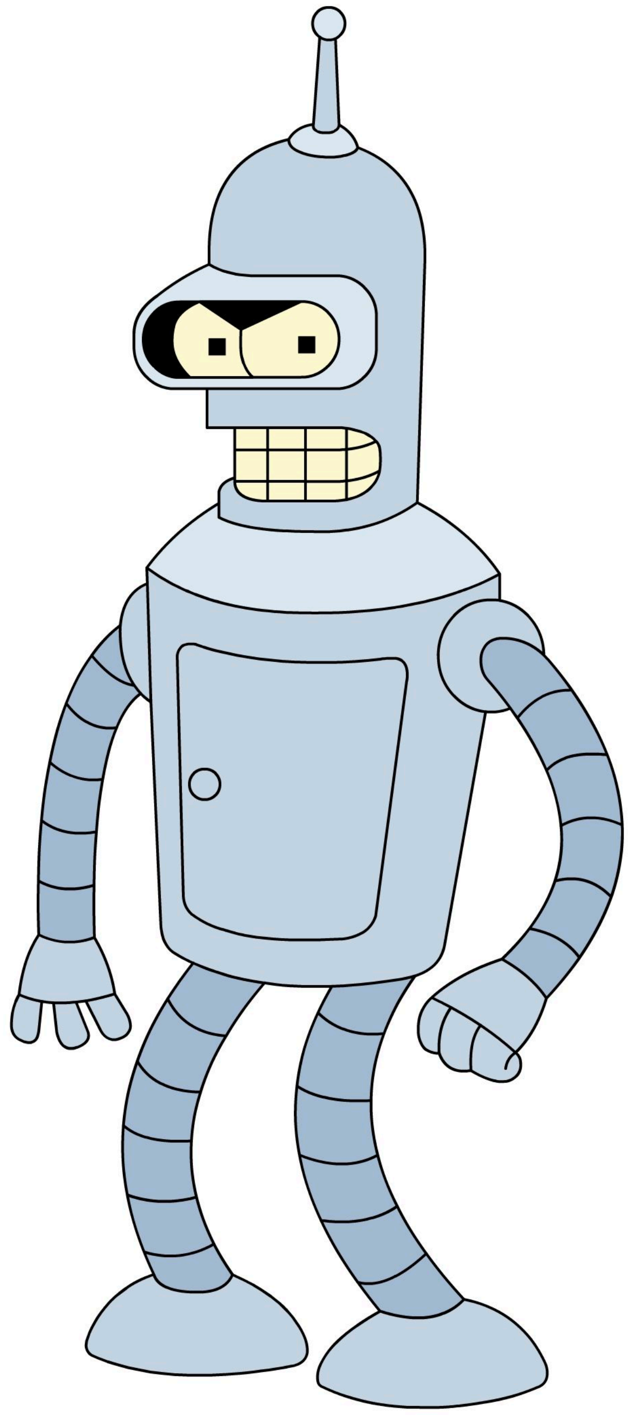
Reinforcement Learning

HSE, winter - spring 2024

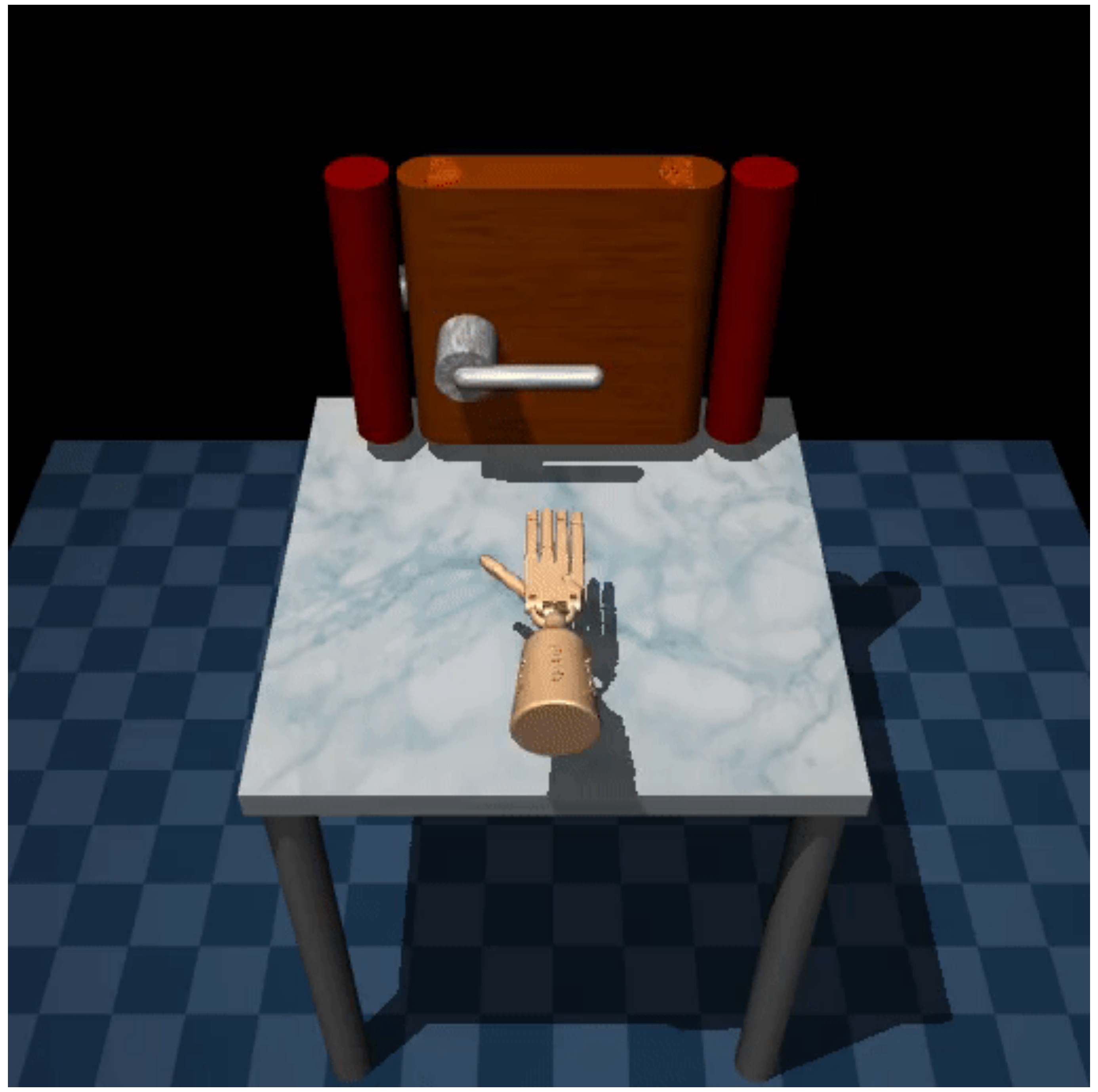
Lecture 6: Continuous Control



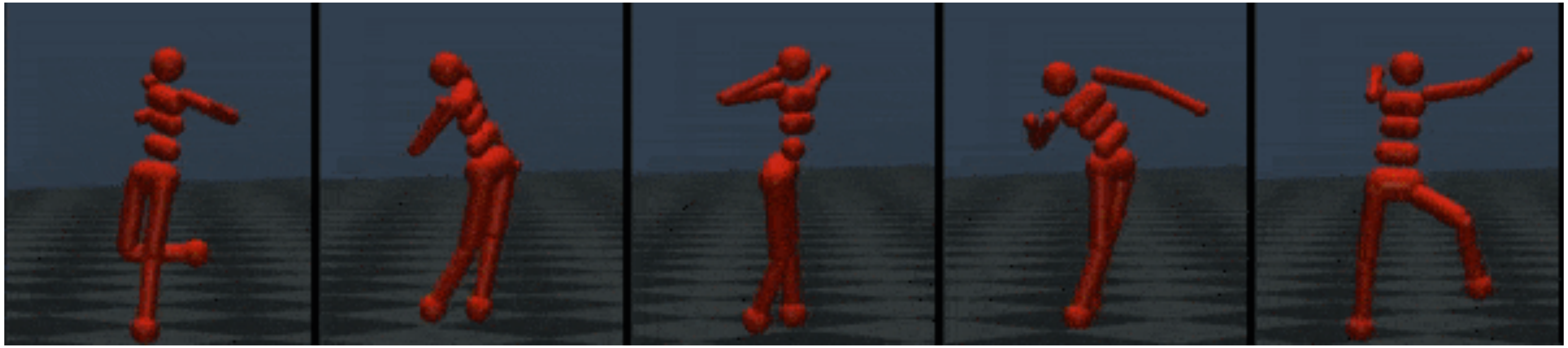
Sergei Laktionov
slaktionov@hse.ru
LinkedIn

