CS446: Machine Learning **Spring 2017**

Problem Set 1

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1. **Neural Network**

   a. The input, hidden and output layers are represented by i,j,k respectively.

$$\frac{\partial E_d}{\partial w_{jk}} = \frac{\partial E_d}{\partial o_k}\frac{\partial o_k}{\partial net_k}\frac{\partial net_k}{\partial w_{jk}}$$

$$\frac{\partial E_d}{\partial o_k} = -(t_k - o_k)$$

$$\frac{\partial o_k}{\partial net_k} = \begin{cases} 0 & net_k \leq 0 \\ 1 & net_k > 0 \end{cases}$$

$$\frac{\partial net_k}{\partial w_{jk}} = x_j$$

$$w_{jk} \leftarrow w_{jk} + R\delta_k x_j$$

where R is learning rate, $\delta_k = -\frac{\partial E_d}{\partial net_k} = \begin{cases} 0 & net_k \leq 0 \\ -(t_k - o_k) & net_k > 0 \end{cases}$

$$\frac{\partial E_d}{\partial w_{ij}} = \frac{\partial E_d}{\partial net_j}\frac{\partial net_j}{\partial w_{ij}}$$

$$= \sum_{k \in downstream(j)} \frac{\partial E_d}{\partial net_k}\frac{\partial net_k}{\partial net_j}\frac{\partial net_j}{\partial w_{ij}}$$

$$= \sum_{k \in downstream(j)} -\delta_k \frac{\partial net_k}{\partial net_j} x_i$$

where,$\delta_k = -\frac{\partial E_d}{\partial net_j}$,$net_j = \sum w_{ij}x_i$

$$\frac{\partial net_k}{\partial net_j} = \frac{\partial net_k}{\partial o_j}\frac{\partial o_j}{\partial net_j}$$

$$\frac{\partial o_j}{\partial net_j} = \begin{cases} 0 & net_j \leq 0 \\ 1 & net_j > 0 \end{cases}$$

$$\frac{\partial net_k}{\partial o_j} = w_{jk}$$

So,

$$\frac{\partial E_d}{\partial w_{ij}} = \begin{cases} 0 & net_j \leq 0 \\ \sum_{k \in downstream(j)} -\delta_k w_{jk} x_i & net_j > 0 \end{cases}$$

So,

$$w_{ij} \leftarrow w_{ij} + R\delta_j x_i$$

$$\text{where,} \delta_j = \{^{0 \quad net_j \leq 0}_{\sum_{k \in downstream(j)} -\delta_k w_j k \quad net_j > 0}$$

b. ii. **CIRCLES:**

$[domain, batch\_size, learning\_rate, activation\_function, hidden\_layer\_width] : accuracy$

$['circles', 10, 0.1, 'relu', 10] : 100.0$
$['circles', 10, 0.1, 'relu', 50] : 100.0$
$['circles', 10, 0.1, 'tanh', 10] : 100.0$
$['circles', 10, 0.1, 'tanh', 50] : 100.0$
$['circles', 10, 0.01, 'relu', 10] : 95.375$
$['circles', 10, 0.01, 'relu', 50] : 100.0$
$['circles', 10, 0.01, 'tanh', 10] : 49.125$
$['circles', 10, 0.01, 'tanh', 50] : 51.0$
$['circles', 50, 0.1, 'relu', 10] : 100.0$
$['circles', 50, 0.1, 'relu', 50] : 100.0$
$['circles', 50, 0.1, 'tanh', 10] : 58.375$
$['circles', 50, 0.1, 'tanh', 50] : 54.75$
$['circles', 50, 0.01, 'relu', 10] : 74.875$
$['circles', 50, 0.01, 'relu', 50] : 94.25$
$['circles', 50, 0.01, 'tanh', 10] : 50.5$
$['circles', 50, 0.01, 'tanh', 50] : 49.75$
$['circles', 100, 0.1, 'relu', 10] : 99.75$
$['circles', 100, 0.1, 'relu', 50] : 100.0$
$['circles', 100, 0.1, 'tanh', 10] : 49.5$
$['circles', 100, 0.1, 'tanh', 50] : 51.875$
$['circles', 100, 0.01, 'relu', 10] : 57.875$
$['circles', 100, 0.01, 'relu', 50] : 78.875$
$['circles', 100, 0.01, 'tanh', 10] : 48.75$
$['circles', 100, 0.01, 'tanh', 50] : 48.875$

