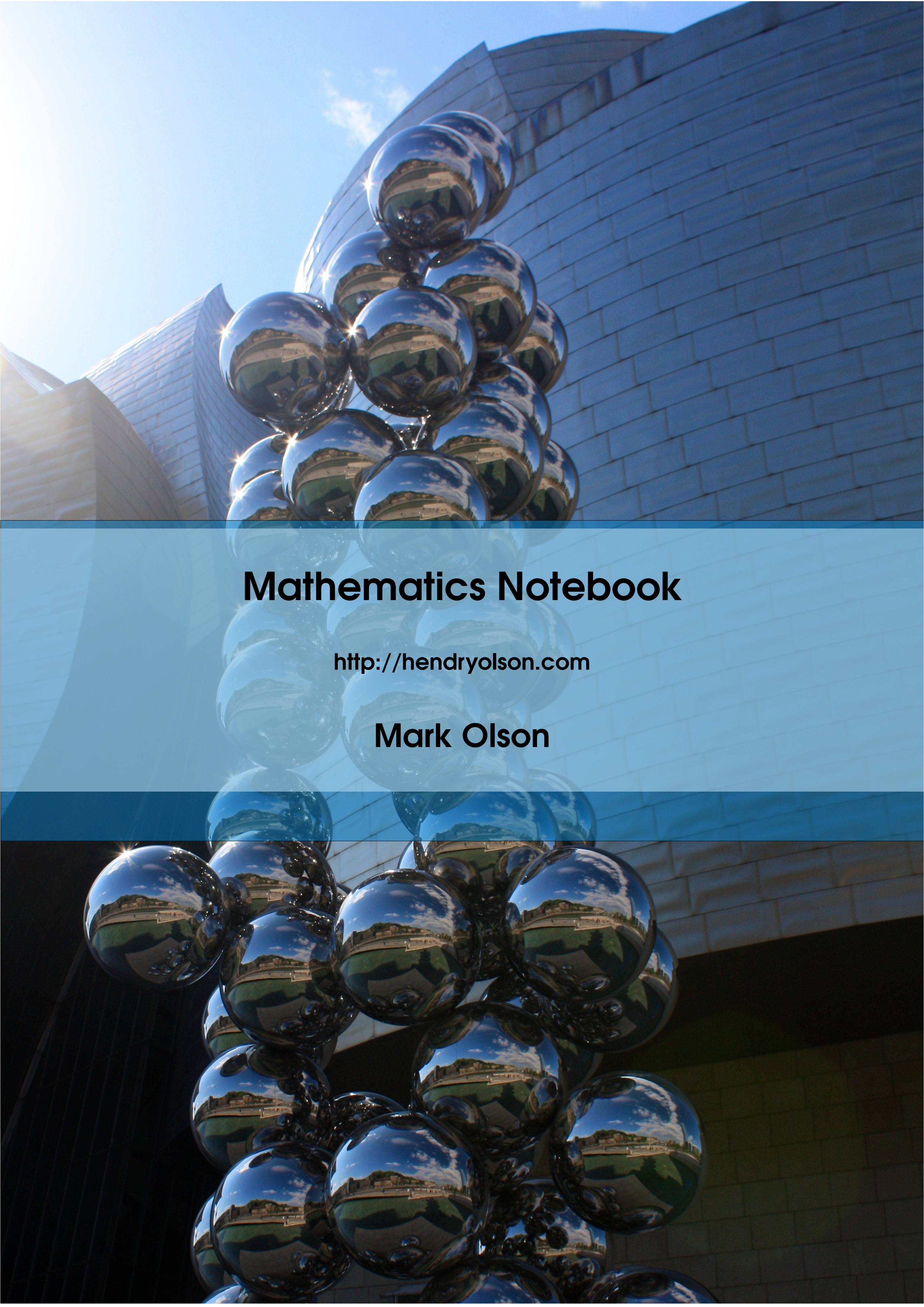

Mathematics Notebook

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Contents

MYP

1. Algebraic Expressions

1.1 Expressions

Essential Questions 1.1

1. What is an algebraic expression?

1.2 Polynomial Expressions

Definition 1.2.1 — Operation of Addition (OOA).

