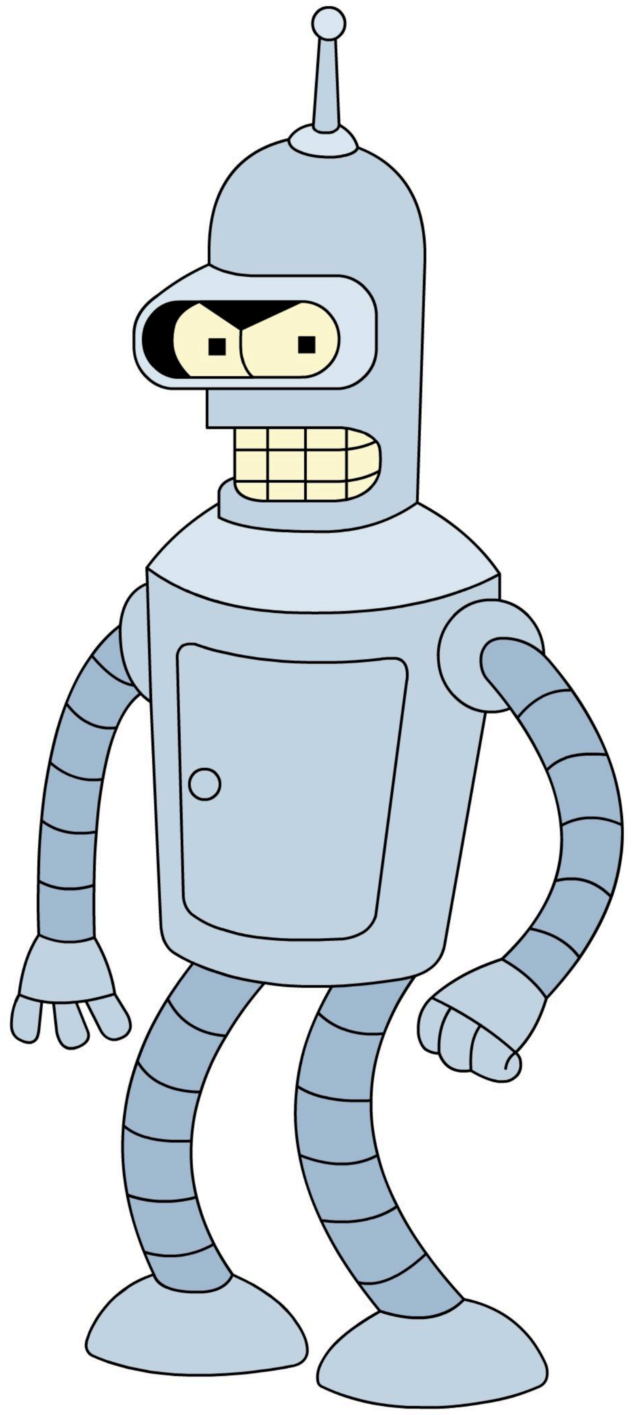
Reinforcement Learning

HSE, winter - spring 2024

Lecture 6: Continuous Control



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Continuous Control Tasks

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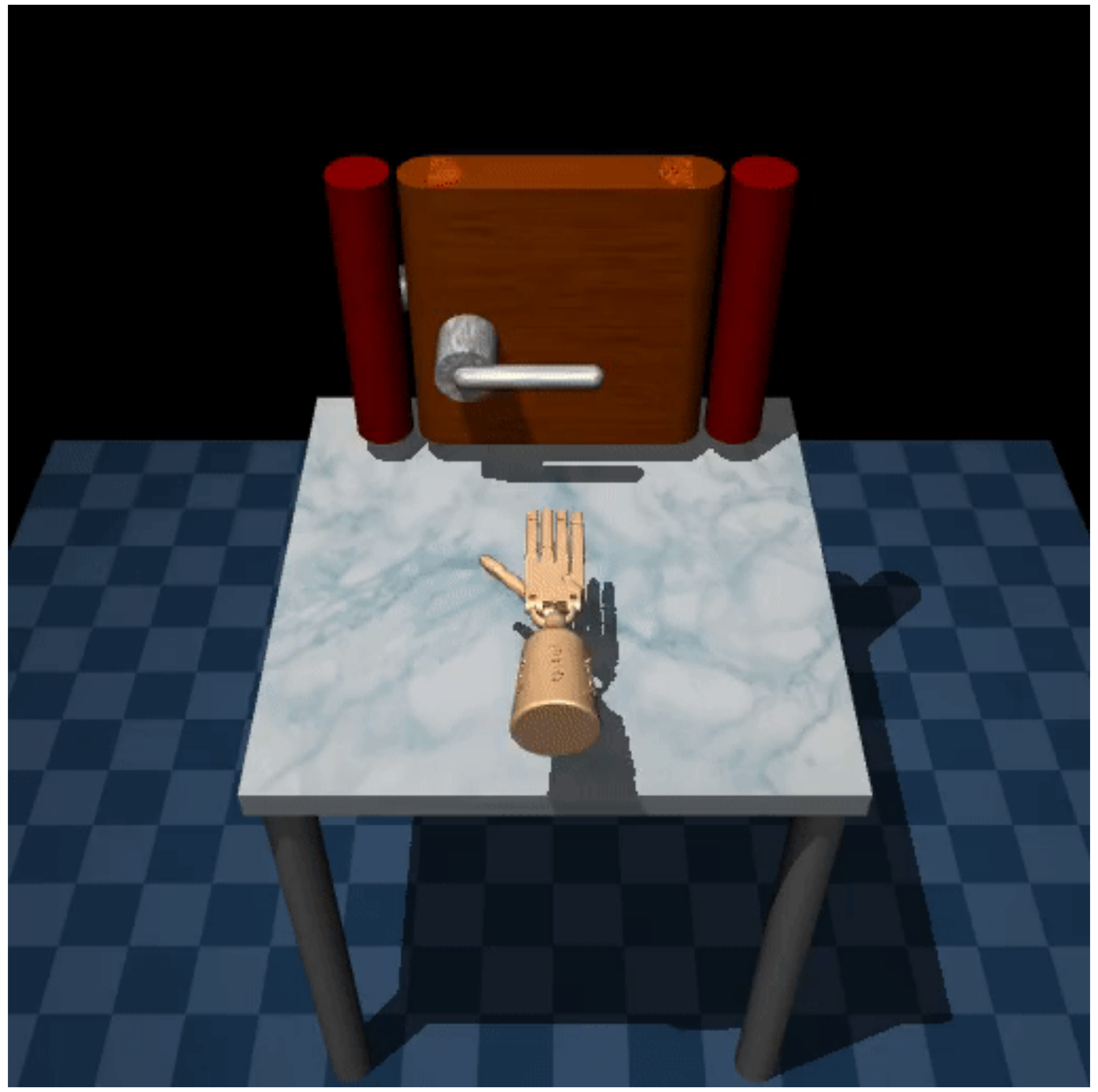
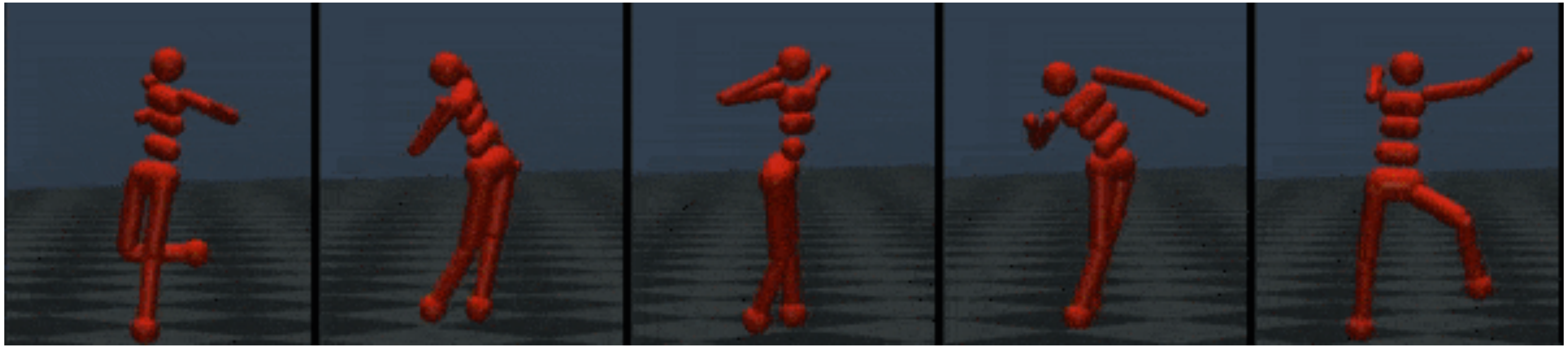


