FACTORIZATION MEETS THE NEIGHBORHOOD
: A MULTIFACETED COLLABORATIVE FILTERING MODEL

MATRIX FACTORIZATION TECHNIQUES FOR RECOMMENDER SYSTEMS

2022-07-05 KyuHyeok Seo

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INTRO

without requiring the creation of explicit profiles

Contents-Based Filtering → **Collaborative Filtering**

creates a profile for each user or product

to characterize its nature.

require gathering external information that

might not be available or easy to collect

analyzes relationships

users - interdependencies

among products to identify

new user-item associations.

INTRO Collaborative Filtering

Neighborhood Model

centered on computing the **relationships between items** OR **between users**

effective at detecting very **localized relationships**

unable to capture the **totality of weak signals**

encompassed in all of a user's ratings.

Item - oriented approach based on ratings of **similar items by the same user**

User - oriented approach based on ratings of **similar Users by the same items**

INTRO

EXAMPLE : User - oriented approach

Better **scalability** and **improved accuracy**

more amenable to explaining

the **reasoning behind predictions**

~~**User - oriented approach**~~

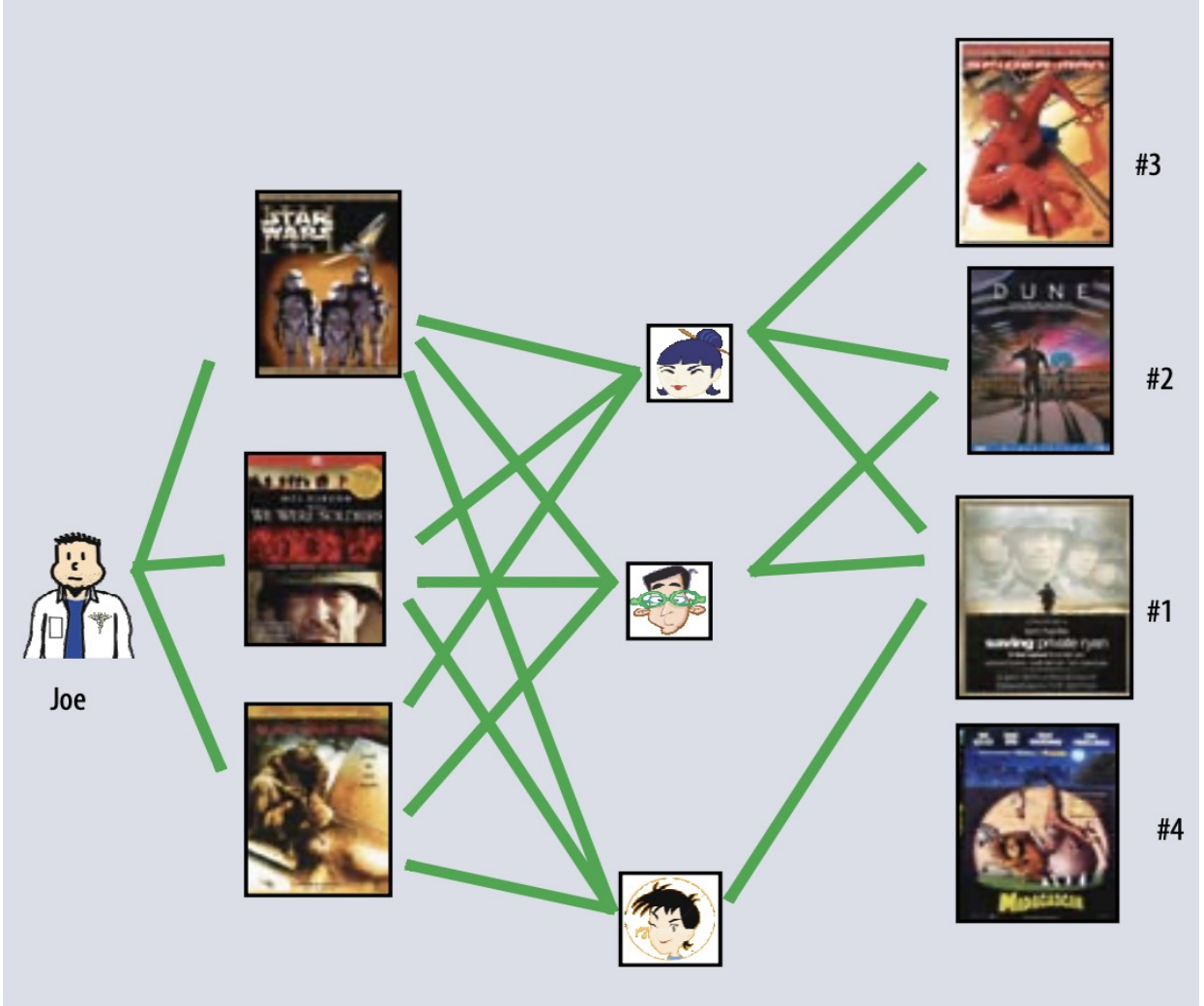


Figure 1. The user-oriented neighborhood method. Joe likes the three movies on the left. To make a prediction for him, the system finds similar users who also liked those movies, and then determines which other movies they liked. In this case, all three liked *Saving Private Ryan*, so that is the first recommendation. Two of them liked *Dune*, so that is next, and so on.