Continuous Control Tasks

Source



- Action space $\mathscr{A} = [-1,1]^A$
- Dense reward



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Recap: Value-based vs Policy-based

- Value-based (DQN):

  1. Policy evaluation:

     Learn $Q*$ using Bellman target $r + \gamma \max_{a'} Q_\theta(s', a')$

  2. Policy improvement:

     Recover policy greedily w.r.t. $Q_\theta(s, a)$

- Policy gradient (REINFORCE, A2C, PPO):

  1. Policy improvement:

     Learn policy directly calculating the gradient using log-derivative trick of $J(\theta)$ w.r.t. policy parameters θ

  2. Policy evaluation:

     Learn critic to estimate the quality of the current policy

Recap: Value-based vs Policy-based

- Value-based (DQN):
  - Only applicable to the discrete action space due to $argmax_a Q(s, a)$
  - Artificial exploration with ε-greedy policies
  - Off-policy algorithm, high sample efficiency thanks to the replay buffer.
  - 1-step target, low signal propagation

- Policy gradient (REINFORCE, A2C, PPO):
  - Applicable to both discrete and continuous action spaces
  - Natural exploration with stochastic policies
  - On-policy, lower sample efficiency, Replay Buffer can not be used
  - N-step target, GAE

Policy Improvement as Optimisation

- Recap: If $\mathbb{E}_{a\sim\pi} Q^{\pi_{old}}(s, a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}
- Optimisation: $\mathbb{E}_s \mathbb{E}_{a\sim\pi} Q^{\pi_{old}}(s, a) \rightarrow \max\limits_{\pi}$

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- Optimisation: $\mathbb{E}_s\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \rightarrow \max_{\pi}$
- $\pi(s) = argmax_a Q^{\pi_{old}}(s,a)$

Policy Improvement as Optimisation

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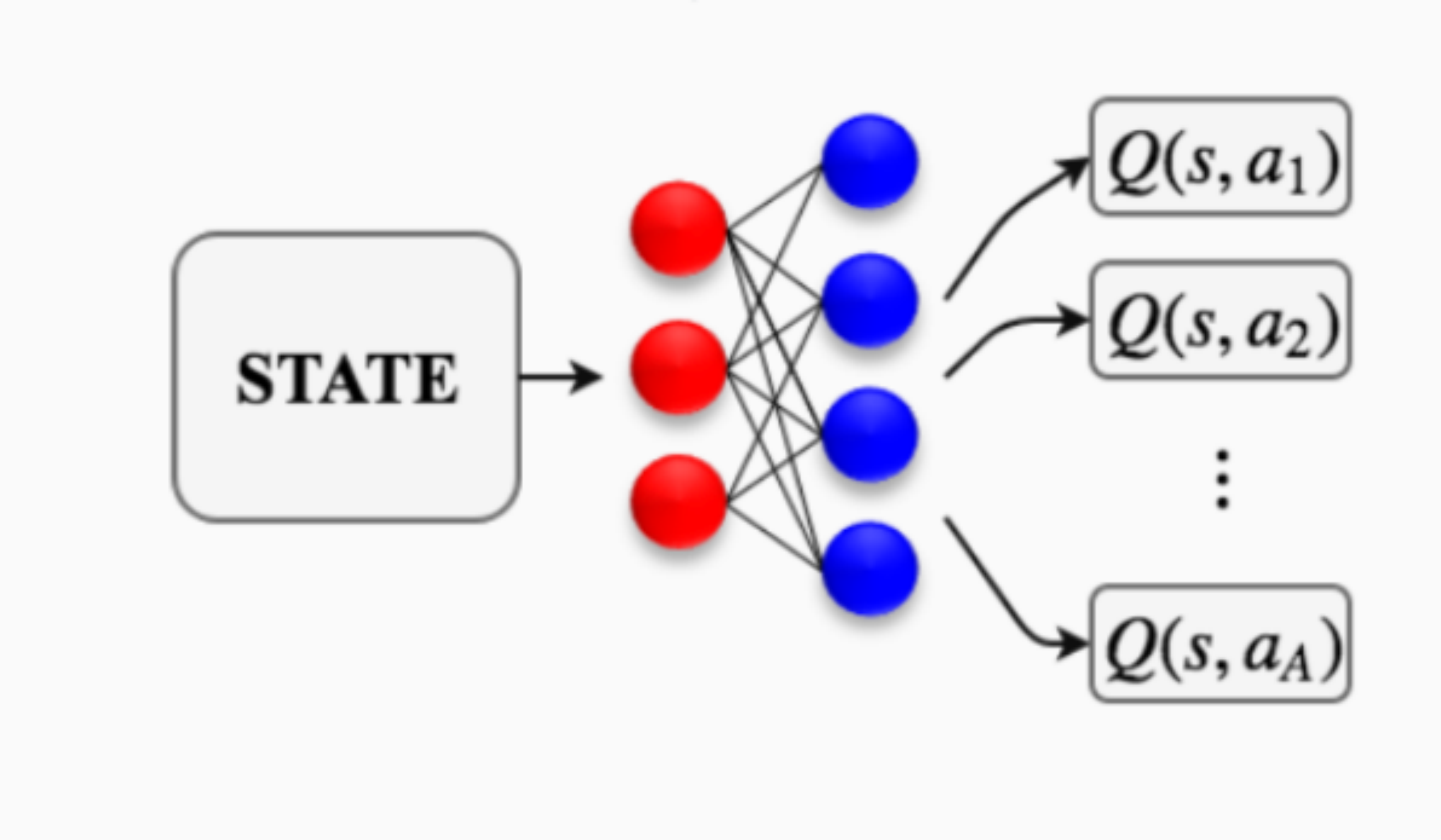

