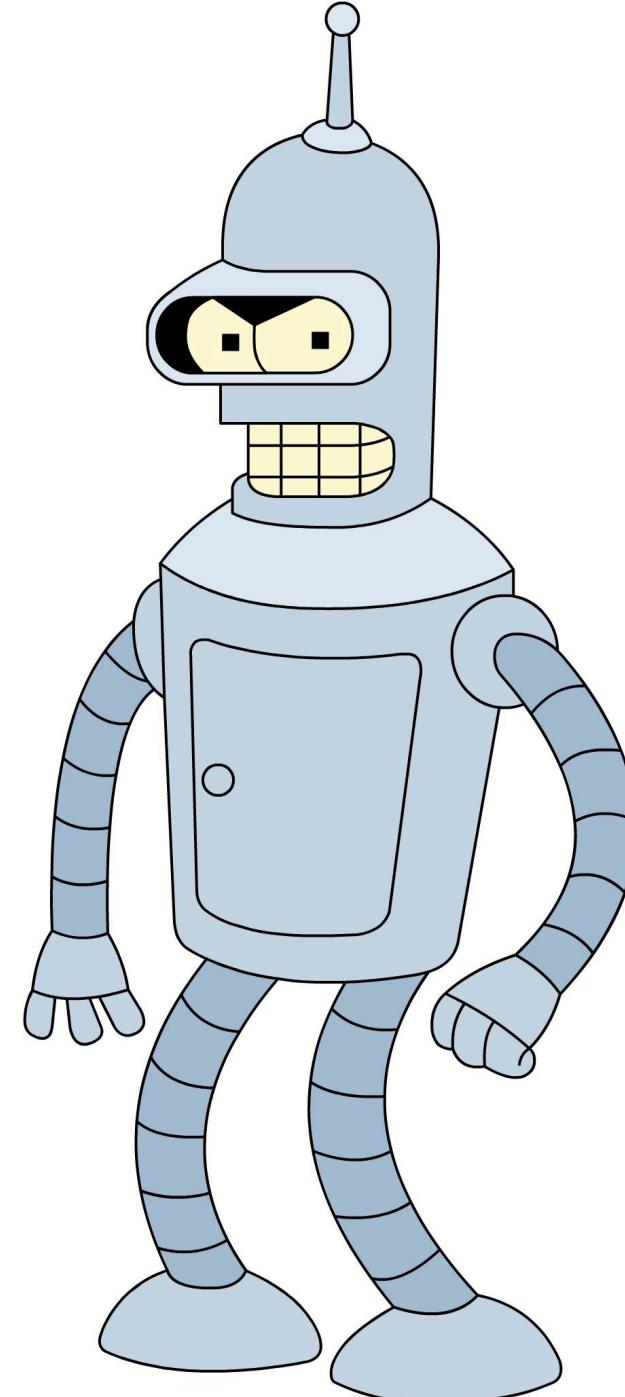


# Reinforcement Learning

HSE, autumn - winter 2022

Lecture 1: Intro, CEM



Sergei Laktionov  
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# Course Staff

Sergei Laktionov

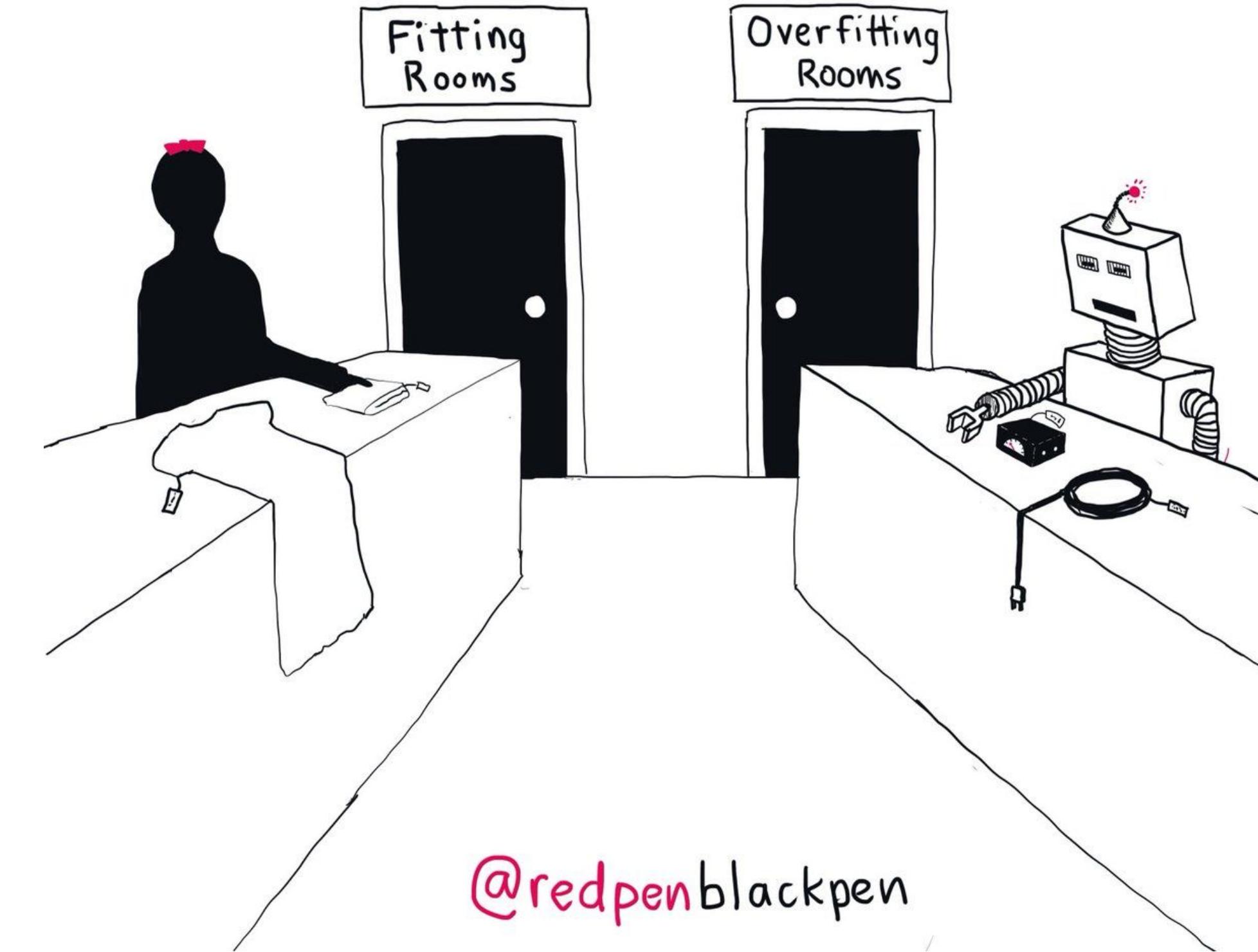
Lectures and practical sections

<https://t.me/lsd4math>

Arsenty Kambalin

Home assignments and grading

[https://t.me/arsentique\\_s](https://t.me/arsentique_s)



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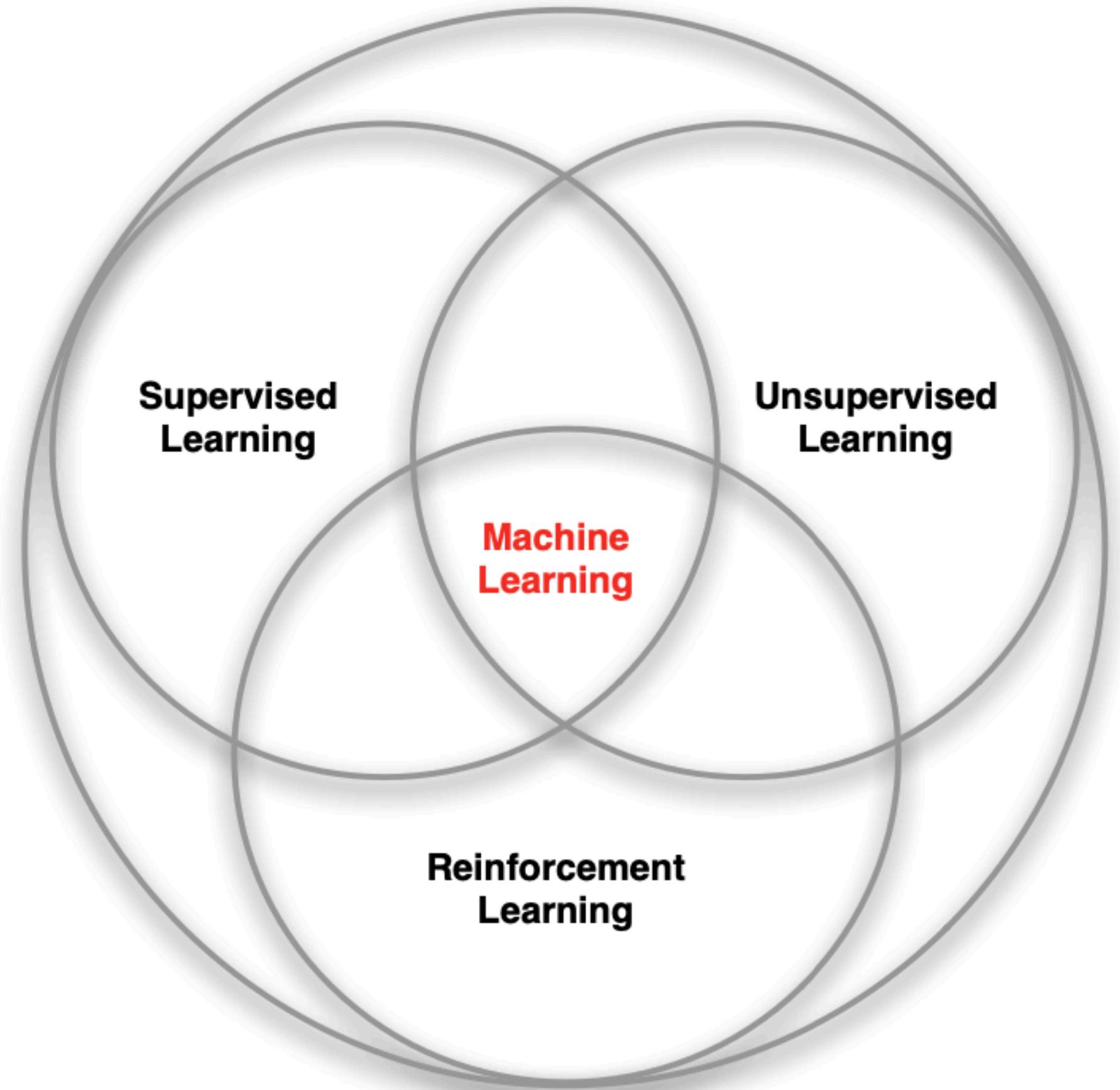
# Course Plan

1. 01.11.2022: RL problem statement. Cross-entropy method. (HW1)
2. 08.11.2022: Bellman equations and Dynamic programming. (HW2)
3. 15.11.2022: Model free algorithms. Tabular methods.
4. 22.11.2022: Intro to deep RL. Value-based methods: DQN and modifications (HW3)
5. 29.11.2022: Policy-based methods
6. 06.12.2022: Actor-critic algorithms (HW4)
7. 13.12.2022: Bandits
8. 20.12.2022: Research club :)

# Useful Reading

1. <http://incompleteideas.net/book/the-book-2nd.html>
2. <http://rail.eecs.berkeley.edu/deeprlcourse/>
3. [https://github.com/yandexdataschool/Practical\\_RL](https://github.com/yandexdataschool/Practical_RL)
4. <https://www.deepmind.com/learning-resources/reinforcement-learning-lecture-series-2021>
5. <https://web.stanford.edu/class/cme241/>
6. <https://github.com/huggingface/deep-rl-class>