INTRO

Latent Model

transforming both items and users

to the **same latent factor space**

effective at estimating **overall structure**

that relates simultaneously to **most items**

poor at detecting **strong associations**

among a **small set of related items**

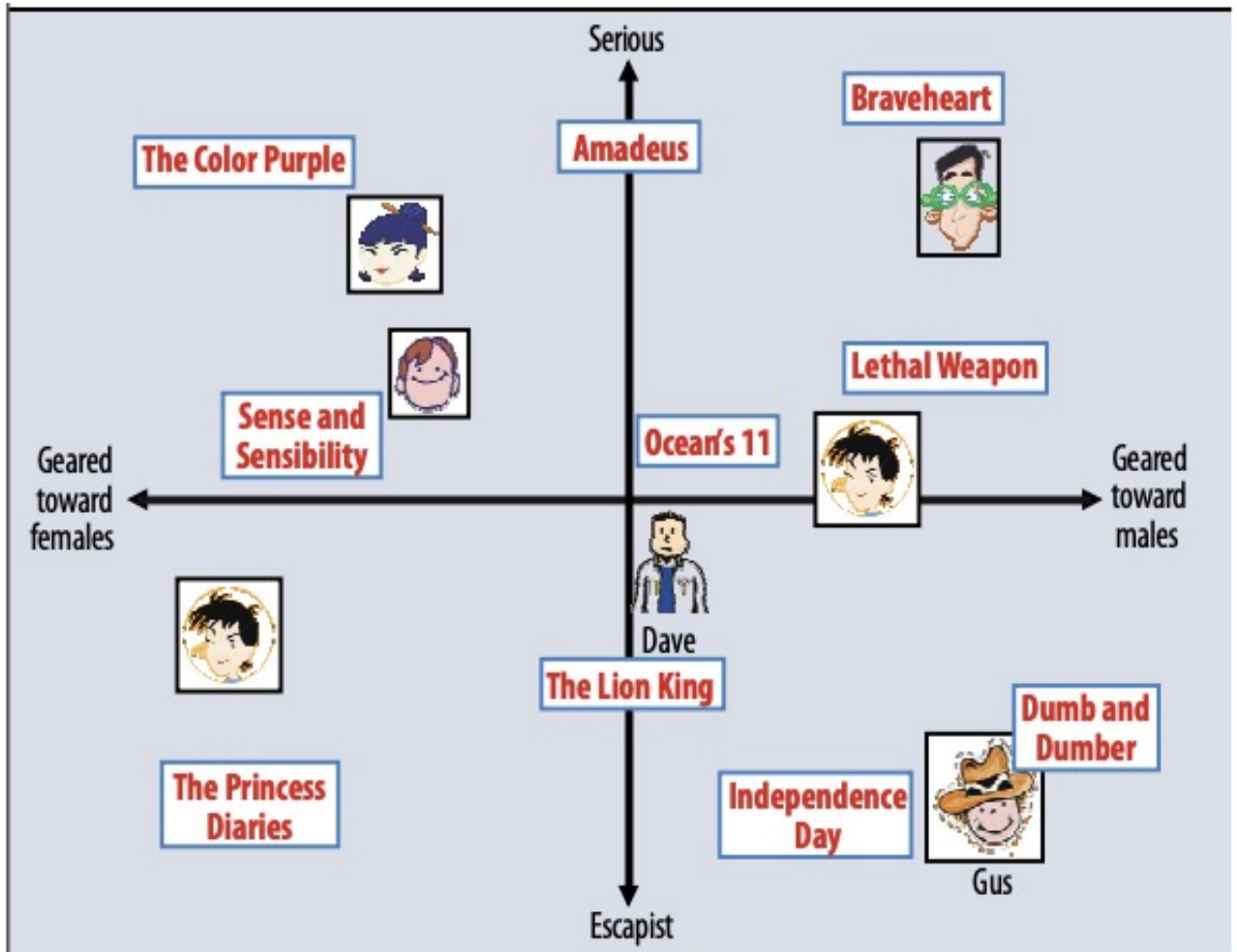


Figure 2. A simplified illustration of the latent factor approach, which characterizes both users and movies using two axes—male versus female and serious versus escapist.

