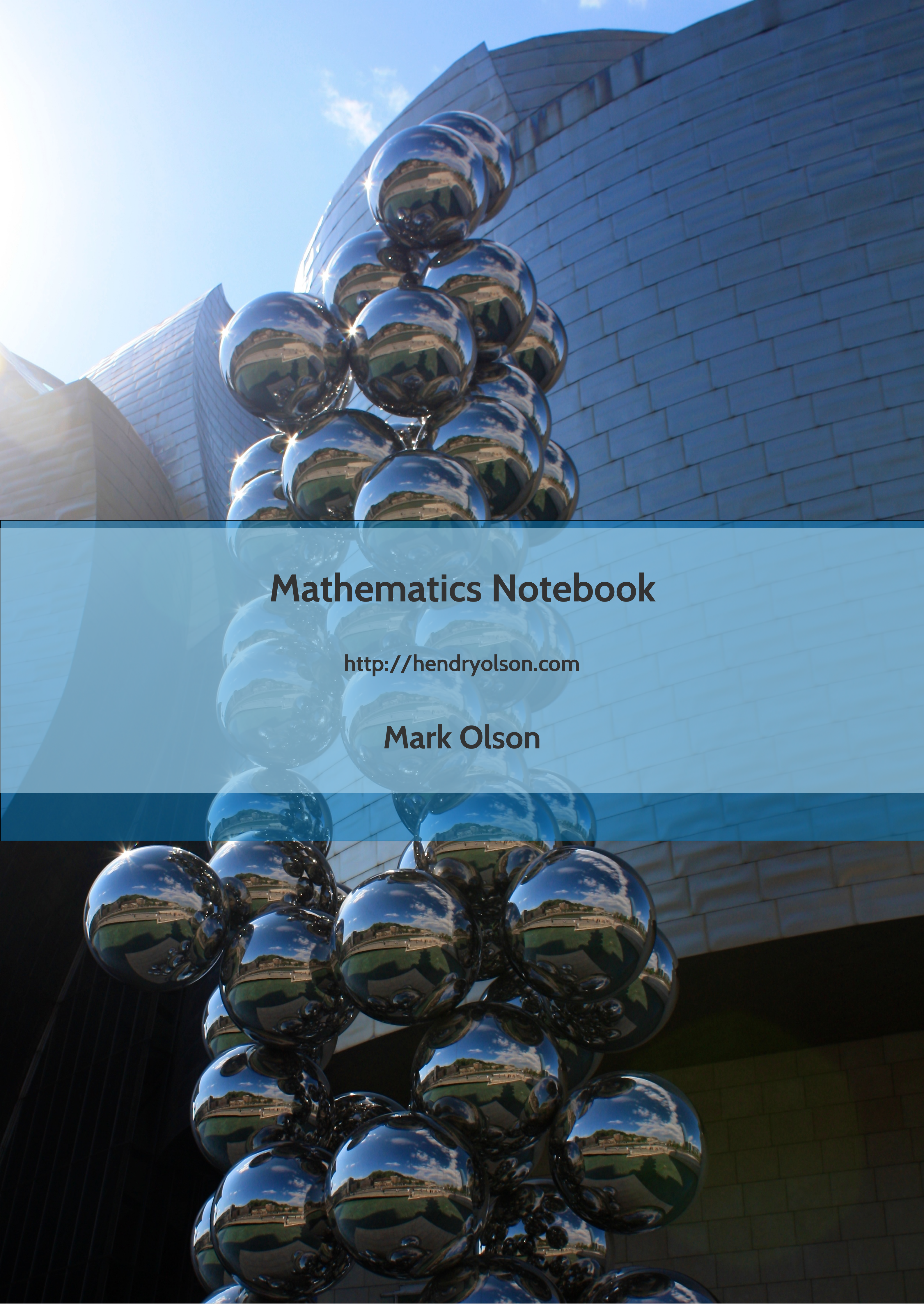

Mathematics Notebook

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Contents

MYP

1. Algebraic Expressions

1.1 Expressions

Essential Questions 1.1

1. What is an algebraic expression?

1.2 Polynomial Expressions

	Arithmetic	Polynomial	Algebraic
Constant	Yes	Yes	Yes
Factorial	Yes	Yes	Yes
Variable: parameter/coefficient	Yes	Yes	Yes
Variable: unknown/indeterminate	No	Yes	Yes
Power with $\mathbb{Z}^+$ exponent	No	Yes	Yes
Power with $\mathbb{Z}$ exponent	No	No	Yes
n-th root	No	No	Yes
Power with $\mathbb{Q}$ exponent	No	No	Yes