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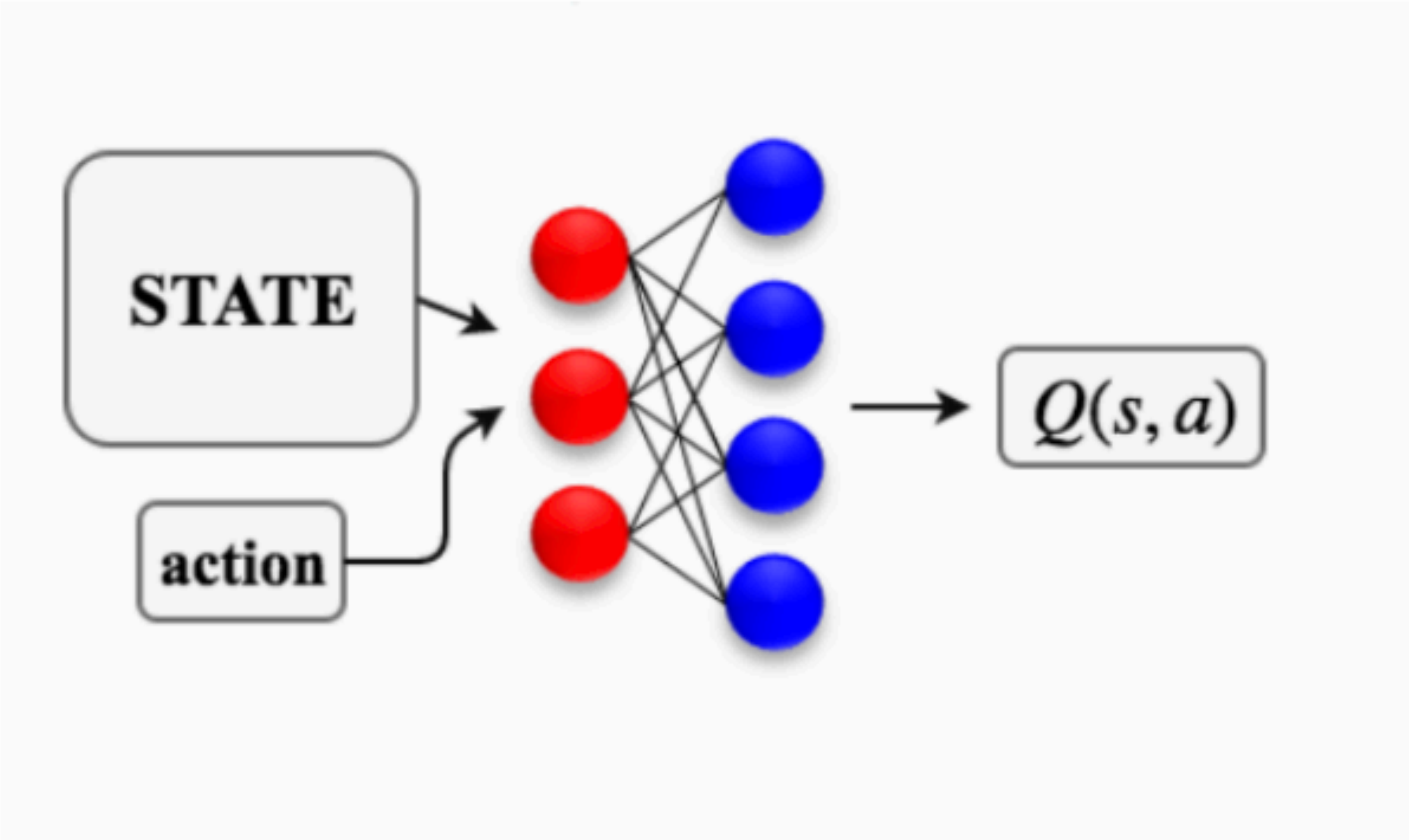

Policy Improvement as Optimisation

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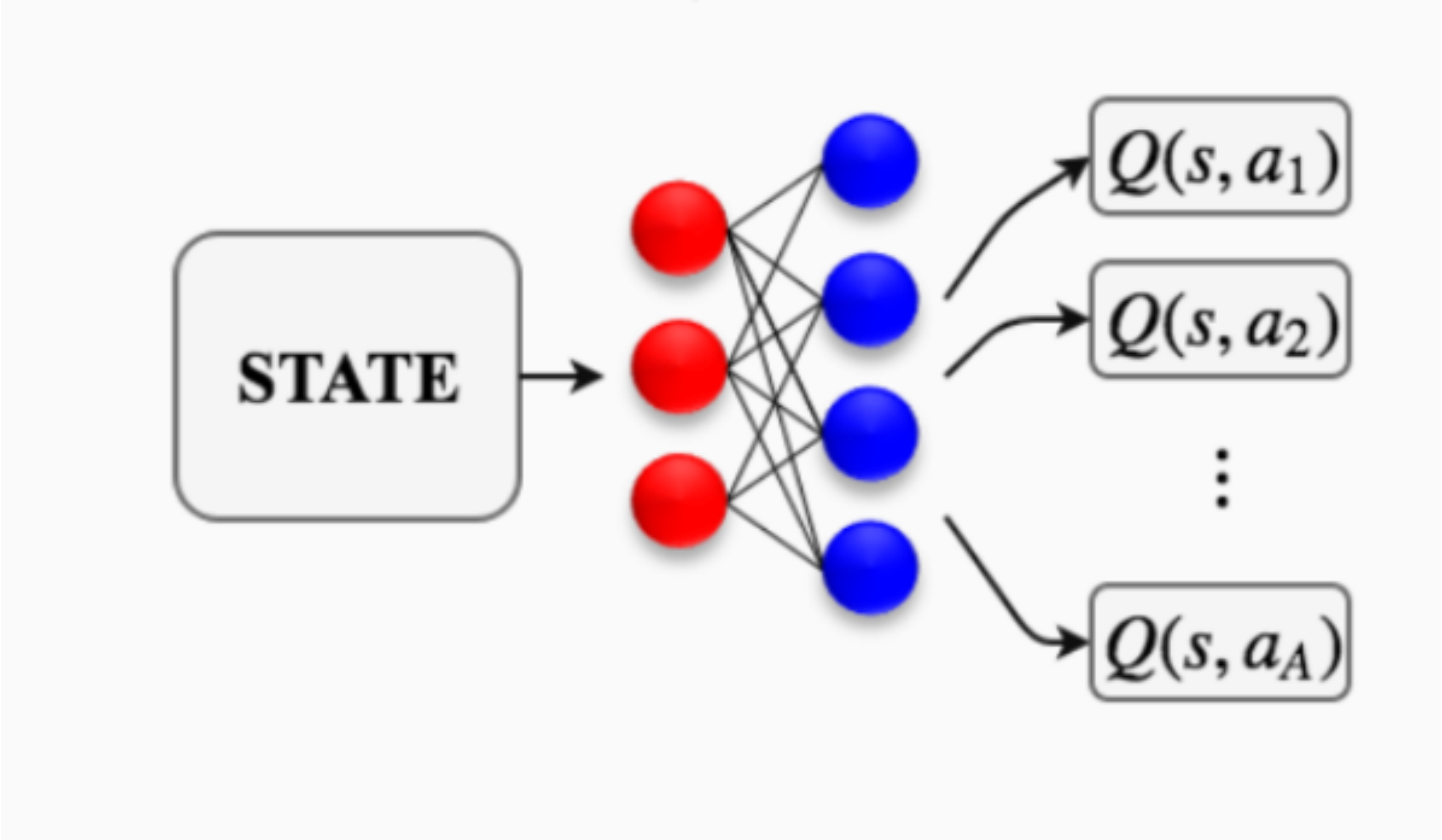

$$Q(s, a) \to \max_a$$

DQN

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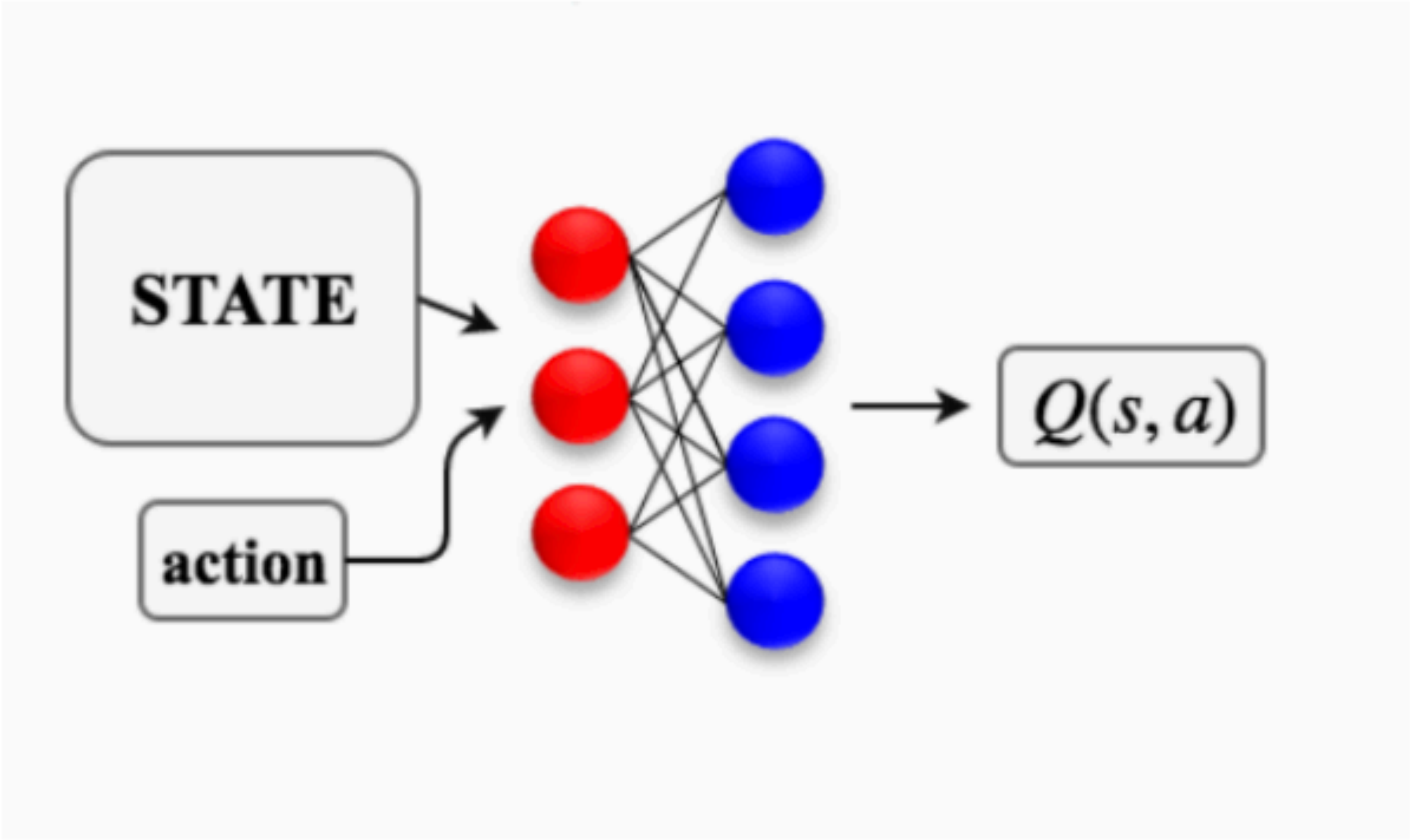