INTRO

Different Forms of USER INPUT

EXPLICIT DATA

explicit input by users

regarding their interest in products

Netflix - star ratings for movies

TiVo - hitting thumbs-up/down buttons

IMPLICIT DATA

indirectly reflect opinion through

observing user behavior

purchase history, browsing history, search

patterns, or even mouse movements

PRELIMINARIES - 01. Baseline estimates

Effect based on user and item

$$b_{ui} = \mu + b_u + b_i$$

Objective function with regularizing term

$$\min_{b_*} \sum_{(u,i)\in\mathcal{K}} (r_{ui} - \mu - b_u - b_i)^2 + \lambda_1 (\sum_u b_u^2 + \sum_i b_i^2)$$

Regularizing term : avoid **overfitting** by penalizing the magnitude of the parameters

PRELIMINARIES - 02. Neighborhood Model : CorNgbr

Measure **Similarity** item *i, j*

$$s_{ij} \overset{\text{def}}{=} \frac{n_{ij}}{n_{ij} + \lambda_2} \rho_{ij}$$

As n increases, s also increases

Predict Value based on k neighbors

$$\hat{r}_{ui} = b_{ui} + \frac{\sum_{j \in S^k(i;u)} s_{ij}(r_{uj} - b_{uj})}{\sum_{j \in S^k(i;u)} s_{ij}}$$

Using **K-neighbors,** reflect **bias (r-b)**

PRELIMINARIES - 02. Neighborhood Model : WgtNgbr

***Concerns** about CorNgbr*

Question suitability of a similarity measure
: relations between two items
: w/o analyzing the interactions full data

neighborhood information is absent
-> rely on baseline estimates.

Predict Value based on k neighbors with weights

$$\hat{r}_{ui} = b_{ui} + \sum_{j \in \mathrm{R}(u)} (r_{uj} - b_{uj}) w_{ij}$$

global weights independent of a specific user

PRELIMINARIES - 03. Latent Model : SVD, SVD w bias

Latent model by **Singular Value Decomposition(SVD)**

$$\hat{r}_{ui} = q_i^T p_u.$$

$$\min_{q^*,p^*} \sum_{(u,i)\in K} (r_{ui} - q_i^T p_u)^2 + \lambda(\| q_i \|^2 + \| p_u \|^2)$$

Latent model by **SVD** with **Base Estimate**

$$\hat{r}_{ui} = \mu + b_i + b_u + q_i^T p_u$$

$$\min_{p_*,q_*,b_*} \sum_{(u,i)\in K} (r_{ui} - \mu - b_u - b_i - p_u^T q_i)^2 + \lambda_3(\|p_u\|^2 + \|q_i\|^2 + b_u^2 + b_i^2)$$

PRELIMINARIES - 04. RMSE & 05. Implicit Feedback

Evaluation using **RMSE**

$$\text{(RMSE): } \sqrt{\sum_{(u,i)\in TestSet}(r_{ui} - \hat{r}_{ui})^2 / |TestSet|}.$$

How to reflect **Implicit Feedback**

Consider **whether movies users rate or not**, regardless of how they rated these movies

user implicitly show their preferences : **choosing and rating**

PRELIMINARIES - 06. Optimizing Algorithm

Stochastic Gradient Descent (SGD)

gradient descent optimization

ease with a relatively fast running time.

- $q_i \leftarrow q_i + \gamma \cdot (e_{ui} \cdot p_u - \lambda \cdot q_i)$
- $p_u \leftarrow p_u + \gamma \cdot (e_{ui} \cdot q_i - \lambda \cdot p_u)$

Alternating Least Squares (ALS)

Fix p, Recompute q by **solving a least-squares problem only for q**

favorable in two cases

system can use parallelization — *computes each q independently of other parameters*

systems centered on implicit data — *the training set cannot be considered sparse*

loop over each training case : not practical for SGD

Neighborhood Model - Integrating Implicit feedback

Reflect Implicit Feedback

$$\hat{r}_{ui} = b_{ui} + \sum_{j \in \mathrm{R}(u)} (r_{uj} - b_{uj}) w_{ij} + \sum_{j \in \mathrm{N}(u)} c_{ij}$$

Formed not only by what he rated, but also by what he did not rate

Reflect Implicit Feedback with penalizing input size

$$\hat{r}_{ui} = \mu + b_u + b_i + |\mathrm{R}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}(u)} (r_{uj} - b_{uj}) w_{ij} + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} c_{ij}$$

Prevent to overemphasize the dichotomy between heavy raters and those that rarely rate