$$\underbrace{\underbrace{a}_{\text{Augend}} + \underbrace{b}_{\text{Addend}}}_{\text{Sum}} \tag{1.1}$$

	Arithmetic	Polynomial	Algebraic
Constant	Yes	Yes	Yes
Factorial	Yes	Yes	Yes
Variable: parameter/coefficient	Yes	Yes	Yes
Variable: unknown/indeterminate	No	Yes	Yes
Power with $\mathbb{Z}^+$ exponent	No	Yes	Yes
Power with $\mathbb{Z}$ exponent	No	No	Yes
n-th root	No	No	Yes
Power with $\mathbb{Q}$ exponent	No	No	Yes

Table 1.1: Names of different types of expressions

Definition 1.2.2 — Common Denominator (CD).

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b} \tag{1.2a}$$

$$\frac{a+c}{b}=\frac{a}{b}+\frac{c}{b} \tag{1.2b}$$

Rule 1.2.3 — Fraction Operation of Addition (FOOA).

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd} \tag{1.3a}$$

$$\frac{ad+bc}{bd}=\frac{a}{b}+\frac{c}{d} \tag{1.3b}$$

Definition 1.2.4 — Operation of Multiplication (OOM).

$$\underbrace{\underbrace{a}_{\text{Multiplicand}} \cdot \underbrace{b}_{\text{Multiplier}}}_{\text{Product}} \tag{1.4}$$

Definition 1.2.5 — Operation of Exponentiation (OOE).

$$\underbrace{\underbrace{b}_{base}{}^{\overbrace{m}^{Exponent}}}_{Power} \tag{1.5}$$

Definition 1.2.6 — Juxtaposition to Center-Dot (JTC).

$$ab = a \cdot b \tag{1.6}$$

Definition 1.2.7 — Center-Dot to Justapostion (CTJ).

$$a \cdot b = ab \tag{1.7}$$

Definition 1.2.8 — Commutative Property of Multiplication (CPM).

$$a \cdot b = b \cdot a \tag{1.8}$$

Definition 1.2.9 — Multiplicative Inverse (MI).

$$a \cdot \frac{1}{a} = 1 \tag{1.9a}$$

$$a \cdot a^{-1} = 1 \tag{1.9b}$$

Definition 1.2.10 — Associative Property of Multiplication (APM).

$$a \cdot b \cdot c = (a \cdot b) \cdot c \tag{1.10a}$$

$$a \cdot b \cdot c = a \cdot (b \cdot c) \tag{1.10b}$$

Powers

Rule 1.2.11 — Power of a Quotient of Powers (PoQPo).

$$\left(\frac{a^m}{b^n}\right)^k = \frac{a^{m \cdot k}}{b^{n \cdot k}} \tag{1.11a}$$

$$\frac{a^{m \cdot k}}{b^{n \cdot k}} = \left(\frac{a^m}{b^n}\right)^k \tag{1.11b}$$