$$Q(s, \mu_\theta(s)) \to \max_\theta$$

$\mu_\theta(s)$ is a deterministic parametrised policy

DDPG

Exploration

An advantage of off-policy algorithms is that we can treat the problem of exploration independently from the learning algorithm.

$a = \mu_\theta(s) + \varepsilon$, where:

1. Gaussian noise: $\varepsilon \sim \mathcal{N}(0, \sigma^2)$

2. Ornstein-Uhlenbeck process: $\varepsilon_t = \alpha \varepsilon_{t-1} + \nu, \nu \sim \mathcal{N}(0, \sigma^2)$

Deep Deterministic Policy Gradient (2015)

Actor $\pi_\theta(s)$, critic $Q_\phi(s, a)$, target actor $\pi_{\theta^-}(s)$, target critic $Q_{\phi^-}(s, a)$.

On each step:

- Observe s, choose $a = \pi_\theta(s) + \varepsilon$, get s', r, $done$, put the transition into the buffer
- On the batch of transitions $(s_i, a, r_i, s_i', done_i)_{i=1}^B$, sampled from the replay buffer, perform:
  1. Policy evaluation: $\frac{1}{B}\sum_{i=1}^{B}(y_i - Q_\phi(s_i, a_i)) \to \min_\phi$, where $y_i = r_i + \gamma(1 - done_i)Q_{\phi^-}(s_i', \pi_{\theta^-}(s_i'))$
  2. Policy improvement: $\frac{1}{B}\sum_{i=1}^{B} Q_\phi(s_i, \pi_\theta(s_i))) \to \max_\theta$
- Soft-update the actor and critic:
  - $\theta^- = \tau\theta + (1 - \tau)\theta^-$, $\phi^- = \phi\theta + (1 - \tau)\phi^-$

Twin Delayed DDPG

Clipped Double-Q Learning:

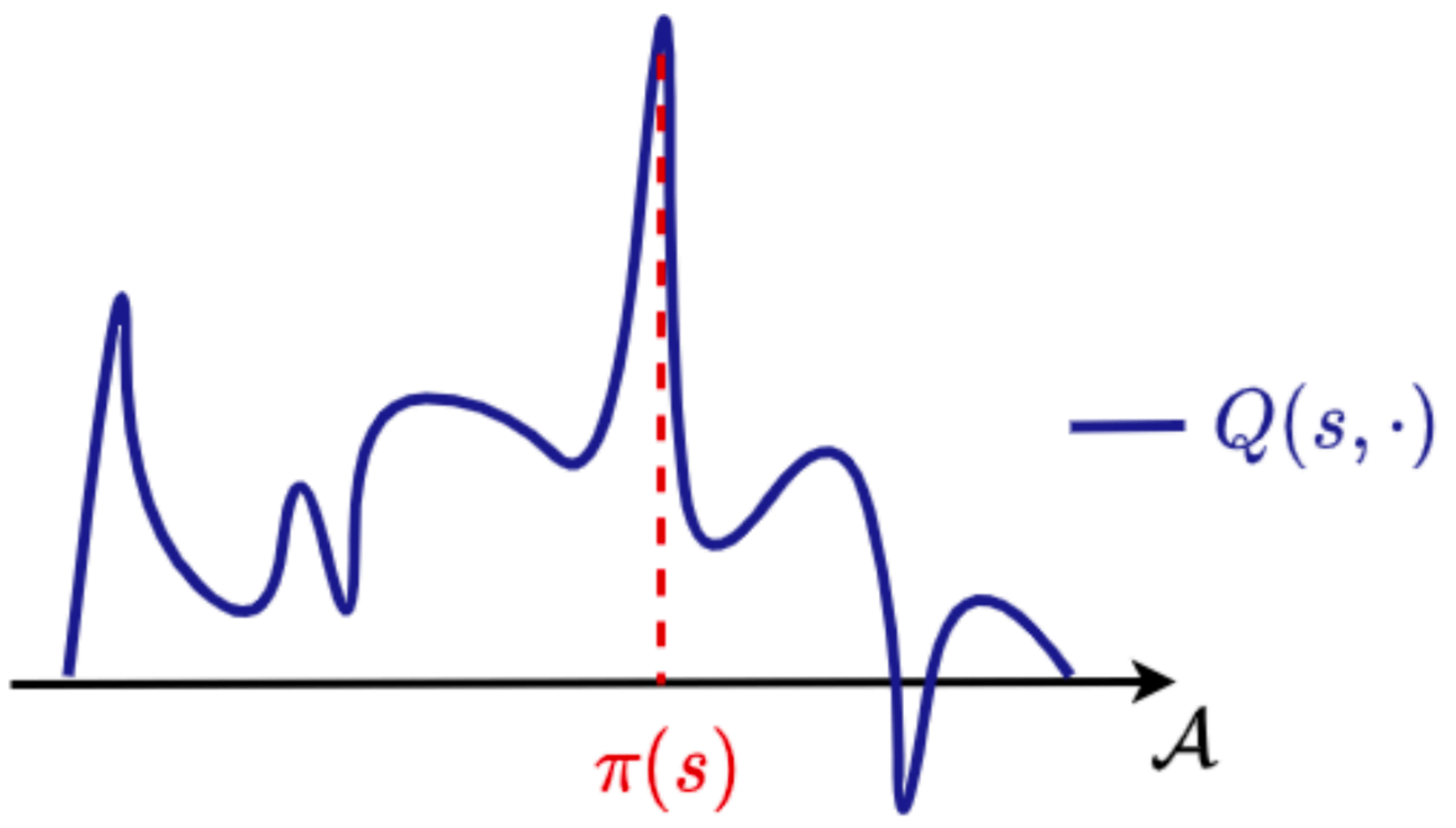
$$y = r + \gamma \min_{i=1,2} Q_{\phi_i^-}(s', \mu_{\theta^-}(s'))$$

Delayed Policy Updates: Update the policy (and target networks) less frequently than the Q-function.

