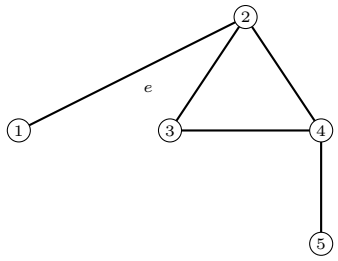

Figure 1: Vertices are ordered as labeled.

The graph in Figure 1 is reducible.

Proof. We need to handle all boards that are nearly colorable for edge e up to permutation of colors, so it will suffice to handle the following 86 boards: 01|012|012|012|01, 01|012|012|012|02, 01|012|012|012|03, 01|012|012|012|23, 01|012|012|013|03, 01|012|012|013|23, 01|012|012|023|03, 01|012|012|023|13, 01|012|012|023|23, 01|012|013|012|01, 01|012|013|012|02, 01|012|013|012|03, 01|012|013|013|01, 01|012|013|013|02, 01|012|013|013|03, 01|012|013|023|01, 01|012|013|023|02, 01|012|013|023|03, 01|012|013|023|12, 01|012|013|023|13, 01|012|013|023|23, 01|012|023|012|01, 01|012|023|012|02, 01|012|023|012|03, 01|012|023|012|12, 01|012|023|012|23, 01|012|023|013|01, 01|012|023|013|02, 01|012|023|013|03, 01|012|023|013|12, 01|012|023|013|13, 01|012|023|013|23, 01|012|023|023|01, 01|012|023|023|02, 01|012|023|023|03, 01|012|023|023|12, 01|012|023|023|23, 01|012|023|123|01, 01|012|023|123|02, 01|012|023|123|03, 01|012|023|123|12, 01|012|023|123|13, 01|012|023|123|23, 01|023|012|012|01, 01|023|012|012|02, 01|023|012|012|03, 01|023|012|012|12, 01|023|012|012|23, 01|023|012|013|01, 01|023|012|013|02, 01|023|012|013|03, 01|023|012|013|12, 01|023|012|013|13, 01|023|012|013|23, 01|023|012|023|01, 01|023|012|023|02, 01|023|012|023|03, 01|023|012|023|12, 01|023|012|023|23, 01|023|012|123|01, 01|023|012|123|02, 01|023|012|123|03, 01|023|012|123|12, 01|023|012|123|13, 01|023|012|123|23, 01|023|023|012|01, 01|023|023|012|12, 01|023|023|012|13, 01|023|023|023|01, 01|023|023|023|02, 01|023|023|023|12, 01|023|023|023|23, 01|023|023|123|01, 01|023|023|123|12, 01|023|123|012|01, 01|023|123|012|02, 01|023|123|012|03, 01|023|123|012|12, 01|023|123|012|13, 01|023|123|012|23, 01|023|123|023|02, 01|023|123|023|12, 01|023|123|023|23, 01|023|123|123|02, 01|023|123|123|12 and 01|023|123|123|23.

Case 1. *B is one of the 59 following boards:* 01|012|012|012|01, 01|012|012|012|02, 01|012|012|012|03, 01|012|012|012|23, 01|012|012|013|03, 01|012|012|013|23, 01|012|012|023|03, 01|012|012|023|13, 01|012|012|023|23, 01|012|013|012|01, 01|012|013|012|02, 01|012|013|012|03, 01|012|013|023|01, 01|012|013|023|02, 01|012|013|023|03, 01|012|013|023|13, 01|012|013|023|23, 01|012|023|012|02, 01|012|023|012|12, 01|012|023|012|23, 01|012|023|013|01, 01|012|023|013|02, 01|012|023|013|03, 01|012|023|013|12, 01|012|023|013|13, 01|012|023|013|23, 01|012|023|023|01, 01|012|023|023|02, 01|012|023|023|03, 01|012|023|023|12, 01|012|023|023|23, 01|012|023|123|01, 01|012|023|123|02, 01|012|023|123|12, 01|012|023|123|13, 01|012|023|123|23, 01|023|012|013|01, 01|023|012|013|02, 01|023|012|013|03, 01|023|012|013|12, 01|023|012|013|13, 01|023|012|013|23, 01|023|012|023|02, 01|023|012|023|12, 01|023|012|023|23, 01|023|012|123|02, 01|023|012|123|12, 01|023|012|123|23, 01|023|023|012|01, 01|023|023|012|12, 01|023|023|012|13, 01|023|023|023|02, 01|023|023|023|12, 01|023|023|023|23, 01|023|123|012|01, 01|023|123|012|02, 01|023|123|012|03, 01|023|123|123|02, 01|023|123|123|12 *and* 01|023|123|123|23.

In all these cases, H is immediately colorable from the lists.

