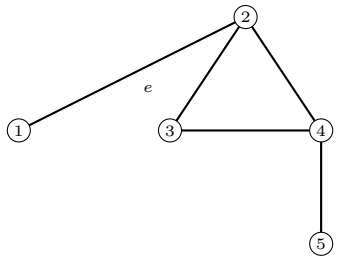

Figure 1: Vertices are ordered as labeled.

The graph in Figure 1 is reducible.

Proof. We need to handle all boards that are nearly colorable for edge e up to permutation of colors, so it will suffice to handle the following 86 boards: $01, 012, 012, 012, 01$, $01, 012, 012, 012, 02$, $01, 012, 012, 012, 03$, $01, 012, 012, 012, 23$, $01, 012, 012, 013, 03$, $01, 012, 012, 013, 23$, $01, 012, 012, 023, 03$, $01, 012, 012, 023, 13$, $01, 012, 012, 023, 23$, $01, 012, 013, 012, 01$, $01, 012, 013, 012, 02$, $01, 012, 013, 012, 03$, $01, 012, 013, 013, 01$, $01, 012, 013, 013, 02$, $01, 012, 013, 013, 03$, $01, 012, 013, 023, 01$, $01, 012, 013, 023, 02$, $01, 012, 013, 023, 03$, $01, 012, 013, 023, 12$, $01, 012, 013, 023, 13$, $01, 012, 013, 023, 23$, $01, 012, 023, 012, 01$, $01, 012, 023, 012, 02$, $01, 012, 023, 012, 03$, $01, 012, 023, 012, 12$, $01, 012, 023, 012, 23$, $01, 012, 023, 013, 01$, $01, 012, 023, 013, 02$, $01, 012, 023, 013, 03$, $01, 012, 023, 013, 12$, $01, 012, 023, 013, 13$, $01, 012, 023, 013, 23$, $01, 012, 023, 023, 01$, $01, 012, 023, 023, 02$, $01, 012, 023, 023, 03$, $01, 012, 023, 023, 12$, $01, 012, 023, 023, 23$, $01, 012, 023, 123, 01$, $01, 012, 023, 123, 02$, $01, 012, 023, 123, 03$, $01, 012, 023, 123, 12$, $01, 012, 023, 123, 13$, $01, 012, 023, 123, 23$, $01, 023, 012, 012, 01$, $01, 023, 012, 012, 02$, $01, 023, 012, 012, 03$, $01, 023, 012, 012, 12$, $01, 023, 012, 012, 23$, $01, 023, 012, 013, 01$, $01, 023, 012, 013, 02$, $01, 023, 012, 013, 03$, $01, 023, 012, 013, 12$, $01, 023, 012, 013, 13$, $01, 023, 012, 013, 23$, $01, 023, 012, 023, 01$, $01, 023, 012, 023, 02$, $01, 023, 012, 023, 03$, $01, 023, 012, 023, 12$, $01, 023, 012, 023, 23$, $01, 023, 012, 123, 01$, $01, 023, 012, 123, 02$, $01, 023, 012, 123, 03$, $01, 023, 012, 123, 12$, $01, 023, 012, 123, 13$, $01, 023, 012, 123, 23$, $01, 023, 023, 012, 01$, $01, 023, 023, 012, 12$, $01, 023, 023, 012, 13$, $01, 023, 023, 023, 01$, $01, 023, 023, 023, 02$, $01, 023, 023, 023, 12$, $01, 023, 023, 023, 23$, $01, 023, 023, 123, 01$, $01, 023, 023, 123, 12$, $01, 023, 123, 012, 01$, $01, 023, 123, 012, 02$, $01, 023, 123, 012, 03$, $01, 023, 123, 012, 12$, $01, 023, 123, 012, 13$, $01, 023, 123, 012, 23$, $01, 023, 123, 023, 02$, $01, 023, 123, 023, 12$, $01, 023, 123, 023, 23$, $01, 023, 123, 123, 02$, $01, 023, 123, 123, 12$ and $01, 023, 123, 123, 23$.