Target Policy Smoothing: Add noise to the target action make it harder for the policy to exploit Q-function errors:

$$y = r + \gamma \min_{i=1,2} Q_{\phi_i^-}(s', a'),\ a' = \mu_{\theta^-}(s) + \varepsilon',$$

$$\varepsilon' \sim clip(\mathcal{N}(0,\sigma' I), -c, c)$$



Deterministic Policy Gradient

$$J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_\theta}}[Q^{\pi_\theta}(s, \pi_\theta(s))]$$

For deterministic policy $\pi_\theta : \mathcal{S} \to \mathcal{A}$:

$$\nabla_\theta J(\pi_\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_\theta}} \nabla_\theta \pi_\theta \nabla_a Q^{\pi_\theta}(s, a)\,|_{a=\pi_\theta(s)}$$

Deterministic Policy Gradient

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Surrogate objective for policy gradient:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_{old}}}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$

Surrogate objective for policy improvement:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$

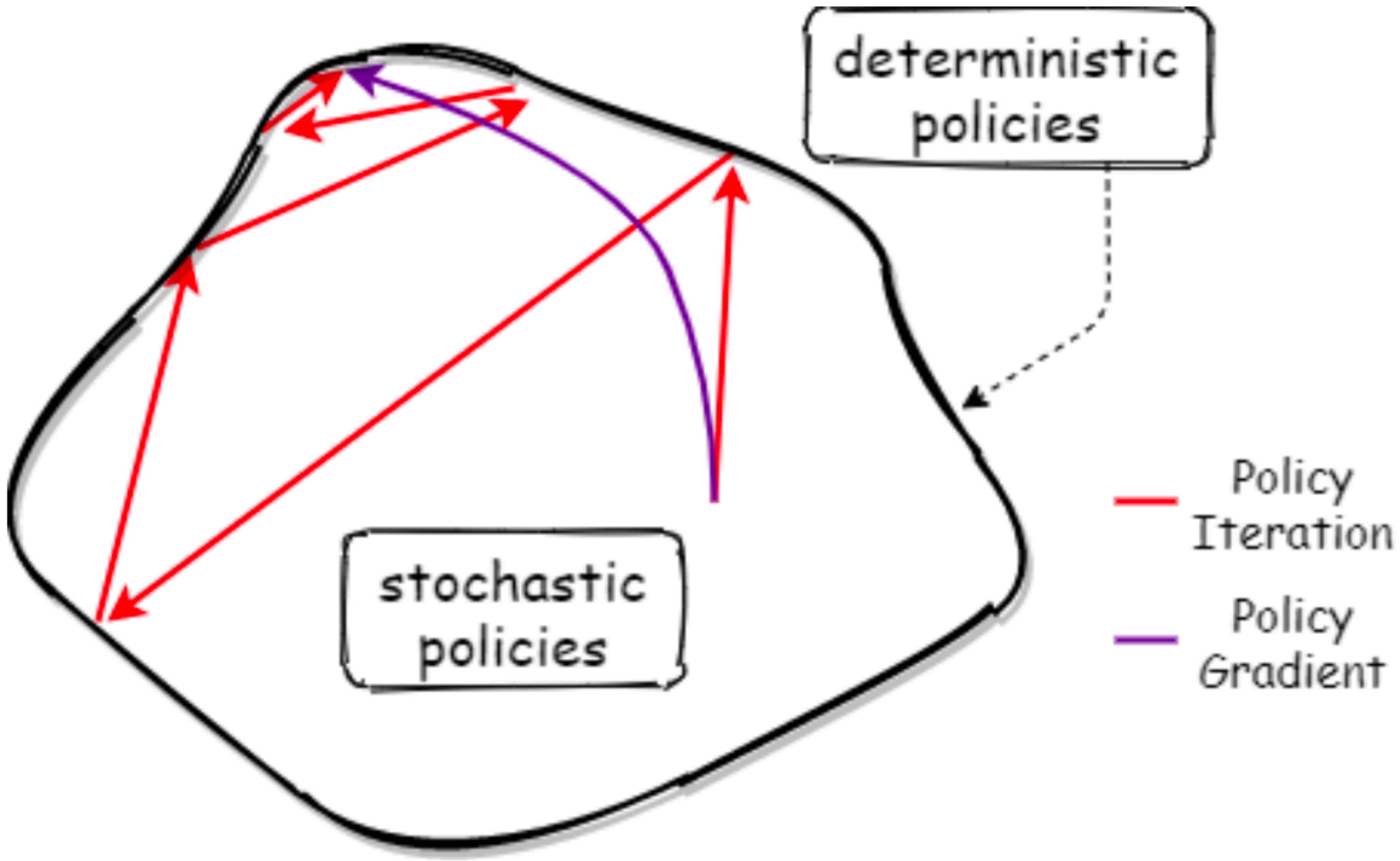
Deterministic Policy Gradient

Surrogate objective for policy gradient:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{\pi_{old}}}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$

Surrogate objective for policy improvement:

$$L_{\pi_{old}}(\theta) = \frac{1}{1-\gamma}\mathbb{E}_{s}[Q^{\pi_{old}}(s, \pi_\theta(s))]$$



Policy Gradient

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[Q^{\pi_{old}}(s, a)] \to \max_\theta$$

Let's take $\mathbb{E}_s \nabla_\theta \mathbb{E}_{a \sim \pi_\theta(.|s)}[Q^{\pi_{old}}(s, a)]$ in two ways:

REINFORCE

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[\nabla_\theta \log \pi_\theta(a \mid s) Q^{\pi_{old}}(s, a)]$$

Reparametrisation Trick

$$\mathbb{E}_s \mathbb{E}_{\varepsilon \sim p(.)}[\nabla_\theta Q^{\pi_{old}}(s, f_\theta(s, \varepsilon))]$$

If $a \sim \pi(\,.\mid s)$ is equivalent to $a = f_\theta(s, \varepsilon)$,
Where f_θ is a deterministic function,
$\varepsilon \sim p(\,.\,)$ is a non-parametric distribution.

Policy Gradient

REINFORCE

$$\mathbb{E}_s \mathbb{E}_{a \sim \pi_\theta(.|s)}[\nabla_\theta \log \pi_\theta(a \mid s) Q^{\pi_{old}}(s, a)]$$

Reparametrisation Trick

$$\mathbb{E}_s \mathbb{E}_{\varepsilon \sim p(.)}[\nabla_\theta Q^{\pi_{old}}(s, f_\theta(s, \varepsilon))]$$

If $a \sim \pi(\,.\mid s)$ is equivalent to $a = f_\theta(s, \varepsilon)$,
where f_θ is a deterministic function,
$\varepsilon \sim p(\,.\,)$ is a non-parametric distribution.