Case 2. *B is one of the 15 following boards:* 01|012|013|013|01*,* 01|012|013|013|02*,* 01|012|013|013|03*,* 01|012|013|023|12*,* 01|012|023|012|01*,* 01|012|023|012|03*,* 01|012|023|123|03*,* 01|023|012|012|12*,* 01|023|012|012|23*,* 01|023|012|023|01*,* 01|023|012|023|03*,* 01|023|012|123|01*,* 01|023|012|123|13*,* 01|023|123|012|12 *and* 01|023|123|023|02*.*

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{12,3}(01|012|013|013|01, 4, 5) \Rightarrow$ 01|012|023|023|01 (Case 1), 01|012|023|013|02 (Case 1) (Case 1).
$\mathbb{K}_{12,4}(01|012|013|013|01, 5) \Rightarrow 01|012|013|023|02$ (Case 1) (Case 1).

$\mathbb{K}_{23,3}(01|012|013|013|02, 4, 5) \Rightarrow 01|012|012|012|02$ (Case 1), 01|012|012|013|03 (Case 1) (Case 1).
$\mathbb{K}_{23,4}(01|012|013|013|02, 5) \Rightarrow 01|012|013|012|03$ (Case 1) (Case 1).

$\mathbb{K}_{13,3}(01|023|012|012|12, \infty, 1, 2, 4) \Rightarrow 01|023|023|012|12$ (Case 1), 01|012|012|023|23 (Case 1), 01|012|023|012|12 (Case 1), 01|023|023|023|12 (Case 1) (Case 1).
$\mathbb{K}_{13,1}(01|023|012|012|12, \infty) \Rightarrow 01|012|023|023|23$ (Case 1) (Case 1).
$\mathbb{K}_{13,2}(01|023|012|012|12, \infty) \Rightarrow 01|012|012|012|02$ (Case 1) (Case 1).
$\mathbb{K}_{13,4}(01|023|012|012|12, \infty) \Rightarrow 01|023|012|023|12$ (Case 1) (Case 1).

$\mathbb{K}_{03,3}(01|023|012|012|23, 4, 5) \Rightarrow 01|023|123|123|23$ (Case 1), 01|023|123|012|02 (Case 1) (Case 1).
$\mathbb{K}_{03,4}(01|023|012|012|23, 5) \Rightarrow 01|023|012|123|02$ (Case 1) (Case 1).

$\mathbb{K}_{12,2}(01|023|012|023|01, 4, 5) \Rightarrow 01|012|013|012|01$ (Case 1), 01|012|013|023|03 (Case 1) (Case 1).
$\mathbb{K}_{12,4}(01|023|012|023|01, 5) \Rightarrow 01|023|012|013|02$ (Case 1) (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{12,\infty}(01|012|013|013|03, 1, 3, 4) \Rightarrow 01|012|023|023|03$ (Case 1), 01|012|023|013|03 (Case 1), 01|012|013|023|03 (Case 1) (Case 1).

$\mathbb{K}_{02,\infty}(01|012|013|023|12, 1, 3, 5) \Rightarrow 01|012|023|123|01$ (Case 1), 01|012|023|123|02 (Case 1), 01|012|013|023|01 (Case 1) (Case 1).

$\mathbb{K}_{12,\infty}(01|012|023|012|01, 1, 3, 5) \Rightarrow 01|012|013|012|02$ (Case 1), 01|012|013|012|01 (Case 1), 01|012|023|012|02 (Case 1) (Case 1).

$\mathbb{K}_{23,\infty}(01|012|023|012|03, 2, 4, 5) \Rightarrow 01|012|023|013|02$ (Case 1), 01|012|023|013|03 (Case 1), 01|012|023|012|02 (Case 1) (Case 1).

$\mathbb{K}_{02,\infty}(01|012|023|123|03, 1, 4, 5) \Rightarrow 01|012|023|013|23$ (Case 1), 01|012|023|013|03 (Case 1), 01|012|023|123|23 (Case 1) (Case 1).

$\mathbb{K}_{12,\infty}(01|023|012|023|03, 1, 2, 4) \Rightarrow 01|012|013|012|02$ (Case 1), 01|012|013|023|02 (Case 1), 01|023|012|013|03 (Case 1) (Case 1).

$\mathbb{K}_{02,\infty}(01|023|012|123|01,1,4,5) \Rightarrow 01|023|012|013|12$ (Case 1), $01|023|012|013|01$ (Case 1), $01|023|012|123|12$ (Case 1) (Case 1).

$\mathbb{K}_{12,\infty}(01|023|012|123|13,1,2,5) \Rightarrow 01|012|013|023|23$ (Case 1), $01|012|013|023|02$ (Case 1), $01|023|012|123|23$ (Case 1) (Case 1).

$\mathbb{K}_{02,\infty}(01|023|123|012|12,1,3,5) \Rightarrow 01|023|012|013|01$ (Case 1), $01|023|012|013|13$ (Case 1), $01|023|123|012|01$ (Case 1) (Case 1).

$\mathbb{K}_{13,\infty}(01|023|123|023|02,1,2,4) \Rightarrow 01|012|023|012|12$ (Case 1), $01|012|023|123|12$ (Case 1), $01|023|123|012|02$ (Case 1) (Case 1).