# Recap



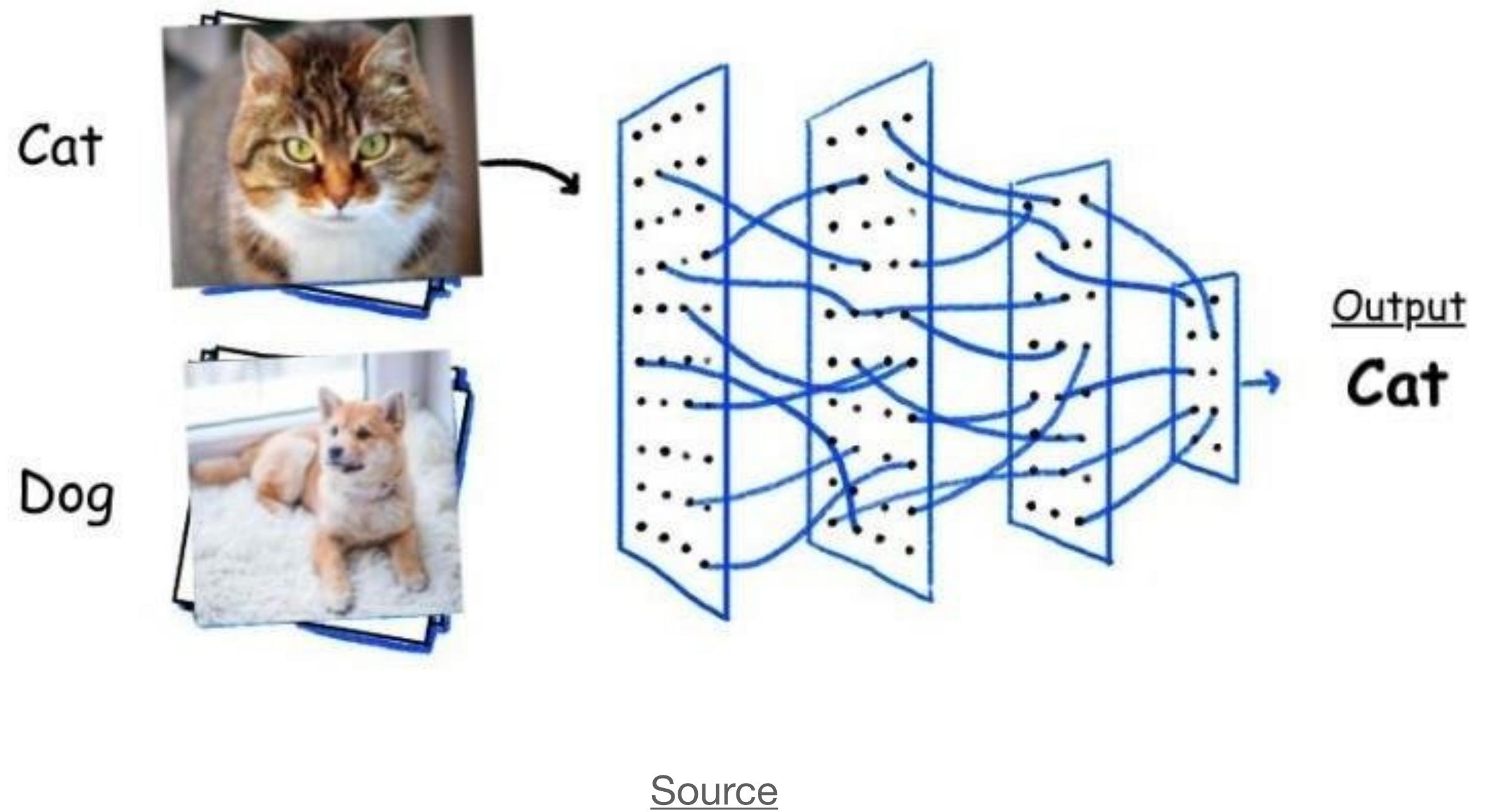
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# Supervised Learning

Train sample:  $(\mathbf{x}_i, y_i) \sim i.i.d.$

Approximation:  $f(\mathbf{x}_i) = \hat{y}_i \approx y_i$

Optimization:  $\frac{1}{n} \sum_{i=1}^n L(f(\mathbf{x}_i), y_i) \rightarrow \min_{f \in \mathcal{F}}$

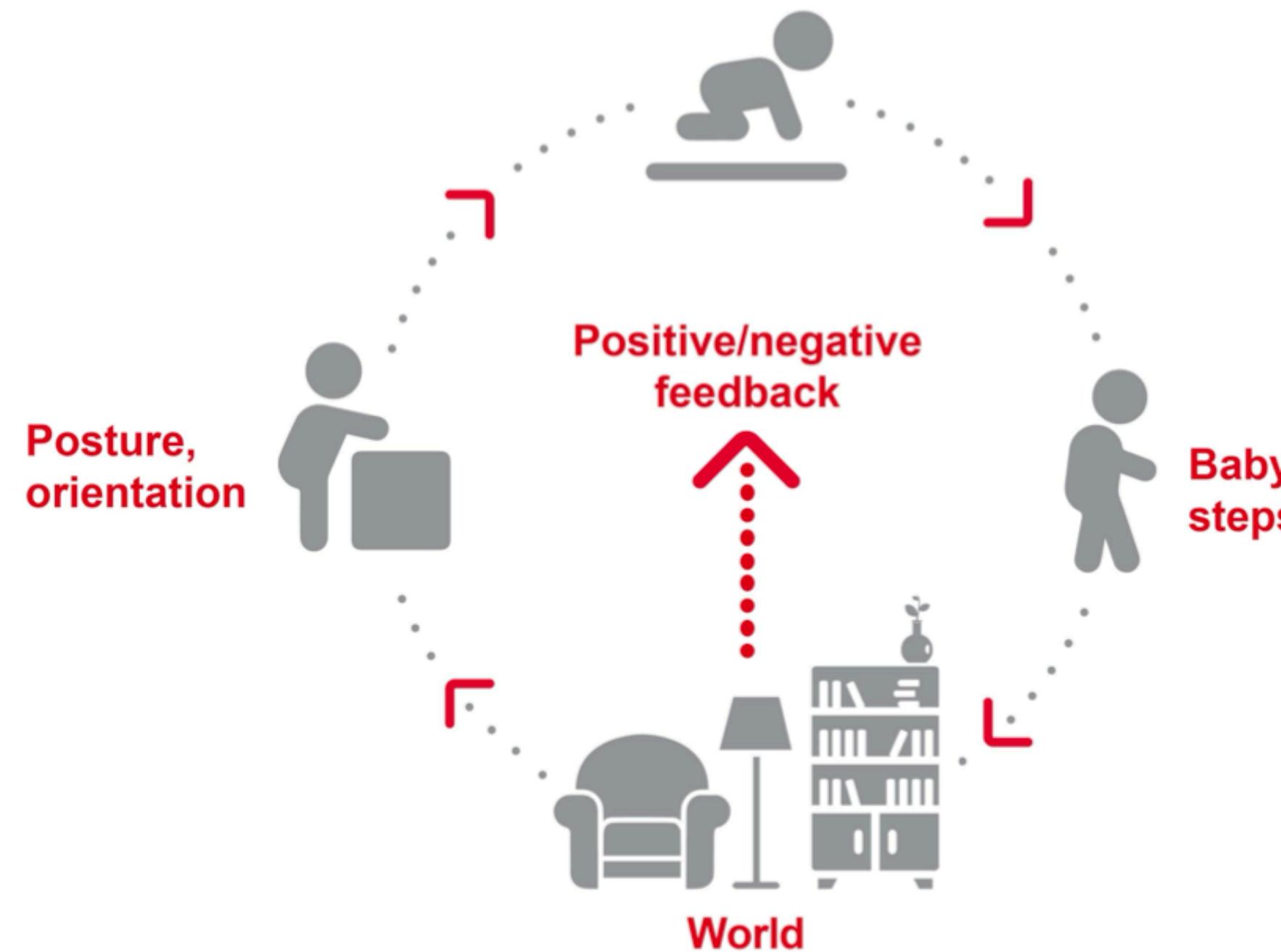


# Background for Lecture 1

1. Sutton & Barto 2018, Chapter 1
2. Rao & Jelvis 2022, Chapter 1

# Decision Making Process

Baby learning



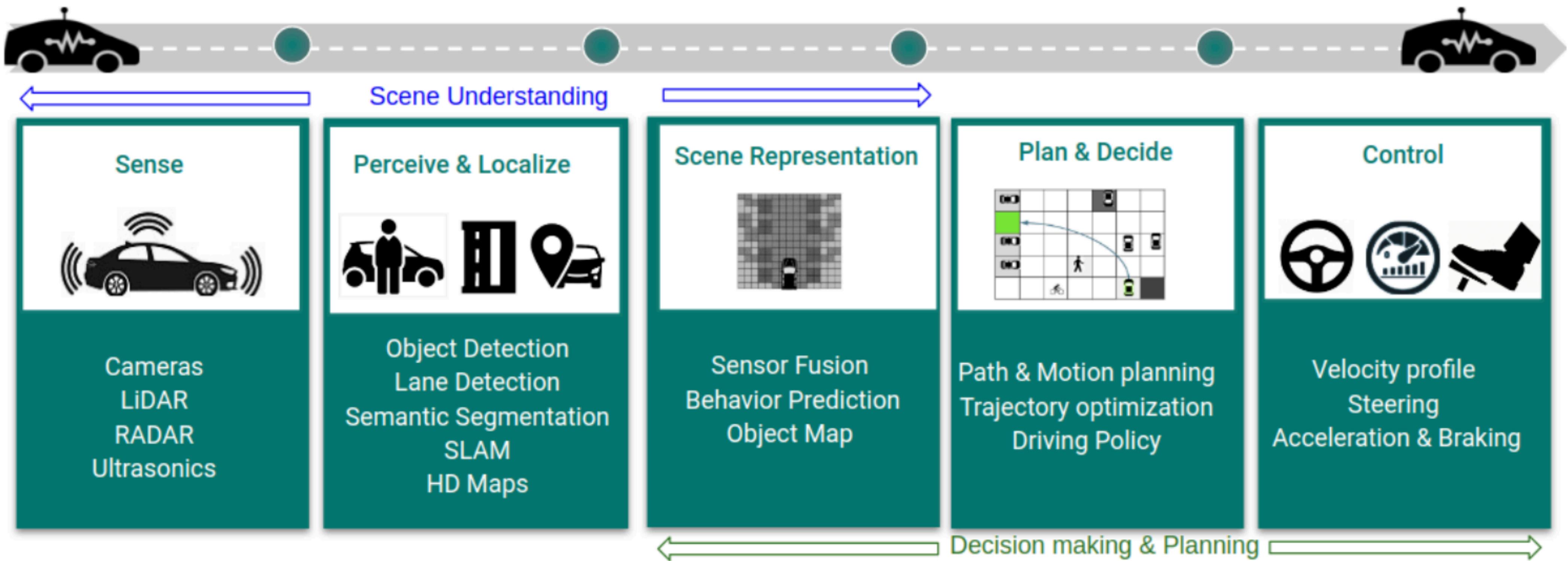
Self-driving car



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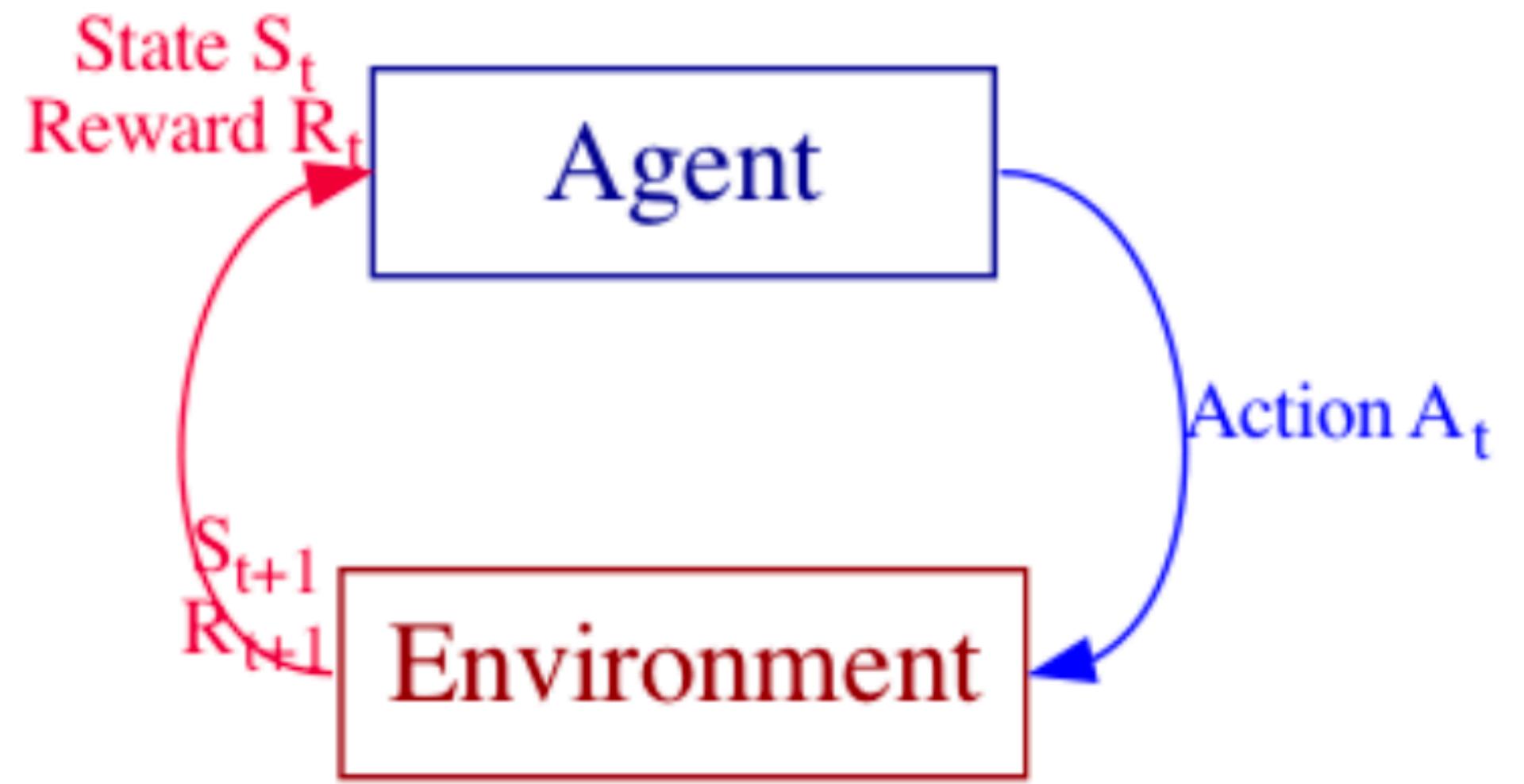
# Self-driving Cars



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# Reinforcement Learning Features

1. Trial-and-error search
2. Delayed rewards
3. Interaction with the environment



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Reward hypothesis (Richard Sutton):

“That all of what we mean by goals and purposes can be well thought of as maximization of the expected value of the cumulative sum of a received scalar signal (reward).”