1. Softmax policy: $a \sim softmax(logit_\theta(s))$

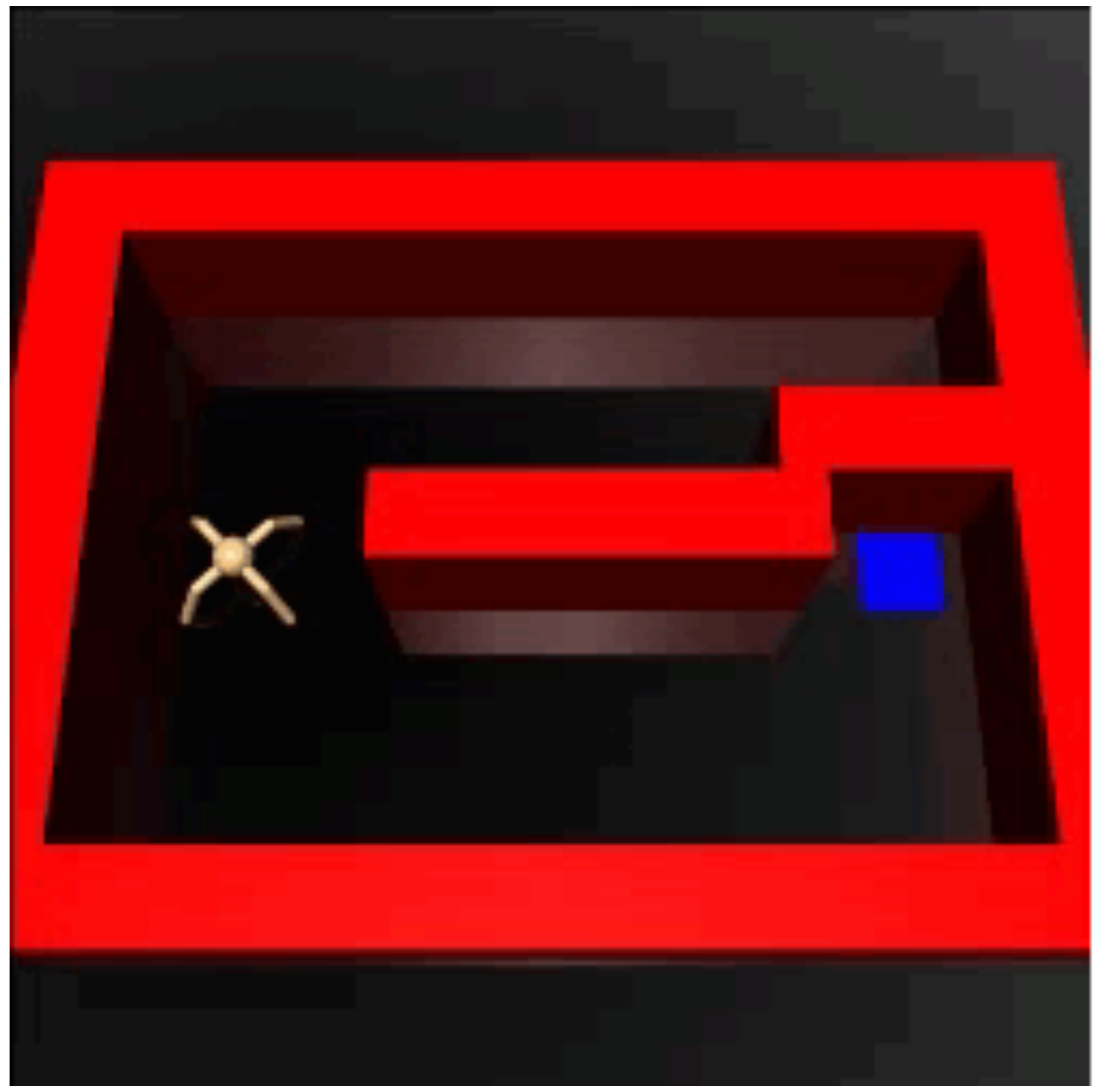
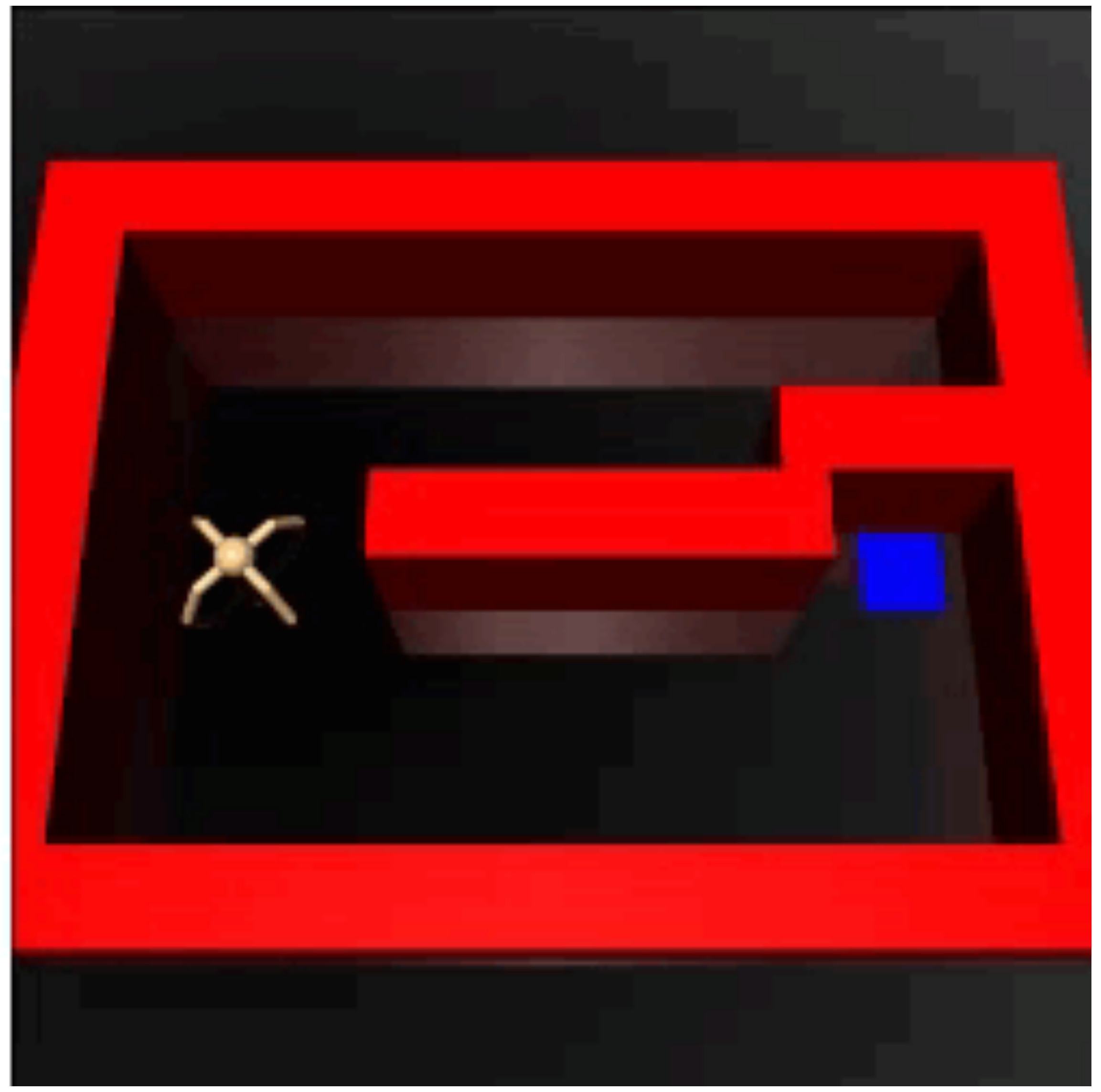
2. Deterministic policy: $a = \pi_\theta(s)$

3. Gaussian policy: $a \sim \mathcal{N}(\mu_\theta(s), \sigma_\theta^2(s)I)$

4. Mixture of gaussian: $a \sim \sum_{i=1}^{K} w_\theta^i(s)\mathcal{N}(\mu_\theta^i(s), (\sigma_\theta^i(s))^2 I)$

Stochastic Policies

- So far, we've introduced two off-policy algorithms learning only deterministic policies, where exploration is artificially maintained by adding noise.
- We want to train stochastic policies to have natural exploration, which prevents our agents from getting stuck in local optima.
- We also want to prevent our stochastic policies from becoming "too deterministic" very quickly.

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Maximum Entropy RL

$J_{soft}(\pi) = \mathbb{E}_{\tau \sim \pi} \sum_{t=0}^{\infty} \gamma^t [r_t + \alpha H(\pi(\,.\,|\,s_t))]$, where $H(\pi(\,.\,|\,s))$ is entropy.

Equivalent form:

$$J_{soft}(\pi) = \mathbb{E}_{\tau \sim \pi} \sum_{t=0}^{\infty} \gamma^t [r_t - \alpha \log \pi(a_t\,|\,s_t)]$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

$$V^{\pi}_{soft}(s) = \mathbb{E}_a[r(s,a) - \log \pi(a\,|\,s) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')]$$

$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')$$

Maximum Entropy RL

To eliminate the reward's dependence on current policy let's fix the following order:

$$s \longrightarrow H(\pi(\,.\,|\,s)) \longrightarrow a \longrightarrow r \longrightarrow s' \longrightarrow H(\pi(\,.\,|\,s')) \longrightarrow a' \longrightarrow \dots$$

$$V^{\pi}_{soft}(s) = \mathbb{E}_a[r(s,a) - \log \pi(a\,|\,s) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')]$$

$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} V^{\pi}_{soft}(s')$$

$$V^{\pi}_{soft}(s) = \mathbb{E}_a[Q^{\pi}_{soft}(s,a) - \alpha \log \pi(a\,|\,s)]$$

$$Q^{\pi}_{soft}(s,a) = r(s,a) + \gamma \mathbb{E}_{s'} \mathbb{E}_{a'}[Q^{\pi}_{soft}(s',a') - \alpha \log \pi(a'\,|\,s')]$$

Soft Policy Evaluation

For transition, (s, a, r, s') define a critic's target:

$$y_Q = r(s,a) + \gamma \mathbb{E}_{\boxed{a' \sim \pi(.|s')}}[Q_\phi(s',a') - \alpha \log \pi(a' \mid s')]$$