- Action space $\mathscr{A} = [-1,1]^A$
- Dense reward



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Recap: Value-based vs Policy-based

- Value-based (DQN):

  1. Policy evaluation:

     Learn $Q*$ using Bellman target $r + \gamma \max_{a'} Q_\theta(s', a')$

  2. Policy improvement:

     Recover policy greedily w.r.t. $Q_\theta(s, a)$

- Policy gradient (REINFORCE, A2C, PPO):

  1. Policy improvement:

     Learn policy directly calculating the gradient using log-derivative trick of $J(\theta)$ w.r.t. policy parameters θ (actor)

  2. Policy evaluation:

     Learn critic to estimate the quality of the current policy (critic)

Recap: Value-based vs Policy-based

- Value-based (DQN):
  - Only applicable to the discrete action space due to $argmax_a Q(s, a)$
  - Artificial exploration with ε-greedy policies
  - Off-policy algorithm, high sample efficiency thanks to the replay buffer.
  - 1-step target, low signal propagation

- Policy gradient (REINFORCE, A2C, PPO):
  - Applicable to both discrete and continuous action spaces
  - Natural exploration with stochastic policies
  - On-policy, lower sample efficiency, Replay Buffer can not be used
  - N-step target, GAE

Policy Improvement as Optimisation

- Recap: If $\mathbb{E}_{a\sim\pi} Q^{\pi_{old}}(s, a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}
- Optimisation: $\mathbb{E}_s \mathbb{E}_{a\sim\pi} Q^{\pi_{old}}(s, a) \to \max_\pi$

Policy Improvement as Optimisation

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- Optimisation: $\mathbb{E}_s \mathbb{E}_{a \sim \pi} Q^{\pi_{old}}(s, a) \rightarrow \max_{\pi}$
- $\pi(s) = argmax_a Q^{\pi_{old}}(s, a)$

Policy Improvement as Optimisation

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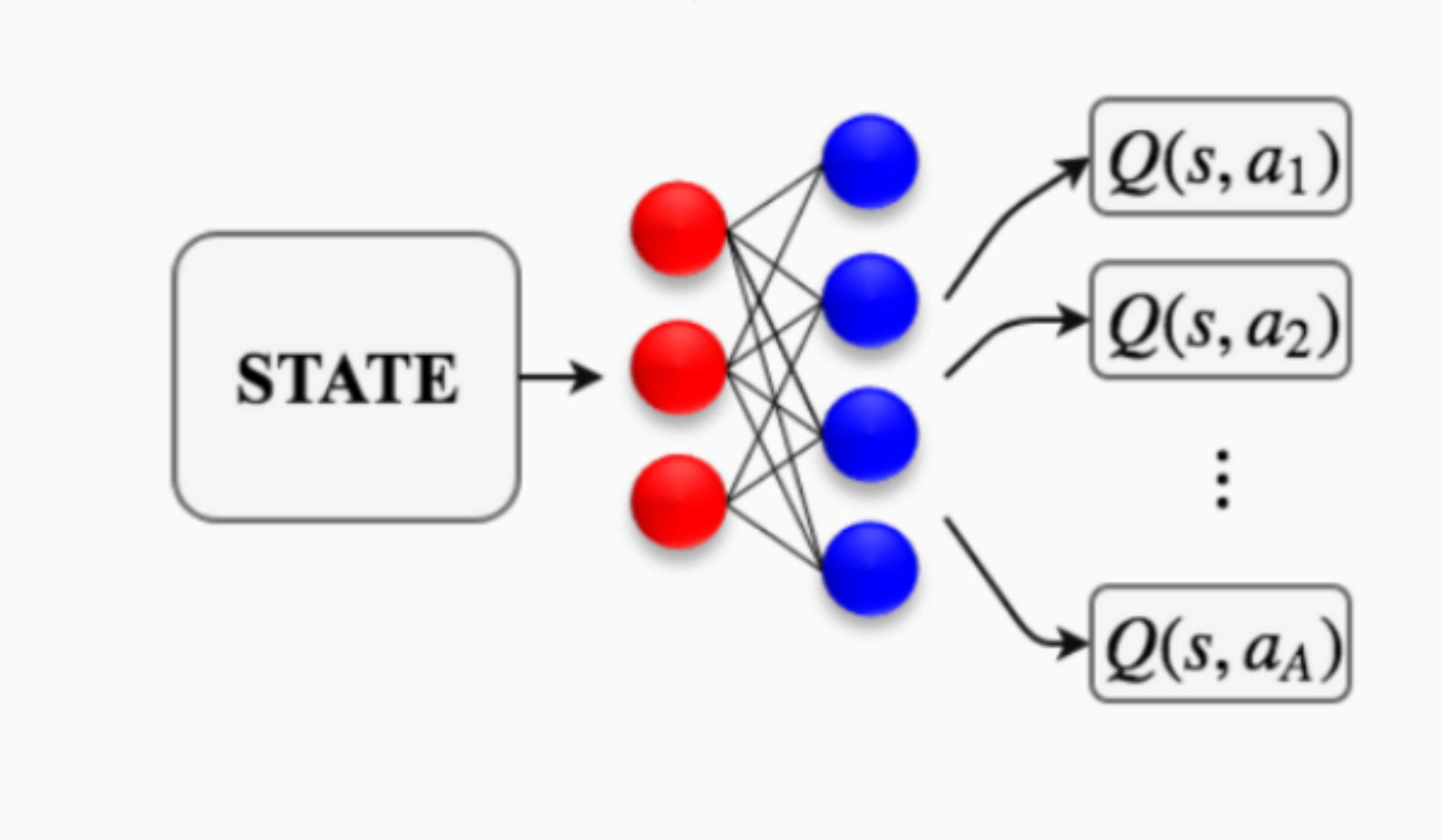

