

$\mathcal{N} = \{n_i\}_{i=1}^N$: population
 $\mathcal{T} = \{t_i\}_{i=1}^T$: training cases
 $e_j(n)$: error on t_j for individual n

Here we define Pareto dominance relations with respect to the training cases.

Definition: n_1 dominates n_2 , i.e., $n_1 \prec n_2$, if $e_j(n_1) \leq e_j(n_2) \forall j \in \{1, \dots, N\}$ and $\exists j \in \{1, \dots, N\}$ for which $e_j(n_1) < e_j(n_2)$.

Theorem 1: Individuals selected by lexicase selection are non-dominated in \mathcal{N} with respect to the training cases \mathcal{T} .

Proof: Let $n_1, n_2 \in \mathcal{N}$ be individuals in a population selected by lexicase selection. Suppose $n_1 \prec n_2$. Then $e_j(n_1) \leq e_j(n_2) \forall j \in \{1, \dots, N\}$ and $\exists j \in \{1, \dots, N\}$ for which $e_j(n_1) < e_j(n_2)$. Therefore n_1 is selected for every case that n_2 is selected, and $\exists t \in \mathcal{T}$ for which n_2 is removed from selection due to n_1 . Therefore n_2 cannot be selected by lexicase selection, the supposition is false, and the theorem is true.