Table 1.1: Names of different types of expressions

Definition 1.2.1 – Operation of Exponentiation (OOE).

$$\underbrace{\underbrace{b}_{base}{}^{\overbrace{m}^{Exponent}}}_{Power} \tag{1.1}$$

Definition 1.2.2 – Juxtaposition to Center-Dot (JTC).

$$ab = a \cdot b \tag{1.2}$$

Definition 1.2.3 – Center-Dot to Justapostion (CTJ).

$$a \cdot b = ab \tag{1.3}$$

Definition 1.2.4 – Commutative Property of Multiplication (CPM).

$$a \cdot b = b \cdot a \tag{1.4}$$

Definition 1.2.5 – Multiplicative Inverse (MI).

$$a \cdot \frac{1}{a} = 1 \tag{1.5a}$$

$$a \cdot a^{-1} = 1 \tag{1.5b}$$

Definition 1.2.6 – Associative Property of Multiplication (APM).

$$a \cdot b \cdot c = (a \cdot b) \cdot c \tag{1.6a}$$

$$a \cdot b \cdot c = a \cdot (b \cdot c) \tag{1.6b}$$

Powers

Rule 1.2.1 – Power of a Quotient of Powers (PoQPo).

$$\left(\frac{a^m}{b^n}\right)^k = \frac{a^{m\cdot k}}{b^{n\cdot k}} \tag{1.7a}$$

$$\frac{a^{m\cdot k}}{b^{n\cdot k}} = \left(\frac{a^m}{b^n}\right)^k \tag{1.7b}$$

Rule 1.2.2 – Power of a Product of Powers (PoPrPo).