Case 3. *B is one of the 5 following boards:* $01|023|012|012|01$*,* $01|023|012|012|03$*,* $01|023|023|023|01$*,* $01|023|023|123|12$ *and* $01|023|123|023|23$*.*

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{13,1}(01|023|012|012|01,\infty,2,3)$ $01|012|023|023|03$ (Case 1), $01|023|023|023|02$ (Case 1), $01|012|012|023|03$ (Case 1) (Case 1).

$\mathbb{K}_{13,2}(01|023|012|012|01,\infty,3) \Rightarrow 01|012|012|012|01$ (Case 1), $01|012|023|012|01$ (Case 2) (Case 1 and 2).

$\mathbb{K}_{13,3}(01|023|012|012|01,\infty) \Rightarrow 01|023|023|012|01$ (Case 1) (Case 1).

$\mathbb{K}_{13,4}(01|023|012|012|01,\infty) \Rightarrow 01|023|012|023|01$ (Case 2) (Case 2).

$\mathbb{K}_{13,2}(01|023|012|012|03,\infty,3,4,5) \Rightarrow 01|012|012|012|03$ (Case 1), $01|012|023|012|03$ (Case 2), $01|012|012|023|03$ (Case 1), $01|012|012|012|01$ (Case 1) (Case 1 and 2).

$\mathbb{K}_{13,4}(01|023|012|012|03,\infty,3,5) \Rightarrow 01|023|012|023|03$ (Case 2), $01|023|023|023|02$ (Case 1), $01|023|012|023|01$ (Case 2) (Case 1 and 2).

$\mathbb{K}_{13,1}(01|023|012|012|03,\infty) \Rightarrow 01|012|023|023|01$ (Case 1) (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{13,\infty}(01|023|023|023|01,1,2,3,4,5) \Rightarrow 01|012|012|012|03$ (Case 1), $01|012|023|023|01$ (Case 1), $01|023|012|023|01$ (Case 2), $01|023|023|012|01$ (Case 1), $01|023|023|023|02$ (Case 1) (Case 1 and 2).

$\mathbb{K}_{12,\infty}(01|023|023|123|12,1,2,3) \Rightarrow 01|012|012|023|03$ (Case 1), $01|012|023|123|13$ (Case 1), $01|023|012|123|13$ (Case 2) (Case 1 and 2).

$\mathbb{K}_{03,\infty}(01|023|123|023|23,1,3,5) \Rightarrow 01|023|012|023|02$ (Case 1), $01|023|012|023|23$ (Case 1), $01|023|123|023|02$ (Case 2) (Case 1 and 2).

Case 4. *B is one of the 4 following boards:* $01|023|012|012|02$*,* $01|023|012|123|03$*,* $01|023|023|123|01$ *and* $01|023|123|023|12$*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{23,\infty}(01|023|012|012|02,3,4,5) \Rightarrow 01|023|012|013|03$ (Case 1), $01|023|012|013|02$ (Case 1), $01|023|012|012|03$ (Case 3) (Case 1 and 3).

$\mathbb{K}_{03,\infty}(01|023|012|123|03, 1, 3, 4) \Rightarrow 01|023|123|012|03$ (Case 1), $01|023|123|123|02$ (Case 1), $01|023|012|012|03$ (Case 3) (Case 1 and 3).

$\mathbb{K}_{03,\infty}(01|023|023|123|01, 1, 4, 5) \Rightarrow 01|023|023|012|13$ (Case 1), $01|023|023|012|01$ (Case 1), $01|023|023|123|12$ (Case 3) (Case 1 and 3).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{01,3}(01|023|123|023|12, 4, 5) \Rightarrow 01|023|023|123|12$ (Case 3), $01|023|023|023|02$ (Case 1) (Case 1 and 3).
$\mathbb{K}_{01,4}(01|023|123|023|12, 5) \Rightarrow 01|023|123|123|02$ (Case 1) (Case 1).

Case 5. *B is one of the 3 following boards:* $01|023|012|013|23$*,* $01|023|123|012|13$ *and* $01|023|123|012|23$*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{02,\infty}(01|023|012|013|23, 1, 4, 5) \Rightarrow 01|023|012|123|03$ (Case 4), $01|023|012|123|23$ (Case 1), $01|023|012|013|03$ (Case 1) (Case 1 and 4).

$\mathbb{K}_{13,\infty}(01|023|123|012|13, 1, 2, 4) \Rightarrow 01|012|023|123|03$ (Case 2), $01|012|023|012|03$ (Case 2), $01|023|123|023|12$ (Case 4) (Case 2 and 4).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{03,3}(01|023|123|012|23, 4, 5) \Rightarrow 01|023|012|123|23$ (Case 1), $01|023|012|012|02$ (Case 4) (Case 1 and 4).
$\mathbb{K}_{03,4}(01|023|123|012|23, 5) \Rightarrow 01|023|123|123|02$ (Case 1) (Case 1).

□