Crystal Symmetry Primer*

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†https://lan496.github.io/

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0 Preliminary

0.1 Recommended books and lecture notes

0.2 International table for crystallography

0.3 Spglib

1 Symmetry operation and space group

1.1 Symmetry operation, isometry

1.2 Affine group, Euclidean group

1.3 Matrix-column pair, Setiz symbol, augmented matrix, short-hand notation

Note: hexagonal axis for $p3$

1.4 Screw and glide

1.5 Working example from plane group

2 Group theory primer

2.1 Definition

2.2 Abelian group

translation subgroup

2.3 Isomorphism, subgroup

2.4 Coset decomposition, normal subgroup, factor group

$\mathbb{Z}_2$ in $\mathbb{Z}_4$

2.5 Homomorphism, kernel, image

point group

2.6 Conjugation

3 Group structure of space groups

3.1 Definition of space group

3.2 Point group

3.3 Vector system

symmorphic and nonsymmorphic space groups

3.4 Working example from plane group

4 Classification of space groups

4.1 Transformation matrix, origin shift

4.2 219 affine space-group types

4.3 230 space-group types

11 enantiomorphic pairs

4.4 32 geometric crystal classes

4.5 14 Bravais types of lattices

limiting case
    special metric

4.6 7 crystal systems

4.7 73 arithmetic crystal classes

5 Conventions for space groups

5.1 Conventional cell

5.2 Helmann–Mauguin symbol

Principal directions
    How to read H-M symbols

5.3 Standard settings in ITA
5.4 Hall symbol
6 Magnetic space group
6.1 Definition
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7 Site symmetry group and normalizer
7.1 Action, orbit, stabilizer
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