$$(a^m \cdot b^n)^k = a^{m\cdot k} \cdot b^{n\cdot k} \tag{1.8a}$$

$$a^{m\cdot k} \cdot b^{n\cdot k} = (a^m \cdot b^n)^k \tag{1.8b}$$

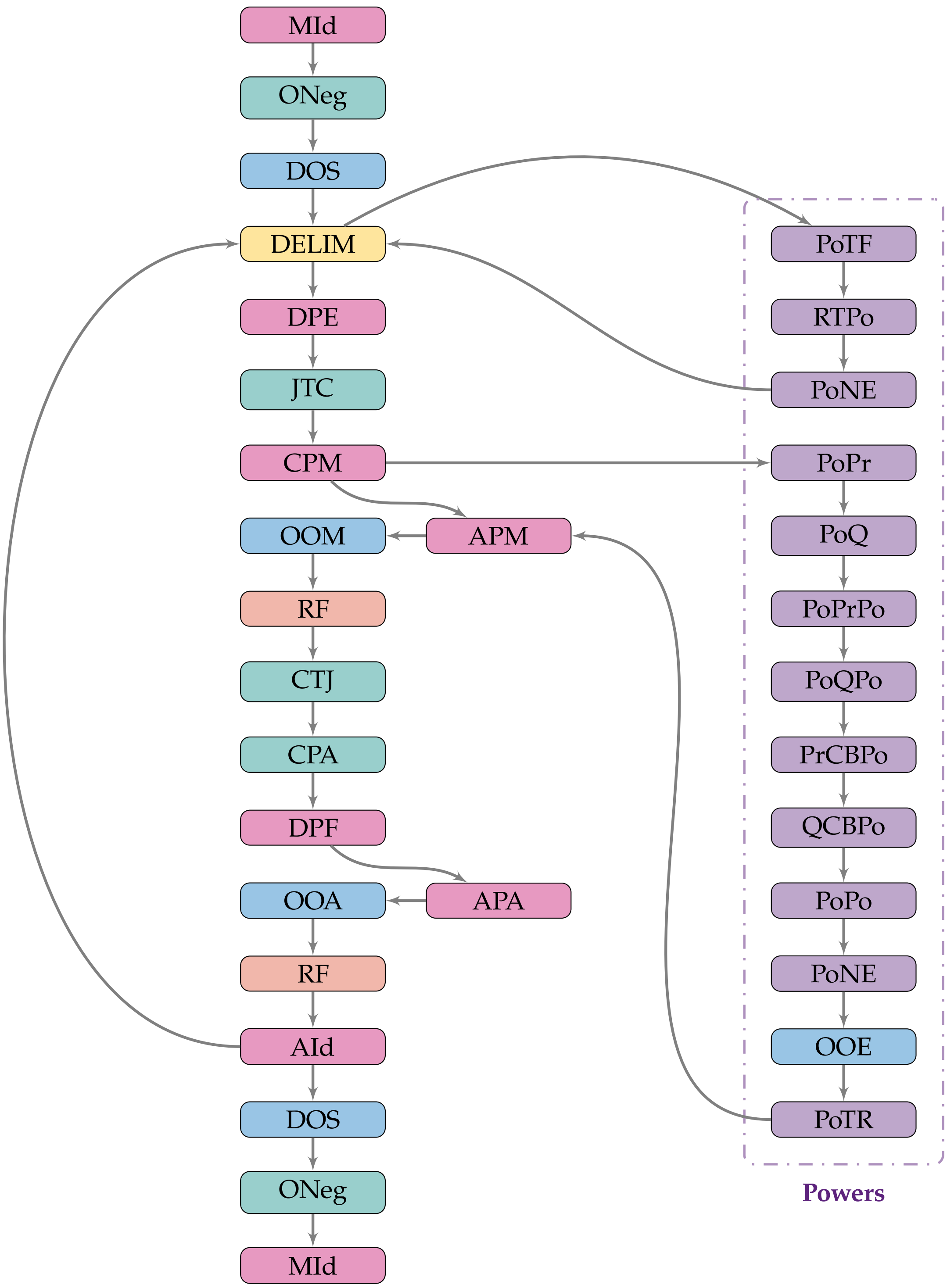

Figure 1.1: Simplifying Expressions Workflow:
■ Property, ■ Operation, ■ Notation, ■ Powers, ■ Delimiters, ■ Process, ■ Not Used

Definition 1.2.7 – Power To Factor (PoTF).

$$a^{n} = a_1 \cdot a_2 \cdot \ldots \cdot a_{n-1} \cdot a_n \tag{1.9}$$

Definition 1.2.8 – Factor To Power (FTPo).

$$a_1 \cdot a_2 \cdot \ldots \cdot a_{n-1} \cdot a_n = a^{n} \tag{1.10}$$

Definition 1.2.9 – Power Inverse (PoI).

$$(b^m)^{\frac{1}{m}} = b \tag{1.11a}$$

Definition 1.2.10 – Power Inverse (PoId).

$$1 = b^0 \tag{1.12a}$$

$$b^0 = 1 \tag{1.12b}$$

Notation 1.1 (Radical To Power (RTPo)).

$$\sqrt[m]{b^n} = b^{\frac{n}{m}} \tag{1.13}$$

Notation 1.2 (Power To Radical (PoTR)).

$$b^{\frac{n}{m}} = \sqrt[m]{b^n} \tag{1.14}$$

1.2.1 Monomials of Like Terms

Definition 1.2.11 – Additive Inverse (AI).

$$a + (-a) = 0 \tag{1.15a}$$

1.2.2 Surds

Example 1.1 – id:20141108-085327.