Case 1. *B is one of the 59 following boards:* $01, 012, 012, 012, 01$, $01, 012, 012, 012, 02$, $01, 012, 012, 012, 03$, $01, 012, 012, 012, 23$, $01, 012, 012, 013, 03$, $01, 012, 012, 013, 23$, $01, 012, 012, 023, 03$, $01, 012, 012, 023, 13$, $01, 012, 012, 023, 23$, $01, 012, 013, 012, 01$, $01, 012, 013, 012, 02$, $01, 012, 013, 012, 03$, $01, 012, 013, 023, 01$, $01, 012, 013, 023, 02$, $01, 012, 013, 023, 03$, $01, 012, 013, 023, 13$, $01, 012, 013, 023, 23$, $01, 012, 023, 012, 02$, $01, 012, 023, 012, 12$, $01, 012, 023, 012, 23$, $01, 012, 023, 013, 01$, $01, 012, 023, 013, 02$, $01, 012, 023, 013, 03$, $01, 012, 023, 013, 12$, $01, 012, 023, 013, 13$, $01, 012, 023, 013, 23$, $01, 012, 023, 023, 01$, $01, 012, 023, 023, 02$, $01, 012, 023, 023, 03$, $01, 012, 023, 023, 12$, $01, 012, 023, 023, 23$, $01, 012, 023, 123, 01$, $01, 012, 023, 123, 02$, $01, 012, 023, 123, 12$, $01, 012, 023, 123, 13$, $01, 012, 023, 123, 23$, $01, 023, 012, 013, 01$, $01, 023, 012, 013, 02$, $01, 023, 012, 013, 03$, $01, 023, 012, 013, 12$, $01, 023, 012, 013, 13$, $01, 023, 012, 023, 01$, $01, 023, 012, 023, 02$, $01, 023, 012, 023, 12$, $01, 023, 012, 023, 23$, $01, 023, 012, 123, 02$, $01, 023, 012, 123, 12$, $01, 023, 012, 123, 23$, $01, 023, 023, 012, 01$, $01, 023, 023, 012, 12$, $01, 023, 023, 012, 13$, $01, 023, 023, 023, 01$, $01, 023, 023, 023, 02$, $01, 023, 023, 023, 12$, $01, 023, 023, 023, 23$, $01, 023, 123, 012, 01$, $01, 023, 123, 012, 02$, $01, 023, 123, 012, 03$, $01, 023, 123, 123, 02$, $01, 023, 123, 123, 12$ *and* $01, 023, 123, 123, 23$.

In all these cases, H is immediately colorable from the lists.

Case 2. *B is one of the 15 following boards:* $01, 012, 013, 013, 01$, $01, 012, 013, 013, 02$, $01, 012, 013, 013, 03$, $01, 012, 013, 023, 12$, $01, 012, 023, 012, 01$, $01, 012, 023, 012, 03$, $01, 012, 023, 123, 03$, $01, 023, 012, 012, 12$, $01, 023, 012, 012, 23$, $01, 023, 012, 023, 01$, $01, 023, 012, 023, 03$, $01, 023, 012, 123, 01$, $01, 023, 012, 123, 13$, $01, 023, 123, 012, 12$ *and* $01, 023, 123, 023, 02$.

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{12,3}(01, 012, 013, 013, 01, 4, 5) \Rightarrow 01, 012, 023, 023, 01$ (Case 1), $01, 012, 023, 013, 02$ (Case 1).

$\mathbb{K}_{12,4}(01, 012, 013, 013, 01, 5) \Rightarrow 01, 012, 013, 023, 02$ (Case 1).

$\mathbb{K}_{23,3}(01, 012, 013, 013, 02, 4, 5) \Rightarrow 01, 012, 012, 012, 02$ (Case 1), $01, 012, 012, 013, 03$ (Case 1).

$\mathbb{K}_{23,4}(01, 012, 013, 013, 02, 5) \Rightarrow 01, 012, 013, 012, 03$ (Case 1).

$\mathbb{K}_{13,3}(01, 023, 012, 012, 12, \infty, 1, 2, 4) \Rightarrow 01, 023, 023, 012, 12$ (Case 1), $01, 012, 012, 023, 23$ (Case 1), $01, 012, 023, 012, 12$ (Case 1), $01, 023, 023, 023, 12$ (Case 1).

$\mathbb{K}_{13,1}(01, 023, 012, 012, 12, \infty) \Rightarrow 01, 012, 023, 023, 23$ (Case 1).

