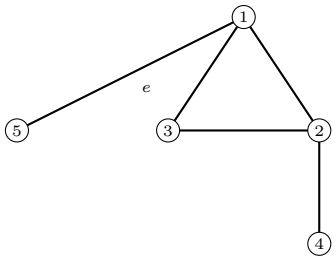

Figure 1: Vertices are ordered as labeled.

The graph in Figure 1 is reducible.

Proof. We need to handle all boards that are nearly colorable for edge e up to permutation of colors, so it will suffice to handle the following 86 boards: 012|012|012|01|01, 012|012|012|01|02, 012|012|012|01|03, 012|012|012|01|23, 012|012|012|03|01, 012|012|012|03|03, 012|012|012|03|12, 012|012|012|03|13, 012|012|013|01|01, 012|012|013|01|02, 012|012|013|01|03, 012|012|013|01|23, 012|012|013|02|01, 012|012|013|02|02, 012|012|013|02|03, 012|012|013|02|12, 012|012|013|02|13, 012|012|013|02|23, 012|012|013|03|01, 012|012|013|03|02, 012|012|013|03|03, 012|012|013|03|12, 012|012|013|03|13, 012|012|013|03|23, 012|013|012|03|01, 012|013|012|03|02, 012|013|012|03|03, 012|013|012|03|12, 012|013|012|03|13, 012|013|012|03|23, 012|013|012|23|01, 012|013|012|23|02, 012|013|012|23|03, 012|013|012|23|23, 012|013|013|01|01, 012|013|013|01|02, 012|013|013|01|03, 012|013|013|01|23, 012|013|013|02|01, 012|013|013|02|02, 012|013|013|02|03, 012|013|013|02|12, 012|013|013|02|13, 012|013|013|02|23, 012|013|013|03|01, 012|013|013|03|02, 012|013|013|03|03, 012|013|013|03|12, 012|013|013|03|13, 012|013|013|03|23, 012|013|023|01|01, 012|013|023|01|02, 012|013|023|01|03, 012|013|023|01|12, 012|013|023|01|13, 012|013|023|01|23, 012|013|023|02|01, 012|013|023|02|02, 012|013|023|02|03, 012|013|023|02|12, 012|013|023|02|13, 012|013|023|02|23, 012|013|023|03|01, 012|013|023|03|02, 012|013|023|03|03, 012|013|023|03|12, 012|013|023|03|13, 012|013|023|03|23, 012|013|023|12|01, 012|013|023|12|02, 012|013|023|12|03, 012|013|023|12|12, 012|013|023|12|13, 012|013|023|12|23, 012|013|023|13|01, 012|013|023|13|02, 012|013|023|13|03, 012|013|023|13|12, 012|013|023|13|13, 012|013|023|13|23, 012|013|023|23|01, 012|013|023|23|02, 012|013|023|23|03, 012|013|023|23|12, 012|013|023|23|13 and 012|013|023|23|23.

Case 1. *B is one of the 59 following boards:* 012|012|012|01|01, 012|012|012|01|02, 012|012|012|01|03, 012|012|012|01|23, 012|012|012|03|01, 012|012|012|03|12, 012|012|012|03|13, 012|012|013|01|01, 012|012|013|01|02, 012|012|013|01|03, 012|012|013|02|01, 012|012|013|02|12, 012|012|013|02|13, 012|012|013|03|01, 012|012|013|03|12, 012|012|013|03|13, 012|013|012|03|01, 012|013|012|03|02, 012|013|012|03|03, 012|013|012|03|12, 012|013|012|03|13, 012|013|012|23|01, 012|013|012|23|02, 012|013|012|23|03, 012|013|013|01|02, 012|013|013|01|23, 012|013|013|02|02, 012|013|013|02|12, 012|013|013|02|23, 012|013|013|03|02, 012|013|013|03|12, 012|013|013|03|23, 012|013|023|01|01, 012|013|023|01|02, 012|013|023|01|03, 012|013|023|01|12, 012|013|023|01|13, 012|013|023|01|23, 012|013|023|02|01, 012|013|023|02|02, 012|013|023|02|03, 012|013|023|02|12, 012|013|023|02|23, 012|013|023|03|01, 012|013|023|03|02, 012|013|023|03|03, 012|013|023|03|12, 012|013|023|03|23, 012|013|023|12|01, 012|013|023|12|12, 012|013|023|12|13, 012|013|023|13|01, 012|013|023|13|02, 012|013|023|13|03, 012|013|023|13|12, 012|013|023|13|13, 012|013|023|23|01, 012|013|023|23|02 *and* 012|013|023|23|03.