The best set for circle is:

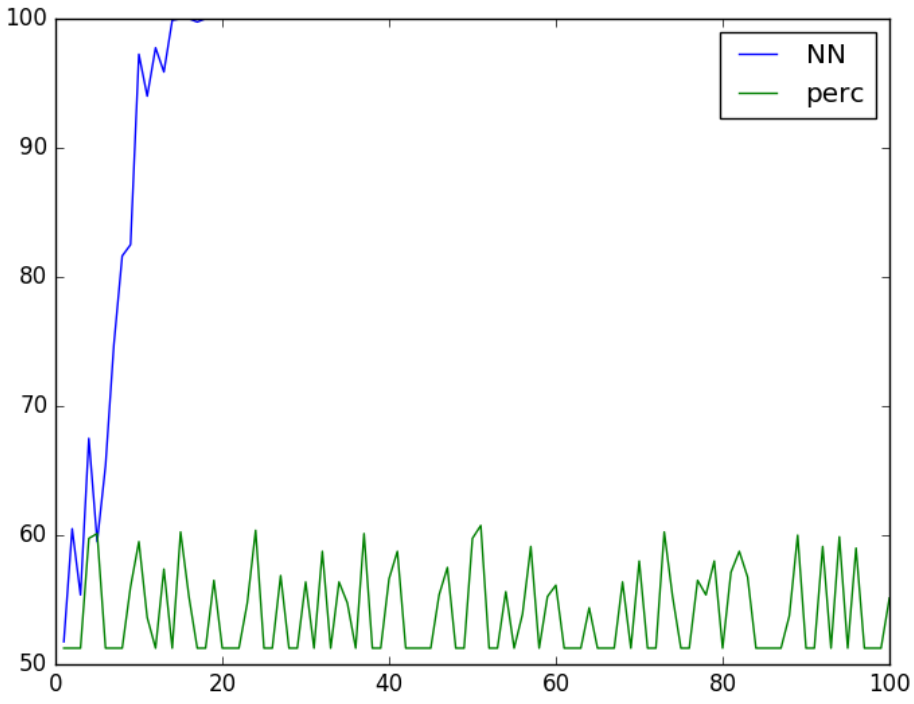
$['circles', 10, 0.1, 'relu', 10] : 100.0$

MNIST:

$['mnist', 10, 0.1, 'relu', 10] : 95.6266212383$
$['mnist', 10, 0.1, 'relu', 50] : 96.2108866459$
$['mnist', 10, 0.1, 'tanh', 10] : 96.9119717182$
$['mnist', 10, 0.1, 'tanh', 50] : 96.8785966603$
$['mnist', 10, 0.01, 'relu', 10] : 96.5364152932$
$['mnist', 10, 0.01, 'relu', 50] : 96.5447625386$
$['mnist', 10, 0.01, 'tanh', 10] : 96.7700615844$
$['mnist', 10, 0.01, 'tanh', 50] : 96.2108727223$
$['mnist', 50, 0.1, 'relu', 10] : 96.5197138406$