$\mathbb{K}_{13,2}(01, 023, 012, 012, 12, \infty) \Rightarrow 01, 012, 012, 012, 02$ (Case 1).

$\mathbb{K}_{13,4}(01, 023, 012, 012, 12, \infty) \Rightarrow 01, 023, 012, 023, 12$ (Case 1).

$\mathbb{K}_{03,3}(01, 023, 012, 012, 23, 4, 5) \Rightarrow 01, 023, 123, 123, 23$ (Case 1), $01, 023, 123, 012, 02$ (Case 1).

$\mathbb{K}_{03,4}(01, 023, 012, 012, 23, 5) \Rightarrow 01, 023, 012, 123, 02$ (Case 1).

$\mathbb{K}_{12,2}(01, 023, 012, 023, 01, 4, 5) \Rightarrow 01, 012, 013, 012, 01$ (Case 1), $01, 012, 013, 023, 03$ (Case 1).

$\mathbb{K}_{12,4}(01, 023, 012, 023, 01, 5) \Rightarrow 01, 023, 012, 013, 02$ (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{12,\infty}(01, 012, 013, 013, 03, 1, 3, 4) \Rightarrow 01, 012, 023, 023, 03$ (Case 1), $01, 012, 023, 013, 03$ (Case 1), $01, 012, 013, 023, 03$ (Case 1).

$\mathbb{K}_{02,\infty}(01, 012, 013, 023, 12, 1, 3, 5) \Rightarrow 01, 012, 023, 123, 01$ (Case 1), $01, 012, 023, 123, 02$ (Case 1), $01, 012, 013, 023, 01$ (Case 1).

$\mathbb{K}_{12,\infty}(01, 012, 023, 012, 01, 1, 3, 5) \Rightarrow 01, 012, 013, 012, 02$ (Case 1), $01, 012, 013, 012, 01$ (Case 1), $01, 012, 023, 012, 02$ (Case 1).

$\mathbb{K}_{23,\infty}(01, 012, 023, 012, 03, 2, 4, 5) \Rightarrow 01, 012, 023, 013, 02$ (Case 1), $01, 012, 023, 013, 03$ (Case 1), $01, 012, 023, 012, 02$ (Case 1).

$\mathbb{K}_{02,\infty}(01, 012, 023, 123, 03, 1, 4, 5) \Rightarrow 01, 012, 023, 013, 23$ (Case 1), $01, 012, 023, 013, 03$ (Case 1), $01, 012, 023, 123, 23$ (Case 1).

$\mathbb{K}_{12,\infty}(01, 023, 012, 023, 03, 1, 2, 4) \Rightarrow 01, 012, 013, 012, 02$ (Case 1), $01, 012, 013, 023, 02$ (Case 1), $01, 023, 012, 013, 03$ (Case 1).

$\mathbb{K}_{02,\infty}(01,023,012,123,01,1,4,5) \Rightarrow 01,023,012,013,12$ (Case 1), $01,023,012,013,01$ (Case 1), $01,023,012,123,12$ (Case 1).

$\mathbb{K}_{12,\infty}(01,023,012,123,13,1,2,5) \Rightarrow 01,012,013,023,23$ (Case 1), $01,012,013,023,02$ (Case 1), $01,023,012,123,23$ (Case 1).

$\mathbb{K}_{02,\infty}(01,023,123,012,12,1,3,5) \Rightarrow 01,023,012,013,01$ (Case 1), $01,023,012,013,13$ (Case 1), $01,023,123,012,01$ (Case 1).

$\mathbb{K}_{13,\infty}(01,023,123,023,02,1,2,4) \Rightarrow 01,012,023,012,12$ (Case 1), $01,012,023,123,12$ (Case 1), $01,023,123,012,02$ (Case 1).

Case 3. *B is one of the 5 following boards:* $01,023,012,012,01$*,* $01,023,012,012,03$*,* $01,023,023,023,01$*,* $01,023,023,123,12$ *and* $01,023,123,023,23$*.*

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{13,1}(01,023,012,012,01,\infty,2,3$ $01,012,023,023,03$ (Case 1), $01,023,023,023,02$ (Case 1), $01,012,012,023,03$ (Case 1).

$\mathbb{K}_{13,2}(01,023,012,012,01,\infty,3) \Rightarrow 01,012,012,012,01$ (Case 1), $01,012,023,012,01$ (Case 2).