In all these cases, H is immediately colorable from the lists.

Case 2. *B is one of the 15 following boards:* 012|012|013|02|02*,* 012|012|013|02|03*,* 012|012|013|02|23*,* 012|012|013|03|02*,* 012|012|013|03|03*,* 012|013|013|01|01*,* 012|013|013|02|01*,* 012|013|013|02|13*,* 012|013|013|03|01*,* 012|013|013|03|13*,* 012|013|023|03|13*,* 012|013|023|12|02*,* 012|013|023|13|23*,* 012|013|023|23|12 *and* 012|013|023|23|23*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{12,\infty}(012|012|013|02|02, 3, 4, 5) \Rightarrow$ 012|012|013|01|01, 012|012|013|01|02, 012|012|013|02|01 (Case 1).

$\mathbb{K}_{13,\infty}(012|012|013|02|03, 1, 2, 5) \Rightarrow 012|013|023|01|02, 012|013|023|01|03, 012|012|013|02|01$ (Case 1).

$\mathbb{K}_{13,\infty}(012|012|013|02|23, 1, 2, 5) \Rightarrow 012|013|023|01|12, 012|013|023|01|13, 012|012|013|02|12$ (Case 1).

$\mathbb{K}_{13,\infty}(012|012|013|03|02, 1, 2, 4) \Rightarrow 012|013|023|02|01, 012|013|023|03|01, 012|012|013|01|02$ (Case 1).

$\mathbb{K}_{12,\infty}(012|013|013|03|01, 2, 3, 5) \Rightarrow 012|013|023|03|02, 012|013|023|03|01, 012|013|013|03|02$ (Case 1).

$\mathbb{K}_{01,\infty}(012|013|023|03|13, 3, 4, 5) \Rightarrow 012|013|023|13|03, 012|013|023|13|13, 012|013|023|03|03$ (Case 1).

$\mathbb{K}_{01,\infty}(012|013|023|12|02, 3, 4, 5) \Rightarrow 012|013|023|02|12, 012|013|023|02|02, 012|013|023|12|12$ (Case 1).

$\mathbb{K}_{03,\infty}(012|013|023|13|23, 1, 4, 5) \Rightarrow 012|013|023|01|02, 012|013|023|01|23, 012|013|023|13|02$ (Case 1).

$\mathbb{K}_{02,\infty}(012|013|023|23|12, 2, 4, 5) \Rightarrow 012|013|023|03|01, 012|013|023|03|12, 012|013|023|23|01$ (Case 1).

$\mathbb{K}_{02,\infty}(012|013|023|23|23, 2, 4, 5) \Rightarrow 012|013|023|03|03, 012|013|023|03|23, 012|013|023|23|03$ (Case 1).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{13,2}(012|012|013|03|03, 4, 5) \Rightarrow$ 012|013|023|02|03, 012|013|023|03|02 (Case 1).
$\mathbb{K}_{13,4}(012|012|013|03|03, 5) \Rightarrow 012|012|013|01|01$ (Case 1).

$\mathbb{K}_{12,3}(012|013|013|01|01, 4, 5) \Rightarrow$ 012|013|023|02|01, 012|013|023|01|02 (Case 1).
$\mathbb{K}_{12,4}(012|013|013|01|01, 5) \Rightarrow 012|013|013|02|02$ (Case 1).

$\mathbb{K}_{23,2}(012|013|013|02|01, 3, 4) \Rightarrow$ 012|012|012|01|02, 012|012|013|03|01 (Case 1).
$\mathbb{K}_{23,3}(012|013|013|02|01, 4) \Rightarrow 012|013|012|03|01$ (Case 1).

$\mathbb{K}_{12,3}(012|013|013|02|13,4,5) \Rightarrow 012|013|023|01|13,\ 012|013|023|02|23$ (Case 1).
$\mathbb{K}_{12,4}(012|013|013|02|13,5) \Rightarrow 012|013|013|01|23$ (Case 1).

$\mathbb{K}_{23,2}(012|013|013|03|13,\infty,3,5) \Rightarrow 012|012|013|03|13,\ 012|012|012|03|13,\ 012|012|013|03|12$ (Case 1).
$\mathbb{K}_{23,3}(012|013|013|03|13,\infty,5) \Rightarrow 012|013|012|03|13,\ 012|013|012|03|12$ (Case 1).
$\mathbb{K}_{23,1}(012|013|013|03|13,\infty) \Rightarrow 012|012|012|01|02$ (Case 1).
$\mathbb{K}_{23,5}(012|013|013|03|13,\infty) \Rightarrow 012|013|013|03|12$ (Case 1).

