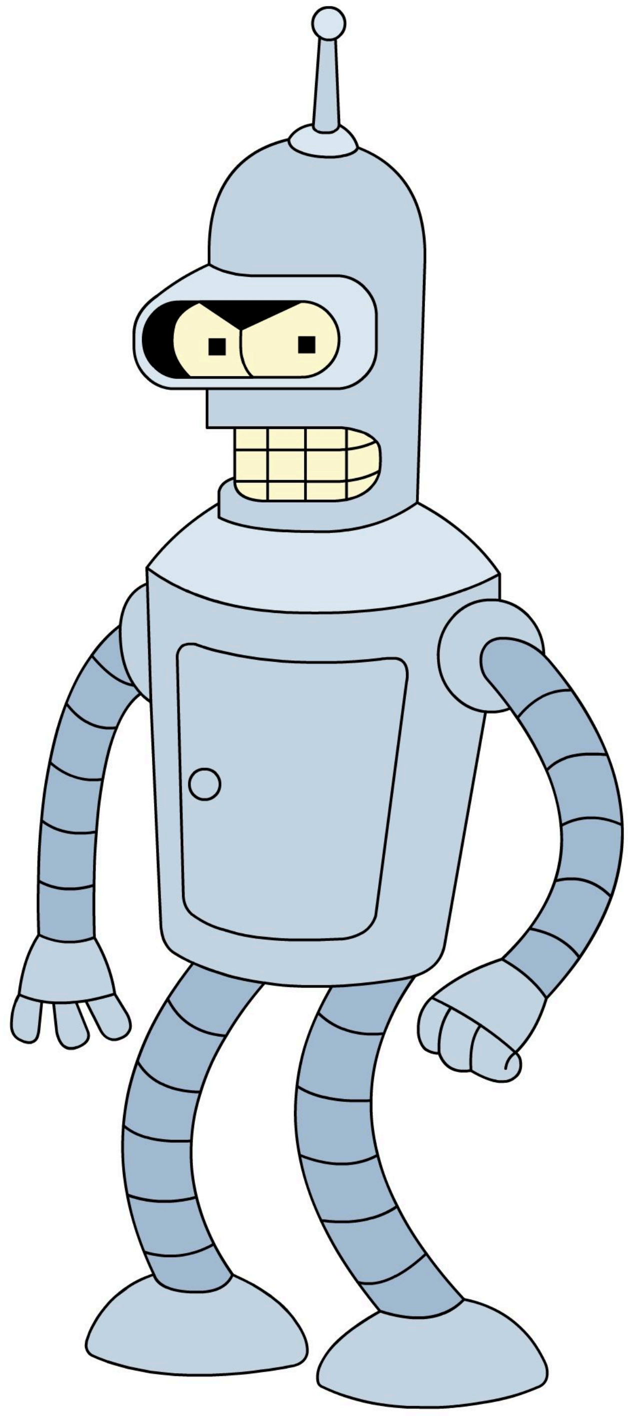
Reinforcement Learning

HSE, autumn - winter 2022

Lecture 6: Advanced Policy Optimisation



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Background

1. Practical RL course by YSDA, week 9
2. https://spinningup.openai.com/en/latest/algorithms/trpo.html
3. https://spinningup.openai.com/en/latest/algorithms/ppo.html

Recap: Policy Gradient

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_\theta(\tau)}\Big[\sum_{t=0}^{T} \nabla \log \pi_\theta(A_t \mid S_t) \sum_{k=t}^{T} \gamma^{k-t} R_k\Big]$$

or

$$\nabla J_{PG}(\theta) = \mathbb{E}_{p_\theta(\tau)}\Big[\sum_{t=0}^{T} \nabla \log \pi_\theta(A_t \mid S_t)[Q_{\pi_\theta}(S_t, A_t) - b(S_t)]\Big]$$

or

$$\nabla J_{AC}(\theta) = \mathbb{E}_{p_\theta(\tau)}\Big[\sum_{t=0}^{T} \nabla \log \pi_\theta(A_t \mid S_t)[A_{\pi_\theta}(S_t, A_t)]\Big]$$

Recap: A2C

- Generate trajectories $\{\tau_i\}$ following $\pi_\theta(a \mid s)$
- Policy improvement:

  Estimate gradient and make gradient ascent step:

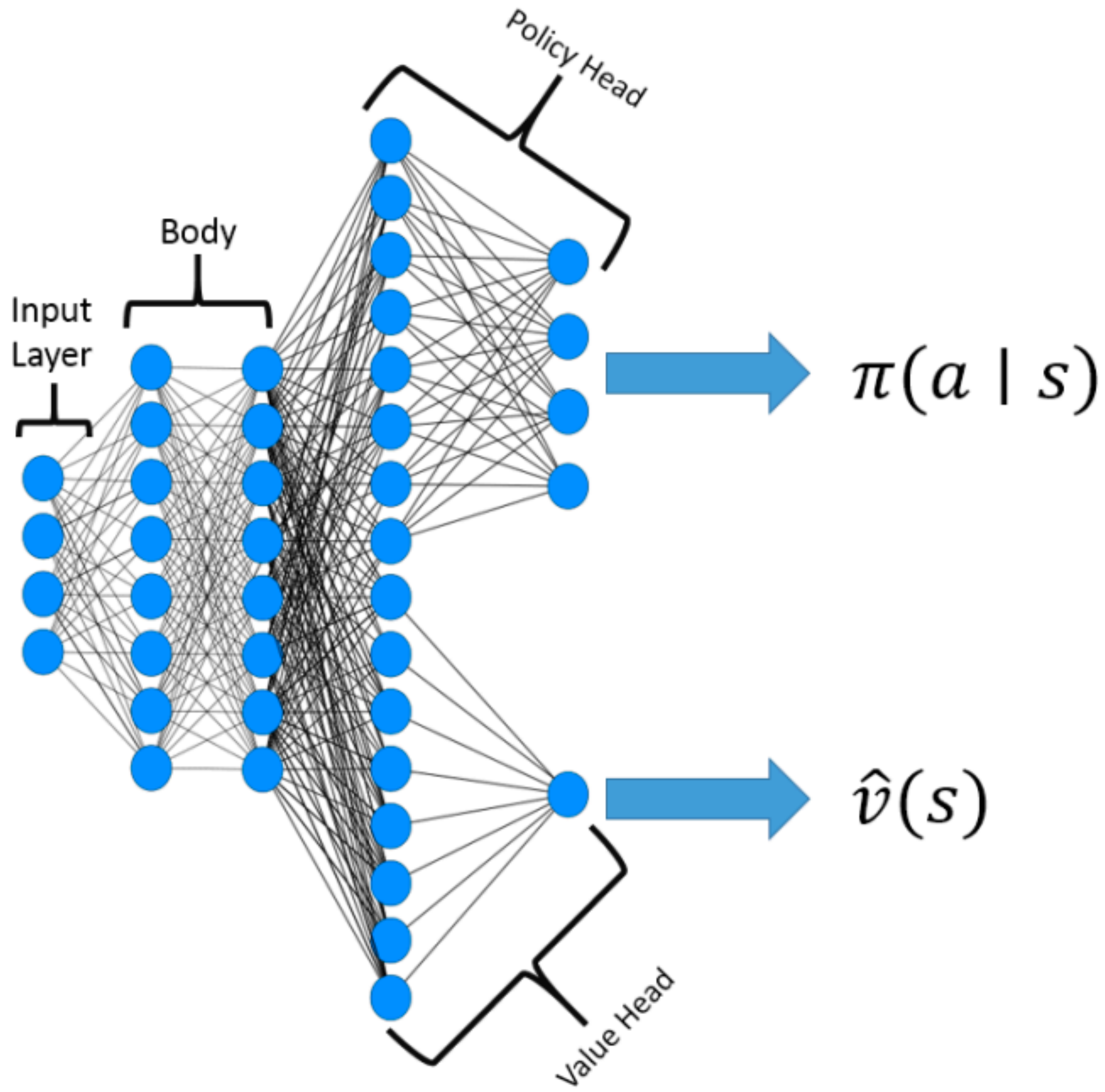
$$\nabla_\theta J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \left[\sum_{t=0}^{T} \nabla \log \pi_\theta(a_{i,t} \mid s_{i,t}) A_{\pi_\theta}(s_{i,t}, a_{i,t}) \right]$$

- Policy evaluation:

  Estimate gradient and make gradient descent step:

$$\nabla_\phi L(\phi) \approx \frac{1}{N} \sum_{i=1}^{N} \left[\sum_{t=0}^{T} \nabla_\phi (r_{i,t} + \gamma \boxed{V_{\phi^-}}(s_{i,t+1}) - V_\phi(s_{i,t}))^2 \right]$$

Not target network, just frozen parameters



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Recap

Policy Gradients and Actor-Critic algorithms are on-policy algorithms so we can not use experience replay. Thus, our sample efficiency is quite low.