In general intractable

Soft Policy Evaluation

For transition, (s, a, r, s') define a critic's target:

$$y_Q = r(s, a) + \gamma \mathbb{E}_{\boxed{a' \sim \pi(.|s')}}[Q_\phi(s', a') - \alpha \log \pi(a' \,|\, s')]$$

In general intractable

- We can estimate the expectation using a sample from the policy
- Can learn V_ψ to approximate the expectation:

$$y_V = Q_\phi(s, a_\pi) - \alpha \log \pi(a_\pi \,|\, s), a_\pi \sim \pi(\,.\,|\,s)$$

$$y_Q = r(s, a) + \gamma V_\psi(s')$$

Policy Improvement

Policy Improvement in traditional RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}

- Optimisation:

$$\mathbb{E}_s\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \to \max_{\pi}$$

- $\pi(s) = argmax_a Q^{\pi_{old}}(s,a)$

Policy Improvement in max entropy RL

Policy Improvement

Policy Improvement in traditional RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}(s,a) \geq V^{\pi_{old}}(s)$ then π is not worse than π_{old}
- Optimisation:

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- $\pi(s) = argmax_a Q^{\pi_{old}}(s,a)$

Policy Improvement in max entropy RL

- If $\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}_{soft}(s,a) + \alpha H(\pi(\,.\,|\,s)) \geq V^{\pi_{old}}_{soft}(s)$ then π is not worse than π_{old}
- Optimisation:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi}Q^{\pi_{old}}_{soft}(s,a) + \alpha H(\pi(\,.\,|\,s))] \rightarrow \max_{\pi}$$

- $\pi(a\,|\,s) \propto \exp\left(\frac{Q^{\pi_{old}}(s,a)}{\alpha}\right)$

Soft Policy Improvement

Actor π_θ learning:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi_\theta} Q_\phi(s, a) + \alpha H(\pi_\theta(\,.\, | \, s))] \to \max_\theta$$

Soft Policy Improvement

Actor π_θ learning:

$$\mathbb{E}_s[\mathbb{E}_{a\sim\pi_\theta} Q_\phi(s,a) + \alpha H(\pi_\theta(\,.\,|\,s))] \to \max_\theta$$

Example:

- $\pi_\theta(\,.\,|\,s) = \mathcal{N}(\mu_\theta(s), \sigma_\theta^2(s)I)$
- $a \sim \pi_\theta(\,.\,|\,s) \iff a = \mu_\theta(s) + \sigma_\theta(s)\varepsilon, \varepsilon \sim \mathcal{N}(0,I)$
- $H(\pi_\theta(\,.\,|\,s))] = \sum_{i=1}^{A} \log \sigma_\theta^i(s)$
- $\mathbb{E}_s[\mathbb{E}_{\varepsilon\sim\mathcal{N}(0,I)} Q_\phi(s, \mu_\theta(s) + \sigma_\theta(s)\varepsilon) + \alpha \sum_{i=1}^{A} \log \sigma_\theta^i(s)] \to \max_\theta$

Soft Actor-Critic

- Actor $\pi_\theta(\,.\,|\,s)$, critics $Q_{\phi_1}(s,a)$, $Q_{\phi_2}(s,a)$, target critics $Q_{\phi_1^-}(s,a)$, $Q_{\phi_2^-}(s,a)$.

On each step:

- Observe s, choose $a \sim \pi_\theta(\,.\,|\,s)$, get s', r, $done$, put the transition into the buffer
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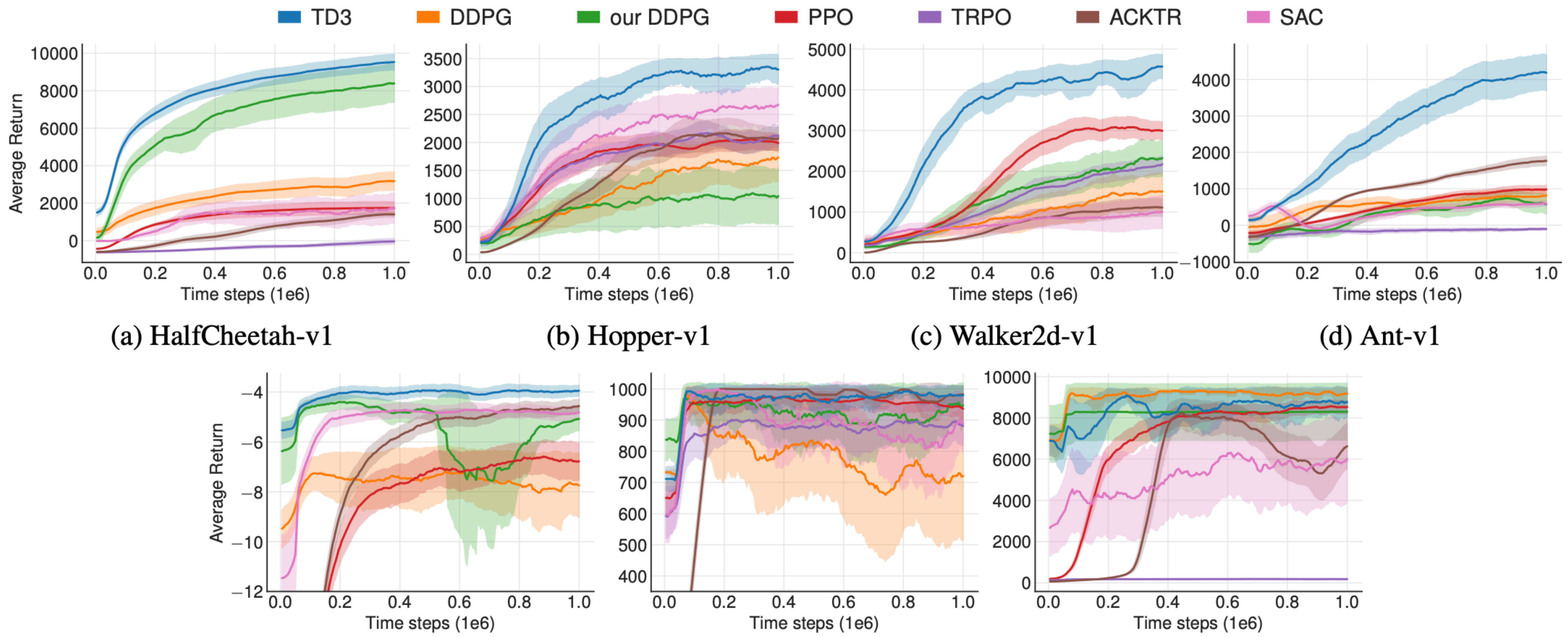
1. Soft Policy evaluation: $\frac{1}{B}\sum_{i=1}^{B}(y_i - Q_{\phi_k}(s_i,a_i)) \to \min\limits_{\phi_k}$, where
   $y_i = r_i + \gamma(1 - done_i)(\min\limits_{k=1,2} Q_{\phi_k^-}(s_i', a_i') - \alpha \log \pi(a_i'\,|\,s_i'))$, $a_i' \sim \pi_\theta(\,.\,|\,s_i')$

2. Policy improvement: $\frac{1}{B}\sum_{i=1}^{B}\min\limits_{k=1,2} Q_{\phi_k}(s_i, a_\theta(s_i)) - \alpha \log \pi(a_\theta(s_i)\,|\,s_i) \to \max\limits_{\theta}$

   Where $a_\theta(s_i)$ is a sample from $\pi_\theta(\,.\,|\,s)$ which is differentiable wrt θ via reparametrisation trick

