Capstone report 1

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Introduction

Solving evolution partial differential equations (PDEs) typically requires ad-hoc methods and special treatments. The recently discovered "Fokas method"[1] allows solving many of these equations algorithmically. The first goal of this project concerns implementing the Fokas method in the Julia mathematical programming language. The first step in the implementation is to construct a valid adjoint boundary condition from a given boundary condition.

We begin by defining the objects involved in the construction.

Preliminary definitions[2]

A linear differential operator L of order n (n¿1) is defined by

$$Lx = p_0 x^{(n)} + p_1 x^{(n-1)} + \cdots + p_{n-1} x' + p_n x,$$

where the p_k are complex-valued functions of class C^{n-k} on a closed bounded interval $[a, b]$ (i.e., derivatives $p_k, p_k', \ldots, p_k^{(n-k)}$ exist on $[a, b]$ and are continuous) and $p_0(t) \neq 0$ on $[a, b]$.

A homogeneous boundary condition is a set of equations of the type

$$\sum_{k=1}^{n} (M_{jk} x^{(k-1)}(a) + N_{jk} x^{(k-1)}(b)) = 0 \quad (j = 1, \ldots, m) \tag{1}$$

where M_{jk}, N_{jk} are complex constants.

A homogeneous boundary-value problem concerns finding the solutions of

$$Lx = 0$$

on $[a, b]$ which satisfy some homogeneous boundary conditions defined above.

A semibilinear form is a complex-valued function $\mathcal{S}$ defined for pairs of vectors $f = (f_1, \ldots, f_k)$, $g = (g_1, \ldots, g_k)$ satisfying

$$\mathcal{S}(\alpha f + \beta g, h) = \alpha \mathcal{S}(f, h) + \beta \mathcal{S}(g, h)$$
$$\mathcal{S}(f, \alpha g + \beta h) = \bar{\alpha} \mathcal{S}(f, g) + \bar{\beta} \mathcal{S}(f, h)$$

for any complex numbers α, β and vectors f, g, h.

With these definitions, we now outline the construction, introducing relevant theorems as per need.

Construction outline

References

[1] Emine Kesici, Beatrice Pelloni, Tristan Pryer, and David Smith. A numerical implementation of the unified Fokas transform for evolution problems on a finite interval. 2016.

[2] Earl A. Coddington and Norman Levinson. *Theory of Ordinary Differential Equations.* McGraw-Hill Publishing, New York, 1977.