Policy Optimisation via Gradient Ascent

Several issues:

- We make gradient step in the space of parameters, get new parameters θ and policy π_θ from θ_{old} and old policy $\pi_{\theta_{old}}$. However, it's difficult to measure the impact of change in parameters on change in policy.
- Apply only first-order optimisation methods
- Low sample efficiency

$$\theta = \theta_{old} + \alpha \nabla J(\theta_{old})$$

Optimisation

$$J(\theta) \approx J(\theta_{old}) + \nabla J(\theta_{old})(\theta - \theta_{old})$$

$$J(\theta) \to \max_{\theta} \quad \Longleftrightarrow \quad \begin{array}{c} \nabla J(\theta_{old})(\theta - \theta_{old}) \to \max\limits_{\theta} \\ \text{s.t. } (\theta - \theta_{old})^T(\theta - \theta_{old}) \leq \delta \end{array}$$

Let's $d = \theta - \theta_{old}$, then $d^* \propto \nabla J(\theta_{old})$

$$\theta = \theta_{old} + \alpha \nabla J(\theta_{old})$$

Optimisation

$$J(\theta_{old})(\theta - \theta_{old}) \rightarrow \max_{\theta} \text{ s.t.}$$

$$(\theta - \theta_{old})^T K(\theta - \theta_{old}) \leq \delta$$

K is symmetric, positive-definite matrix

Let's $d = \theta - \theta_{old}$, then $d^* \propto K^{-1} \nabla J(\theta_{old})$

$$\theta = \theta_{old} + \alpha K^{-1} \nabla J(\theta_{old})$$

Natural Gradient

$KL(\pi_{\theta_{old}}||\pi_\theta) \approx \frac{1}{2}(\theta - \theta_{old})^T K(\theta_{old})(\theta - \theta_{old})$, where $K(\theta_{old}) = \nabla_\theta^2 KL(\pi_{old}||\pi_\theta)|_{\theta_{old}}$

$\theta = \theta_{old} + \alpha s$, where $s = K^{-1}\nabla J(\theta_{old})$, α is a step size.

Choose the largest step:

$$\frac{1}{2}(\theta - \theta_{old})^T K(\theta_{old})(\theta - \theta_{old}) = \delta \iff \alpha = \sqrt{\frac{2\delta}{g^T K^{-1} g}}$$

$$\theta = \theta_{old} + \alpha K^{-1}\nabla J(\theta_{old})$$

Natural Gradient

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Natural Gradient

$KL(\pi_{\theta_{old}} || \pi_\theta) \approx \frac{1}{2}(\theta - \theta_{old})^T K(\theta_{old})(\theta - \theta_{old})$, where $K(\theta_{old}) = \nabla^2_\theta KL(\pi_{old} || \pi_\theta)|_{\theta_{old}}$

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Choose the largest step:

$$\frac{1}{2}(\theta - \theta_{old})^T K(\theta_{old})(\theta - \theta_{old}) = \delta \iff \alpha = \sqrt{\frac{2\delta}{g^T K g}}$$

$$\theta = \theta_{old} + \alpha K^{-1} \nabla J(\theta_{old})$$

$K \in \mathbb{R}^{|\theta| \times |\theta|}$, K^{-1} computation takes $O(|\theta|^3)$

Conjugate Gradient Method

Paper

K is symmetric, positive-definite matrix

In order to find $K^{-1}\nabla J(\theta_{old})$ we can solve system $Kx = \nabla J(\theta)$ iteratively.