- Soft-update for the target critics

Comparison

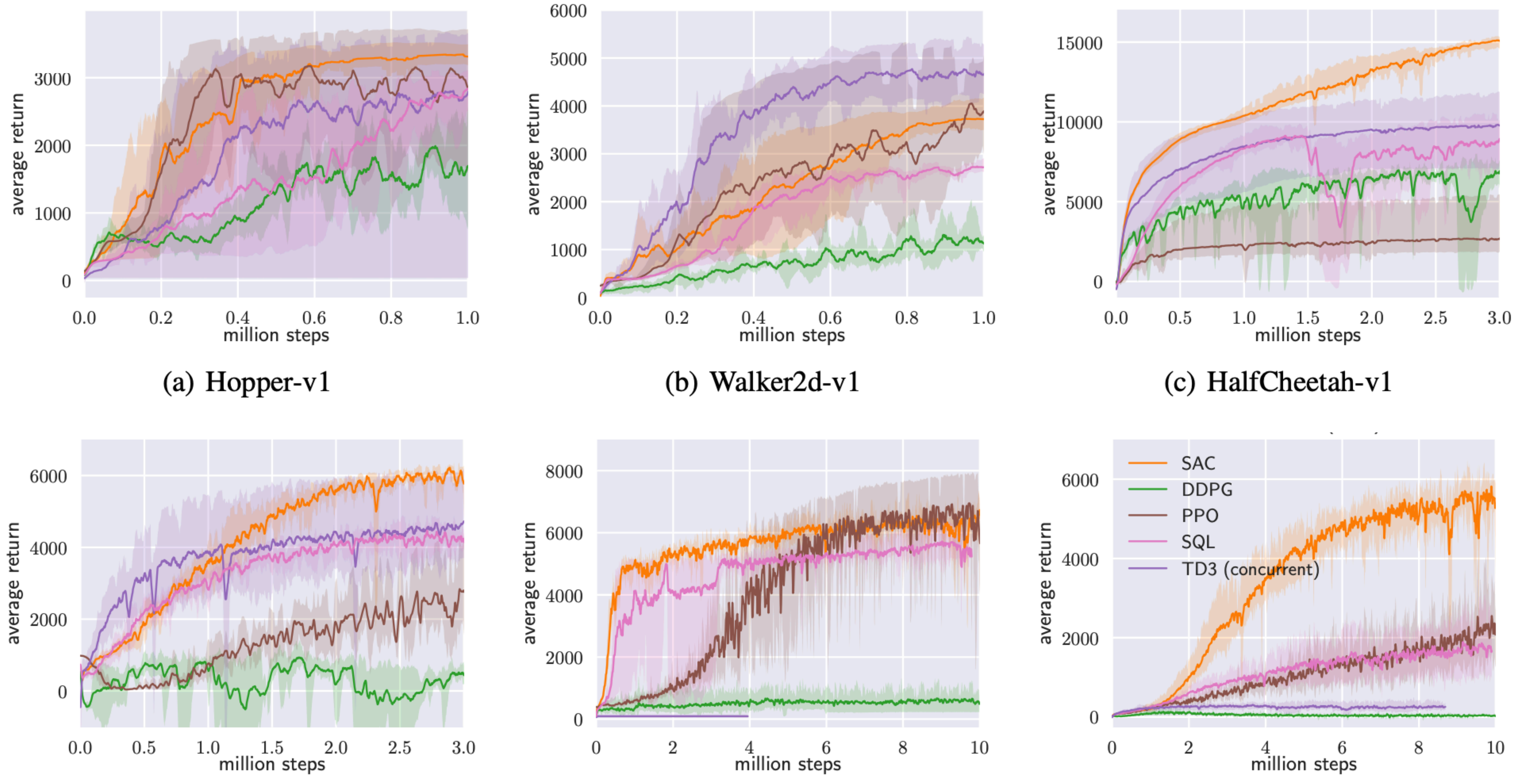


(a) HalfCheetah-v1 (b) Hopper-v1 (c) Walker2d-v1 (d) Ant-v1

(e) Reacher-v1 (f) InvertedPendulum-v1 (g) InvertedDoublePendulum-v1

TD3 paper

SAC paper

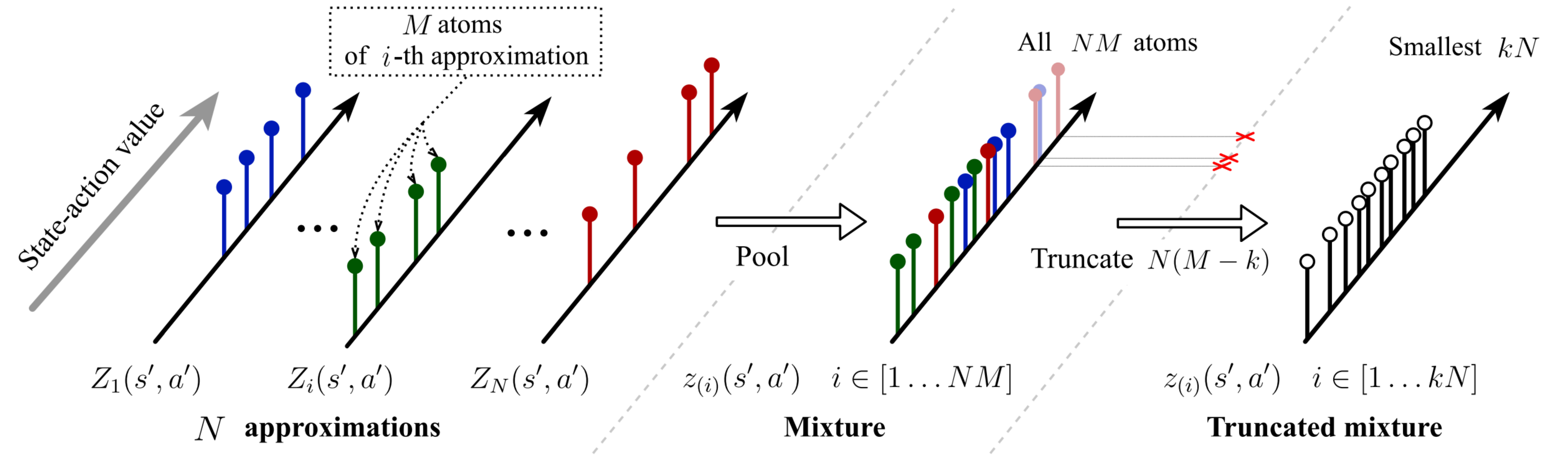


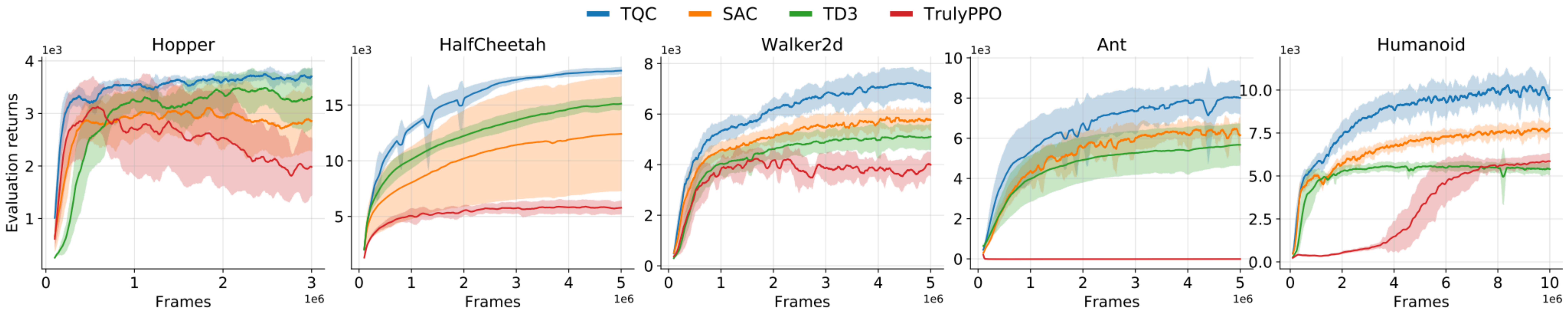
(a) Hopper-v1 (b) Walker2d-v1 (c) HalfCheetah-v1

(d) Ant-v1 (e) Humanoid-v1 (f) Humanoid (rllab)

Truncated Quantile Critics

Controlling Overestimation Bias with Truncated Mixture of Continuous Distributional Quantile Critics





Background

1. Reinforcement Learning Textbook (in Russian): 6
2. Lecture 19: Connection between Inference and Control
3. Soft Actor-Critic Algorithms and Applications

Thank you for your attention!