Simplify $2\sqrt{2} - \frac{\left(\sqrt{2}\right)^3}{3} - \left(2\left(-\sqrt{2}\right) - \frac{\left(-\sqrt{2}\right)^3}{3}\right)$

S

Solution:

$$2\sqrt{2} - \frac{1\left(1\sqrt{2}\right)^3}{3} - 1\left(2\left(-1\sqrt{2}\right) - \frac{1\left(-1\sqrt{2}\right)^3}{3}\right) \qquad \text{MId(??)}$$

$$2\sqrt{2} - \frac{1\left(1\sqrt{2}\right)^3}{3} - 1\left(2\left(\neg 1\sqrt{2}\right) - \frac{1\left(\neg 1\sqrt{2}\right)^3}{3}\right) \qquad \text{ONeg(??)}$$

$$2\sqrt{2} + \frac{\neg 1\left(1\sqrt{2}\right)^3}{3} + \neg 1\left(2\left(\neg 1\sqrt{2}\right) + \frac{\neg 1\left(\neg 1\sqrt{2}\right)^3}{3}\right) \qquad \text{DOS(??)}$$

$$2\cdot 2^{1/2} + \frac{\neg 1\left(1\cdot 2^{1/2}\right)^3}{3} + \neg 1\left(2\left(\neg 1\cdot 2^{1/2}\right) + \frac{\neg 1\left(\neg 1\cdot 2^{1/2}\right)^3}{3}\right) \qquad \text{RTPo(1.13)}$$

$$2\cdot 2^{1/2} + \frac{\neg 1\left(1\cdot 2^{1/2}\right)^3}{3} + \neg 1\left(2\cdot\neg 1\cdot 2^{1/2} + \frac{\neg 1\left(\neg 1\cdot 2^{1/2}\right)^3}{3}\right) \qquad \text{JTC(1.2)}$$

$$2\cdot 2^{1/2} + \frac{\neg 1\cdot 1\cdot 2^{3/2}}{3} + \neg 1\left(2\cdot\neg 1\cdot 2^{1/2} + \frac{\neg 1\cdot\neg 1\cdot 2^{3/2}}{3}\right) \qquad \text{PoPrPo(1.8a)}$$

$$2\cdot 2^{1/2} + \frac{\neg 1\cdot 1\cdot 2^{2/2}\cdot 2^{1/2}}{3} + \neg 1\left(2\cdot\neg 1\cdot 2^{1/2} + \frac{\neg 1\cdot\neg 1\cdot 2^{2/2}\cdot 2^{1/2}}{3}\right) \qquad \text{PrCBPo(??)}$$

$$2\cdot 2^{1/2} + \frac{\neg 1\cdot 1\cdot 2\cdot 2^{1/2}}{3} + \neg 1\left(2\cdot\neg 1\cdot 2^{1/2} + \frac{\neg 1\cdot\neg 1\cdot 2\cdot 2^{1/2}}{3}\right) \qquad \text{MId(??)}$$

$$2\cdot\sqrt{2} + \frac{\neg 1\cdot 1\cdot 2\cdot\sqrt{2}}{3} + \neg 1\left(2\cdot\neg 1\cdot\sqrt{2} + \frac{\neg 1\cdot\neg 1\cdot 2\cdot\sqrt{2}}{3}\right) \qquad \text{PoTR(1.14)}$$

$$2\cdot\sqrt{2} + \frac{\neg 1\cdot 1\cdot 2\cdot\sqrt{2}}{3} + \neg 1\cdot 2\cdot\neg 1\cdot\sqrt{2} + \frac{\neg 1\cdot\neg 1\cdot\neg 1\cdot 2\cdot\sqrt{2}}{3} \qquad \text{DPE(??)}$$

$$2\cdot\sqrt{2} + \frac{\neg 2\cdot\sqrt{2}}{3} + 2\cdot\sqrt{2} + \frac{\neg 2\cdot\sqrt{2}}{3} \qquad \text{OOM(??)}$$

$$2\sqrt{2} + \frac{\neg 2\sqrt{2}}{3} + 2\sqrt{2} + \frac{\neg 2\sqrt{2}}{3} \qquad \text{CTJ(1.3)}$$

$$\left(2 + \frac{\neg 2}{3} + 2 + \frac{\neg 2}{3}\right)\sqrt{2} \qquad \text{DPF(??)}$$

$$\frac{8}{3}\sqrt{2} \qquad \text{OOA(??)}$$

D

Dependencies:example **??**-20141108-083108

II

DP-SL

2. Differentiation

2.1 Limit of the Difference Quotient

Definition 2.1.1 – Derivative. The derivative of a function $f(x)$ with respect to the variable x is defined as

$$f'(x) \equiv \lim_{\Delta x \to 0} \underbrace{\frac{f(x+\Delta x) - f(x)}{\Delta x}}_{\text{Difference Quotient}} \tag{2.1}$$