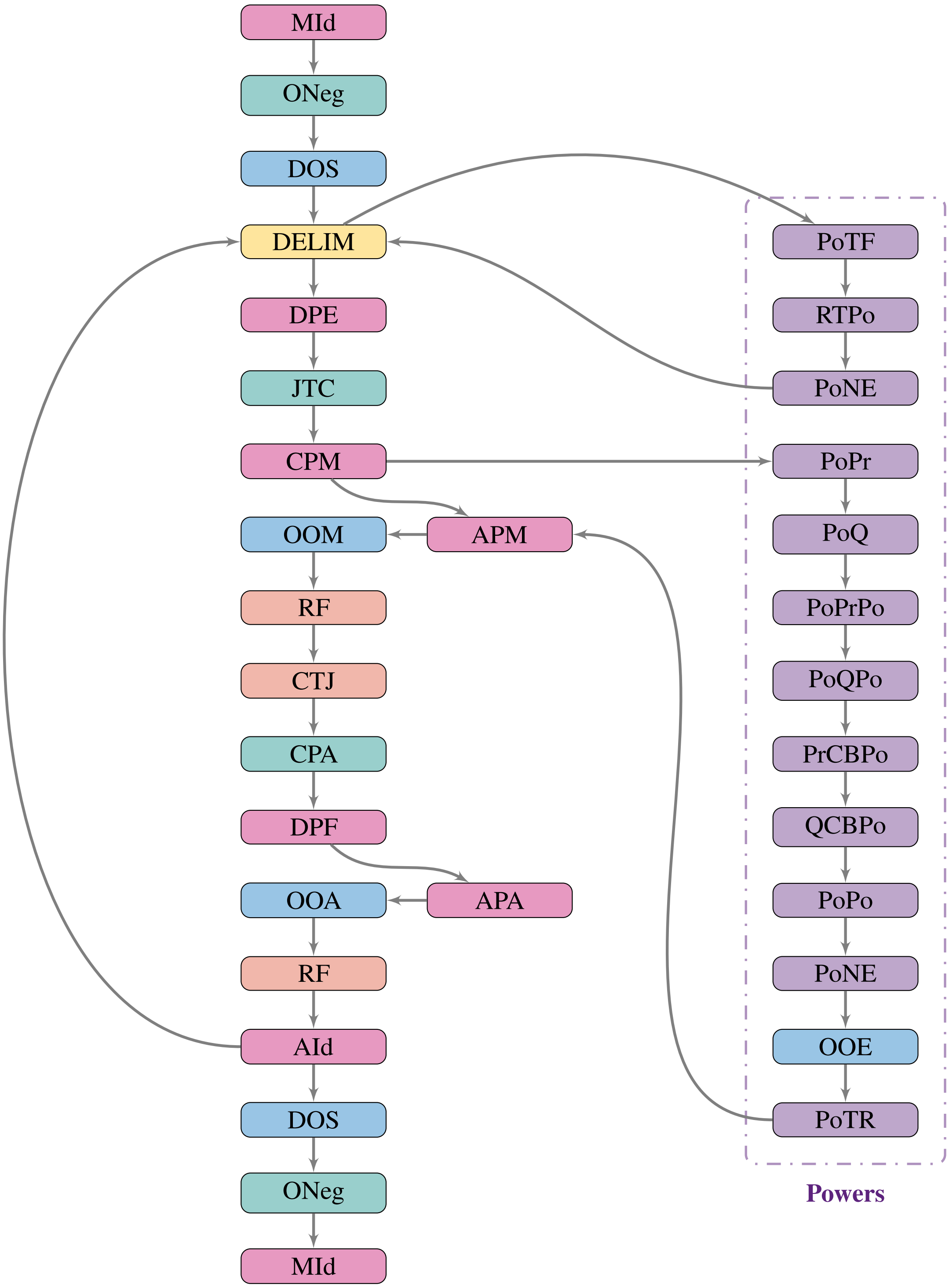


Figure 1.1: Simplifying Expressions Workflow:
Property, Operation, Notation, Powers, Delimiters, Process, Not Used

Rule 1.2.12 — Power of a Product of Powers (PoPrPo).

$$(a^m \cdot b^n)^k = a^{m\cdot k} \cdot b^{n\cdot k} \tag{1.12a}$$

$$a^{m\cdot k} \cdot b^{n\cdot k} = (a^m \cdot b^n)^k \tag{1.12b}$$

Definition 1.2.13 — Power To Factor (PoTF).

$$a^n = a_1 \cdot a_2 \cdot \ldots \cdot a_{n-1} \cdot a_n \tag{1.13}$$

Definition 1.2.14 — Factor To Power (FTPo).