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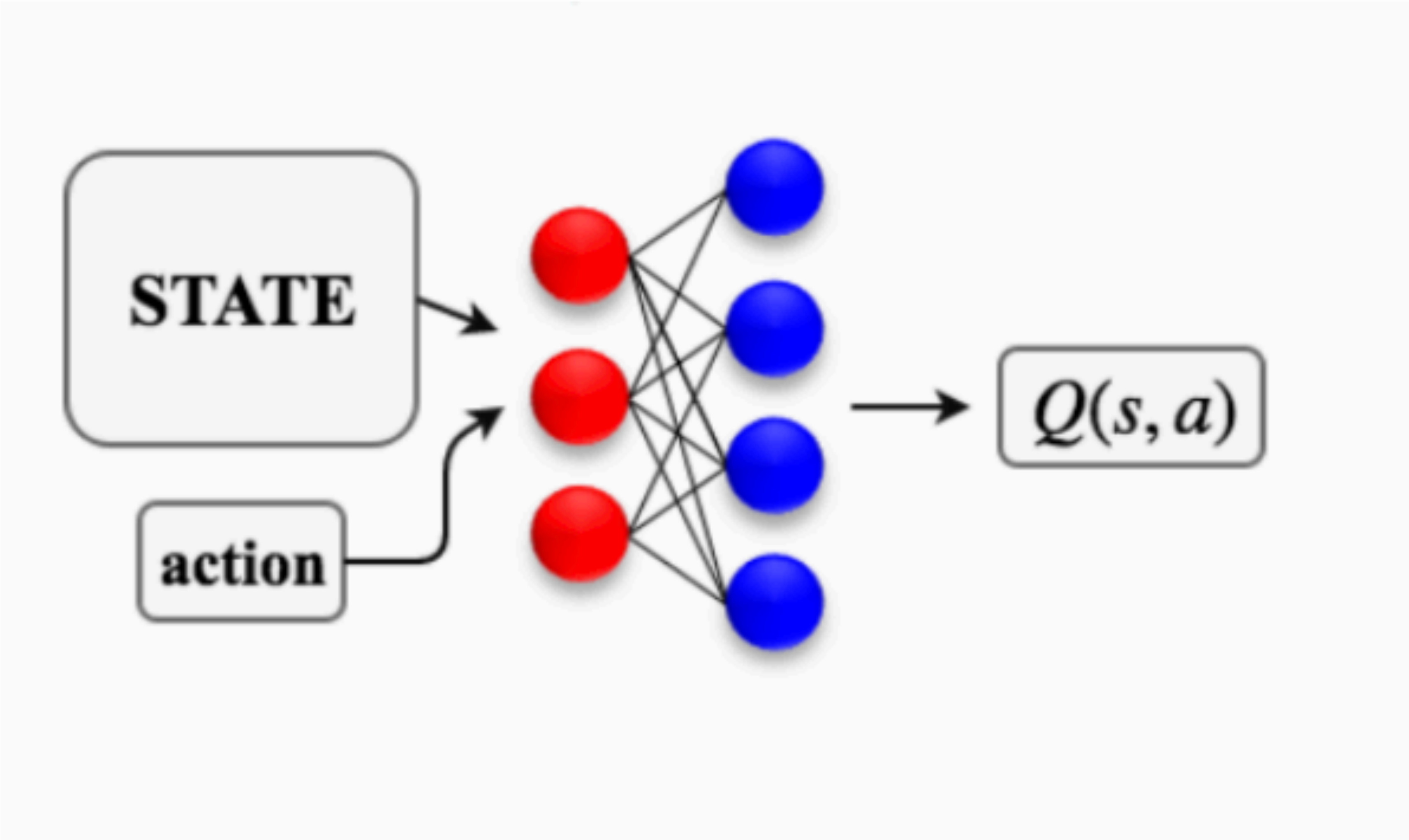

Policy Improvement as Optimisation

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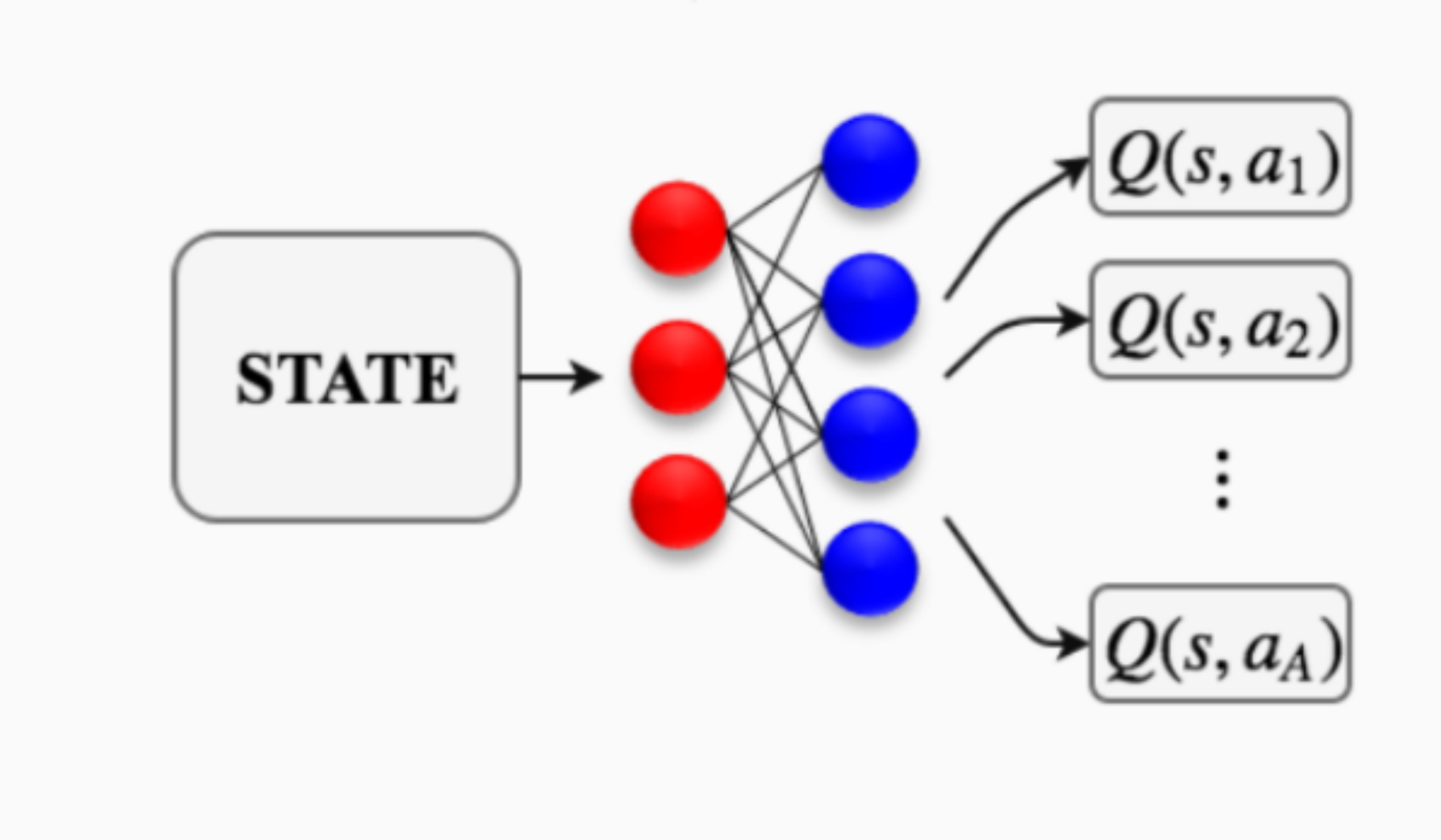