Neighborhood Model - Integrating Implicit feedback

Pruning parameters

$$\hat{r}_{ui} = \mu + b_u + b_i + |\mathrm{R}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}^k(i;u)} (r_{uj} - b_{uj}) w_{ij} + |\mathrm{N}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}^k(i;u)} c_{ij}$$

Trade-off k : accuracy and computational Cost

New NEIGHBORHOOD MODEL

$$\min_{b_*, w_*, c_*} \sum_{(u,i) \in \mathcal{K}} \left(r_{ui} - \mu - b_u - b_i - |\mathrm{N}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}^k(i;u)} c_{ij} - |\mathrm{R}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}^k(i;u)} (r_{uj} - b_{uj}) w_{ij} \right)^2 + \lambda_4 \left(b_u^2 + b_i^2 + \sum_{j \in \mathrm{R}^k(i;u)} w_{ij}^2 + \sum_{j \in \mathrm{N}^k(i;u)} c_{ij}^2 \right) \tag{11}$$

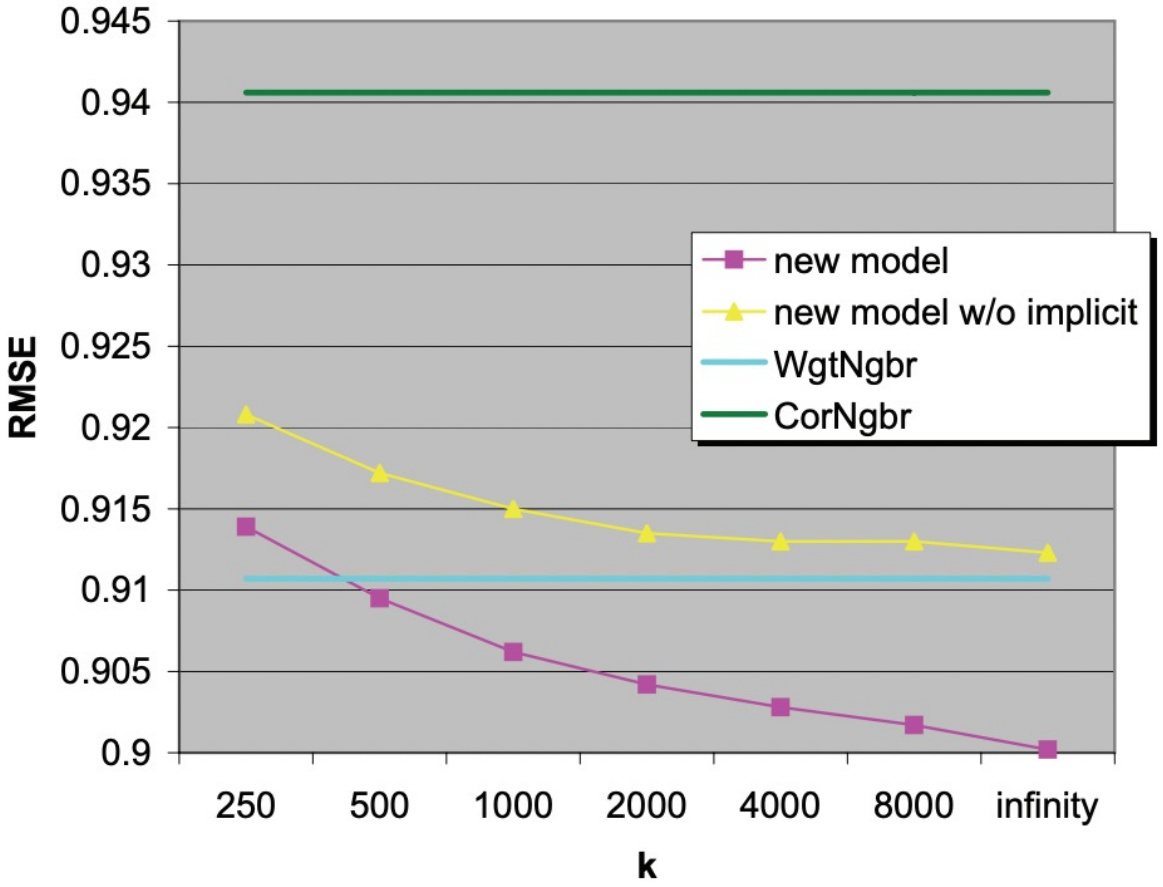
Neighborhood Model - Optimization

Optimized by modifying the parameters
by moving in the **opposite direction of the gradient**