# Reinforcement Learning Features

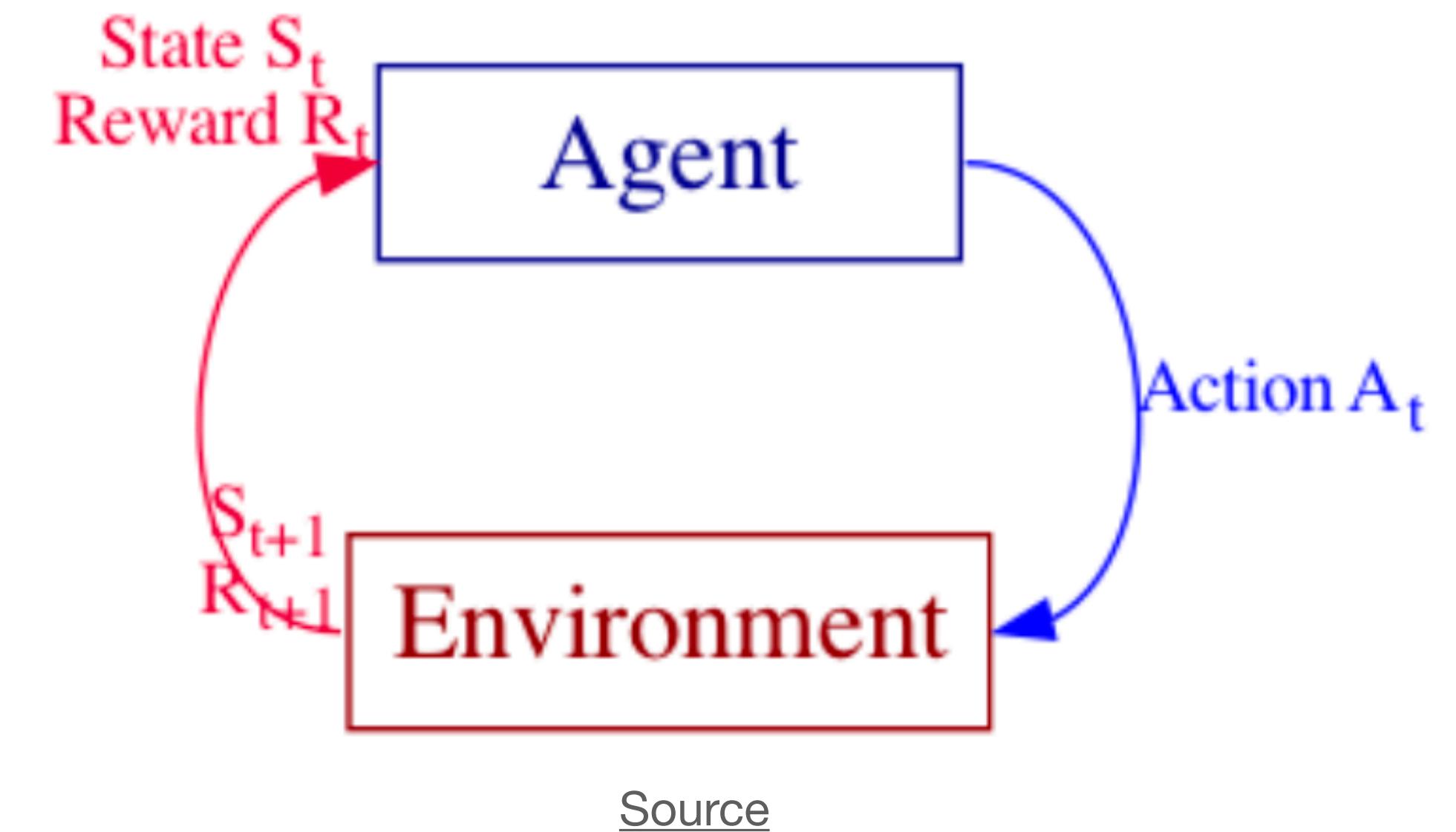
The agent chooses the action by using a **policy**  $\pi$ .

Deterministic policy

$$a := \pi(s)$$

Stochastic policy

$$a \sim \pi(a | s)$$



# Imitation Learning and Behavioural Cloning

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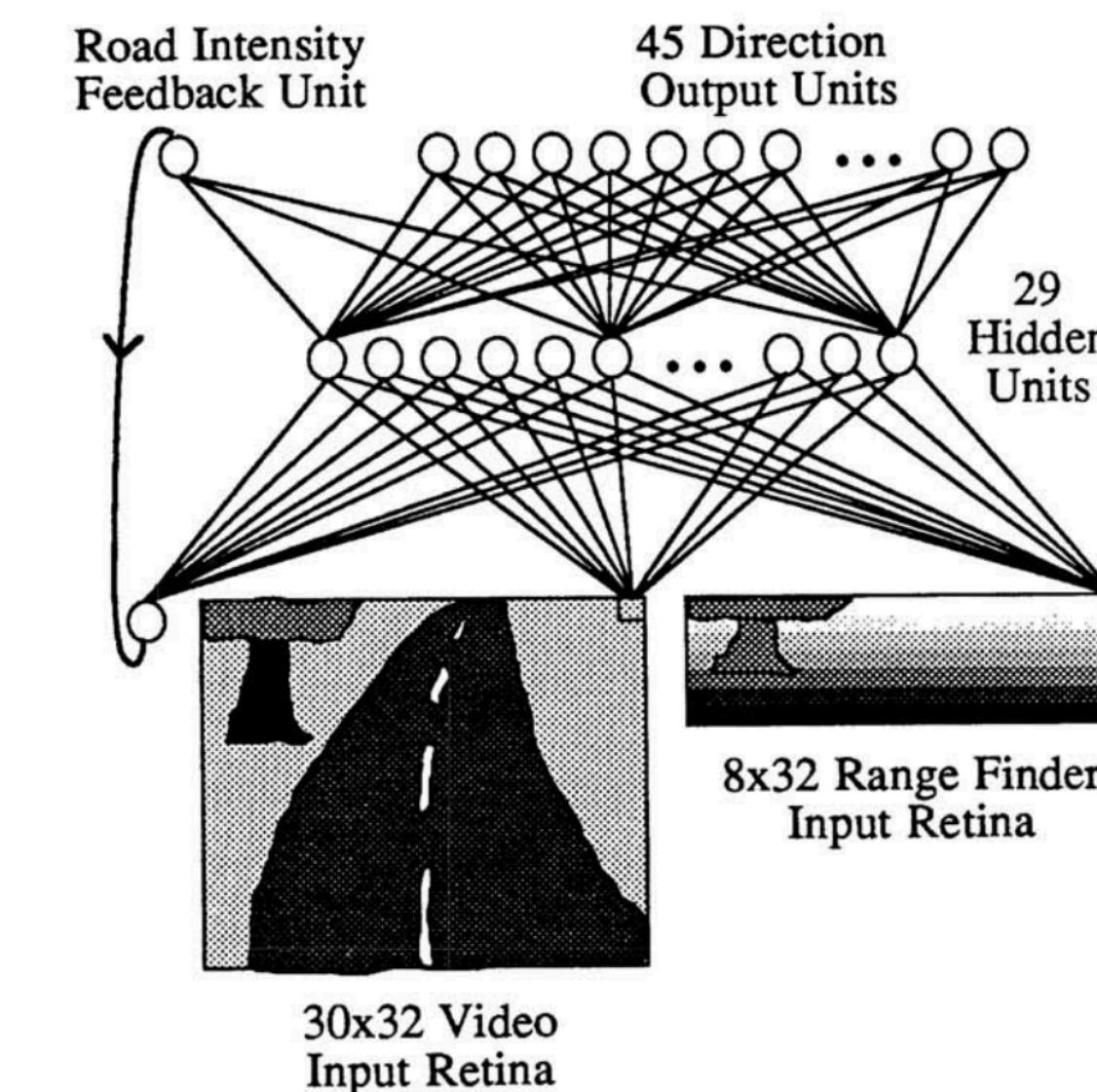
Imitation Learning is useful when it is easier for an expert to demonstrate the desired behaviour rather than to specify a reward function which would generate the same behaviour or to directly learn the policy.

The simplest form of imitation learning is Behavioural Cloning, which focuses on learning the expert's policy using supervised learning.

## ALVINN: AN AUTONOMOUS LAND VEHICLE IN A NEURAL NETWORK

Dean A. Pomerleau  
Computer Science Department  
Carnegie Mellon University  
Pittsburgh, PA 15213

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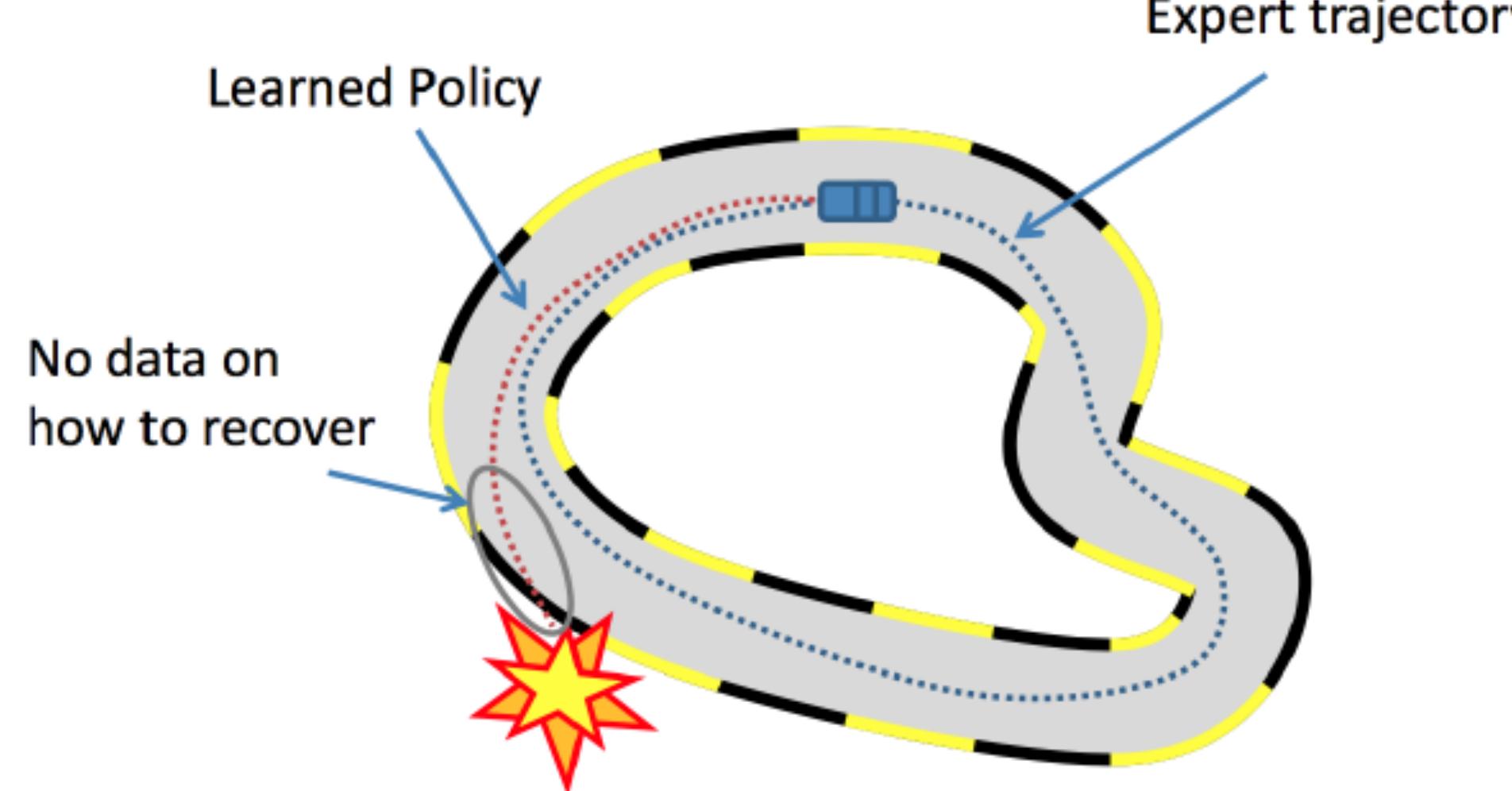
# Behavioral Cloning for Self-driving Cars

1. Collect demonstrations (trajectories  $\tau^*$ ) from expert
2. Treat the demonstrations as i.i.d. state-action pairs:  $(s_i, a_i)$
3. Learn the policy in a supervised mode: 
$$\frac{1}{n} \sum_{i=1}^n L(\pi(s_i), a_i) \rightarrow \min_{\pi \in \mathcal{P}}$$