$\mathbb{K}_{13,3}(01,023,012,012,01,\infty) \Rightarrow 01,023,023,012,01$ (Case 1).

$\mathbb{K}_{13,4}(01,023,012,012,01,\infty) \Rightarrow 01,023,012,023,01$ (Case 2).

$\mathbb{K}_{13,2}(01,023,012,012,03,\infty,3,4,5) \Rightarrow 01,012,012,012,03$ (Case 1), $01,012,023,012,03$ (Case 2), $01,012,012,023,03$ (Case 1), $01,012,012,012,01$ (Case 1).

$\mathbb{K}_{13,4}(01,023,012,012,03,\infty,3,5) \Rightarrow 01,023,012,023,03$ (Case 2), $01,023,023,023,02$ (Case 1), $01,023,012,023,01$ (Case 2).

$\mathbb{K}_{13,1}(01,023,012,012,03,\infty) \Rightarrow 01,012,023,023,01$ (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{13,\infty}(01,023,023,023,01,1,2,3,4,5) \Rightarrow 01,012,012,012,03$ (Case 1), $01,012,023,023,01$ (Case 1), $01,023,012,023,01$ (Case 2), $01,023,023,012,01$ (Case 1), $01,023,023,023,02$ (Case 1).

$\mathbb{K}_{12,\infty}(01,023,023,123,12,1,2,3) \Rightarrow 01,012,012,023,03$ (Case 1), $01,012,023,123,13$ (Case 1), $01,023,012,123,13$ (Case 2).

$\mathbb{K}_{03,\infty}(01,023,123,023,23,1,3,5) \Rightarrow 01,023,012,023,02$ (Case 1), $01,023,012,023,23$ (Case 1), $01,023,123,023,02$ (Case 2).

Case 4. *B is one of the 4 following boards:* $01,023,012,012,02$*,* $01,023,012,123,03$*,* $01,023,023,123,01$ *and* $01,023,123,023,12$*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{23,\infty}(01,023,012,012,02,3,4,5) \Rightarrow 01,023,012,013,03$ (Case 1), $01,023,012,013,02$ (Case 1), $01,023,012,012,03$ (Case 3).

$\mathbb{K}_{03,\infty}(01, 023, 012, 123, 03, 1, 3, 4) \Rightarrow 01, 023, 123, 012, 03$ (Case 1), $01, 023, 123, 123, 02$ (Case 1), $01, 023, 012, 012, 03$ (Case 3).

$\mathbb{K}_{03,\infty}(01, 023, 023, 123, 01, 1, 4, 5) \Rightarrow 01, 023, 023, 012, 13$ (Case 1), $01, 023, 023, 012, 01$ (Case 1), $01, 023, 023, 123, 12$ (Case 3).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{01,3}(01, 023, 123, 023, 12, 4, 5) \Rightarrow 01, 023, 023, 123, 12$ (Case 3), $01, 023, 023, 023, 02$ (Case 1).
$\mathbb{K}_{01,4}(01, 023, 123, 023, 12, 5) \Rightarrow 01, 023, 123, 123, 02$ (Case 1).

Case 5. *B is one of the 3 following boards:* $01, 023, 012, 013, 23$*,* $01, 023, 123, 012, 13$ *and* $01, 023, 123, 012, 23$*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{02,\infty}(01, 023, 012, 013, 23, 1, 4, 5) \Rightarrow 01, 023, 012, 123, 03$ (Case 4), $01, 023, 012, 123, 23$ (Case 1), $01, 023, 012, 013, 03$ (Case 1).

$\mathbb{K}_{13,\infty}(01, 023, 123, 012, 13, 1, 2, 4) \Rightarrow 01, 012, 023, 123, 03$ (Case 2), $01, 012, 023, 012, 03$ (Case 2), $01, 023, 123, 023, 12$ (Case 4).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{03,3}(01, 023, 123, 012, 23, 4, 5) \Rightarrow 01, 023, 012, 123, 23$ (Case 1), $01, 023, 012, 012, 02$ (Case 4).
$\mathbb{K}_{03,4}(01, 023, 123, 012, 23, 5) \Rightarrow 01, 023, 123, 123, 02$ (Case 1).

□