.938    0.000
Variances:
   .sw                0.336    0.017   19.442    0.000

Group 2 [female]:

Regressions:
  sw ~
    overt            -0.232    0.081   -2.871    0.004
    covert           -0.558    0.045  -12.295    0.000
Intercepts:
   .sw                5.062    0.106   47.703    0.000
Variances:
   .sw                0.318    0.019   17.044    0.000
```

Multigroup regression: Test equality of coefficients

```
# Model specification
model.MGregr_equal <- '
  # model with labeled parameters
  sw ~ c(b1_m,b1_f)*overt + c(b2_m,b2_f)*covert
  sw ~~ sw
  sw ~ 1
'

# Fit model
fit_MGregr_equal <- lavaan(model = model.MGregr_equal,
                           data = data_regr,
                           group = "gender") # multigroup specification


# Test equality of regression coefficients
lavTestWald(fit_MGregr_equal,
            constraints = 'b1_m == b1_f; b2_m == b2_f')
```

Comparing nested models

Note: $constraints =' b1_m == b1_f; b2_m == b2'_f$,
so comparison of two nested models.
Thus, one can perform a χ^2_Δ test.

To compare fit of two nested models, you need:

1. χ^2 of the constrained model (χ^2_C)
2. df_C of the constrained model
3. χ^2 of the unconstrained model(χ^2_U)
4. df_U of the unconstrained model

Then, subtract: χ^2_C - $\chi^2_u = \chi^2_\Delta$,
where χ^2_Δ follows a Chi-square distribution with $df = df_C$-df_U.

The function lavTestWald() does this for you!

Comparing nested models Ctd.

Hypotheses
H_0: Unconstrained model fit $=$ constrained model fit
H_A: Unconstrained model fit $\neq$ constrained model fit

High p-values: No significant difference between the model fits: No evidence that the coefficients differ across groups.

Low p-values: Significant difference between the model fits: Evidence that the coefficients differ across groups.

Multigroup regression: Test equality of coefficients Ctd.

```
# Test equality of regression coefficients \ Wald test
lavTestWald(fit_MGregr_equal,
            constraints = 'b1_m == b1_f; b2_m == b2_f')