$$a_1 \cdot a_2 \cdot \ldots \cdot a_{n-1} \cdot a_n = a^n \tag{1.14}$$

Definition 1.2.15 — Power Inverse (PoI).

$$(b^m)^{\frac{1}{m}} = b \tag{1.15a}$$

Definition 1.2.16 — Power Inverse (PoId).

$$1 = b^0 \tag{1.16a}$$

$$b^0 = 1 \tag{1.16b}$$

Notation 1.1 (Radical To Power (RTPo))**.**

$$\sqrt[m]{b^n} = b^{\frac{n}{m}} \tag{1.17}$$

Notation 1.2 (Power To Radical (PoTR))**.**

$$b^{\frac{n}{m}} = \sqrt[m]{b^n} \tag{1.18}$$

1.2.1 Monomials of Like Terms

Definition 1.2.17 — Additive Inverse (AI).

$$a + (-a) = 0 \tag{1.19a}$$

1.2.2 Surds

Example 1.1 — id:20141108-085327.

Simplify $2\sqrt{2} - \dfrac{\left(\sqrt{2}\right)^3}{3} - \left(2\left(-\sqrt{2}\right) - \dfrac{\left(-\sqrt{2}\right)^3}{3}\right)$

S

Solution:

$$2\sqrt{2}-\frac{1\left(1\sqrt{2}\right)^{3}}{3}-1\left(2\left(-1\sqrt{2}\right)-\frac{1\left(-1\sqrt{2}\right)^{3}}{3}\right) \qquad \text{MId(2.1a)}$$

$$2\sqrt{2}-\frac{1\left(1\sqrt{2}\right)^{3}}{3}-1\left(2\left(\neg1\sqrt{2}\right)-\frac{1\left(\neg1\sqrt{2}\right)^{3}}{3}\right) \qquad \text{ONeg(2.6a)}$$

$$2\sqrt{2}+\frac{\neg1\left(1\sqrt{2}\right)^{3}}{3}+\neg1\left(2\left(\neg1\sqrt{2}\right)+\frac{\neg1\left(\neg1\sqrt{2}\right)^{3}}{3}\right) \qquad \text{DOS(2.5a)}$$

$$2\cdot2^{1/2}+\frac{\neg1\left(1\cdot2^{1/2}\right)^{3}}{3}+\neg1\left(2\left(\neg1\cdot2^{1/2}\right)+\frac{\neg1\left(\neg1\cdot2^{1/2}\right)^{3}}{3}\right) \qquad \text{RTPo(1.17)}$$

$$2\cdot2^{1/2}+\frac{\neg1\left(1\cdot2^{1/2}\right)^{3}}{3}+\neg1\left(2\cdot\neg1\cdot2^{1/2}+\frac{\neg1\left(\neg1\cdot2^{1/2}\right)^{3}}{3}\right) \qquad \text{JTC(1.6)}$$

$$2\cdot2^{1/2}+\frac{\neg1\cdot1\cdot2^{3/2}}{3}+\neg1\left(2\cdot\neg1\cdot2^{1/2}+\frac{\neg1\cdot\neg1\cdot2^{3/2}}{3}\right) \qquad \text{PoPrPo(1.12a)}$$

$$2\cdot2^{1/2}+\frac{\neg1\cdot1\cdot2^{2/2}\cdot2^{1/2}}{3}+\neg1\left(2\cdot\neg1\cdot2^{1/2}+\frac{\neg1\cdot\neg1\cdot2^{2/2}\cdot2^{1/2}}{3}\right) \qquad \text{PrCBPo(2.10b)}$$

$$2\cdot2^{1/2}+\frac{\neg1\cdot1\cdot2\cdot2^{1/2}}{3}+\neg1\left(2\cdot\neg1\cdot2^{1/2}+\frac{\neg1\cdot\neg1\cdot2\cdot2^{1/2}}{3}\right) \qquad \text{MId(2.1b)}$$