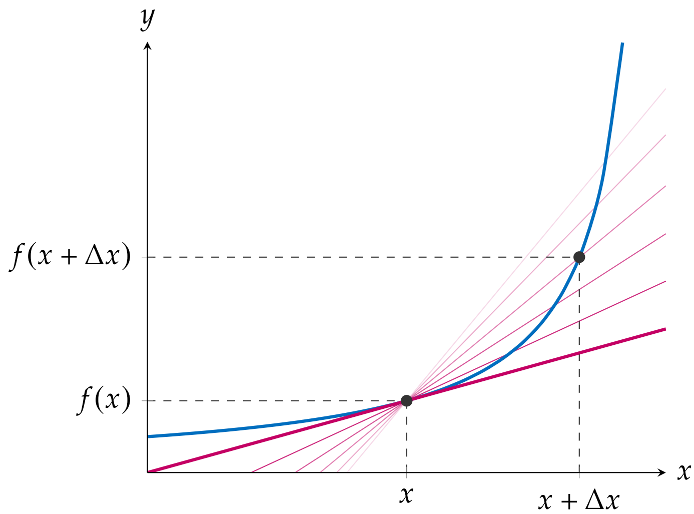

Figure 2.1: [**mooculus:textbook**]

Example 2.1 – id:20141219-212546.

Differentiate the function $f(x) = 5$

S

Solution:

$$
\begin{aligned}
f(x) &= 5x^0 && \text{PoID(1.12a)} \\
f'(x) &= \lim_{\Delta x \to 0} \frac{5[x + \Delta x]^0 - 5[x]^0}{\Delta x} && \text{SPE(??)\&DBFP(2.1)} \\
f'(x) &= \lim_{\Delta x \to 0} \frac{5(1) - 5(1)}{\Delta x} && \text{PoID(1.12b)} \\
f'(x) &= \lim_{\Delta x \to 0} 0 && \text{OOM(??)} \\
f'(x) &= 0
\end{aligned}
$$

■

2.2 Derivative of a Monomial Functions

Rule 2.2.1 – Derivative of a Constant (DC).

$$[c]' = 0 \tag{2.2}$$

$$\frac{d}{dx}[c] = 0 \tag{2.3}$$

Rule 2.2.2 – Derivative of a Constant Multiple (DCM).

$$[cf(x)]' = c[f(x)]' \tag{2.4}$$

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)] \tag{2.5}$$

Rule 2.2.3 – Derivative of a Power (DPo).

$$[x^n]' = nx^{n-1} \tag{2.6}$$

$$\frac{d}{dx}[x^n] = nx^{n-1} \tag{2.7}$$

Example 2.2 – id:20141124-153017.

Differentiate $f(x) = -3$

S

Solution:

$f'(x) = [-3]'$ SPE(**??**)
$f'(x) = 0$ DC(2.2)

D

Dependencies:
example 2.4-20141124-152503

Example 2.3 – id:20141124-141850.

Differentiate $f(x) = x^2$

S

Solution:

$f'(x) = [x^2]'$ SPE(**??**)
$f'(x) = 2x^{2-1}$ DPo(2.6)
$f'(x) = 2x^1$ OOA(**??**)
$f'(x) = 2x$ MId(**??**)

S

Less Steps Solution:

$f'(x) = 2x$ DPo(2.6)

D

Dependencies:
example 2.4-20141124-152503

2.3 Derivative of Polynomial Functions

Rule 2.3.1 – Derivative of a Sum (DS).

$$[f(x) + g(x)]' = f'(x) + g'(x) \tag{2.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x) + g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}[f(x)] + \frac{\mathrm{d}}{\mathrm{d}x}[g(x)] \tag{2.9}$$

Example 2.4 – id:20141124-152503.