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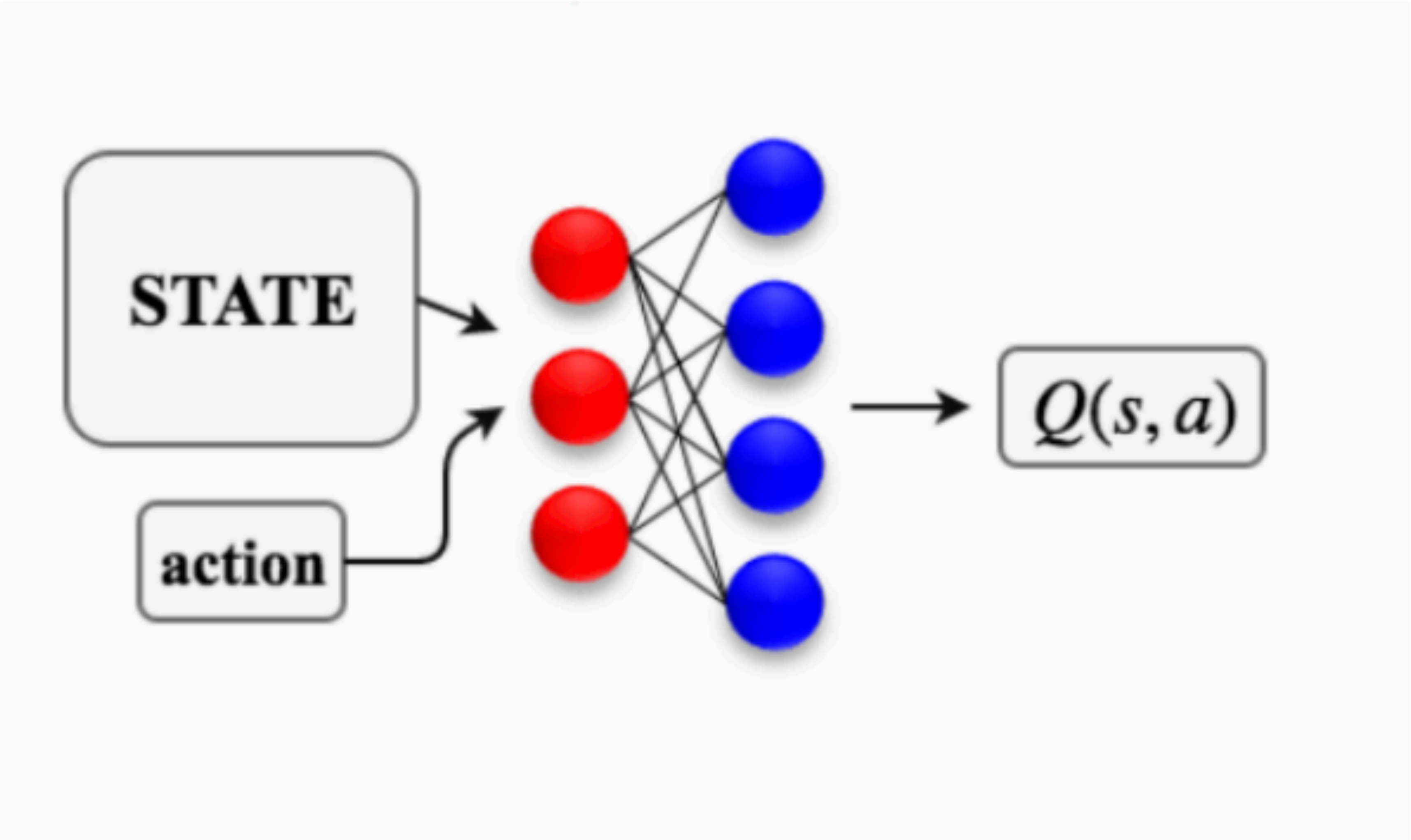


$$Q(s, a) \to \max_{a}$$

$$Q(s, \mu_\theta(s)) \to \max_{\theta}$$

$\mu_\theta(s)$ is a deterministic parametrised policy

DQN

DDPG

Exploration

An advantage of off-policy algorithms is that we can treat the problem of exploration independently from the learning algorithm.

$a = \mu_\theta(s) + \varepsilon$, where:

1. Gaussian noise: $\varepsilon \sim \mathcal{N}(0, \sigma^2)$

2. Ornstein-Uhlenbeck process: $\varepsilon_t = \alpha \varepsilon_{t-1} + \nu, \nu \sim \mathcal{N}(0, \sigma^2)$

Deep Deterministic Policy Gradient (2015)

Actor $\pi_\theta(s)$, critic $Q_\phi(s, a)$, target actor $\pi_{\theta^-}(s)$, target critic $Q_{\phi^-}(s, a)$.

On each step:

- Observe s, choose $a = \pi_\theta(s) + \varepsilon$, get s', r, $done$, put the transition into the buffer
- On the batch of transitions $(s_i, a, r_i, s_i', done_i)_{i=1}^B$, sampled from the replay buffer, perform:

1. Policy evaluation: $\frac{1}{B}\sum_{i=1}^{B}(y_i - Q_\phi(s_i, a_i)) \to \min_\phi$, where $y_i = r_i + \gamma(1 - done_i)Q_{\phi^-}(s_i', \pi_{\theta^-}(s_i'))$
2. Policy improvement: $\frac{1}{B}\sum_{i=1}^{B} Q_\phi(s_i, \pi_\theta(s_i))) \to \max_\theta$

- Soft-update the actor and critic:
  - $\theta^- = \tau\theta + (1 - \tau)\theta^-$, $\phi^- = \phi\theta + (1 - \tau)\phi^-$

Twin Delayed DDPG

Clipped Double-Q Learning:

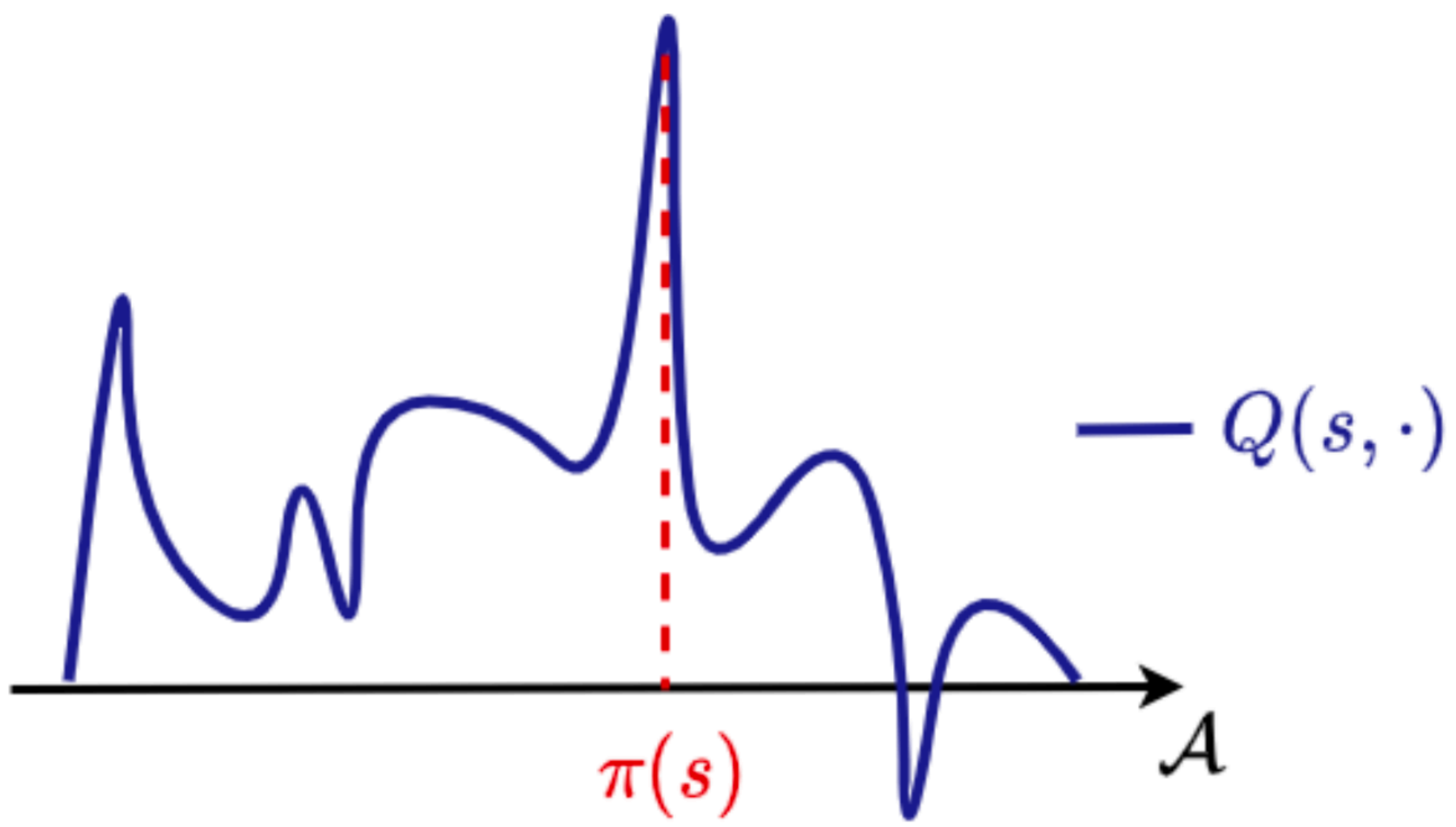
$$y = r + \gamma \min_{i=1,2} Q_{\phi_i^-}(s', \mu_{\theta^-}(s'))$$