Case 3. *B is one of the 5 following boards:* 012|012|012|03|03*,* 012|012|013|01|23*,* 012|013|012|03|23*,* 012|013|013|02|03 *and* 012|013|013|03|03*.*

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{23,\infty}(012|012|012|03|03,1,2,3,4,5) \Rightarrow 012|013|013|02|02,\ 012|013|012|03|03,$ 012|012|013|03|03, 012|012|012|01|03, 012|012|012|03|01 (Case 1 and 2).

$\mathbb{K}_{12,\infty}(012|012|013|01|23,3,4,5) \Rightarrow 012|012|013|02|13,\ 012|012|013|02|23,\ 012|012|013|01|03$ (Case 1 and 2).

$\mathbb{K}_{03,\infty}(012|013|012|03|23,1,3,5) \Rightarrow 012|013|023|13|12,\ 012|013|023|13|23,\ 012|013|012|03|02$ (Case 1 and 2).

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{23,2}(012|013|013|02|03,\infty,1,3,$ 012|012|013|02|03, 012|013|012|03|02, 012|012|012|01|03, 012|012|013|03|03, 012|012|013|02|02 (Case 1 and 2).
$\mathbb{K}_{23,5}(012|013|013|02|03,\infty,4) \Rightarrow 012|013|013|02|02,\ 012|013|013|03|02$ (Case 1).
$\mathbb{K}_{23,1}(012|013|013|02|03,\infty) \Rightarrow 012|012|012|03|01$ (Case 1).

$\mathbb{K}_{23,5}(012|013|013|03|03,\infty,1,2,3) \Rightarrow 012|013|013|03|02,\ 012|012|012|01|03,\ 012|012|013|03|02,$ 012|013|012|03|02 (Case 1 and 2).
$\mathbb{K}_{23,1}(012|013|013|03|03,\infty) \Rightarrow 012|012|012|01|01$ (Case 1).
$\mathbb{K}_{23,2}(012|013|013|03|03,\infty) \Rightarrow 012|012|013|03|03$ (Case 2).
$\mathbb{K}_{23,3}(012|013|013|03|03,\infty) \Rightarrow 012|013|012|03|03$ (Case 1).

Case 4. *B is one of the 4 following boards:* 012|012|013|03|23*,* 012|013|012|23|23*,* 012|013|013|01|03 *and* 012|013|023|12|23*.*

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{23,2}(012|012|013|03|23,3,4) \Rightarrow$ 012|013|012|03|23, 012|013|013|02|23 (Case 1 and 3).
$\mathbb{K}_{23,3}(012|012|013|03|23,4) \Rightarrow 012|012|012|01|03$ (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{12,\infty}(012|013|012|23|23,2,4,5) \Rightarrow 012|013|012|03|03,\ 012|013|012|03|23,$ 012|013|012|23|03 (Case 1 and 3).

$\mathbb{K}_{12,\infty}(012|013|013|01|03, 2, 3, 4) \Rightarrow 012|013|023|02|03, 012|013|023|01|03, 012|013|013|02|03$ (Case 1 and 3).

$\mathbb{K}_{12,\infty}(012|013|023|12|23, 2, 3, 5) \Rightarrow 012|013|013|02|03, 012|013|013|02|23, 012|013|023|12|13$ (Case 1 and 3).

Case 5. *B is one of the 3 following boards:* $012|013|023|02|13$*,* $012|013|023|12|03$ *and* $012|013|023|23|13$*.*

Each of the following boards can be handled by a single Kempe change. $\mathbb{K}_{12,3}(012|013|023|02|13, 4, 5) \Rightarrow 012|013|013|01|03$, $012|013|013|02|23$ (Case 1 and 4).
$\mathbb{K}_{12,4}(012|013|023|02|13, 5) \Rightarrow 012|013|023|01|23$ (Case 1).

Each of the following boards can be handled by a single Kempe change that has an endpoint at infinity. $\mathbb{K}_{02,\infty}(012|013|023|12|03, 2, 4, 5) \Rightarrow 012|013|023|01|23$, $012|013|023|01|03$, $012|013|023|12|23$ (Case 1 and 4).

$\mathbb{K}_{23,\infty}(012|013|023|23|13, 1, 2, 5) \Rightarrow 012|012|013|03|02, 012|012|013|03|23, 012|013|023|23|12$ (Case 2 and 4).

□