## $stat
## [1] 1.035523
##
## $df
## [1] 2
##
## $p.value
## [1] 0.5958527
##
## $se
## [1] "standard"

# Could also specify both models and use anova()
```

Hence, $\Delta\chi^2 = 1.035, \chi^2(2) = 1.035, p = .596$.

Multigroup regression: Test conclusion

Test result: $\chi^2(2) = 1.035, p = .596$.

Test conclusion:

- No evidence for a difference in model fit for the constrained and the unconstrained models.
- No evidence for a difference in the regression coefficients.
- The overt coefficients are the same for males and females.

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Measurement invariance

Important in case of latent variables and multiple groups.

Is my measurement model the same in different groups?

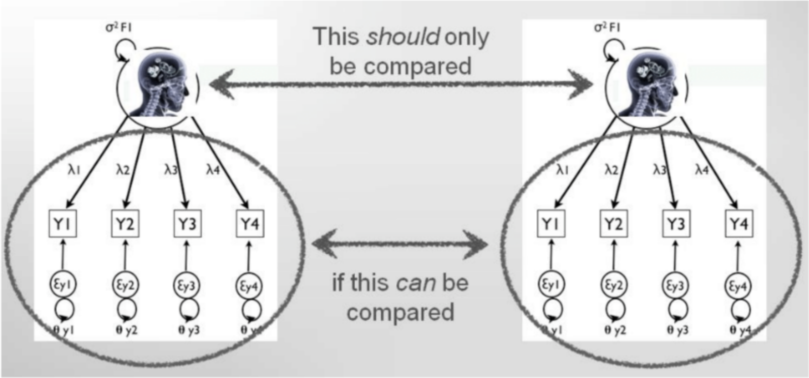
Construct validity:
Is the model measuring the same thing for boys and girls? Or across different countries?

Can we make a fair comparison between groups?
Did the groups understand the questions in the same way?

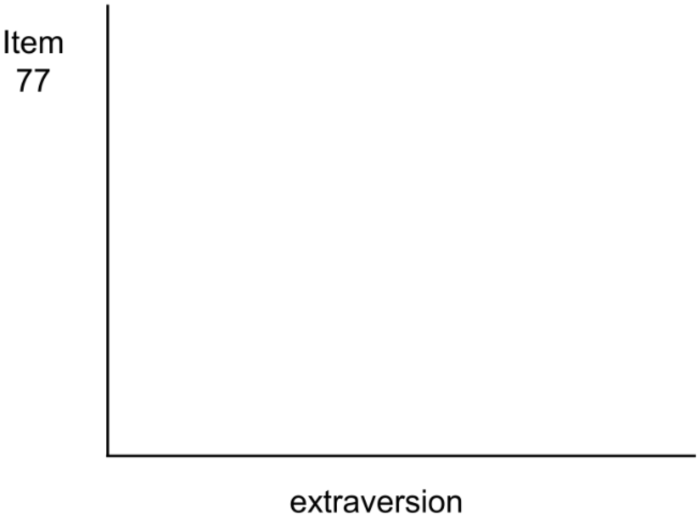
Same latent score should result in the same observed scores.