Delayed Policy Updates: Update the policy (and target networks) less frequently than the Q-function.

Target Policy Smoothing: Add noise to the target action make it harder for the policy to exploit Q-function errors:

$$y = r + \gamma \min_{i=1,2} Q_{\phi_i^-}(s', a'), \; a' = \mu_{\theta^-}(s) + \varepsilon',$$

$$\varepsilon' \sim clip(\mathcal{N}(0, \sigma' I), -c, c)$$



Deterministic Policy Gradient

$$J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_\theta}}[Q^{\pi_\theta}(s, \pi_\theta(s))]$$

For deterministic policy $\pi_\theta : \mathcal{S} \to \mathcal{A}$:

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_\theta}} \nabla_\theta \pi_\theta \nabla_a Q^{\pi_\theta}(s, a)\,|_{a=\pi_\theta(s)}$$

Deterministic Policy Gradient

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Surrogate objective for policy gradient:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_{old}}}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$

Surrogate objective for policy improvement:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$

Deterministic Policy Gradient

Surrogate objective for policy gradient:

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Surrogate objective for policy improvement:

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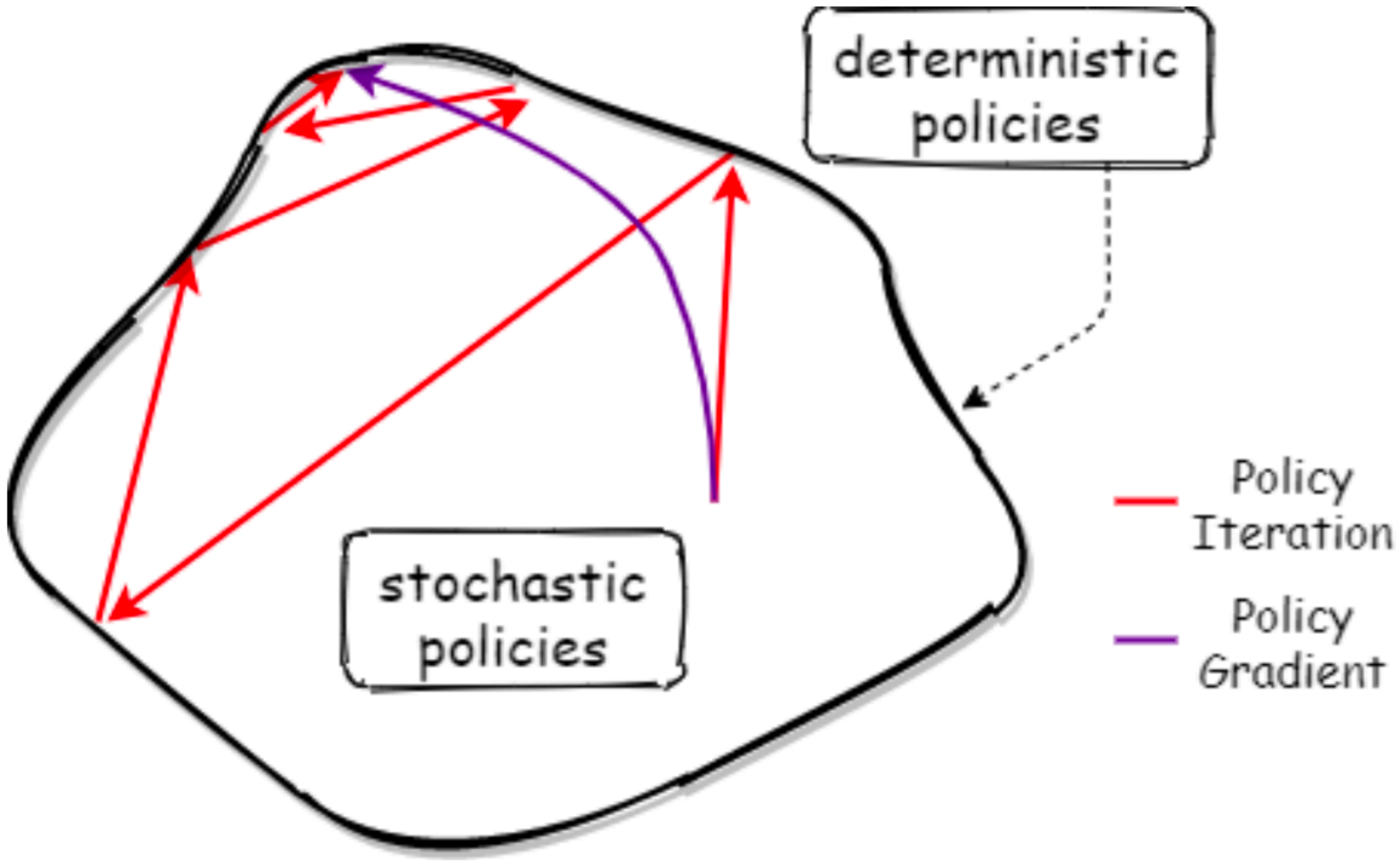


Policy Gradient

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[Q^{\pi_{old}}(s, a)] \to \max_\theta$$

Let's take $\mathbb{E}_s \nabla_\theta \mathbb{E}_{a \sim \pi_\theta(.|s)}[Q^{\pi_{old}}(s, a)]$ in two ways:

REINFORCE

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[\nabla_\theta \log \pi_\theta(a \mid s) Q^{\pi_{old}}(s, a)]$$

Reparametrisation Trick

$$\mathbb{E}_s \mathbb{E}_{\varepsilon \sim p(.)}[\nabla_\theta Q^{\pi_{old}}(s, f_\theta(s, \varepsilon))]$$

If $a \sim \pi(\,.\mid s)$ is equivalent to $a = f_\theta(s, \varepsilon)$,
Where f_θ is a deterministic function,
$\varepsilon \sim p(\,.\,)$ is a non-parametric distribution.