$$2\cdot\sqrt{2}+\frac{\neg1\cdot1\cdot2\cdot\sqrt{2}}{3}+\neg1\left(2\cdot\neg1\cdot\sqrt{2}+\frac{\neg1\cdot\neg1\cdot2\cdot\sqrt{2}}{3}\right) \qquad \text{PoTR(1.18)}$$

$$2\cdot\sqrt{2}+\frac{\neg1\cdot1\cdot2\cdot\sqrt{2}}{3}+\neg1\cdot2\cdot\neg1\cdot\sqrt{2}+\frac{\neg1\cdot\neg1\cdot\neg1\cdot2\cdot\sqrt{2}}{3} \qquad \text{DPE(2.9a)}$$

$$2\cdot\sqrt{2}+\frac{\neg2\cdot\sqrt{2}}{3}+2\cdot\sqrt{2}+\frac{\neg2\cdot\sqrt{2}}{3} \qquad \text{OOM(1.4)}$$

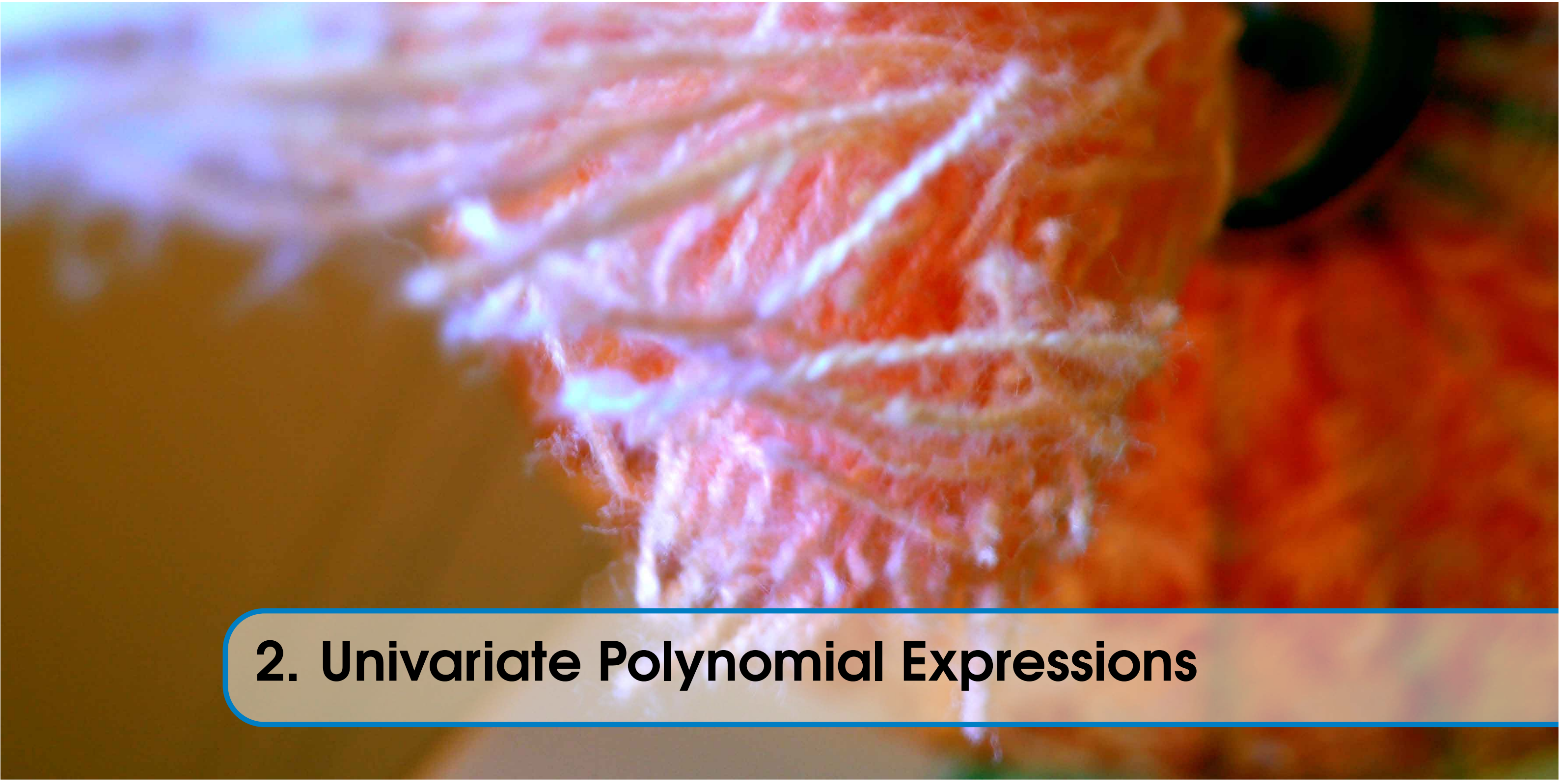
$$2\sqrt{2}+\frac{\neg2\sqrt{2}}{3}+2\sqrt{2}+\frac{\neg2\sqrt{2}}{3} \qquad \text{CTJ(1.7)}$$