Differentiate $f(x) = x^2 - 3$

S

Solution:

$f(x) = x^2 + \neg 3$	DOS(**??**)
$f'(x) = [x^2 + \neg 3]'$	SPE(**??**)
$f'(x) = [x^2]' + [\neg 3]'$	DS(2.8)
$f'(x) = [x^2]' + 0$	DC(2.2)
$f'(x) = [x^2]'$	AId(**??**)
$f'(x) = 2x$	DPo(2.6) goto 2.3

S

Less Steps Solution:

$f(x) = 2x^2$	DPo(2.6)&DC(2.6)

D

Dependencies:
example 2.14-20141124-205219

Example 2.5 – id:20141128-151834.

Differentiate $f(x) = 3x^2 - 6x + 4$

S

Solution:

$$
\begin{aligned}
f(x) &= 3x^2 + \neg 6x + 4 && \text{DOS(??)} \\
f'(x) &= [3x^2 + \neg 6x + 4]' && \text{SPE(??)} \\
f'(x) &= [3x^2]' + [6x]' + [4]' && \text{DS(2.8)} \\
f'(x) &= [3x^2]' + [6x]' + 0 && \text{DC(2.2)} \\
f'(x) &= [3x^2]' + [6x]' && \text{AId(??)} \\
f'(x) &= 3[x^2]' + 6[x]' && \text{DCM(2.4)} \\
f'(x) &= 3\,(2x) + 6\,(1) && \text{DPo(2.6)} \\
f'(x) &= 6x + 6 && \text{OOM(??)}
\end{aligned}
$$

S

$$
f'(x) = 6x + 6 \qquad \text{DS(2.8)}
$$

■

2.3.1 Finding the vertex of a quadratic function using differentiation.

We can find the vertex of a quadratic function, $f(x)$ using differentiation by:

1. Differentiate the function: Find $f'(x)$.
2. Set the derivative equal to zero: $f'(x) = 0$.
3. Find the abscissa of the vertex by solving the equation $f'(x) = 0$ for x to find the critical x value: $x = k$.
4. Find the ordinate of the vertex by substituting the value of critical value $x = k$ into the function $f(x)$: Evaluate $f(k)$

Example 2.6 – id:20150923-152515.
Find the vertex of the parabola $y = x^2 - 2x - 6$ using differentiation.

S

1. Differentiate the function.

$$
\begin{aligned}
f(x) &= x^2 - 2x - 6 && \\
f(x) &= x^2 + \neg 2x + \neg 6 && \text{DOS(??)} \\
[f(x)]' &= [x^2 + \neg 2x + \neg 6]' && \text{SPE(??)} \\
f'(x) &= [x^2]' + [\neg 2x]' + [\neg 6]' && \text{DS(2.8)} \\
f'(x) &= [x^2]' + \neg 2[x]' + [\neg 6]' && \text{DCM(2.4)} \\
f'(x) &= 2x + \neg 2 + [\neg 6]' && \text{DPo(2.6)} \\
f'(x) &= 2x + \neg 2 + 0 && \text{DC(2.2)} \\
f'(x) &= 2x + \neg 2 && \text{AId(??)} \\
f'(x) &= 2x - 2 && \text{DOS(??)}
\end{aligned}
$$

2 and 3. Set the derivative equal to zero and solve for x

$$\begin{aligned} 2x - 2 &= 0 \\ x &= 1 \end{aligned}$$

4. Find the value of $f(1)$

$$\begin{aligned} f(x) &= x^2 - 2x - 6 \\ f(1) &= [1]^2 - 2[1] - 6 && \text{SPE(??)} \\ f(1) &= -7 && \text{Evaluate} \end{aligned}$$

The vertex of this parabola is the point $(1, -7)$ ■

Rule 2.3.2 – Derivative of a Product (DPr).