Policy Gradient

REINFORCE

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[\nabla_\theta \log \pi_\theta(a \mid s) Q^{\pi_{old}}(s, a)]$$

Reparametrisation Trick

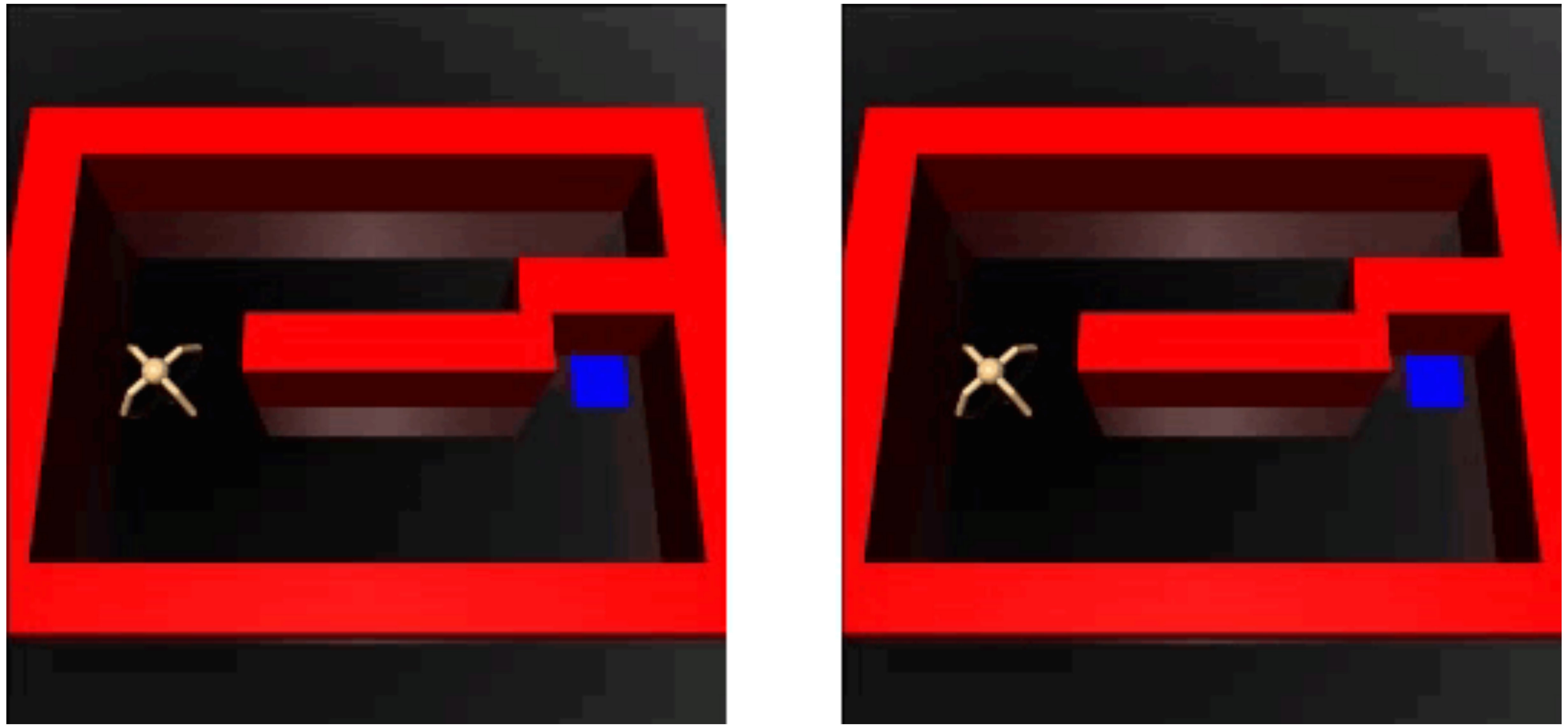
$$\mathbb{E}_s \mathbb{E}_{\varepsilon \sim p(.)}[\nabla_\theta Q^{\pi_{old}}(s, f_\theta(s, \varepsilon))]$$

If $a \sim \pi(\,.\mid s)$ is equivalent to $a = f_\theta(s, \varepsilon)$,
where f_θ is a deterministic function,
$\varepsilon \sim p(\,.\,)$ is a non-parametric distribution.

1. Softmax policy: $a \sim softmax(logit_\theta(s))$
2. Deterministic policy: $a = \pi_\theta(s)$
3. Gaussian policy: $a \sim \mathcal{N}(\mu_\theta(s), \sigma_\theta^2(s)I)$
4. Mixture of gaussian: $a \sim \sum_{i=1}^{K} w_\theta^i(s)\mathcal{N}(\mu_\theta^i(s), (\sigma_\theta^i(s))^2 I)$

Stochastic Policies

- So far, we've introduced two off-policy algorithms learning only deterministic policies, where exploration is artificially maintained by adding noise.
- We want to train stochastic policies to have natural exploration, which prevents our agents from getting stuck in local optima.
- We also want to prevent our stochastic policies from becoming "too deterministic" very quickly.



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Maximum Entropy RL

$$J_{soft}(\pi) = \mathbb{E}_{\tau \sim \pi} \sum_{t=0}^{\infty} \gamma^t [r_t + \alpha H(\pi(\,.\,|\,s_t))]$$, where $H(\pi(\,.\,|\,s))$ is entropy.

Equivalent form:

$$J_{soft}(\pi) = \mathbb{E}_{\tau \sim \pi} \sum_{t=0}^{\infty} \gamma^t [r_t - \alpha \log \pi(a_t\,|\,s_t)]$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

$$V^{\pi}_{soft}(s) = \mathbb{E}_a[r(s,a) - \log \pi(a\,|\,s) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')]$$

$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

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$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')$$

$$V^{\pi}_{soft}(s) = \mathbb{E}_a[Q^{\pi}_{soft}(s,a) - \alpha \log \pi(a\,|\,s)]$$

$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} \mathbb{E}_{a'}[Q^{\pi}_{soft}(s',a') - \alpha \log \pi(a'\,|\,s')]$$

Soft Policy Evaluation

For transition, (s, a, r, s') define a critic's target:

$$y_Q = r(s, a) + \gamma \mathbb{E}_{\boxed{a' \sim \pi(.|s')}}[Q_\phi(s', a') - \alpha \log \pi(a' \mid s')]$$

In general intractable

Soft Policy Evaluation

For transition, (s, a, r, s') define a critic's target:

$$y_Q = r(s, a) + \gamma \mathbb{E}_{\boxed{a' \sim \pi(.|s')}}[Q_\phi(s', a') - \alpha \log \pi(a' \mid s')]$$

In general intractable

- We can estimate the expectation using a sample from the policy
- Can learn V_ψ to approximate the expectation:

$$y_V = Q_\phi(s, a_\pi) - \alpha \log \pi(a_\pi \mid s), a_\pi \sim \pi(. \mid s)$$

$$y_Q = r(s, a) + \gamma V_\psi(s')$$

Policy Improvement

Policy Improvement in traditional RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}
- Optimisation:

$$\mathbb{E}_s\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \to \max_{\pi}$$

- $\pi(s) = argmax_a Q^{\pi_{old}}(s,a)$

Policy Improvement in max entropy RL

Policy Improvement

Policy Improvement in traditional RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}
- Optimisation:

$$\mathbb{E}_s\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \rightarrow \max_{\pi}$$