$$\left(2+\frac{\neg2}{3}+2+\frac{\neg2}{3}\right)\sqrt{2} \qquad \text{DPF(2.4a)}$$

$$\frac{8}{3}\sqrt{2} \qquad \text{OOA(1.1)}$$

D

Dependencies:example 6.5-20141108-083108



2. Univariate Polynomial Expressions

2.1 Classification of Univariate Polynomial Expressions

Definition 2.1.1 — Indeterminate.

$$x$$

An indeterminate is a symbol that is treated as a variable, but does not stand for anything else but itself and is used as a placeholder.

- it does **not** designate a constant or a parameter
- it is **not** an unknown that could be solved for
- it is **not** a variable designating a function argument

[**wikipediate:indeterminate**]

Definition 2.1.2 — Coefficient.

$$Cx^k$$

A coefficient, C is a real number multiplicative factor.

Definition 2.1.3 — Univariate Monomial.

$$C_k x^k$$

A univariate monomial is made up of two factors. The first factor of a monomial, C_k, is the coefficient. The second factor of each monomial, x^k, is an indeterminate raised to a non-negative integer power k.

Definition 2.1.4 — Multiplicative Identity (MId).

$$1a = a \tag{2.1a}$$
$$a = 1a \tag{2.1b}$$

R If the coefficient of a univariate monomial is the multiplicative identity 2.1a, 1, then it is not shown in it's canonical form.

$$\begin{aligned} C_k x^k &= C_k x^k \\ &= 1x^k \\ &= x^k \end{aligned}$$

Example 2.1 — id:20141120-202842.

Express $1x^2$ in canonical form.

S

Solution:

x^2 MId(2.1b)

Definition 2.1.5 — Univariate Polynomial Expression.

$$\sum_{k=0}^{n} C_k x^n = C_n x^n + C_{n-1} x^{n-1} + \cdots + C_k x^k + \cdots + C_2 x^2 + C_1 x^1 + \underbrace{C_0}_{C_0 x^0} \tag{2.2}$$

A univariate polynomial in an indeterminate x is an expression made up of one or more summands of the form $C_k x^k$, which are called monomials. The first factor of each monomial, C_k, is a numerical factor called the coefficient where $C_k \in \mathbb{R}$. The second factor of each monomial, x^k, is an indeterminate raised to a non-negative integer power i.

[**wikipedia:polynomial**]

Definition 2.1.6 — Degree of the Indeterminate.

$$x^k$$

The exponent of an indeterminate power, k is called the degree of the indeterminate.

[**wikipedia:polynomial**]

Definition 2.1.7 — Degree of the Univarite Polynomial.

$$C_n x^n + C_{n-1} x^{n-1} + \cdots + C_k x^k + \cdots + C_2 x^2 + C_1 x^1 + \underbrace{C_0}_{C_0 x^0}$$

The degree of the univariate polynomial is determined by the monomial with the largest degree of the indeterminate.