We want (at least):

- Equal intercepts (= item means)
- Equal slopes (factor loadings)

Stated differently



Stated differently...



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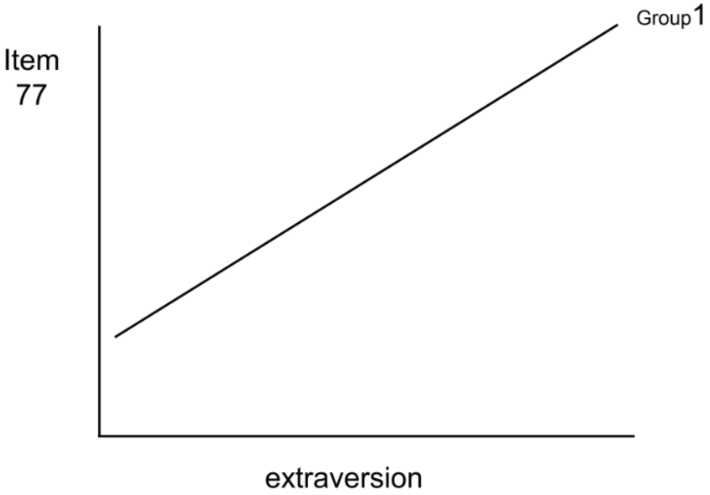
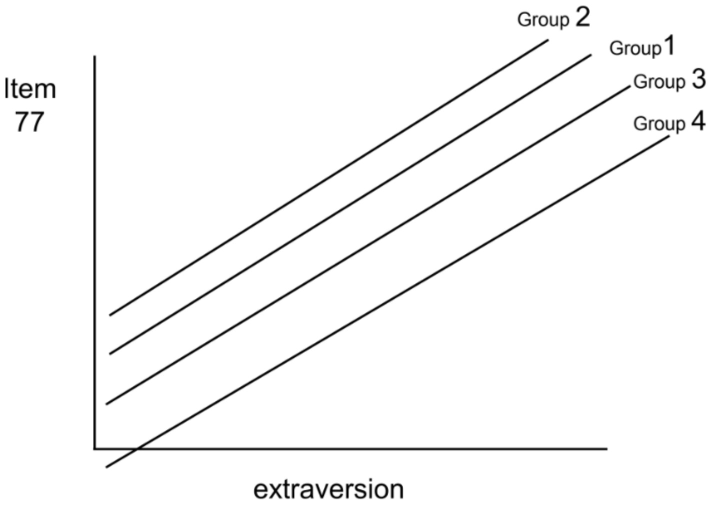


Stated differently...



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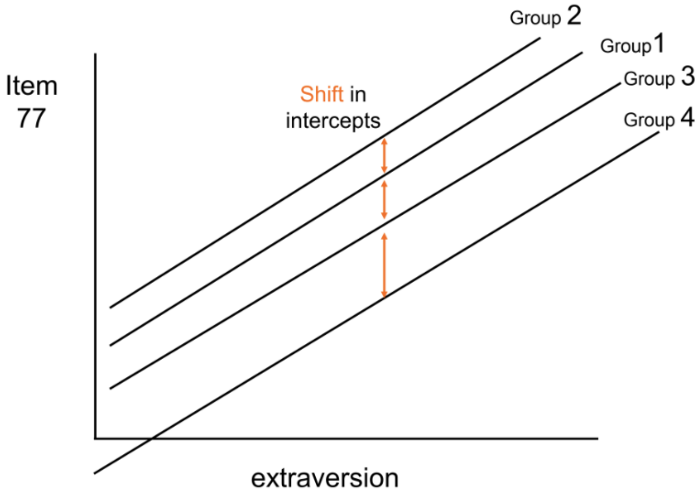
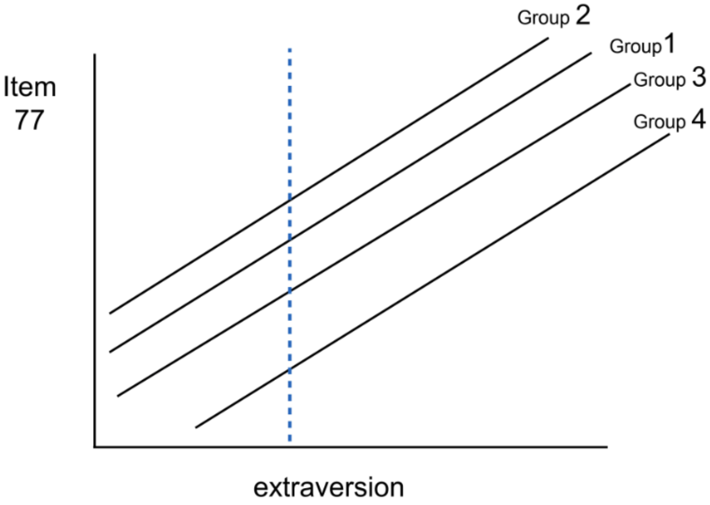
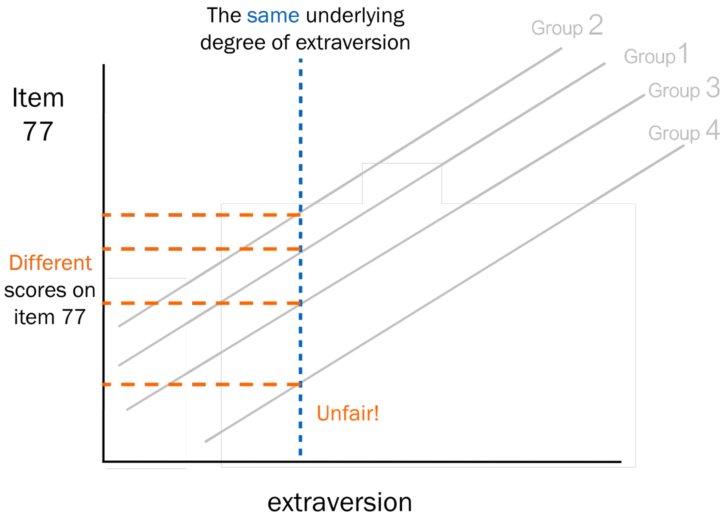


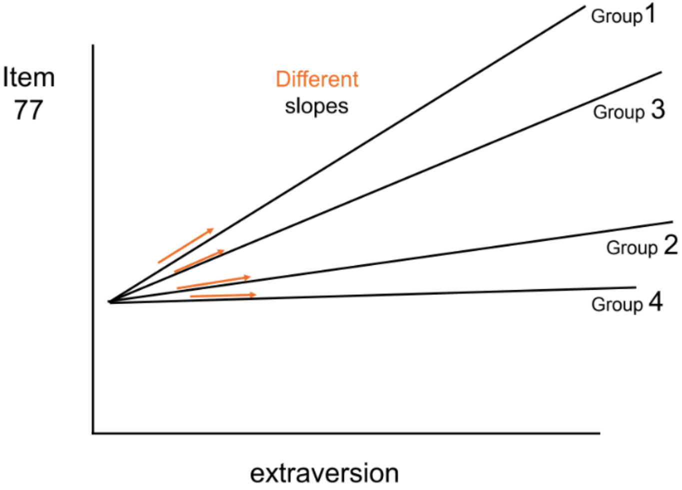
Stated differently...



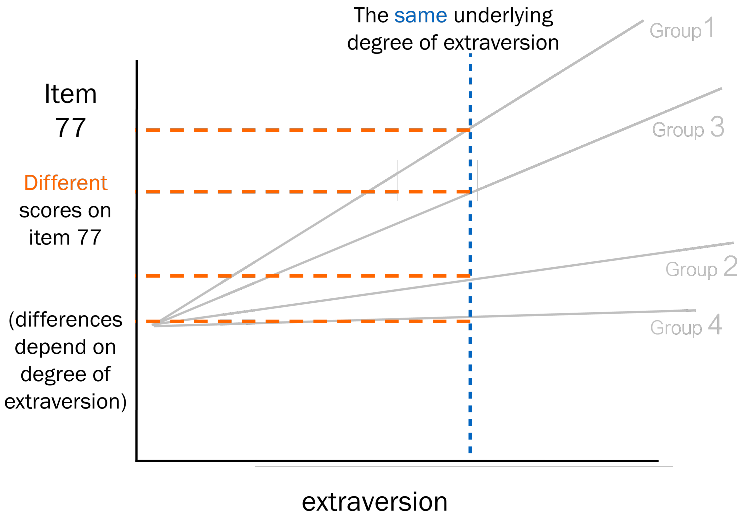
Stated differently...



Stated differently...



Stated differently...



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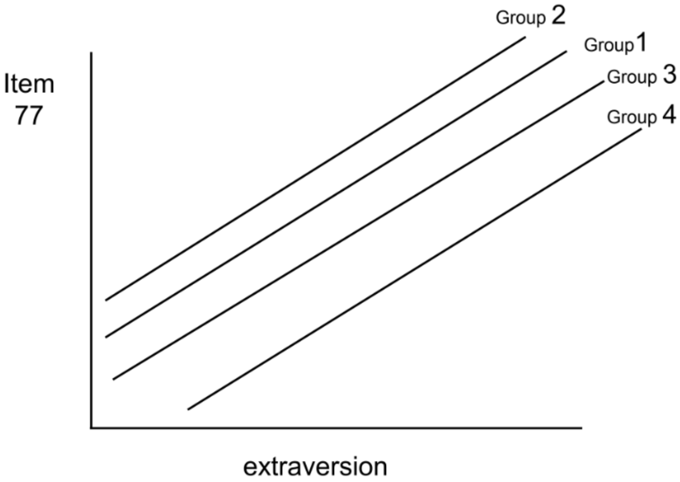

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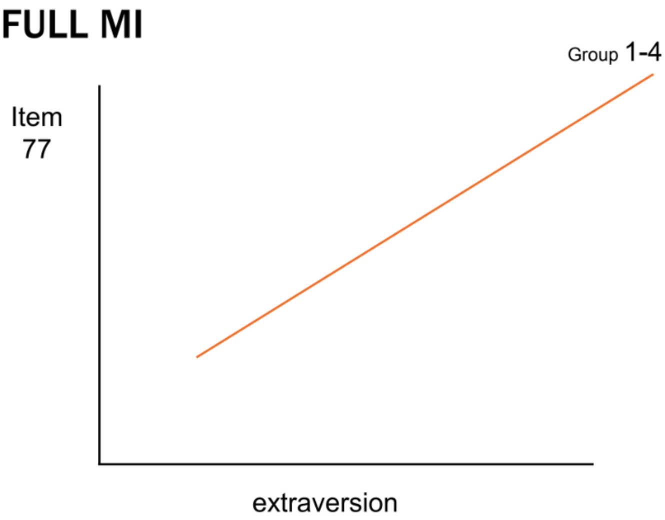

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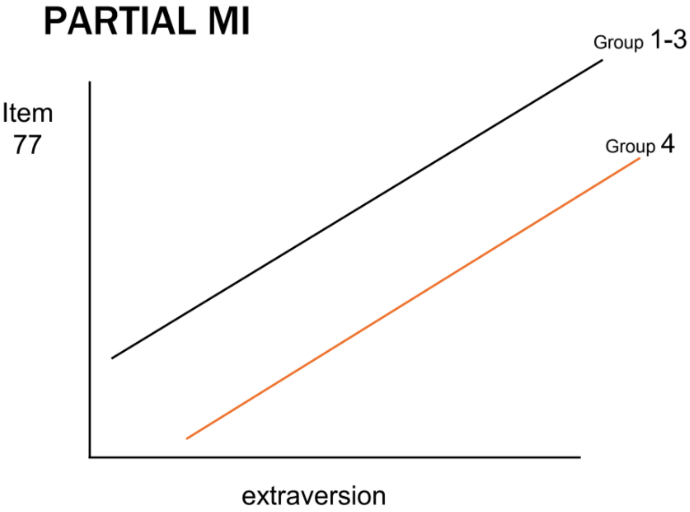


Measurement invariance

Exploratory approach:

- Find a good factor structure.
- Impose constraints one-by-one.

Confirmatory approaches:

1. Bottom-up:
    - Start with a CFA in each group separately.
    - Impose equality constrained one by one.
      Note: multiple variations on the procedure possible.
2. Top-down:
    - Assume complete measurement invariance.
    - Check in modification indices for constrains that if released would significantly improve the model.

Confirmatory bottom-up approach

Outline:

1. Have the same factor structure across groups.
   No constraints, except same model structure.
2. Have equal loadings across groups.
3. Have equal intercepts across groups.

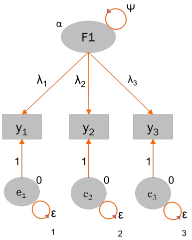
Equivalence is vital for valid comparisons across groups.
Partial equivalence may be acceptable.

Confirmatory bottom-up approach

1. Test overall model both groups combined.
2. Test model separately for each group (**configural invariance**).
   - Must fit the data
3. Test equality of loadings across groups (**metric/weak invariance**).
   - Must be equal
4. Test equality of intercepts across groups (**scalar/strong invar.**).
   - Must be equal
5. Test equality of measurement error variances (**strict invariance**).
   - Not essential, often overly restrictive
6. Test equality of factor means / factor variances.

Testing measurement invariance (MI)

1. Test overall model.
2. Test model for groups separately.
3. Test equal factor loadings (λ_1, λ_2, λ_3,).
4. Test equal intercepts / item means (ν_1, ν_2, ν_3,).
5. Test equal residual/error variances (ϵ_1, ϵ_2, ϵ_3,).
6. Test factor means (α) and/or variances (Ψ).



Note: Steps 3, 4, and 5, that is, testing for configural invariance, metric/weak invariance, and scalar/strong invariance, respectively, are quite easily incorporated in lavaan.

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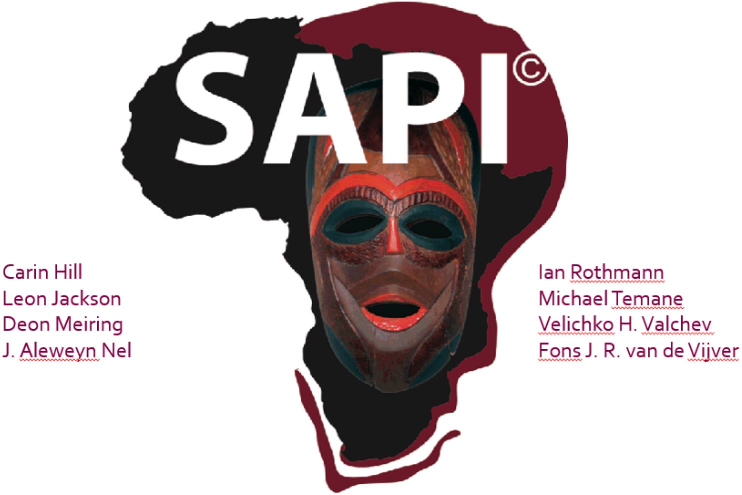
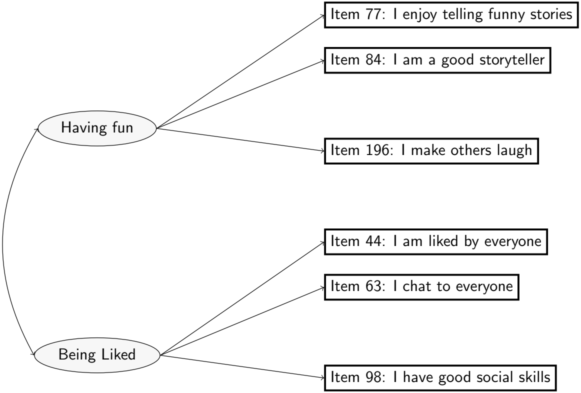
Example: South African Personality Inventory Project (SAPI)



Nel, J. A., Valchev, V. H., Rothmann, S., van de Vijver, F. J. R., Meiring, D., & de Bruin, G. P. (2012). Exploring the personality structure in the 11 languages of South Africa. *Journal of Personality*, *80*, 915–948.

CFA: only hypothesized loadings



In addition: group = 'Gender'

SAPI Example: MI models with lavaan