- $\pi(s) = argmax_a Q^{\pi_{old}}(s,a)$

Policy Improvement in max entropy RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}_{soft}(s,a) + H(\pi(\,.\,|\,s)) \geq V^{\pi_{old}}_{soft}(s)$ then π is not worse than π_{old}
- Optimisation:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}_{soft}(s,a) + H(\pi(\,.\,|\,s))] \rightarrow \max_{\pi}$$

- $\pi(\,.\,|\,s) = softmax(\frac{Q^{\pi_{old}}(s,\,.\,)}{\alpha})$

Soft Policy Improvement

Actor π_θ learning:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi_\theta}Q_\phi(s,a) + H(\pi_\theta(\,.\,|\,s))] \to \max_\theta$$

Soft Policy Improvement

Actor π_θ learning:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi_\theta}Q_\phi(s,a) + H(\pi_\theta(\,.\,|\,s))] \to \max_\theta$$

Example:

- $\pi_\theta(\,.\,|\,s) = \mathcal{N}(\mu_\theta(s), \sigma_\theta^2(s)I)$
- $a \sim \pi_\theta(\,.\,|\,s) \iff a = \mu_\theta(s) + \sigma_\theta(s)\varepsilon, \varepsilon \sim \mathcal{N}(0,I)$
- $H(\pi_\theta(\,.\,|\,s))] = \sum_{i=1}^{A} \log \sigma_\theta^i(s)$
- $\mathbb{E}_s[\mathbb{E}_{\varepsilon\sim\mathcal{N}(0,I)}Q_\phi(s, \mu_\theta(s) + \sigma_\theta(s)\varepsilon) + \sum_{i=1}^{A} \log \sigma_\theta^i(s)] \to \max_\theta$

Soft Actor-Critic

- Actor $\pi_\theta(\,.\,|\,s)$, critics $Q_{\phi_1}(s,a)$, $Q_{\phi_2}(s,a)$, target critics $Q_{\phi_1^-}(s,a)$, $Q_{\phi_2^-}(s,a)$.

On each step:

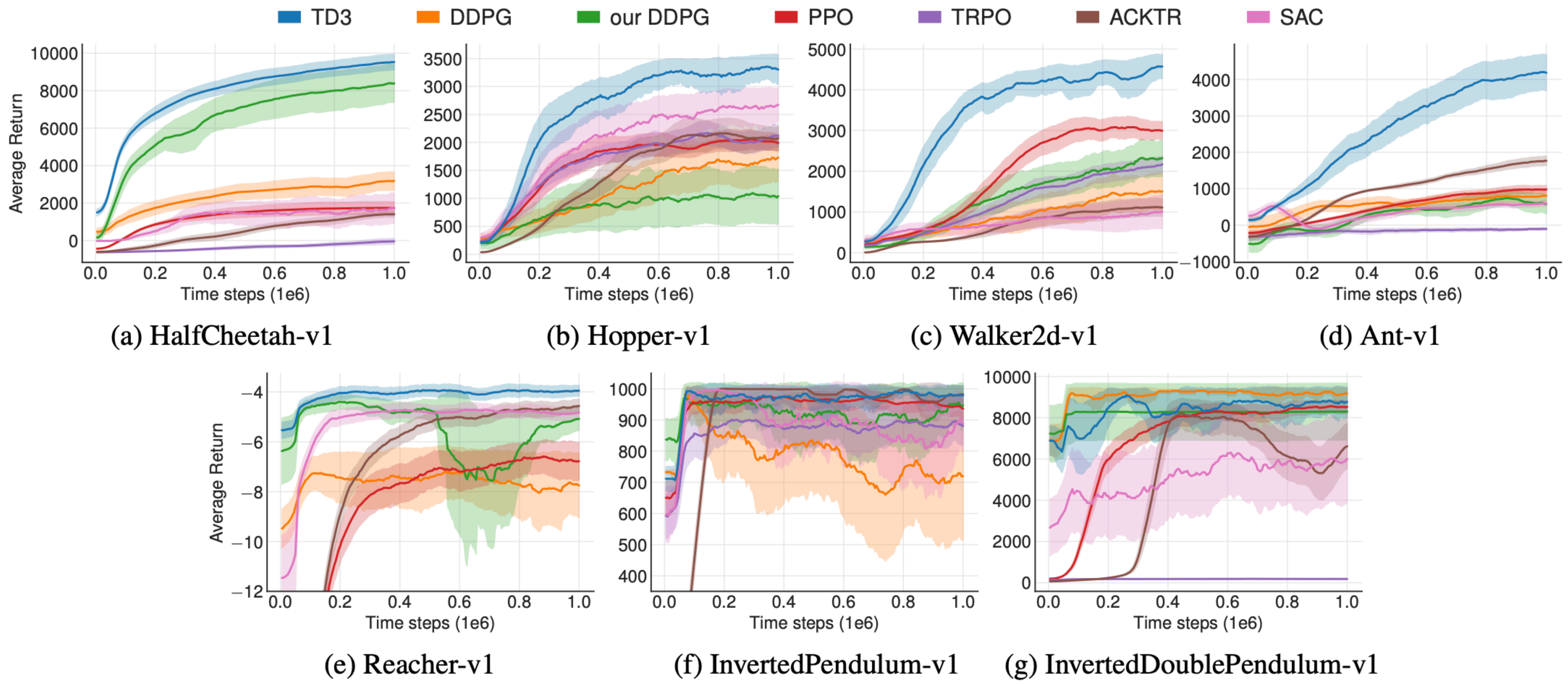
- Observe s, choose $a \sim \pi_\theta(\,.\,|\,s)$, get $s', r, done$, put the transition into the buffer
- On the batch of transitions $(s_i, a, r_i, s'_i, done_i)_{i=1}^B$ sampled from the replay buffer, perform:

1. Soft Policy evaluation: $\frac{1}{B}\sum_{i=1}^{B}(y_i - Q_{\phi_k}(s_i, a_i)) \to \min\limits_{\phi_k}$, where
   $y_i = r_i + \gamma(1 - done_i)(\min\limits_{k=1,2} Q_{\phi_k^-}(s'_i, a'_i) - \alpha \log \pi(s'_i\,|\,a'_i)),\ a'_i \sim \pi_\theta(\,.\,|\,s'_i)$
2. Policy improvement: $\frac{1}{B}\sum_{i=1}^{B}\min\limits_{k=1,2} Q_{\phi_k}(s_i, a_\theta(s_i)) - \alpha \log \pi(s_i\,|\,a_\theta(s_i)) \to \max\limits_{\theta}$

   Where $a_\theta(s_i)$ is a sample from $\pi_\theta(\,.\,|\,s)$ which is differentiable wrt θ via reparametrisation trick