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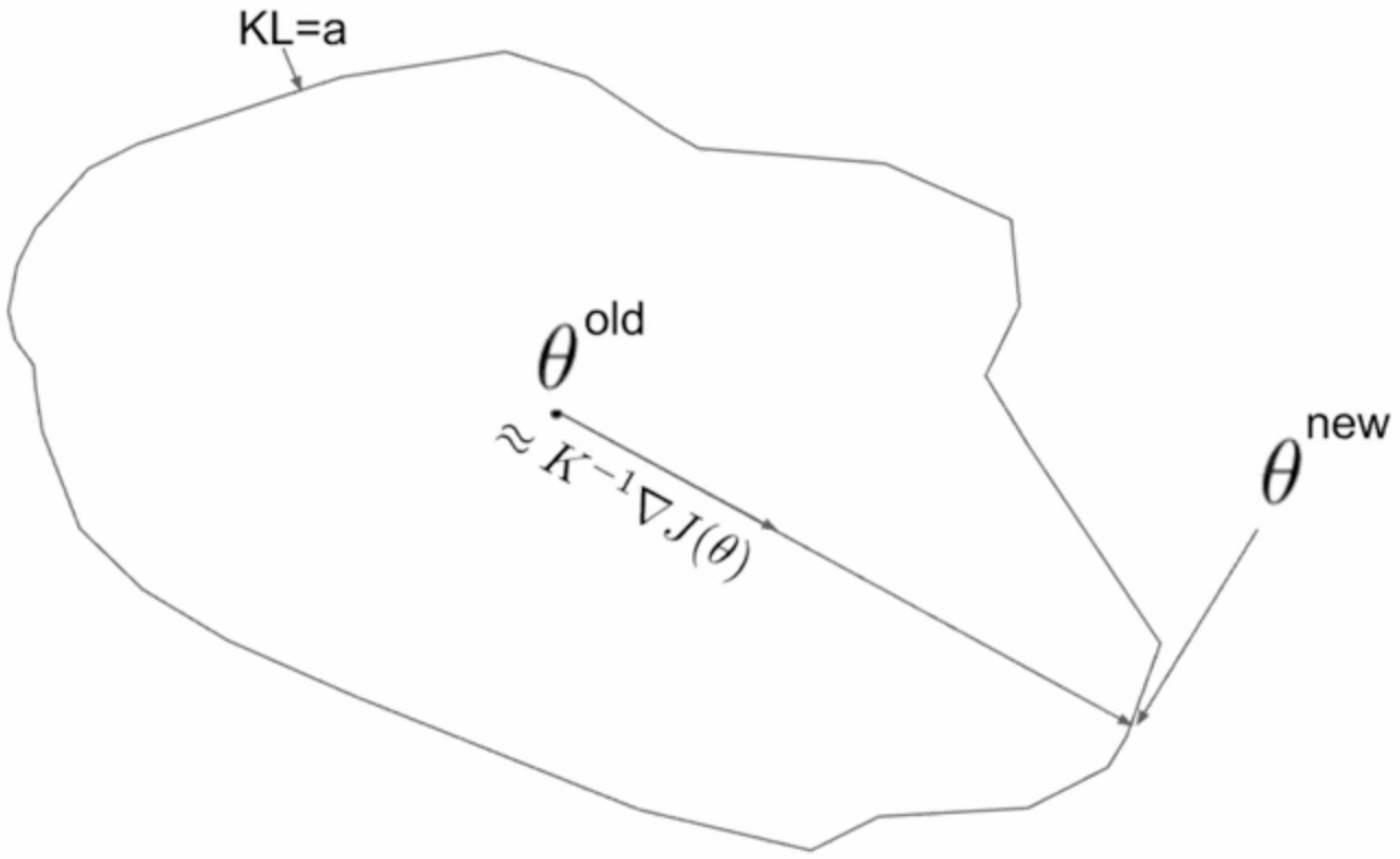

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Optimisation in Policy Space

<u>Lemma</u>:

$$J(\pi) = J(\pi_{old}) + \mathbb{E}_{\tau\sim\pi}[\sum_{t=0}^{T} \gamma^t A_{\pi_{old}}(S_t, A_t)], \text{ where } A_{\pi_{old}}(S_t, A_t) = Q_{\pi_{old}}(S_t, A_t) - V_{\pi_{old}}(S_t)$$

Let's rewrite it as a sum over states instead of timesteps:

$$J(\pi) = J(\pi_{old}) + \sum_s \rho_\pi(s) \sum_a \pi(a \,|\, s) A_{\pi_{old}}(s, a), \text{ where } \rho_\pi(s) = \mathbb{P}(s_0 = s) + \gamma \mathbb{P}(s_1 = s) + \dots$$

Optimisation in Policy Space

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If this term is nonnegative than the policy improvement is guaranteed

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Since we don't know π this expression is intractable…

Optimisation in Policy Space

$$J(\pi) = J(\pi_{old}) + \sum_s \rho_\pi(s) \sum_a \pi(a \mid s) A_{\pi_{old}}(s, a)$$

$$J(\pi) \approx J(\pi_{old}) + \sum_s \rho_{\pi_{old}}(s) \sum_a \pi(a \mid s) A_{\pi_{old}}(s, a) = L_{\pi_{old}}(\pi)$$