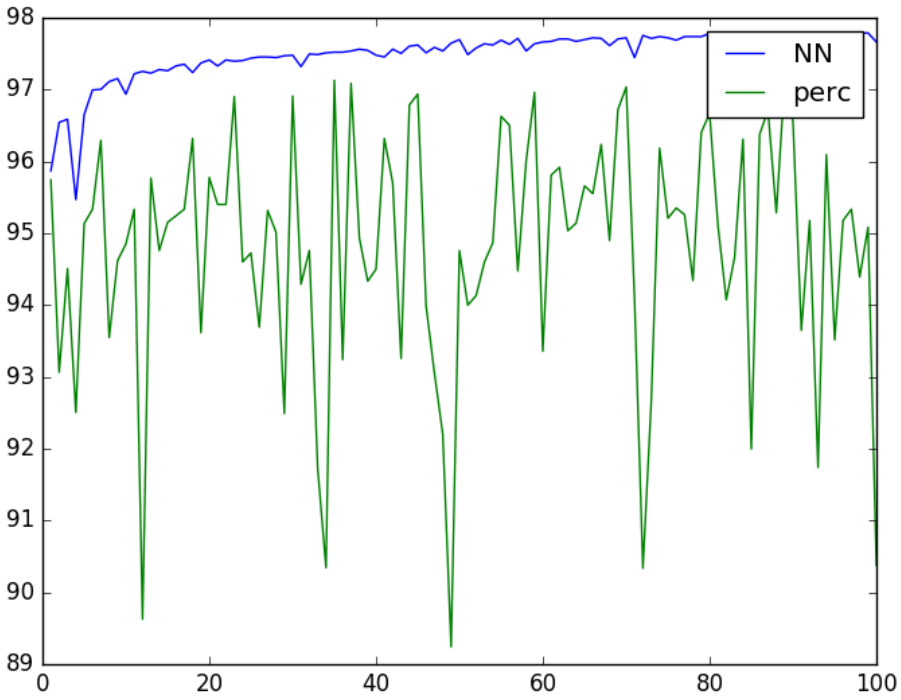
$['mnist', 50, 0.1, 'relu', 50]$: 96.4279080647
$['mnist', 50, 0.1, 'tanh', 10]$: 96.886929982
$['mnist', 50, 0.1, 'tanh', 50]$: 96.553068013
$['mnist', 50, 0.01, 'relu', 10]$: 96.7868048081
$['mnist', 50, 0.01, 'relu', 50]$: 96.7283671284
$['mnist', 50, 0.01, 'tanh', 10]$: 96.3444147251
$['mnist', 50, 0.01, 'tanh', 50]$: 96.1775185498
$['mnist', 100, 0.1, 'relu', 10]$: 96.6365613526
$['mnist', 100, 0.1, 'relu', 50]$: 96.4612831227
$['mnist', 100, 0.1, 'tanh', 10]$: 96.7784157917
$['mnist', 100, 0.1, 'tanh', 50]$: 96.2359005348
$['mnist', 100, 0.01, 'relu', 10]$: 96.6365613526
$['mnist', 100, 0.01, 'relu', 50]$: 96.6699642579
$['mnist', 100, 0.01, 'tanh', 10]$: 96.2108796841
$['mnist', 100, 0.01, 'tanh', 50]$: 96.1441295682
The best set for mnist:
$['mnist', 10, 0.1, 'tanh', 10]$: 96.9119717182

iii. **Circles:**



test accuracy of NN:100%
test accuracy of perceptron:49.5%

Mnist:



test accuracy of NN:96.673
test accuracy of perceptron:88.86%

Circle:
Perceptron basically fails to learn the Circle, because it's a linear separator and there is no way it could generate a good hypothesis separating the circle data in the primary feature space. Relatively, the Neural network learns perfectly on the circle, since theoratically neural network could learn any non-linear function. This is perfectly revealed in the learning curves. The curve_NN increases singularly until it reaches the top, while the curve_perceptron stays low below 50%.
Mnist:
The perceptron and NN's performances are close on this data set, with NN's slightly higher. Perhaps the minst data is more linearly separable than the Circle. Interestingly, perceptron's learning curve fluctuates dramatically, although it gives a suprisingly decent result in the end, while the learning curve of NN grows almost singularly until it reaches the local maximum.