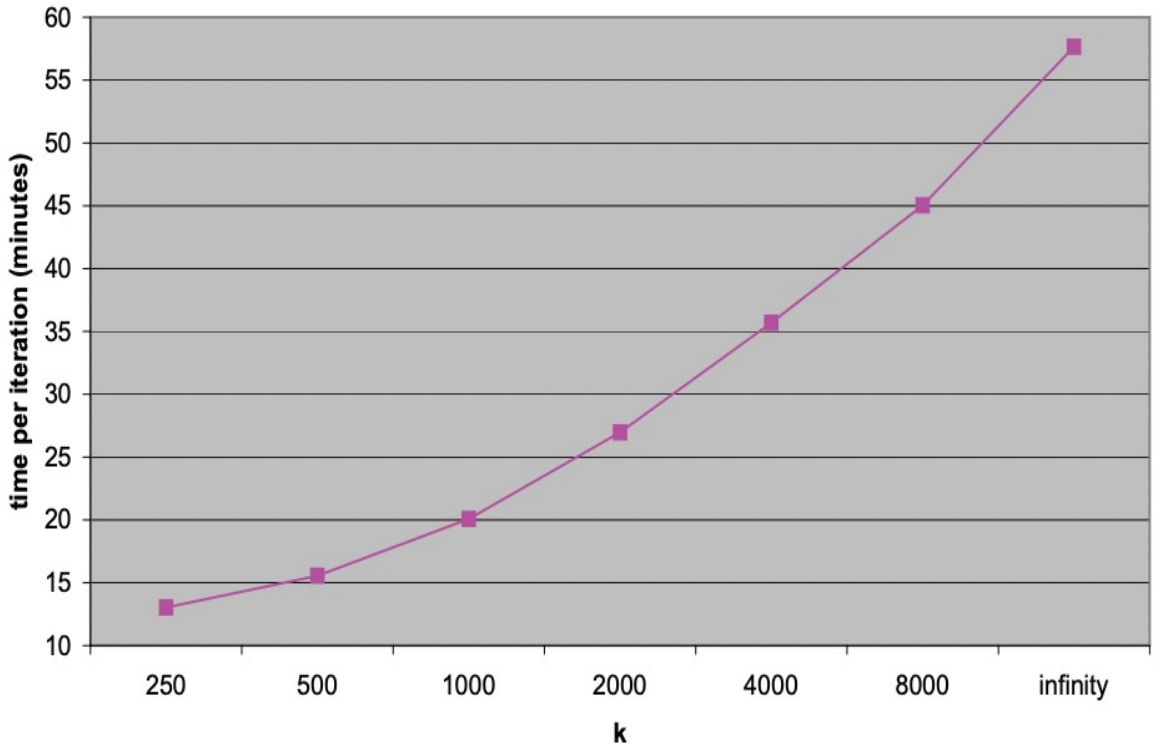
- $b_u \leftarrow b_u + \gamma \cdot (e_{ui} - \lambda_4 \cdot b_u)$
- $b_i \leftarrow b_i + \gamma \cdot (e_{ui} - \lambda_4 \cdot b_i)$
- $\forall j \in \mathrm{R}^k(i;u)$:
  $w_{ij} \leftarrow w_{ij} + \gamma \cdot \left(|\mathrm{R}^k(i;u)|^{-\frac{1}{2}} \cdot e_{ui} \cdot (r_{uj} - b_{uj}) - \lambda_4 \cdot w_{ij} \right)$
- $\forall j \in \mathrm{N}^k(i;u)$:
  $c_{ij} \leftarrow c_{ij} + \gamma \cdot \left(|\mathrm{N}^k(i;u)|^{-\frac{1}{2}} \cdot e_{ui} - \lambda_4 \cdot c_{ij} \right)$

Neighborhood Model - Experimental results

Comparison of neighborhood-based models



Running times per iteration



Latent Model - Asymmetric SVD

Extend SVD model by considering implicit information

$$\hat{r}_{ui} = b_{ui} + q_i^T \left(|\mathrm{R}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}(u)} (r_{uj} - b_{uj}) x_j + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j \right)$$

Replace previous user factor p

Rated by user u : (Rated Item by user u) (Rated item -> factors)

Implicit feedback by user u : (Implicit feedback by user u -> factors)

Latent Model - Asymmetric SVD

Benefits

$$\hat{r}_{ui} = b_{ui} + q_i^T \left(|\mathrm{R}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}(u)} (r_{uj} - b_{uj}) x_j + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j \right)$$

Fewer parameters	the number of users is much larger than the number of products
New users	handle new users without needing to re-train the model
Explainability	abstracting users via an intermediate layer of user factors complicates explanations
Efficient integration of implicit feedback	Prevent to reflect more implicit feedback compare to importance of explicit feedback

Latent Model - SVD++

Question : **predictive accuracy** OR **benefits of Asymmetric-SVD**

SVD++ model

$$\hat{r}_{ui} = b_{ui} + q_i^T \left(p_u + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j \right)$$