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If π_θ is quite close to $\pi_{\theta_{old}}$ ($\mathbb{E}_{s \sim \rho_{old}}[KL(\pi_{\theta_{old}} \mid\mid \pi_\theta)] \leq \delta$), then

$$L_{\pi_{\theta_{old}}}(\pi_{\theta_{old}}) = J(\pi_{\theta_{old}})$$

$$\nabla_\theta L_{\pi_{\theta_{old}}}(\pi_\theta) \mid_{\theta_{old}} = \nabla_\theta J(\pi_\theta) \mid_{\theta_{old}}$$

Optimisation in Policy Space

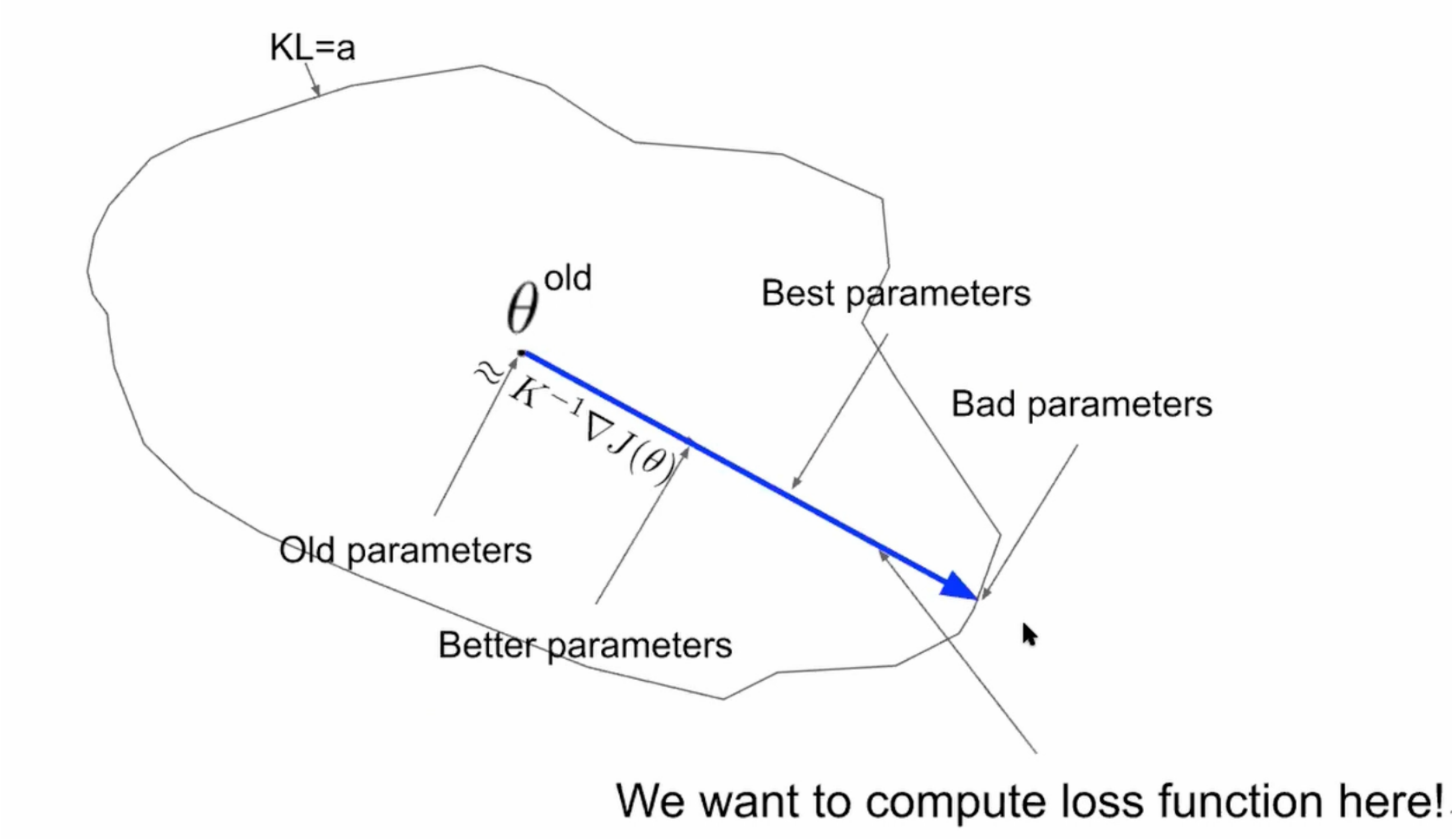
$$J(\pi) = J(\pi_{old}) + \sum_s \rho_\pi(s) \sum_a \pi(a \mid s) A_{\pi_{old}}(s, a)$$