2.2 Degree -1 Univariate Polynomials

Monomials

2.3 Degree 0 Univariate Polynomials

$$C_n x^n + C_{n-1}x^{n-1} + \cdots + C_k x^k + \cdots + C_2 x^2 + C_1 x^1 + \underbrace{C_0}_{C_0 x^0}$$

Degree 0 univariate polynomial expressions are made up of univariate monomials, C_0, called constants. The power identity is an indeterminate raised to a power of 0 has a value of 1. Thus, $x^0 = 1$ and results in the monomial $C_0 \cdot 1$. The canonical form of a product does not show the multiplicative identity factor, so what remains of this monomial product is only the coefficient factor C_0 and from now on will be referred to as a constant.

Example 2.2 — id:20141121-093747.

Express $13x^0$ in canonical form.

S

Solution:

13

Monomials

Degree 0 univariate polynomial expressions are usually a monomial in their canonical form if C_0 is a non-zero real number. The exception is if $C_0 = 0$, the additive identity, then the result is the zero polynomial, which can be considered a degree -1 polynomial.

The expression can be manipulated into its monomial canonical form by simplifying the expression. Simplifying the expression can be defined as evaluating the expression by following order of operations, which is the same as evaluating an arithmetic expression.

2.4 Degree 1 Univariate Polynomials

$$C_n x^n + C_{n-1}x^{n-1} + \cdots + C_k x^k + \cdots + C_2 x^2 + C_1 x^1 + \underbrace{C_0}_{C_0 x^0}$$

Degree 1 univariate polynomial expressions can be expressed with at most two different terms and consequently this expression in its canonical form has at most two monomial summands – called a binomial.

Definition 2.4.1 — Univariate Like Terms.

$$C_1 x^k = C_2 x^k$$

Two or more univariate monomials are defined as having like terms if each monomial has the same term, which will be the same indeterminate raised to the same positive integer power.

Sometimes the word term is used to describe monomials (including both the coefficient and the term), which may be confusing when trying to define like terms. For this reason, we will refer to the summands of a polynomial as monomials.

The monomials $5x^1$ and $3x^1$ can be described as having like terms because they share the common term x^1. One could also say that $5x^1$ and $3x^1$ are like terms by definition and consequenlty giving the reader an impression that $5x^1$ and $3x^1$ are terms themselves.

R A degree 1 indeterminate does not display the multiplicative identity in the exponent when its in canonical form.

Example 2.3 — id:20141120-202042.

Express $5x^1$ in canonical form

S

Solution:

$5x$ MId(2.1b)

■

Example 2.4 — id:20141121-093439.

Express $7x^1 + 5$ in canonical form.

S

Solution:

$7x + 5$ MId(2.1b)

■

Monomials

Essential Questions 2.1

1. How do we simplify univariate polynomial expressions?

Definition 2.4.2 — Additive Identity (AId).

$$a + 0 = a \tag{2.3a}$$
$$a = a + 0 \tag{2.3b}$$

If the constant monomial is 0, the additive identity, then the canonical form of a degree 1 univariate polynomial is a degree 1 monomial.

Simplifying Univariate Monomial Expressions

The definition of a univariate monomial expression is based on the expanded canonical form of some polynomial expression. It might be that the original expression might not be in the expanded canonical form, so a process called simplifying by expanding will be introduced to manipulate the expression such that it can be written in its expanded canonical form.

This process of simplifying by expanding polynomial expressions will be developed to the extent that it will be used to simplify multivariate polynomials. We will start by simplifying univariate monomial expressions.

Definition 2.4.3 — Distributive Property Factoring (DPF).

$$ba + ca = (b + c)a \tag{2.4a}$$

$$ab + ac = a(b + c) \tag{2.4b}$$

Example 2.5 — id:20141120-203846.

Simplify $6x + 7x$

S

Solution:

Notice that the indeterminate of each monomial is of degree 1; however, the exponent 1 is not shown. The monomials $6x$ and $7x$ have a like term of x.