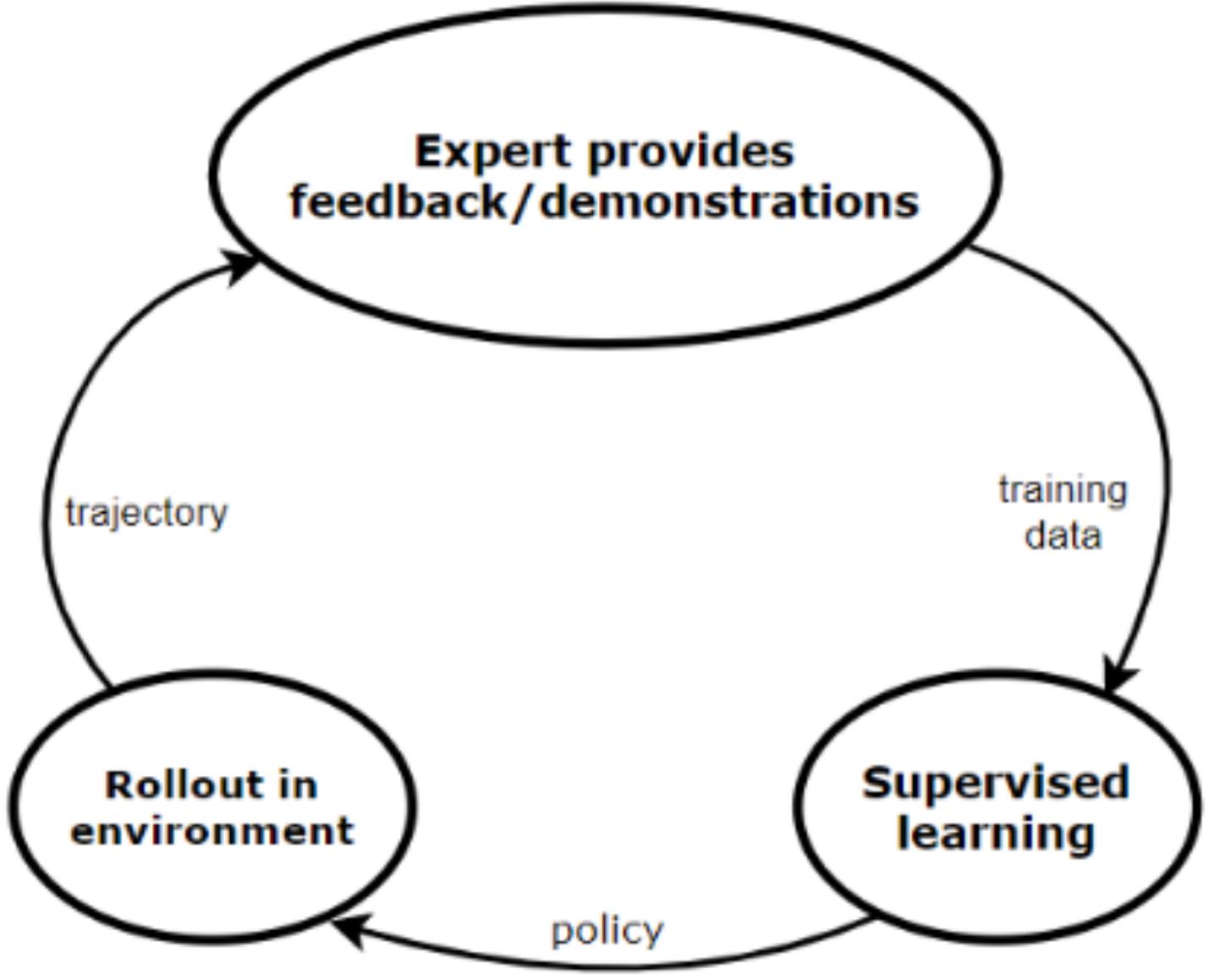
# Behavioral Cloning for Self-driving Cars

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$$\frac{1}{n} \sum_{i=1}^n L(\pi(s_i), a_i) \rightarrow \min_{\pi \in \mathcal{P}}$$



# Direct Policy Learning



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Initial predictor:  $\pi_0$

For  $m = 1$ :

- Collect trajectories  $\tau$  by rolling out  $\pi_{m-1}$
- Estimate state distribution  $P_m$  using  $s \in \tau$
- Collect interactive feedback  $\{\pi^*(s) \mid s \in \tau\}$
- Data Aggregation (e.g. Dagger)
  - Train  $\pi_m$  on  $P_1 \cup \dots \cup P_m$
- Policy Aggregation (e.g. SEARN & SMILe)
  - Train  $\pi'_m$  on  $P_m$
  - $\pi_m = \beta\pi'_m + (1 - \beta)\pi_{m-1}$

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# Demonstration



# More Examples

## Robotics

- Optimal control

Chess, Go

- AlphaGo
- AlphaZero

## Quantitative Finance:

- Market making
- Optimal execution
- Portfolio management

- Healthcare
- Industry automation
- Engineering

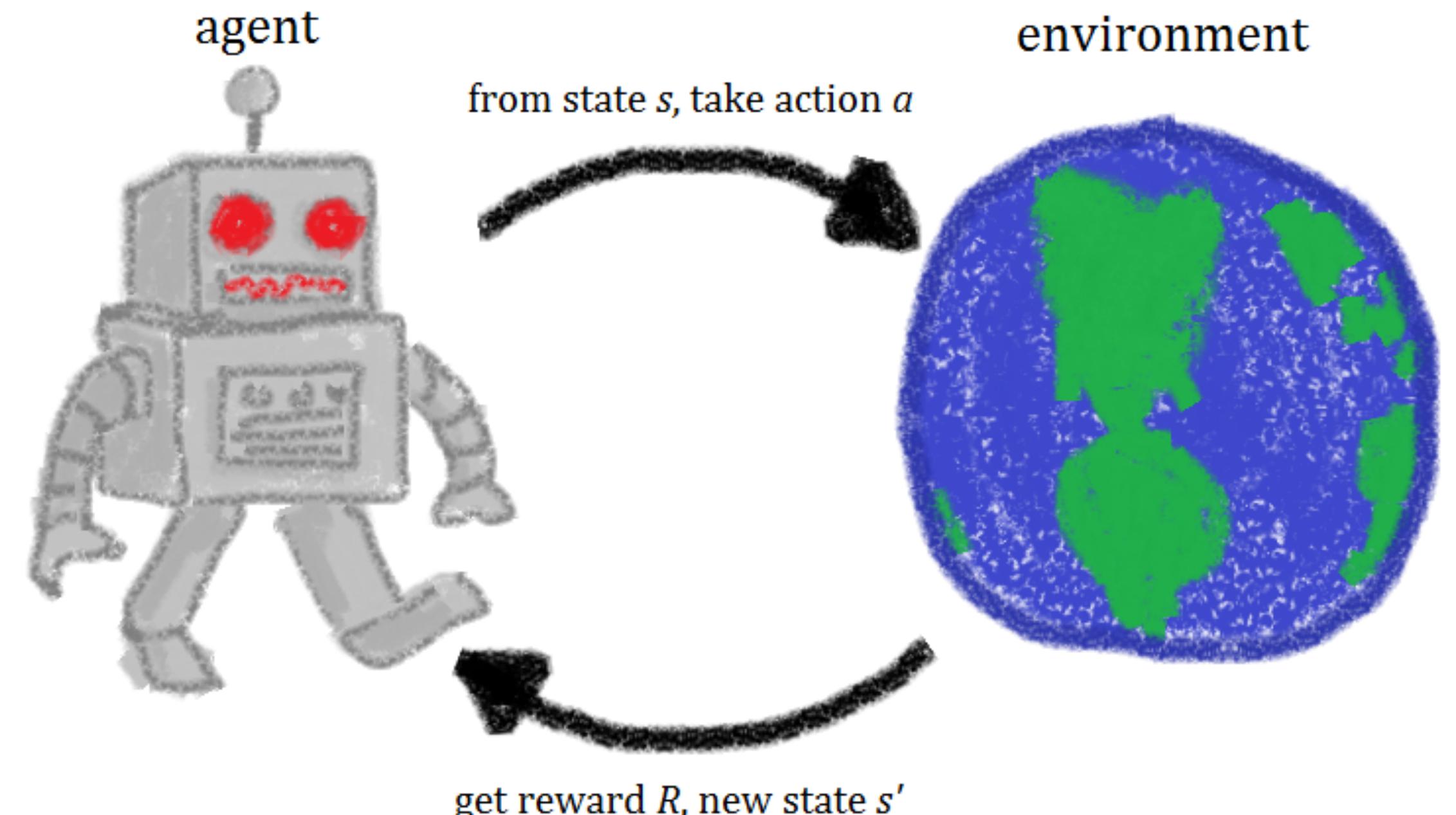
# Reinforcement Learning Framework

At each step  $t$  the agent:

1. Receives state (observation)  $s_t$  ( $o_t$ ) and reward  $r_t$
2. Executes action  $a_t$

The environment:

1. Receives action  $a_t$
2. Emits state (observation)  $s_{t+1}$  ( $o_{t+1}$ ) and reward  $r_{t+1}$



$$\mathbb{E}_{p(\tau|\pi)}\left(\sum_{t=0}^T r_t\right) \rightarrow \max_{\pi}$$

# Reward Design

... is a tricky task



# Discount Factor



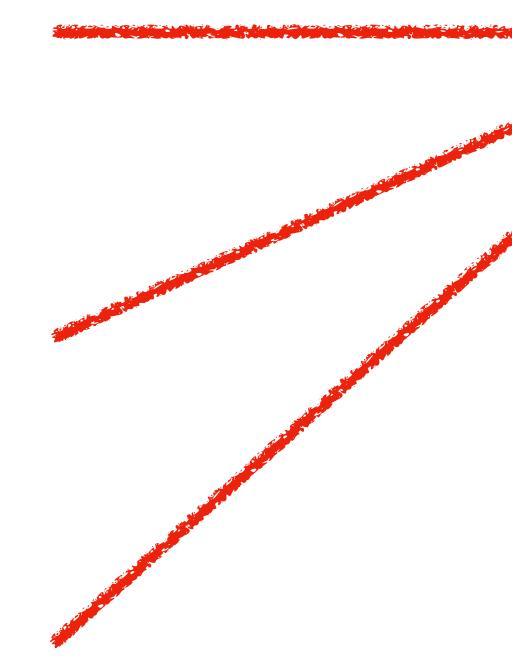
# Discount Factor



For  $\gamma \in [0,1]$  consider  $\mathbb{E}_\pi(\sum_{t=0}^T \gamma^t r_t) \rightarrow \max_\pi$

# Reinforcement Learning Formalism

1.  $a \in \mathcal{A}$  - action space
2.  $s \in \mathcal{S}$  - state space
- 2\*.  $o \in \mathcal{O}$  - observation space for partial observable environment
3.  $r(s, a) : \mathcal{A} \times \mathcal{S} \rightarrow \mathcal{P}(\mathcal{R})$  - reward function
4.  $p(r, s' | s, a)$  - environment dynamics
5.  $p(s_0)$  - initial state distribution
6.  $\pi(a | s) : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{A})$  - policy function



In general, unknown

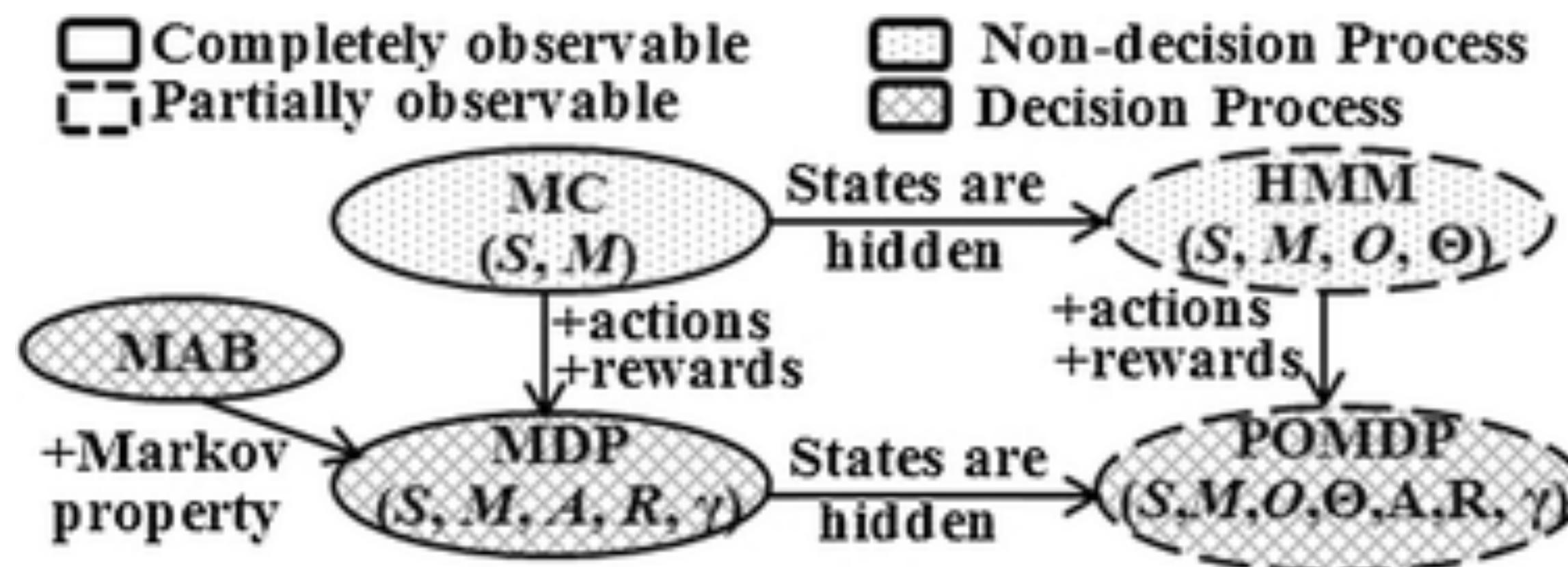


Would like to find optimal

# Markov Decision Process

**Markov property:**

$$p(R_t, S_{t+1} | S_t, A_t, R_{t-1}, S_{t-1}, A_{t-1}, \dots) = p(R_t, S_{t+1} | S_t, A_t)$$