$$J(\pi) \approx J(\pi_{old}) + \sum_s \rho_{\pi_{old}}(s) \sum_a \pi(a \mid s) A_{\pi_{old}}(s, a) =$$

$$= J(\pi_{old}) + \sum_s \rho_{\pi_{old}}(s) \sum_a \pi_{old}(a \mid s) \frac{\pi(a \mid s)}{\pi_{old}(a \mid s)} A_{\pi_{old}}(s, a)$$

$$= J(\pi_{old}) + \mathbb{E}_{\rho_{old}}\Big[\frac{\pi(a \mid s)}{\pi_{old}(a \mid s)} A_{\pi_{old}}(s, a)\Big]$$

Visualisation



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Trust Region Policy Optimisation (TRPO)

Original paper

We have to solve the following optimisation problem to generate a policy update:

$$\max_{\theta} \mathbb{E}_{s\sim\rho_{old}, a\sim\pi_{\theta_{old}}}\left[\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_{old}}(a\,|\,s)} A_{\pi_{\theta_{old}}}(s,a)\right]$$

s.t. $\mathbb{E}_{s\sim\rho_{old}}[KL(\pi_{\theta_{old}}\,||\,\pi_\theta)] \leq \delta$

The authors change the advantage function by the Q-function.

TRPO Algorithm

Repeat until convergence:

1. Collect transitions following current policy $\pi_{\theta_{old}}$
2. Compute $g = \nabla_\theta \frac{1}{N} \sum_{i=1}^{N} \frac{\pi_\theta(a_i \mid s_i)}{\pi_{\theta_{old}}(a_i \mid s_i)} Q_{\pi_{\theta_{old}}}(s_i, a_i)$
3. Compute $K = \nabla_\theta^2 \frac{1}{N} \sum_{i=1}^{N} KL(\pi_{\theta_{old}}(\,.\mid s_i) \,||\, \pi_\theta(\,.\mid s_i))$
4. Find optimal direction via Conjugate Gradients Method (find $s = K^{-1} g$)
5. Do linear search in optimal direction checking the KL constraint and objective value for each new parameter: $\theta_j = \theta_{old} + \alpha_j \sqrt{\frac{2\delta}{g^T K g}} s$

TRPO

+ Extremely stable
+ Prominent results
- Computational expensive
- Require cheap sampling
- Difficult to implement

Conditional vs Unconditional Problem

TRPO problem

$$\max_{\theta} \hat{\mathbb{E}}_t\Big[\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_{old}}(a\,|\,s)}\hat{A}_t\Big]$$

s.t. $\hat{\mathbb{E}}_t[KL(\pi_{\theta_{old}}\,||\,\pi_\theta)] \le \delta$

Equivalent problem

$$\max_{\theta} \hat{\mathbb{E}}_t\Big[\frac{\pi_\theta(a\,|\,s)}{\pi_{\theta_{old}}(a\,|\,s)}\hat{A}_t - \beta KL(\pi_{\theta_{old}}\,||\,\pi_\theta)\Big]$$

$\hat{A}$ is an estimator of the advantage function at timestep t. Here, the expectation $\hat{\mathbb{E}}_t$ indicates the empirical average over a finite batch of samples, in an algorithm that alternates between sampling and optimization.