Reflect **Implicit Feedback** on basic SVD model

Latent Model - Comparison

Comparison of SVD-based Model **(SVD, Asymmetric-SVD, SVD++)**

Model	50 factors	100 factors	200 factors
SVD	0.9046	0.9025	0.9009
Asymmetric-SVD	0.9037	0.9013	0.9000
SVD++	0.8952	0.8924	0.8911

Table 1: Comparison of SVD-based models: prediction accuracy is measured by RMSE on the Netflix test set for varying number of factors (f). Asymmetric-SVD offers practical advantages over the known SVD model, while slightly improving accuracy. Best accuracy is achieved by SVD++, which directly incorporates implicit feedback into the SVD model.

Latent Model - Other models

Temporal Dynamics

$$\hat{r}_{ui}(t) = \mu + b_i(t) + b_u(t) + q_i^T p_u(t)$$

an item's popularity might change over time

users to change their baseline ratings over time.

Inputs with Varying Confidence Levels

$$\min_{p^*,q^*,b^*} \sum_{(u,i)\in\kappa} c_{ui}(r_{ui} - \mu - b_u - b_i - p_u^T q_i)^2 + \lambda(\| p_u \|^2 + \| q_i \|^2 + b_u^2 + b_i^2)$$

a system might face adversarial users that try to tilt the ratings of certain items.

Integrated Model

$$\hat{r}_{ui} = \mu + b_u + b_i + q_i^T \left(p_u + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j \right)$$

$$+ |\mathrm{R}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{R}^k(i;u)} (r_{uj} - b_{uj}) w_{ij} + |\mathrm{N}^k(i;u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}^k(i;u)} c_{ij}$$

general properties of the item and the user

the interaction between the user profile and the item profile

neighborhood tier

Integrated Model - Optimization & Experiment

Optimization by Gradient Descent

- $b_u \leftarrow b_u + \gamma_1 \cdot (e_{ui} - \lambda_6 \cdot b_u)$
- $b_i \leftarrow b_i + \gamma_1 \cdot (e_{ui} - \lambda_6 \cdot b_i)$
- $q_i \leftarrow q_i + \gamma_2 \cdot (e_{ui} \cdot (p_u + |\mathrm{N}(u)|^{-\frac{1}{2}} \sum_{j \in \mathrm{N}(u)} y_j) - \lambda_7 \cdot q_i)$
- $p_u \leftarrow p_u + \gamma_2 \cdot (e_{ui} \cdot q_i - \lambda_7 \cdot p_u)$
- $\forall j \in \mathrm{N}(u)$:
  $y_j \leftarrow y_j + \gamma_2 \cdot (e_{ui} \cdot |\mathrm{N}(u)|^{-\frac{1}{2}} \cdot q_i - \lambda_7 \cdot y_j)$
- $\forall j \in \mathrm{R}^k(i;u)$:
  $w_{ij} \leftarrow w_{ij} + \gamma_3 \cdot \left(|\mathrm{R}^k(i;u)|^{-\frac{1}{2}} \cdot e_{ui} \cdot (r_{uj} - b_{uj}) - \lambda_8 \cdot w_{ij} \right)$
- $\forall j \in \mathrm{N}^k(i;u)$:
  $c_{ij} \leftarrow c_{ij} + \gamma_3 \cdot \left(|\mathrm{N}^k(i;u)|^{-\frac{1}{2}} \cdot e_{ui} - \lambda_8 \cdot c_{ij} \right)$

Performance of the INTEGRATED MODEL

	50 factors	100 factors	200 factors
RMSE	0.8877	0.8870	0.8868
time/iteration	17min	20min	25min

Table 2: Performance of the integrated model. Prediction accuracy is improved by combining the complementing neighborhood and latent factor models. Increasing the number of factors contributes to accuracy, but also adds to running time.

Evaluation through a Top-K Recommender

A solution with a **slightly better RMSE** will lead to

completely different and better recommendations?

common case : providing **“top K recommendations”**

Our goal is to find the relative place of these “interesting movies”