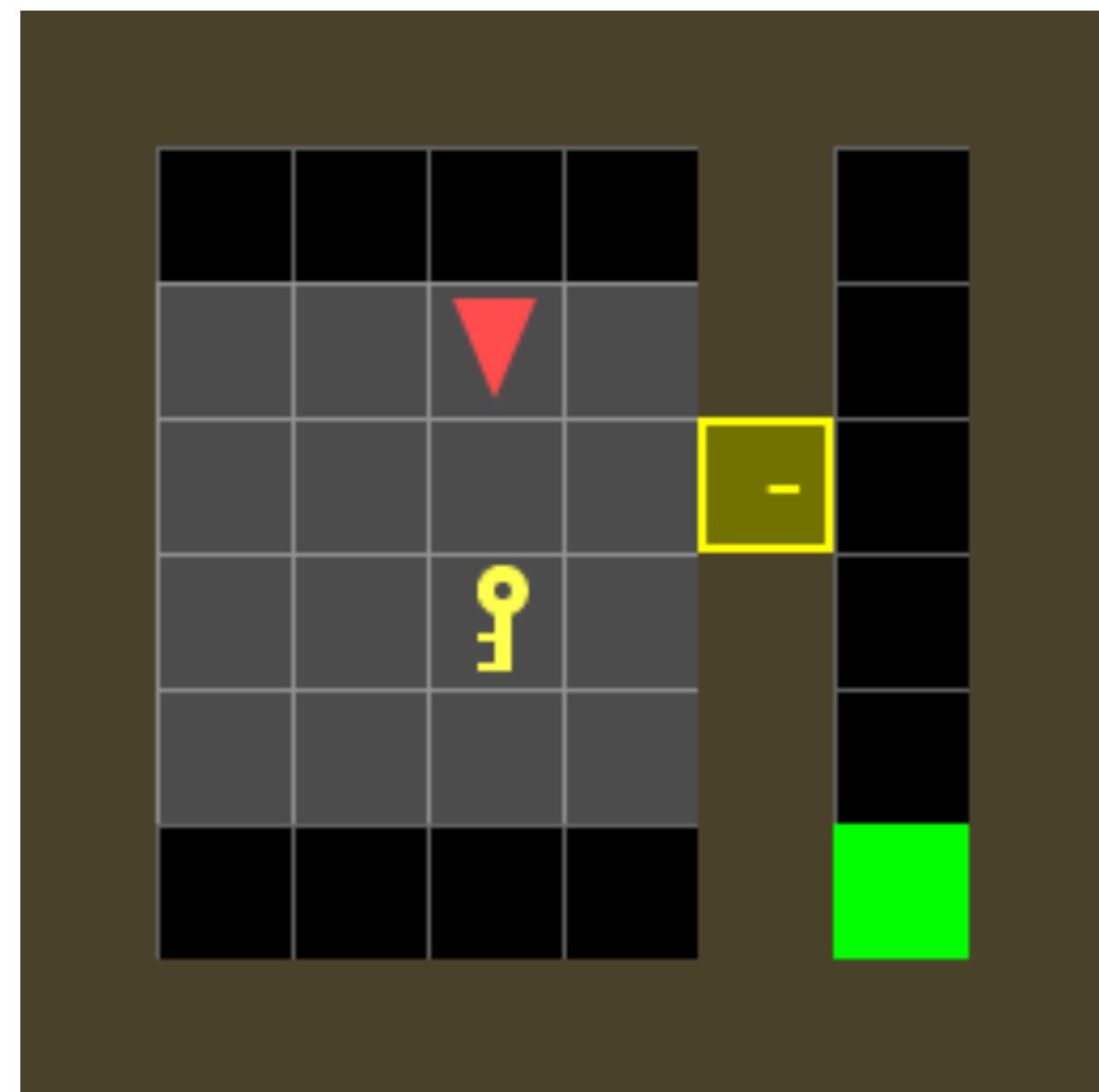


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# Markov Decision Process

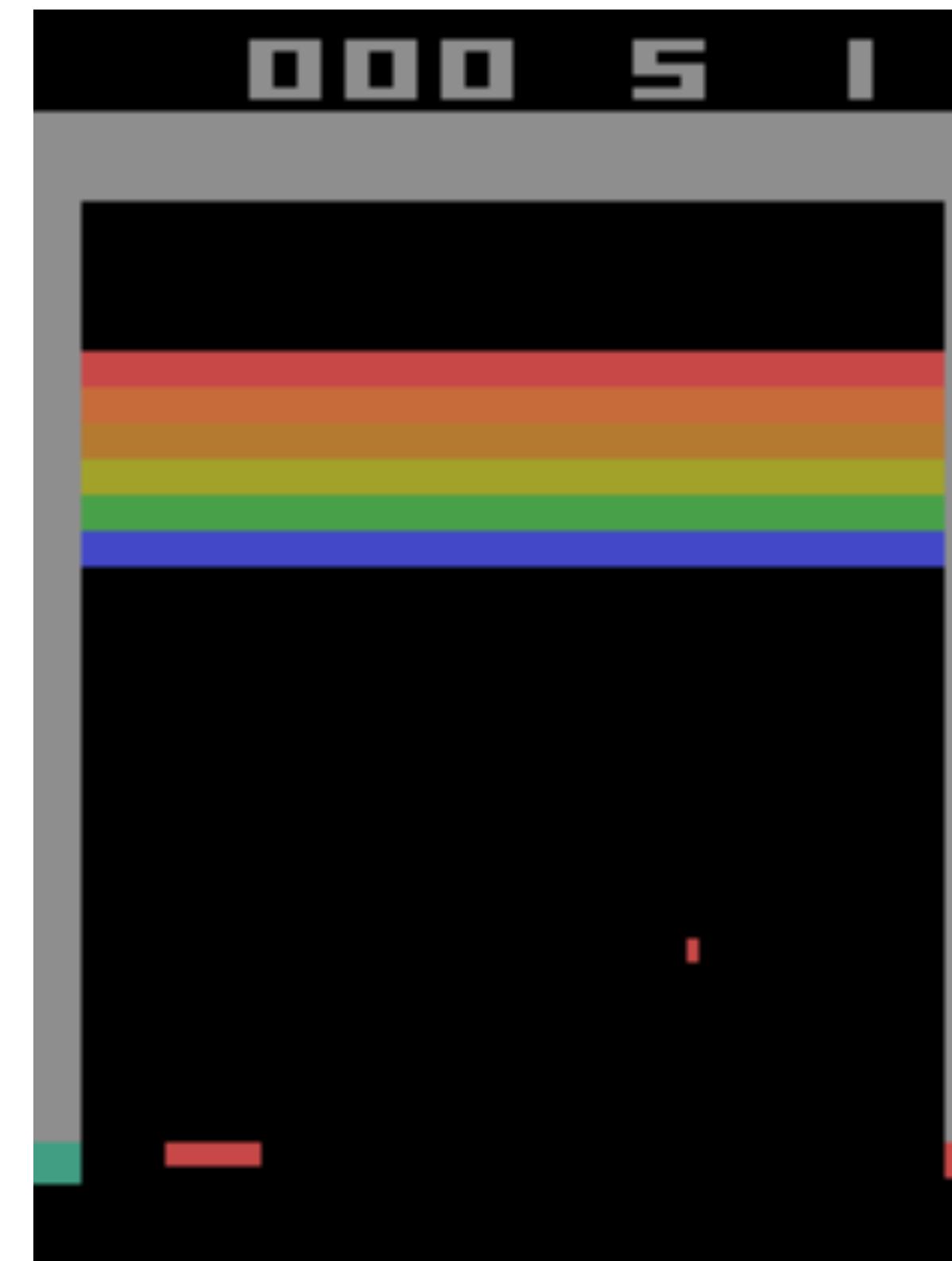
Possible states:

1. Full history
2. Coordinates
3. Coordinates + does the agent have a key
4. Image



# Non Markovian Environment

Can you observe ball's direction  
if only this observation is  
available?



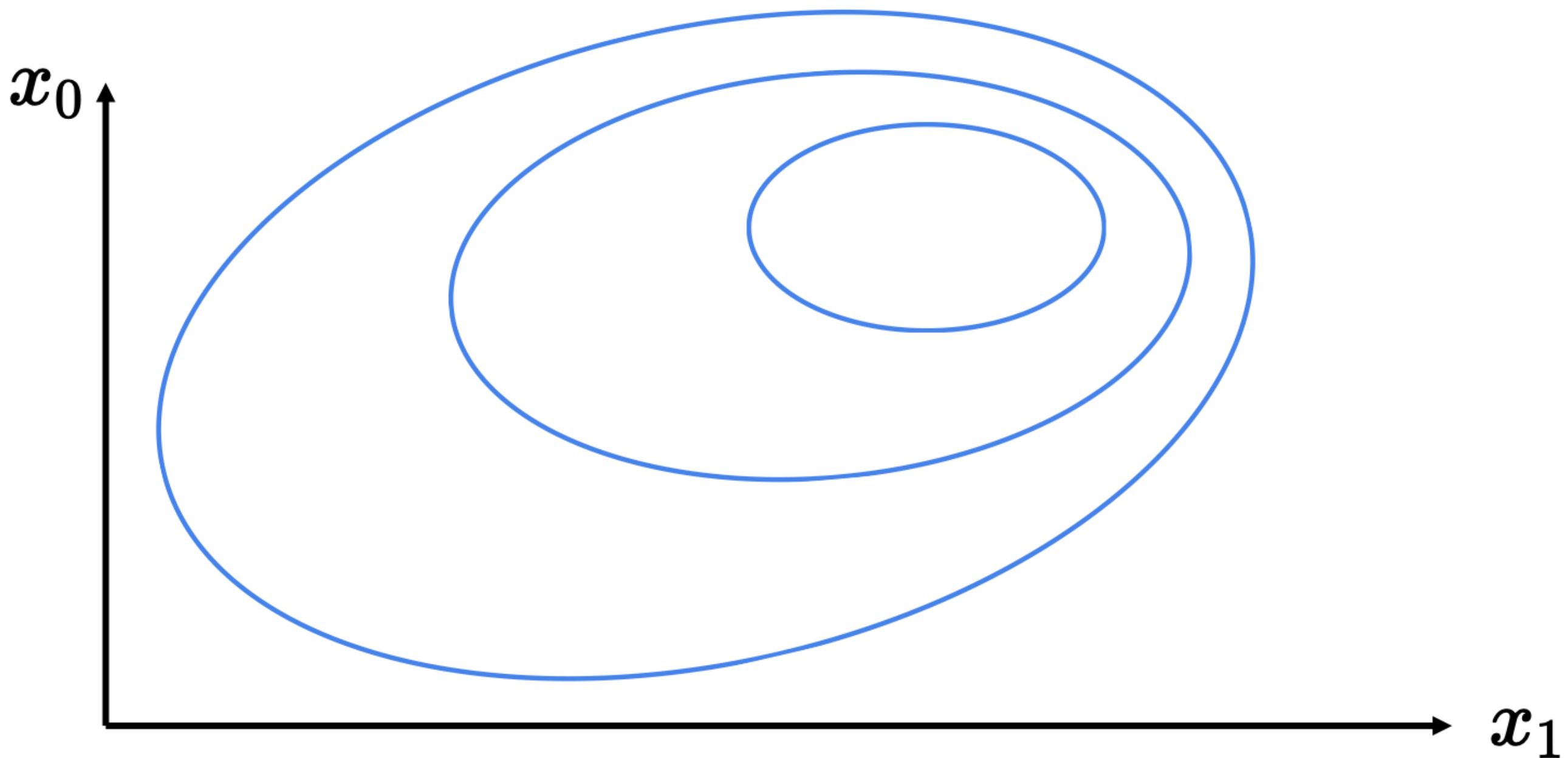
# Reinforcement Learning Problem

1.  $a \in \mathcal{A}$  - action space
2.  $s \in \mathcal{S}$  - state space
- 2\*.  $o \in \mathcal{O}$  - observation space for partial observable environment
3.  $r(s, a) : \mathcal{A} \times \mathcal{S} \rightarrow \mathcal{P}(\mathcal{R})$  - reward function
4.  $p(r, s' | s, a)$  - environment dynamics
5.  $p(s_0)$  - initial state distribution
6.  $\pi(a | s) : \mathcal{S} \rightarrow \mathcal{P}(\mathcal{A})$  - policy function
7.  $\gamma \in [0,1]$  - discount factor
8.  $T$  - time horizon, can be infinite