PPO Objective

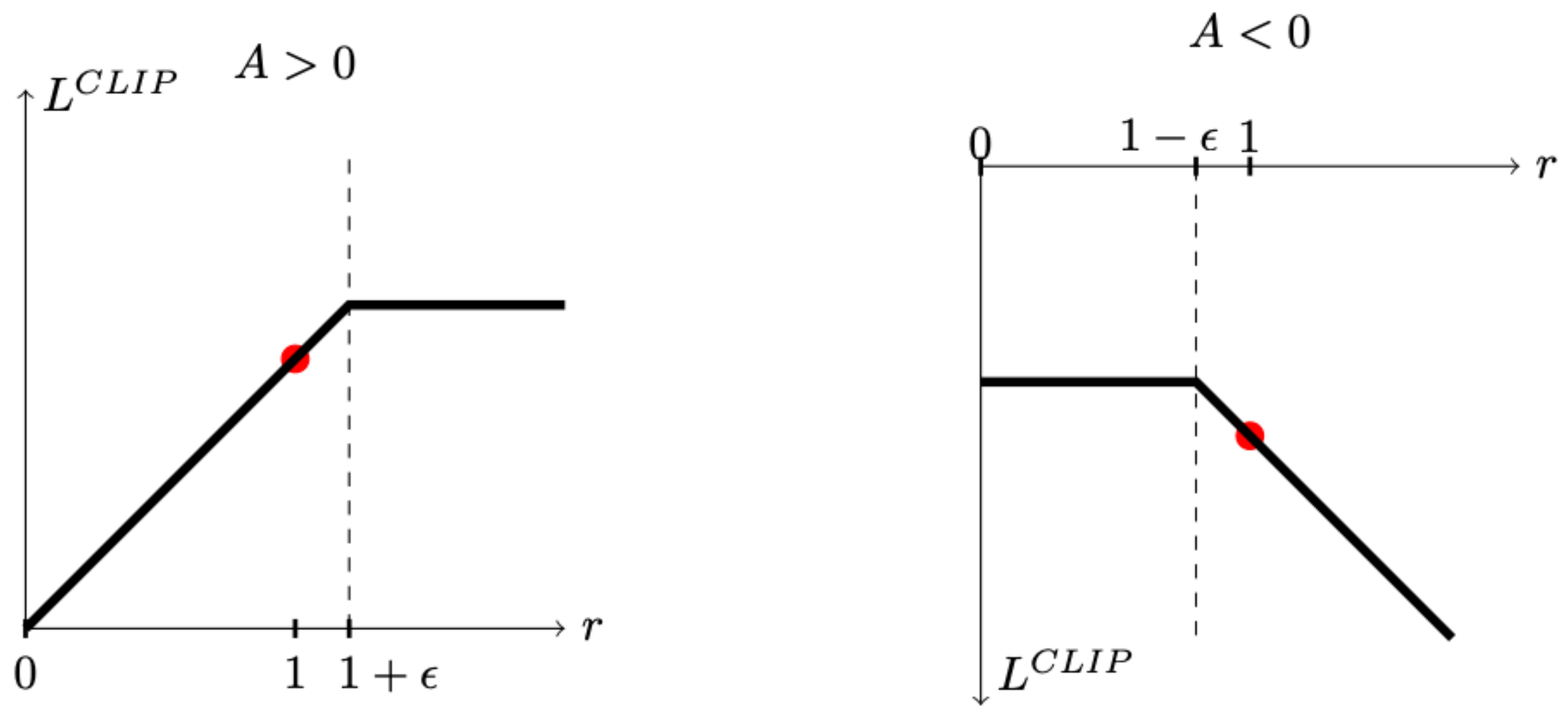
$$r_t(\theta) = \frac{\pi_\theta(a_t \mid s_t)}{\pi_{\theta_{old}}(a_t \mid s_t)}, \text{ so } r_t(\theta_{old}) = 1$$

$$L^{CPI}(\theta) = \hat{\mathbb{E}}_t\Big[\frac{\pi_\theta(a \mid s)}{\pi_{\theta_{old}}(a \mid s)}\hat{A}_t\Big] = \hat{\mathbb{E}}_t\Big[r_t(\theta)\hat{A}_t\Big]$$

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\Big[\min\big(r_t(\theta)\hat{A}_t, clip(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t\big)\Big]$$

PPO Objective

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t\Big[\min\big(r_t(\theta)\hat{A}_t, clip(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t\big)\Big]$$



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$p_t(\theta) > 0$	A_t	**Return Value of** ***min***	**Objective is Clipped**	**Sign of Objective**	**Gradient**
$p_t(\theta) \in [1-\epsilon, 1+\epsilon]$	$+$	$p_t(\theta)A_t$	no	$+$	✓
$p_t(\theta) \in [1-\epsilon, 1+\epsilon]$	$-$	$p_t(\theta)A_t$	no	$-$	✓
$p_t(\theta) < 1-\epsilon$	$+$	$p_t(\theta)A_t$	no	$+$	✓
$p_t(\theta) < 1-\epsilon$	$-$	$(1-\epsilon)A_t$	yes	$-$	**0**
$p_t(\theta) > 1+\epsilon$	$+$	$(1+\epsilon)A_t$	yes	$+$	**0**
$p_t(\theta) > 1+\epsilon$	$-$	$p_t(\theta)A_t$	no	$-$	✓