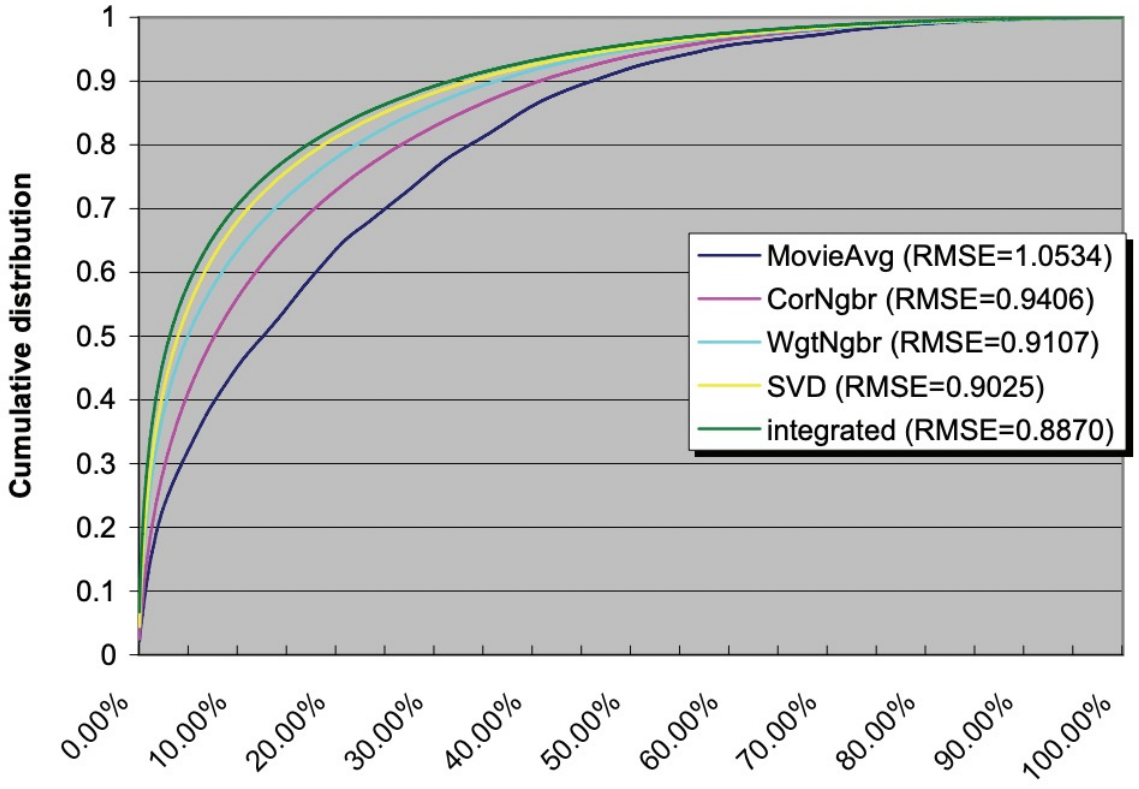
for each such movie i, rated 5-stars by user u, select 1000 additional random movies

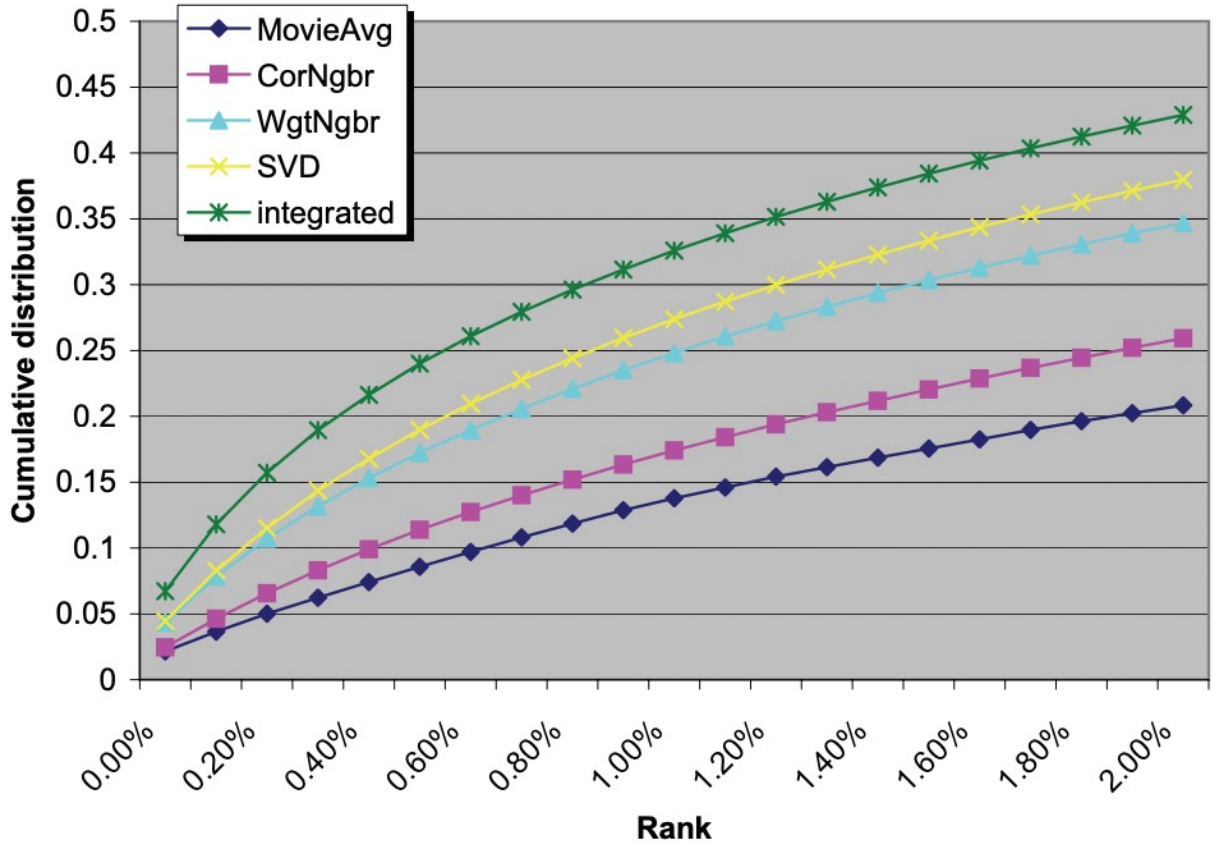
predict the ratings by u for i and for the other 1000 movies