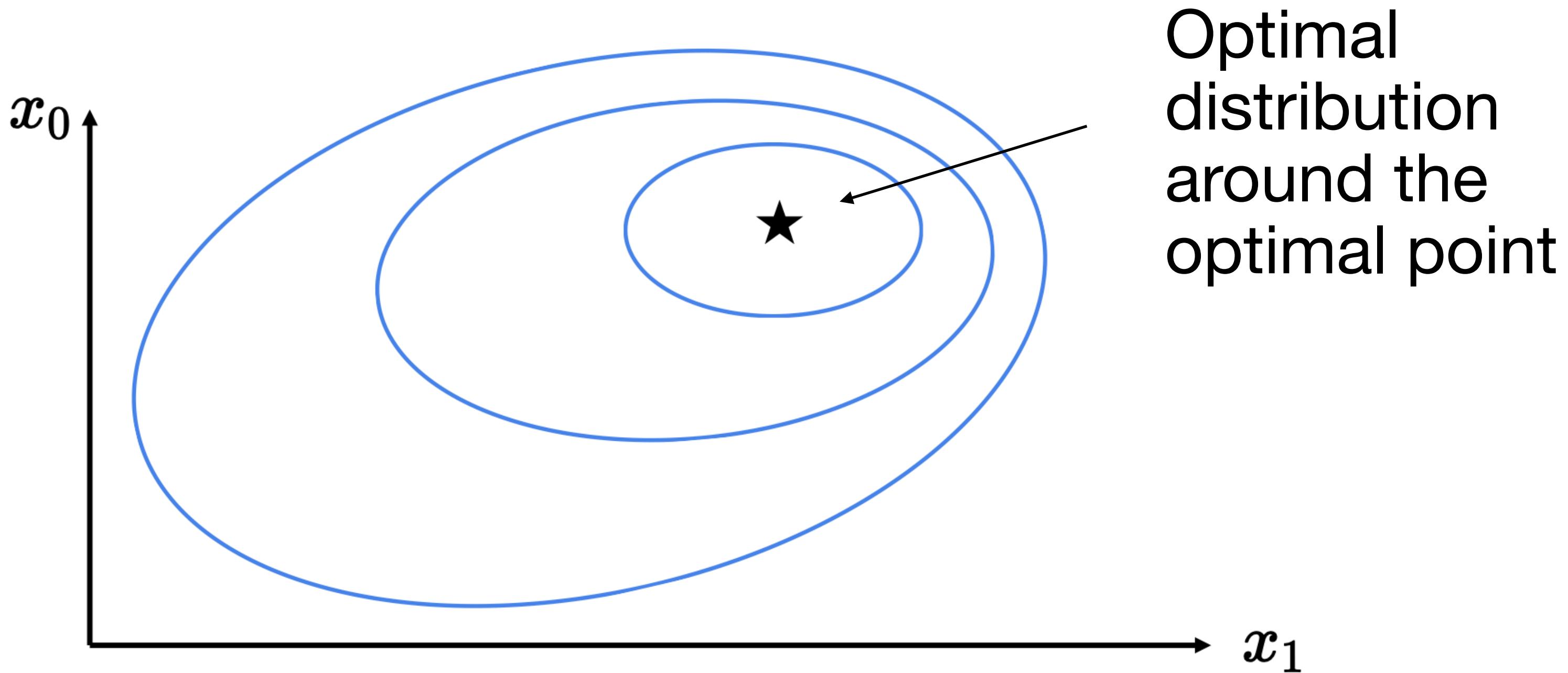
$$J(\pi) = \mathbb{E}_{p(\tau|\pi)}\left(\sum_{t=0}^T \gamma^t R_t\right) = \mathbb{E}_{s_0 \sim p(s_0)} \dots \mathbb{E}_{a_t \sim \pi(a|s_t)} \mathbb{E}_{r_T, s_{T+1} \sim p(r, s'|s_t, a_t)} \left(\sum_{t=0}^T \gamma^t R_t\right) \rightarrow \max_{\pi}$$

# Cross-entropy Method

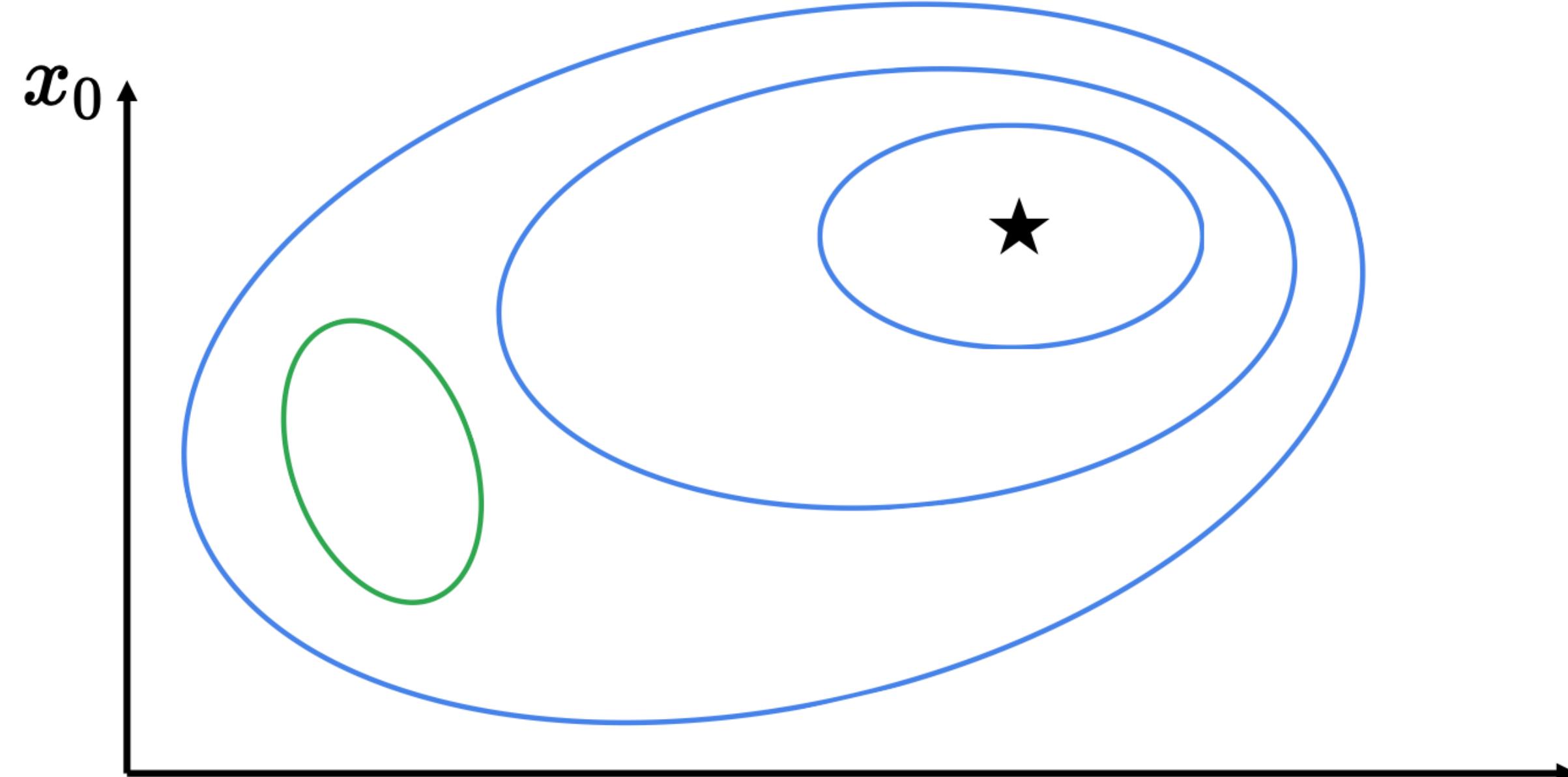
1. Zero-order optimization method
2. Try to find distribution around the optimal point (instead of point itself)