```
# Data
data_sapi <- read.table("Sapi.txt", header = T)

data_sapi[sapply(data_sapi,
    function(x) as.character(x) %in% c("-999") )] <- NA

data_sapi$Gender <- factor(data_sapi$Gender,
                           labels = c("male", "female"))
```

```
# multigroup two-factor CFA (MG2CFA) - group = 'Gender'
model.2CFA <- '
 Havingfun  =~ Q77 + Q84 + Q196
 Beingliked =~ Q44 + Q63 + Q98
'
```

SAPI Example: MI models with lavaan Ctd.

```
# configural invariance
fit_MG2CFA_ci <- cfa(model.2CFA, data=data_sapi,
                     group = 'Gender',
                     missing='fiml', fixed.x=F)

# metric/weak invariance
fit_MG2CFA_wi <- cfa(model.2CFA, data=data_sapi,
                     group = 'Gender',
                     missing='fiml', fixed.x=F,
                     group.equal = "loadings")

# scalar/strong invariance
fit_MG2CFA_si <- cfa(model.2CFA, data=data_sapi,
                     group = 'Gender',
                     missing='fiml', fixed.x=F,
                     group.equal = c("intercepts",
                                     "loadings"))
```

SAPI Example: Testing MI with lavaan

```
# model comparison tests: anova() or lavTestLRT()
lavTestLRT(fit_MG2CFA_ci, fit_MG2CFA_wi,
           fit_MG2CFA_si)[-c(2,3,4)]

##                 Df Chisq diff  RMSEA Df diff Pr(>Chisq)
## fit_MG2CFA_ci 16
## fit_MG2CFA_wi 20     1.3992 0.0000       4     0.8443
## fit_MG2CFA_si 24     7.3791 0.0415       4     0.1172
```

- Metric/Weak invariance model fits just as well as the configural invariance model ($\chi^2(4) = 1.40$, $p = .84$).
- Scalar/Strong invariance model also fits just as well as the metric/weak invariance model ($\chi^2(4) = 7.38$, $p = .12$).

Testing MI with lavaan: IF significant

- If the test for configural invariance against weak invariance is significant, then there is a lack of metric invariance and, thus, there is no need to test for scalar and strict invariance.

- If tests significant: may try to find source of bias with modification indices.
- Then, aim for partial MI: Continue with MI tests with source of bias freely estimated between groups.

SAPI Example: MI - Model comparison