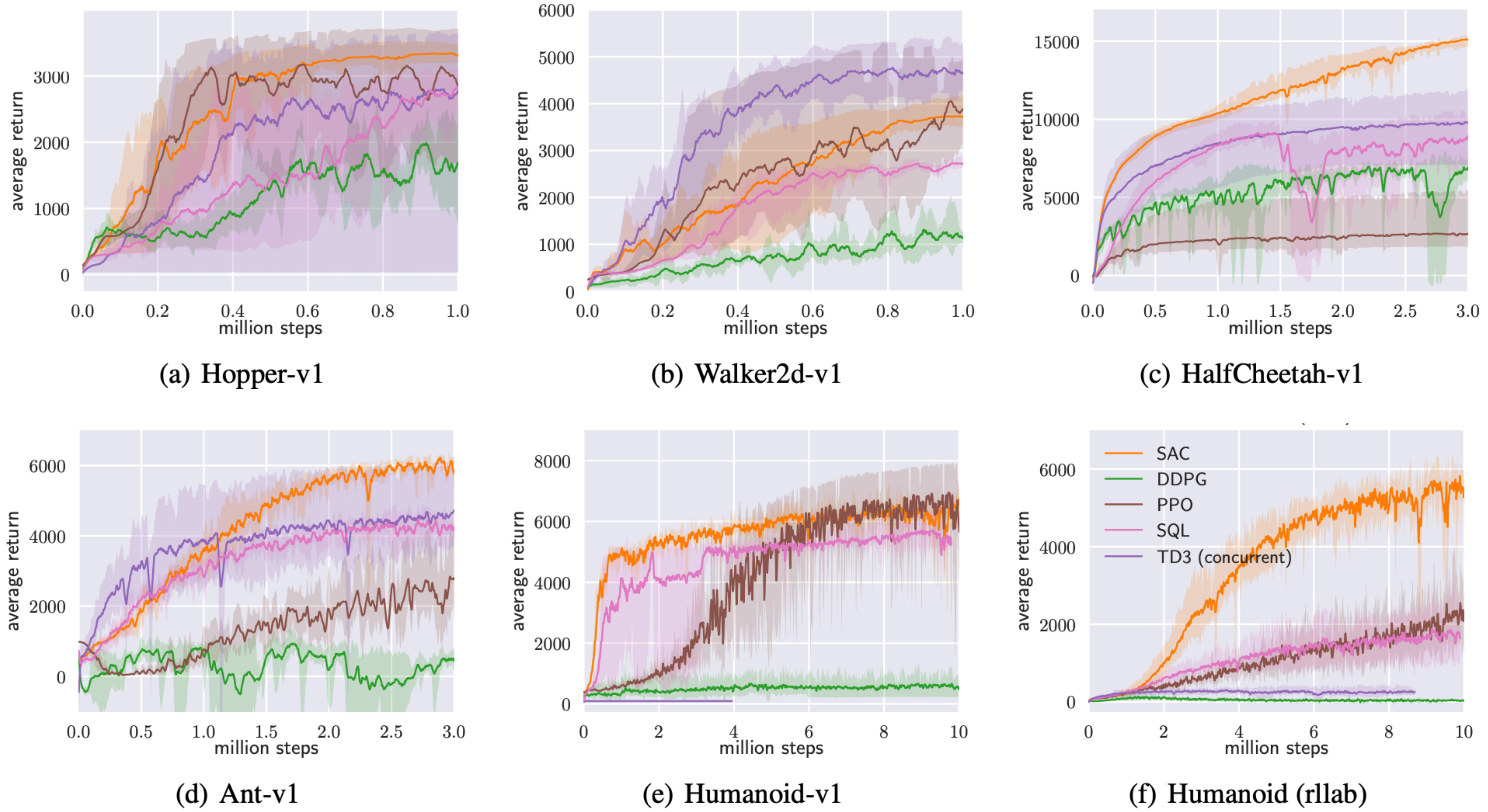
- Soft-update for the target critics

Comparison



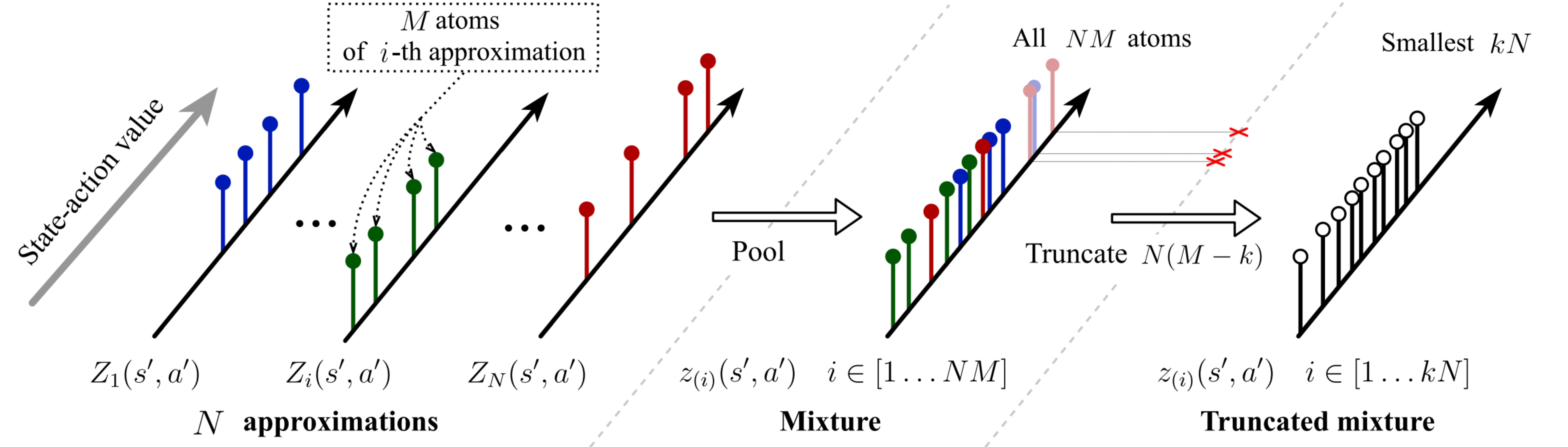
TD3 paper

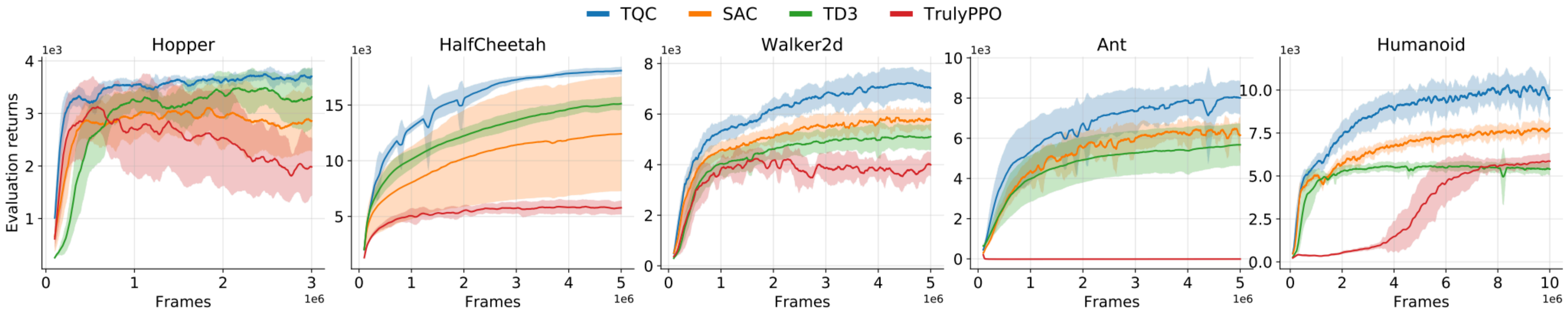
SAC paper



Truncated Quantile Critics

Controlling Overestimation Bias with Truncated Mixture of Continuous Distributional Quantile Critics





Background

1. Reinforcement Learning Textbook (in Russian): 6
2. Lecture 19: Connection between Inference and Control
3. Soft Actor-Critic Algorithms and Applications

Thank you for your attention!