order the 1001 movies based on their predicted rating, in a decreasing order

Evaluation through a Top-K Recommender

Comparing the performance of 5 methods on a top-K recommendation





DISCUSSION

Propose IMPROVEMENTS to two of the most popular approaches

New Neighborhood Based Model

Improve prediction accuracy + maintain explainability, ability to handle new users

Integrated Model (neighborhood model & SVD++ model)

Improve prediction accuracy, integrating implicit feedback

Evaluating top-K recommenders sharpens the differences between the methods

Further experimentation is needed with better sources of implicit feedback

Implement

Modeling

```
def predict_r_ui(mat, u, i, mu, bu, bi, Rk_iu, w_ij, Nk_iu, c_ij, baseline_bu, baseline_bi, qi, pu, N_u, yj):

    Rk_iu_sum = 0
    for j in Rk_iu:
        buj = mu + bu[u] + bi[0,j]
        Rk_iu_sum += (mat[u,j]-buj) * w_ij[i][j]

    Nk_iu_sum = c_ij[i][Rk_iu].sum()
    N_u_sum = yj[N_u].sum(0)

    return mu + bu[u] + bi[0, i] \
          + np.dot(qi[i], (pu[u] + N_u_sum / sqrt(len(N_u)))) + Rk_iu_sum / sqrt(len(Rk_iu)) + Nk_iu_sum / sqrt(len(Nk_iu))
```

Implement

Gradient

```
for u, i, v in zip(cx.row, cx.col, cx.data):

    N_u = bi_index[u]
    Nk_iu = np.flip(np.argsort(S[i,].toarray()))[:k].ravel()
    Rk_iu = Nk_iu

    e_ui = compute_e_ui(mat, u, i, mu, bu, bi, Rk_iu, wij, Nk_iu, cij, baseline_bu, baseline_bi, qi, pu, N_u, yj)
    bu[u] += gamma1 * (e_ui - l_reg6 * bu[u])
    bi[0,i] += gamma1 * (e_ui - l_reg6 * bi[0, i])

    qi[i] += gamma2 * (e_ui * (pu[u] + 1 / sqrt(len(N_u)) * yj[N_u].sum(0)) - l_reg7 * qi[i])
    pu[u] += gamma2 * (e_ui * qi[i] - l_reg7 * pu[u])
    yj[N_u] += gamma2 * (e_ui * 1 / sqrt(len(N_u)) * qi[i] - l_reg7 * yj[N_u])

    buj = []
    for j in Rk_iu :
        buj.append((mu+bu[u]+bi[0,j])[0])

    wij[i][Rk_iu] += gamma3 * ( 1/sqrt(len(Rk_iu)) * e_ui * (mat[u, Rk_iu].toarray().ravel() - buj) - l_reg8 * wij[i][Rk_iu])
    cij[i][Nk_iu] += gamma3 * ( 1/sqrt(len(Nk_iu)) * e_ui - l_reg8 * cij[i][Nk_iu])
```

Implement

Experiment

Comparison of SVD-based Model

	25_factors	50_factors	100_factors
SVD	0.8380	0.7054	0.5602
SVD++	0.8182	0.6969	0.5434
Integrated	0.8074	0.6745	0.5277

Performance of Integrated Model

Integrated	50_factors	100_factors	200_factors
RMSE	0.6745	0.5277	0.4380
Time / Iter	1781.0ms	1789.0ms	1804.2ms