```
# model comparison tests: anova() or lavTestLRT()
lavTestLRT(fit_MG2CFA_ci, fit_MG2CFA_wi,
           fit_MG2CFA_si)[c(2,3)]

##                  AIC   BIC
## fit_MG2CFA_ci 15354 15540
## fit_MG2CFA_wi 15348 15514
## fit_MG2CFA_si 15347 15494
```

- Scalar/Strong invariance model fits best (lowest IC).
- Note: You can also quantify the support for this model versus the other two, using so-called IC weights; see the lab and/or Model Selection lecture.

SAPI Example: MI - Model fit

```
# model comparison tests: anova() or lavTestLRT()
fitMI <- sapply(list(fit_MG2CFA_ci, fit_MG2CFA_wi,
                     fit_MG2CFA_si),
                fitMeasures, c("cfi", "rmsea"))
colnames(fitMI) <- c("config", "weak", "strong")
fitMI

##           config       weak     strong
## cfi   0.93940080 0.94168219 0.93871806
## rmsea 0.09382256 0.08232267 0.07703613

# and many more measures...
```

– Cheung, G. W., & Rensvold, R. B. (2002). Evaluating goodness-of-fit indexes for testing measurement invariance. Structural Equation Modeling, 9(2), 233–255. doi:10.1207/S15328007SEM0902_5

– Chen, F. F. (2007). Sensitivity of goodness of fit indexes to lack of measurement invariance. Structural Equation Modeling, 14(3), 464–504. doi:10.1080/10705510701301834

Further Reading

Van de Schoot, R., Lugtig, P., & Hox. J. (2012). A checklist for testing measurement invariance. European Journal of Developmental Psychology, 9,486-492.

Vandenberg, R.J., & Lance, C.E. (2000). A review and synthesis of the measurement invariance literature: Suggestions, practices, and recommendations for organizational research, Organizational Research Methods, 3, 4-70.

Byrne, B.M., Shavelson, R.J., & Muthén, B.O. (1989). Testing for equivalence of factor covariance and mean structures: The issue of partial measurement invariance. Psychological Bulletin, 105, 456-466.

Foundational Work: Jöreskog, K.G. (1971). Simultaneous factor analysis in several populations. Psychometrika, 36, 409-426.

More interesting reading

– Van De Schoot, R., Kluytmans, A., Tummers, L., Lugtig, P., Hox, J., & Muthén, B. (2013). Facing off with Scyllaand Charybdis: A comparison of scalar, partial, and the novel possibility of approximate measurement invariance. Frontiers inPsychology, 4.
– Fox, J.-P., & Verhagen, A. J. (2010). Random item effects modeling for cross-national survey data.
– Davidov, P. Schmidt, & J.Billiet (Eds.). Cross-cultural analysis: Methods and applications (pp. 467-488). London, UK: Routledge Academic.
– Davidov, E., Dülmer, H., Schlüter, E., Schmidt, P., & Meuleman, B. (2012). Using a multilevel structural equation modeling approach to explain cross cultural measurement noninvariance. Journal of Cross-Cultural Psychology, 43(4), 558–575.
– Muthén, B., and Asparouhov, T.(2013). BSEM Measurement Invariance Analysis. Mplus Web Notes: No. 17.
– www.statmodel.com

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Summary

- Multi-group analysis
- Multi-group regression in R using lavaan
- Measurement invariance
- Measurement invariance in R using lavaan

Thanks & How to proceed

Thanks for listening!

Are there any questions?

- Ask fellow participant on course platform.
- Ask teacher during Q&A (or via course platform).
- See if making the lab exercises help.
- Check the lavaan tutorial: e.g., `https://lavaan.ugent.be/tutorial/index.html`.
- Do not forget that Google is your best friend :-).

You can start working on the lab exercises.