# Cross-entropy Method

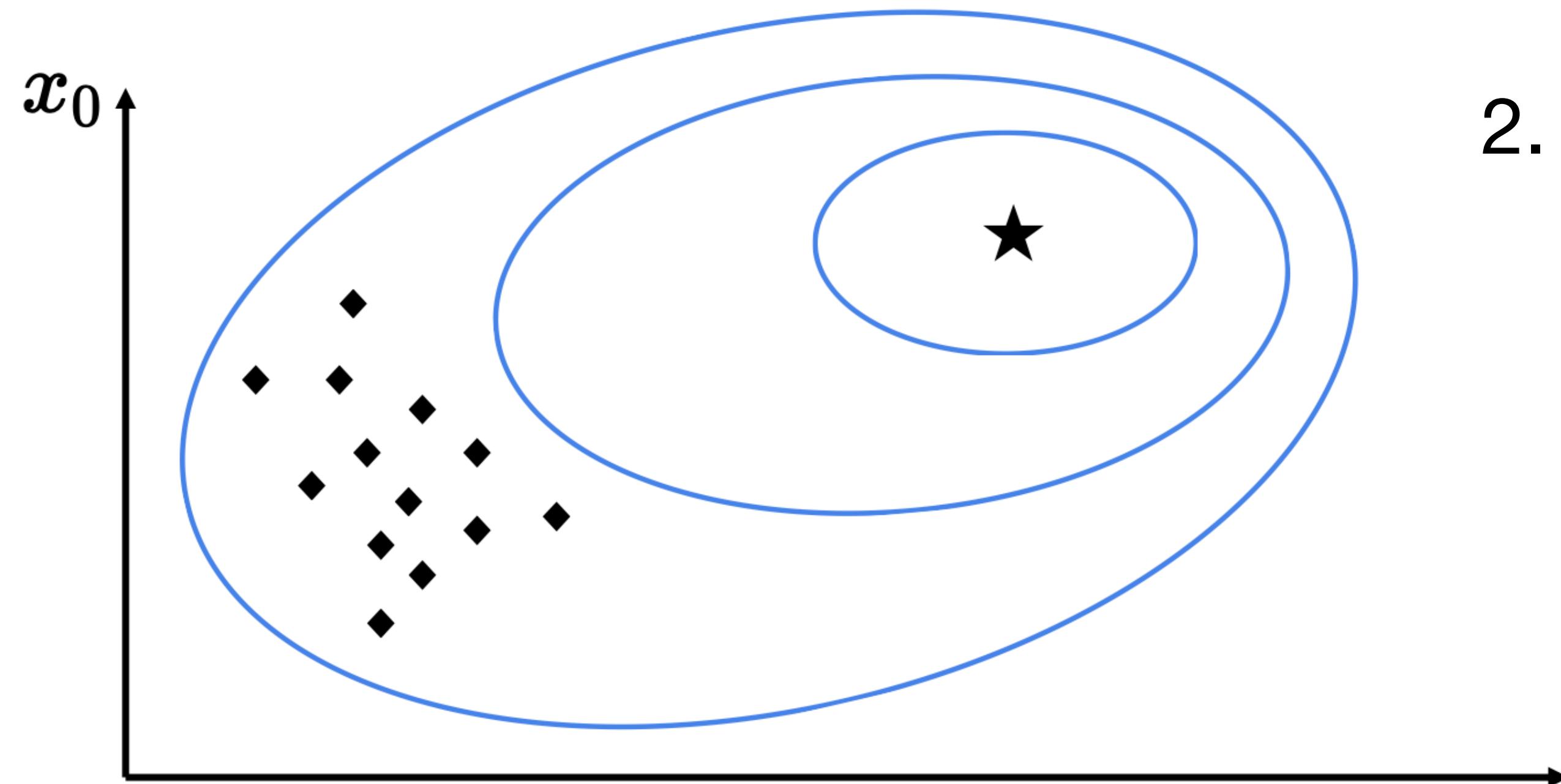


# Cross-entropy Method



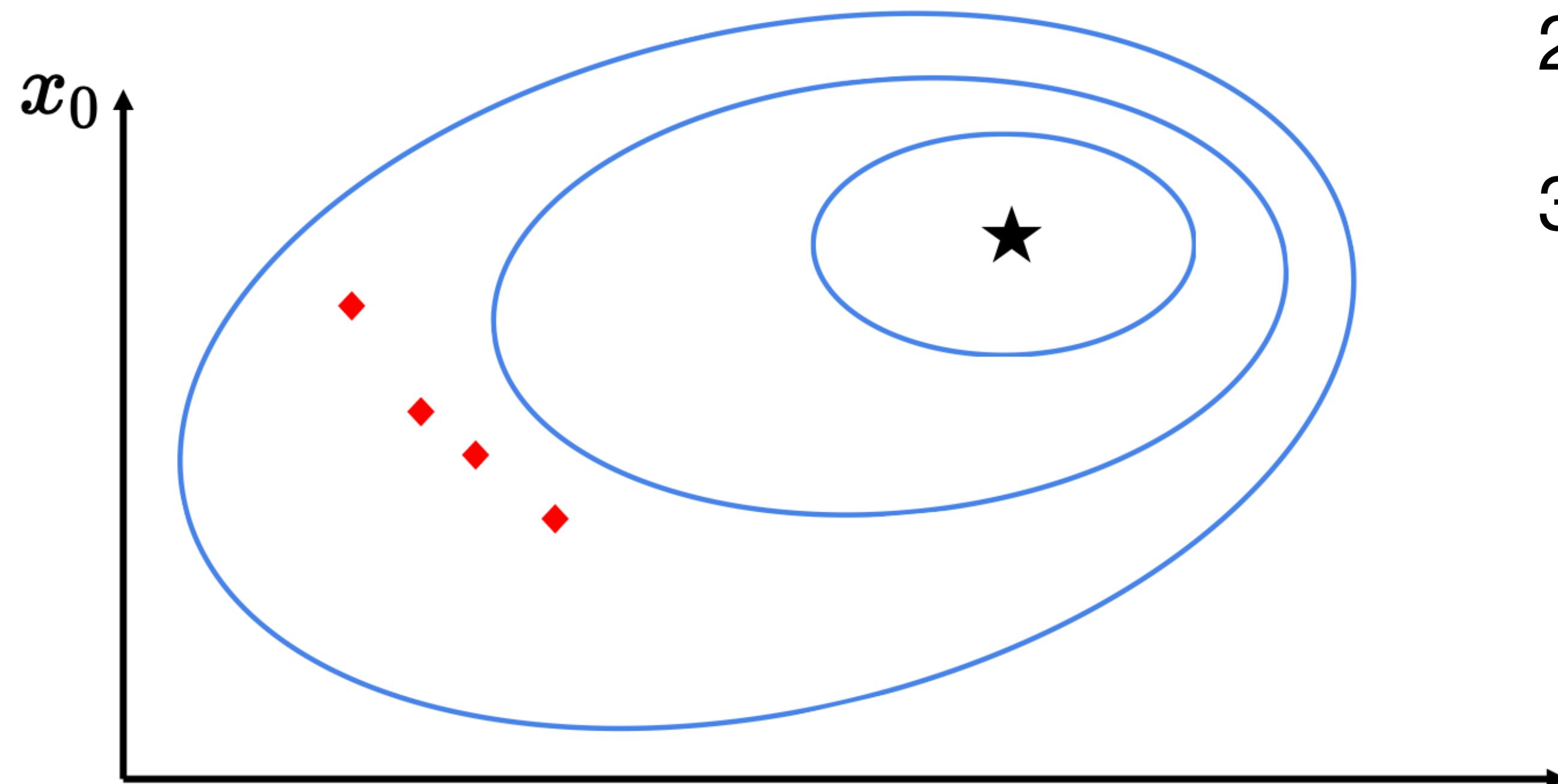
1. Initialise points distribution  $q_0$

# Cross-entropy Method



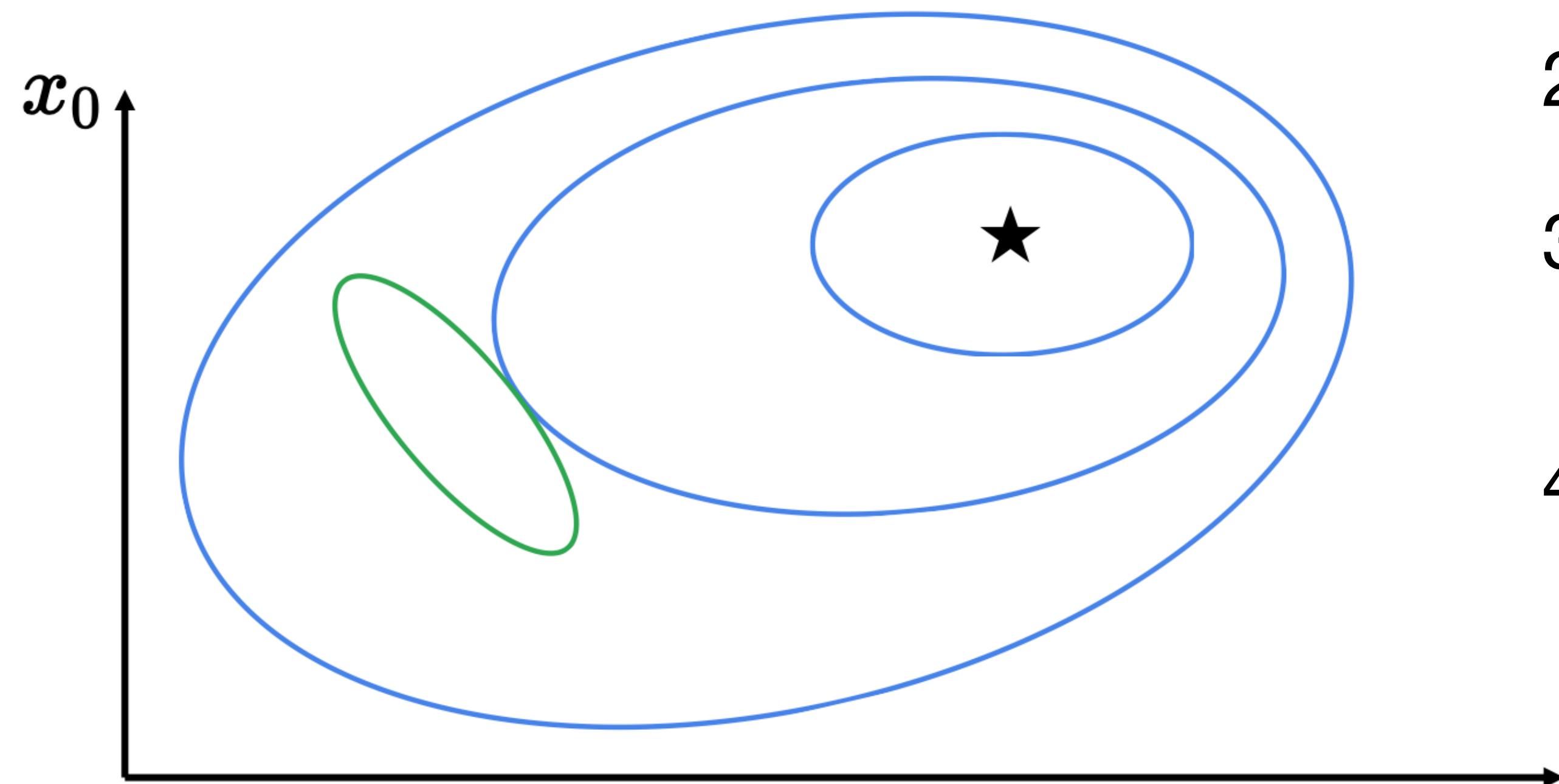
1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$

# Cross-entropy Method



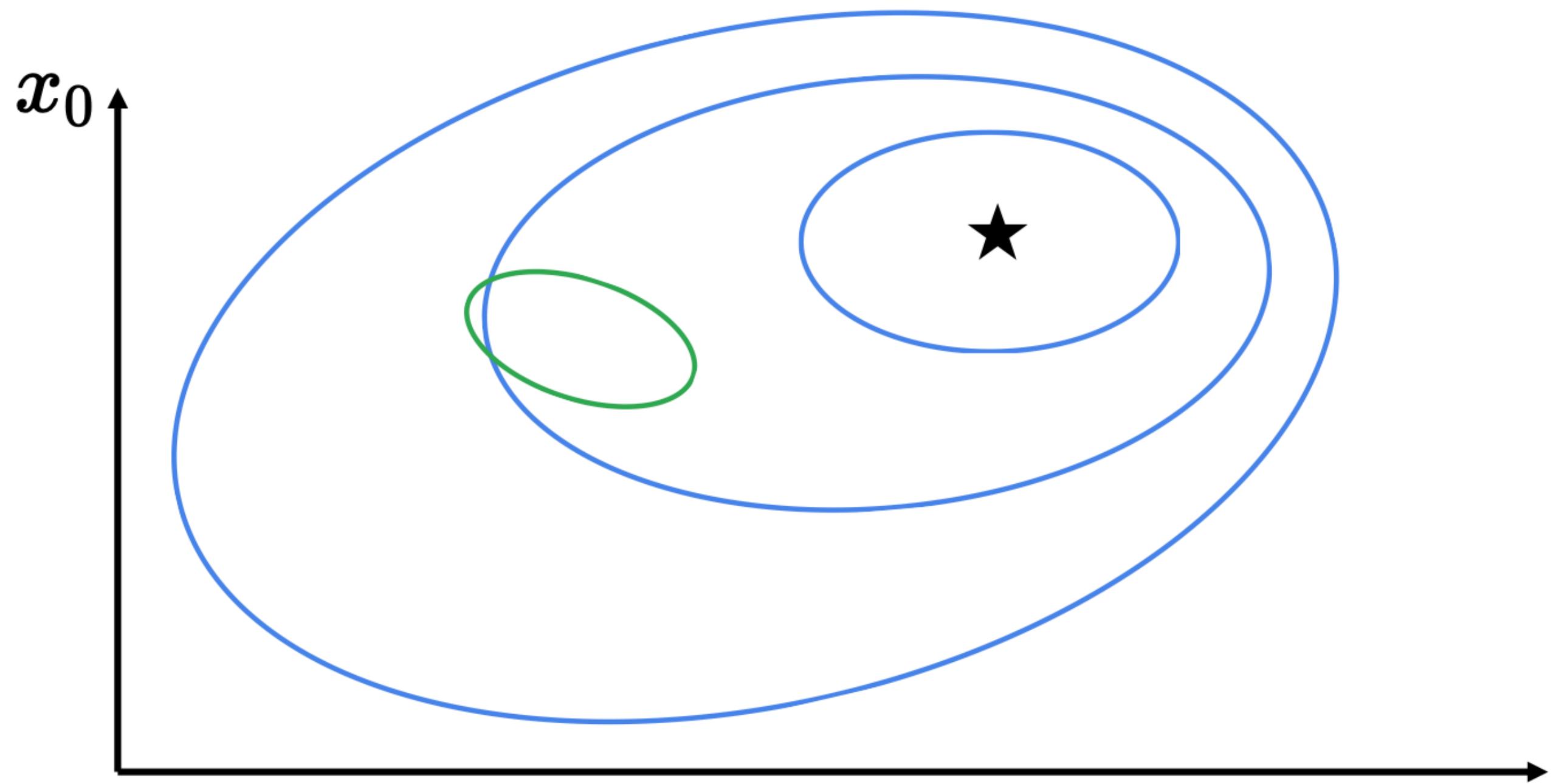
1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$
3. Select M elite point based on criteria

# Cross-entropy Method



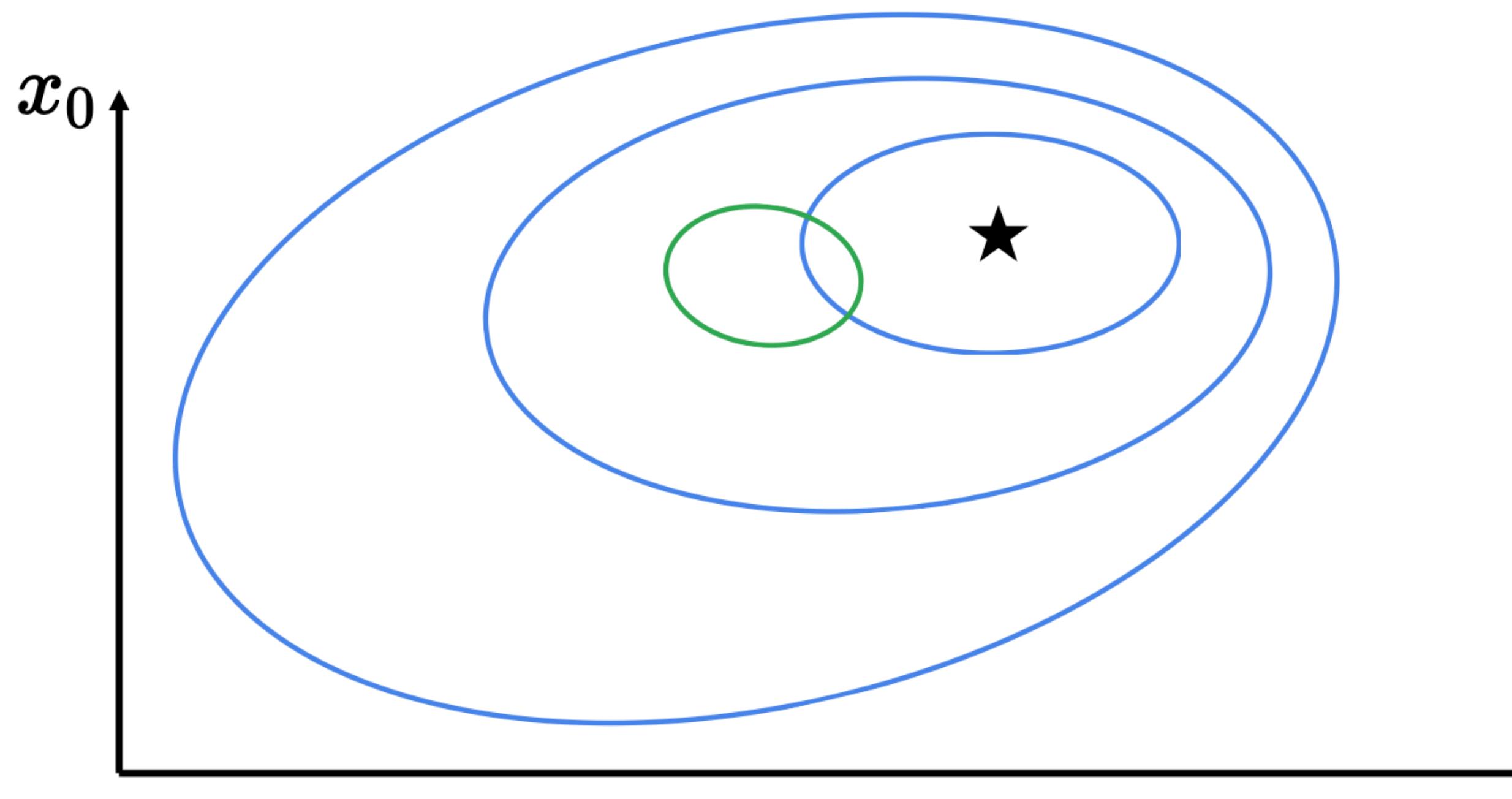
1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$
3. Select M elite point based on criteria
4. Update to get new distribution  $q_1$