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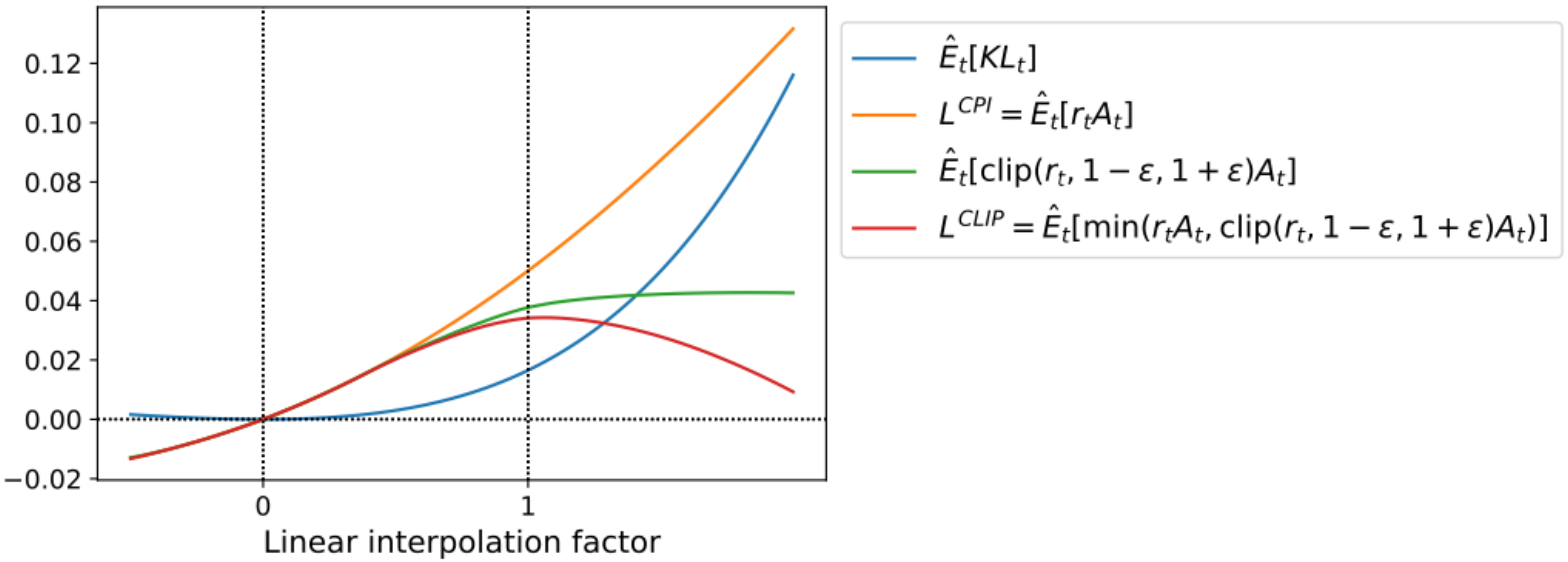
Surrogate Objectives



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Optimisation

Like in A2C, we obtain the following object, which is approximately maximised each iteration:

$$L_t^{CLIP}(\theta) - c_1 L_t^{VF}(\theta) + c_2 H_{\pi_\theta}(s_t),$$

where c_1, c_2 are coefficients, H denotes an entropy bonus,

$L_t^{VF}(\theta)$ is a squared-error loss $\hat{\mathbb{E}}_t[V_\theta(s_t) - V_t^{target}]$

TRPO vs PPO

- Works for smaller models

+ Second-order optimisation

+ Works for big models

- First-order optimisation

Bonus

Published as a conference paper at ICLR 2020

IMPLEMENTATION MATTERS IN DEEP POLICY GRADIENTS: A CASE STUDY ON PPO AND TRPO

Logan Engstrom[1*], Andrew Ilyas[1*], Shibani Santurkar[1], Dimitris Tsipras[1], Firdaus Janoos[2], Larry Rudolph[1,2], and Aleksander Mądry[1]

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ABSTRACT

We study the roots of algorithmic progress in deep policy gradient algorithms through a case study on two popular algorithms: Proximal Policy Optimization (PPO) and Trust Region Policy Optimization (TRPO). Specifically, we investigate the consequences of "code-level optimizations:" algorithm augmentations found only in implementations or described as auxiliary details to the core algorithm. Seemingly of secondary importance, such optimizations turn out to have a major impact on agent behavior. Our results show that they (a) are responsible for most of PPO's gain in cumulative reward over TRPO, and (b) fundamentally change how RL methods function. These insights show the difficulty and importance of attributing performance gains in deep reinforcement learning.

Original paper

Thank you for your attention!