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x) \tag{2.10}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}[f(x)]g(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}[g(x)] \tag{2.11}$$

Example 2.7 – id:20141209-144203.

Differentiate $f(x) = x^2(2x+4)$

S

Solution:

$$\begin{aligned} f'(x) &= [x^2(2x+4)]' && \text{SPE(??)} \\ f'(x) &= [x^2]'(2x+4) + x^2[2x+4]' && \text{DPr(2.10)} \\ f'(x) &= [x^2]'(2x+4) + x^2[2x] + [4]' && \text{DS(2.8)} \\ f'(x) &= [x^2]'(2x+4) + x^2 \cdot 2[x] + [4]' && \text{DCM(2.4)} \\ f'(x) &= 2x(2x+4) + x^2 \cdot 2 \cdot 1 + [4]' && \text{DPo(2.6)} \\ f'(x) &= 2x(2x+4) + x^2 \cdot 2 \cdot 1 + 0 && \text{DC(2.2)} \\ f'(x) &= 6x^2 + 8x && \text{simplify goto ??} \end{aligned}$$

■

Example 2.8 – id:20141209-142321.

Differentiate $f(x) = x^2\cos(x)$

S

Solution:

$$
\begin{aligned}
f'(x) &= [x^2 \cos(x)]' && \text{SPE(??)}\\
f'(x) &= [x^2]' \cos(x) + x^2[\cos(x)]' && \text{DPr(2.10)}\\
f'(x) &= 2x \cos(x) + x^2[\cos(x)]' && \text{DPo(2.6)}\\
f'(x) &= 2x \cos(x) + x^2(-1 \sin(x)) && \text{DCos(2.14)}\\
f'(x) &= 2x \cos(x) - x^2 \sin x && \text{OOM(??)}
\end{aligned}
$$

■

2.4 Derivative of Trigonometric Functions

Rule 2.4.1 – Derivative of Sine (DSin).

$$[\sin(x)]' = \cos(x) \tag{2.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\sin(x)] = \cos(x) \tag{2.13}$$

Rule 2.4.2 – Derivative of Cosine (DCos).

$$[\cos(x)]' = -\sin(x) \tag{2.14}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\cos(x)] = -\sin(x) \tag{2.15}$$

Example 2.9 – id:20150910-115935.

Differentiate $f(x) = \sin(x)\cos(x)$

S

Solution:

$$
\begin{aligned}
f'(x) &= [\sin(x) \cos(x)]' && \text{SPE(??)}\\
f'(x) &= [\sin(x)]' \cos(x) + \sin(x)[\cos(x)]' && \text{DPr(2.10)}\\
f'(x) &= \cos(x)\cos(x) + \sin(x)[\cos(x)]' && \text{DSin(2.12)}\\
f'(x) &= \cos(x)\cos(x) + \sin(x)(-\sin(x)) && \text{DCos(2.14)}\\
f'(x) &= \cos^2(x) - \sin^2(x) && \text{simplify goto ??}
\end{aligned}
$$

■

Example 2.10 – id:20141209-151354.

Differentiate $f(x) = \sin(x)\sin(x)$

S

Solution:

$$
\begin{aligned}
f'(x) &= [\sin(x)\sin(x)]' && \text{SPE(??)}\\
f'(x) &= [\sin(x)]'\sin(x) + \sin(x)[\sin(x)] && \text{DPr(2.10)}\\
f'(x) &= \cos(x)\sin(x) + \sin(x)\cos(x) && \text{DSin(2.12)}\\
f'(x) &= \cos(x)\sin(x) + \cos(x)\sin(x) && \text{CPM(1.4)}\\
f'(x) &= 2\cos(x)\sin(x) && \text{OOA(??)}
\end{aligned}
$$

■

2.5 Derivative of Rational Functions

Rule 2.5.1 – Derivative of a Quotient (DQ).