# Cross-entropy Method



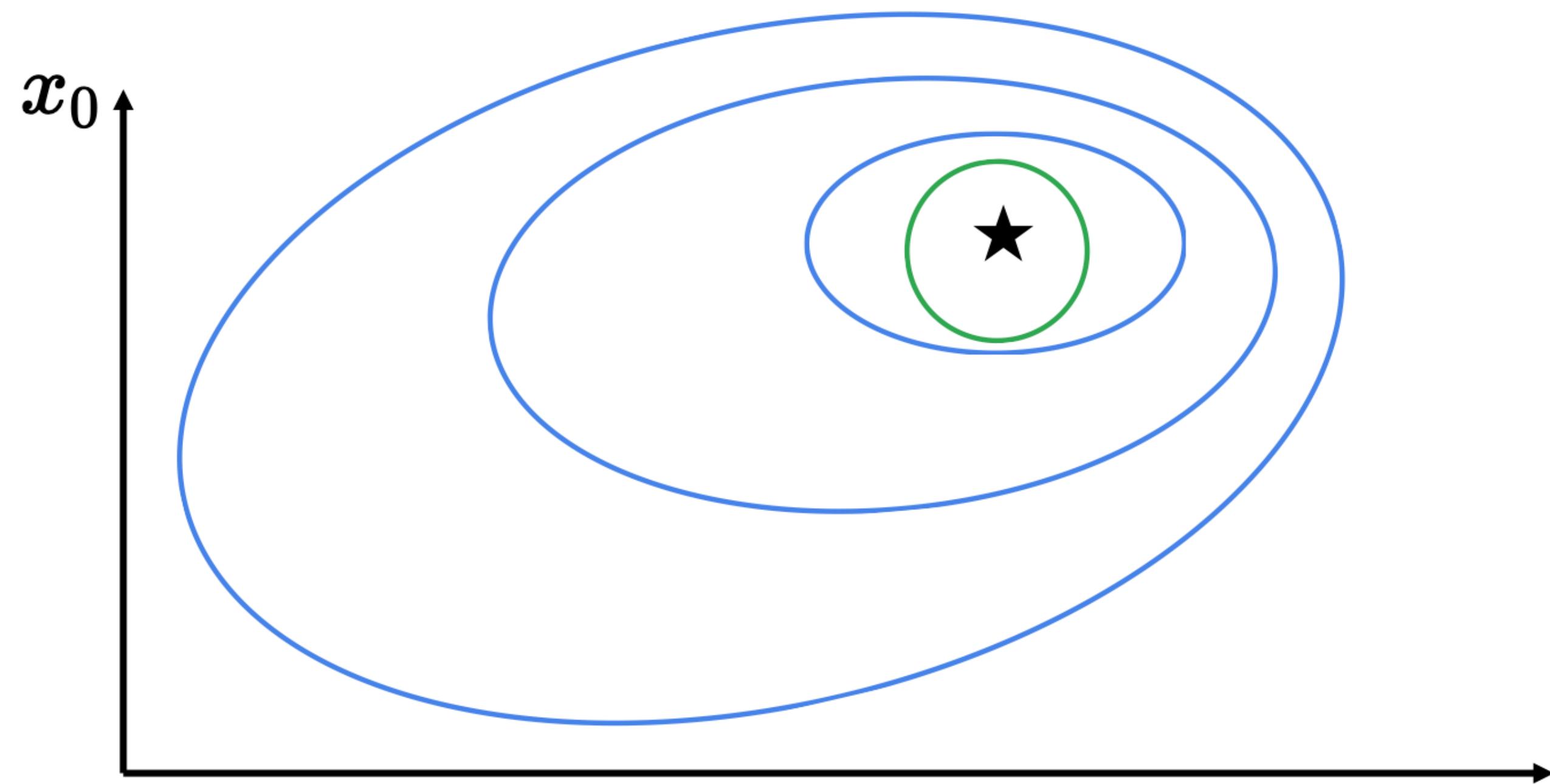
1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$
3. Select M elite point based on criteria
4. Update to get new distribution  $q_1$
5. Repeat 2-4 until convergence

# Cross-entropy Method



1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$
3. Select  $M$  elite points based on some criteria
4. Update to get new distribution  $q_1$
5. Repeat 2-4 until convergence

# Cross-entropy Method



1. Initialise points distribution  $q_0$
2. Sample from  $x_i \sim q_0$
3. Select M elite point based on criteria
4. Update to get new distribution  $q_1$
5. Repeat 2-4 until convergence

# Cross-entropy Method

Distribution update:

$$KL(p_{\text{data}} \parallel q) = \mathbb{E}_{p_{\text{data}}} \log\left(\frac{p_{\text{data}}}{q}\right) = - \mathbb{E}_{x \sim p_{\text{data}}} \log q(x) \rightarrow \min_q$$

Distribution update:  $q^{k+1} = \operatorname{argmin}_q \left[ -\frac{1}{|\mathcal{M}^k|} \sum_{x \in \mathcal{M}^k} \log q(x) \right]$ ,

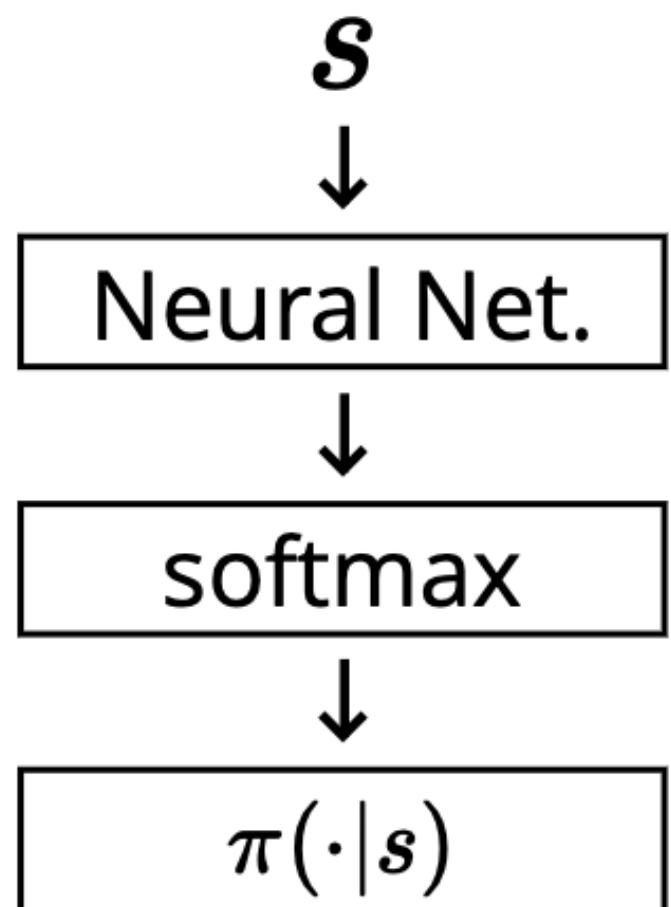
where  $\mathcal{M}^k$  is a set of elites points from the k-th iteration

# Deep CME for RL

$$J(\pi) = \mathbb{E}_{p(\tau|\pi)}\left(\sum_{t=0}^T \gamma^t R_t\right) \rightarrow \max_{\pi}$$

Let  $\pi = \pi_\theta$  is a neural network

Discrete actions



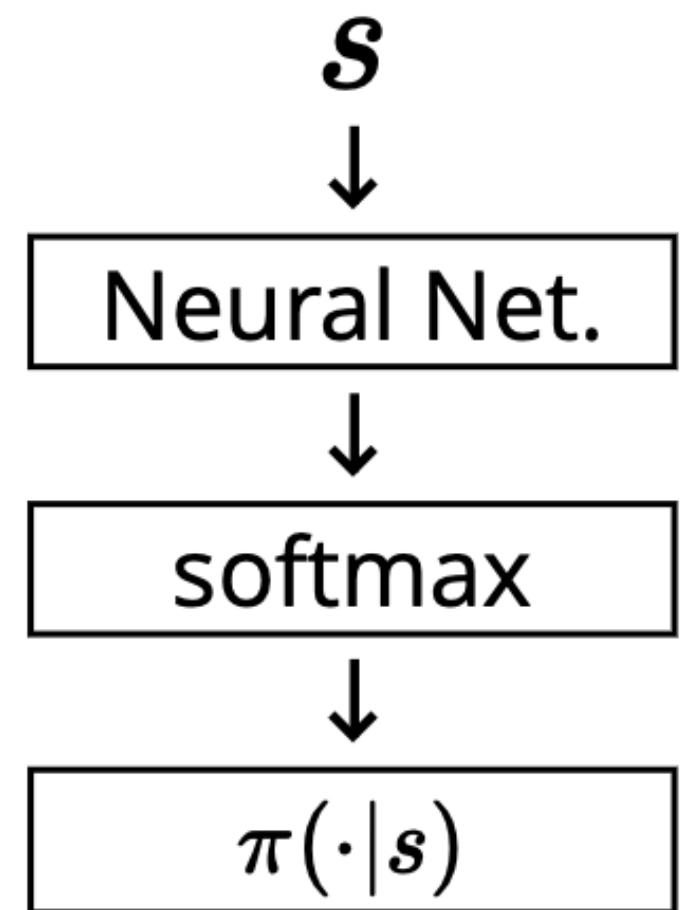
Continuous actions

# Deep CME for RL

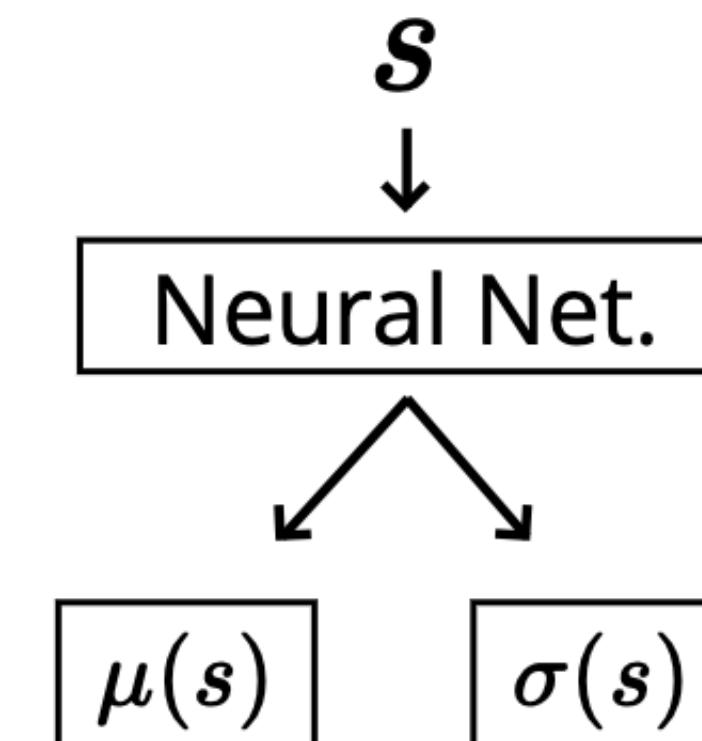
$$J(\pi) = \mathbb{E}_{p(\tau|\pi)}\left(\sum_{t=0}^T \gamma^t R_t\right) \rightarrow \max_{\pi}$$

Let  $\pi = \pi_\theta$  is a neural network

Discrete actions



Continuous actions



# Deep CME for RL

1. Input: elite percentile  $\alpha$
2. Initialize  $\pi = \pi_\theta$  as a neural network
3. Repeat until convergence:
  1. Run agent  $\pi_\theta$  to collect trajectories  $\mathcal{T} = \{\tau_i\}$
  2.  $\delta = \text{percentile}(\mathcal{T}, p = \alpha)$
  3.  $\mathcal{M} = \text{select\_elites}(\mathcal{T}, \delta)$
  4. Partial fit  $\pi_\theta$  to predict  $a_i$  based on  $s_i$ ,  $(a_i, s_i) \in \mathcal{M}$

# Bonus

Evolution Strategies as a Scalable Alternative to Reinforcement Learning

**Thank you for your attention!**