2. **Multi-class classification**

a. i. For One vs All, k separators are learnt.
For All vs All, k^2 separators are learnt.

ii. For One vs All, each of the separators uses m examples.
For All vs All, each of the separators uses only $\frac{2m}{k}$ samples.

iii. For One vs All, I will decide the final label by the rule of Winner Takes All, $f(x) = argmax_i(w_i^T x)$
For All vs All, I will decide the final label by majority, outputing the label that has been predicted by most number of separators.

iv. For One vs All, the complexity of training is $O(km)$
For All vs All, the complexity of training is also $O(km)$

b. I would prefer One vs All. Although it is less expressive than All vs. All, it has more data to train on per classifier, while All vs. All could have very little number of samples to train on, depending on the size of m and k. Also One vs. all is simpler to implement given that it has less number of separators to learn.

c. Yes, when using the kernel perceptron, the complexity for One vs. All is $O(km^2)$ and the complexity for All vs.All is $O(m^2)$. Using All vs. All kernel perceptron allows us to work in the dual space more efficiently and learn with relatively small training set. So I would prefer to use All vs. All.

d. One vs. All:$O(kdm^2)$
All vs. All:$O(dm^2)$
All vs. All is more efficient.

e. One vs. All:$O(kd^2m)$
All vs. All:$O(kd^2m)$
They are equally efficient.

f. Counting:$O(k^2)$
KnockOut:$O(k)$

3. **Probability Review**

a. i.

$$E(A) = 1$$

$$E(B) = \lim_{n\to\infty} \sum_{i=1}^{n} i \cdot (\frac{1}{2})^i = 2$$

ii. Both of the towns boy-girl ratios maintain 1:1 at the end of a generation.

b. i.

$$P(A|B) \cdot P(B) = P(A, B)$$

$$P(B|A) \cdot P(A) = P(A, B)$$

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

ii.

$$P(A, B, C) = P(A|B, C) \cdot P(B|C) \cdot P(C)$$

c.

$$E(X) = 1 \cdot P(A) + 0 \cdot (1 - P(A)) = P(A)$$

d. i. No,because $P(X = 0) = \frac{55}{90}$,$P(Y = 0) = \frac{48}{90}$,$P(X = 0) \cdot P(Y = 0) = \frac{88}{270}$,but according to the table$P(X = 0, Y = 0) = \frac{30}{90}$,which means that $P(X) \neq P(X|Y)$, and the knowledge of Y will influence the probability of the X. By definition, they are not independent.

ii. Yes,since in the table, $P(X, Y|Z) = P(X|Z) \cdot P(Y|Z)$, i.e. $\frac{P(X,Y,Z)}{P(Z)} = \frac{P(X,Z)}{P(Z)} \cdot \frac{P(Y,Z)}{P(Z)} \Rightarrow P(X, Y, Z) = \frac{P(X,Z) \cdot P(Y,Z)}{P(Z)}$. For example, $P(X = 0, Y = 0, Z = 0) = \frac{1}{15}$,$P(X = 0, Z = 0) \cdot P(Y = 0, Z = 0) = \frac{1}{6} \cdot \frac{2}{15} = \frac{1}{45}$,$P(Z = 0) = \frac{1}{3}$ So $P(X = 0, Y = 0, Z = 0) = \frac{P(X=0,Z=0) \cdot P(Y=0,Z=0)}{P(Z=0)}$.

iii.

$$P(X = 0|X + Y > 0) = \frac{P(X = 0, X + Y > 0)}{P(X + Y > 0)}$$

$$P(X = 0, X + Y > 0) = P(X = 0, Y = 1)$$

$$= P(X = 0, Y = 1, Z = 0) + P(X = 0, Y = 1, Z = 1)$$

$$= \frac{1}{10} + \frac{8}{45} = \frac{5}{18}$$

$$P(X + Y > 0) = P(X = 0, Y = 1) + P(X = 1, Y = 0) + P(X = 1, Y = 1)$$

$$= \frac{25}{90} + \frac{3}{15} + \frac{17}{90} = \frac{2}{3}$$

So,

$$P(X = 0|X + Y > 0) = \frac{\frac{5}{18}}{\frac{2}{3}} = \frac{5}{12}$$