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \tag{2.16}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{\mathrm{d}}{\mathrm{d}x}[f(x)]\,g(x) - f(x)\frac{\mathrm{d}}{\mathrm{d}x}[g(x)]}{[g(x)]^2} \tag{2.17}$$

2.6 Derivative of Exponential Functions

2.7 Derivative of Logarithmic Functions

Rule 2.7.1 – Derivative of a Natural Logarithm (DNL).

$$[\ln x]' = \frac{1}{x} \tag{2.18}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln x] = \frac{1}{x} \tag{2.19}$$

2.8 Derivative of Composite Functions

Rule 2.8.1 – Derivative of a Composite Function (DComp).

$$[f(g(x))]' = [g(x)]'\,[f(g(x))]' \tag{2.20}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] = \frac{\mathrm{d}}{\mathrm{d}x}[g(x)]\,\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] \tag{2.21}$$

Example 2.11 – id:20141124-203850.

Differentiate $y = \ln(3x)$

S

Solution:

After identifying that $y = \ln(3x)$ is a composite function, we let $u = 3x$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(2.21)}$$

We need to find the factors $\frac{dy}{du}$ and $\frac{du}{dx}$.

$$y = \ln(u) \qquad u = 3x$$

$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 3$$

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} && \text{DComp(2.21)} \\
\frac{dy}{dx} &= \frac{1}{u} \cdot \frac{du}{dx} && \text{DNL(2.18)} \\
\frac{dy}{dx} &= \frac{1}{u} \cdot 3 && \text{DPo(2.6)} \\
\frac{dy}{dx} &= \frac{1}{3x} \cdot 3 \\
\frac{dy}{dx} &= \frac{3}{3x} && \text{OOM(??)} \\
\frac{dy}{dx} &= \frac{1}{x}
\end{aligned}$$

D

Dependencies:
example 2.14-20141124-205219

Example 2.12 – id:20141128-160248.

Differentiate $y = \ln(3x^2 - 6x + 4)$

S

Solution: After identifying that $y = \ln(3x^2 - 6x + 4)$ is a composite function, we let $u = 3x^2 - 6x + 4$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(2.21)}$$

We need to find the factors $\frac{dy}{du}$ and $\frac{du}{dx}$.

$$y = \ln(u) \qquad u = 3x^2 - 6x + 4$$
$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 6x - 6 \qquad \text{goto } 2.5$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(2.21)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} \qquad \text{DNL(2.18)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot (6x - 6) \qquad \text{DPo(2.6)}$$
$$\frac{dy}{dx} = \frac{1}{3x^2 - 6x + 4} \cdot (6x - 6)$$
$$\frac{dy}{dx} = \frac{6x - 6}{3x^2 - 6x + 4} \qquad \text{OOM(??)}$$

■

Example 2.13 – id:20141128-155506.

Differentiate $y = \ln(\cos x)$

S

Solution: After identifying that $y = \ln(\cos x)$ is a composite function, we let $u = \cos x$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(2.21)}$$

We need to find the factors $\frac{dy}{du}$ and $\frac{du}{dx}$.

$$y = \ln(u) \qquad u = \cos x$$
$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$ DComp(2.21)

$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$ DNL(2.18)

$$\frac{dy}{dx} = \frac{1}{u} \cdot -\sin x$$ DPo(2.6)

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x$$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$ OOM(**??**)

$$\frac{dy}{dx} = -\tan x$$

■

Example 2.14 – id:20141124-205219.

Differentiate $y = (x^2 - 1)\ln(3x)$

S

Solution:

$$y' = [\mathbf{x^2 - 3}]' \cdot \ln x + (x^2 - 3) \cdot [\mathbf{\ln 3x}]'$$ DPr(2.10)

$$y' = 2x \cdot \ln x + (x^2 - 3) \cdot [\mathbf{\ln 3x}]'$$ differentiate goto 2.4

$$y' = 2x \cdot \ln x + (x^2 - 3) \cdot \frac{1}{x}$$ differentiate goto 2.11

$$y' = 2x \cdot \ln x + \frac{x^2 - 3}{x}$$ OOM(**??**)

$$y' = 2x^2 \ln x + \frac{x^2 - 3}{x}$$ JTC(1.2)

$$y' = \frac{2x^2 \ln x + (x^2 - 3)}{x}$$ OOA(**??**)

■

Bibliography