$(6+7)x$	DPF(2.4a)
$13x$	OOA(1.1)

Notice that the sum of two monomials that have like terms can be found by adding the coefficients of the monomials. The distributive property in the factoring direction provides some insight to why we can add the coefficients of monomials that have like terms.

S

Less Steps Solution:

$13x$	OOA(1.1)

■

Example 2.6 — id:20141027-075159.

Simplify $7\,\text{cm} + 8\,\text{cm}$

S

Solution:

$$
\begin{aligned}
(7+8)\,\text{cm} &\qquad \text{DPF(2.4a)}\\
15\,\text{cm} &\qquad \text{OOA(1.1)}
\end{aligned}
$$

∎

R Remember, if a monomial does not have a coefficient factor, then it's implied that the coefficient factor is 1, the multiplicative identity, and consequently its not explicitly shown.

Example 2.7 — id:20141121-185558.

Simplify $x+5x$

S

Solution:

It can be useful when simplifying expressions to make the multiplicative identity (MId) factor explicit.

$$
\begin{aligned}
1x+5x &\qquad \text{MId(2.1a)}\\
(1+5)x &\qquad \text{DPF(2.4a)}\\
6x &\qquad \text{OOA(1.1)}
\end{aligned}
$$

S

Less Steps Solution:

$$
\begin{aligned}
1x+5x &\qquad \text{MId(2.1a)}\\
6x &\qquad \text{OOA(1.1)}
\end{aligned}
$$

As one becomes more experienced, there is no reason to make the multiplicative identity coefficient explicit.

S

Less Steps Solution:

$$6x \qquad \text{OOA(1.1)}$$

∎

Definition 2.4.4 — Definition of Subtraction (DOS).

$$a-b=a+\neg b \tag{2.5a}$$

$$a+\neg b=a-b \tag{2.5b}$$

Notation 2.1 (Operation of Negation (ONeg))**.**

$$-a = \neg a \tag{2.6a}$$

$$\neg a = -a \tag{2.6b}$$

I have used a different symbol, ¬, as the prefix negation operator only to differentiate it from the minus sign infix operator symbol, −, which is also used as the infix operator for the dyadic operation of subtraction. I will refer to this change of symbol as ONeg. This is used only as a teaching tool and should not be confused with the logic negation operator. Another advantage of using this symbol is that it reduces the number of delimiters used in an expression for example, $\neg a$ *versus* $(-a)$*.*

- *Negative five:* -5
- *Negative five:* $\neg 5$
- *Four minus five:* $4-5$
- *Four minus negative five:* $4--5$
- *Four minus negative five:* $4-(-5)$
- *Four minus negative five:* $4-\neg 5$
- *Negative four minus five:* $-4-5$
- *Negative four minus five:* $\neg 4-5$

Example 2.8 — id:20141121-190857.

Simplify $8x-6x$

S

Solution:

$8x+\neg 6x$	DOS(2.5a)
$(8+\neg 6)x$	DPF(2.4a)
$2x$	OOA(1.1)

S

Less Steps Solution:

$8x+\neg 6x$	DOS(2.5a)
$2x$	OOA(1.1)

■

Example 2.9 — id:20141121-193636.

Simplify $3x-5x$

S

Solution:

$3x + \neg 5x$	DOS(2.5a)
$(3 + \neg 5)x$	DPF(2.4a)
$\neg 2x$	OOA(1.1)
$-2x$	ONeg(2.6b)

S

Less Steps Solution:

$3x + \neg 5x$	DOS(2.5a)
$\neg 2x$	OOA(1.1)
$-2x$	ONeg(2.6b)

S

Less Steps Solution:

$-2x$	OOA(1.1)

■

Example 2.10 — id:20141106-150622.

Simplify $13x - x$

S

Solution:

$13x - 1x$	MId(2.1a)
$13x + \neg 1x$	DOS(2.5a)
$(13 + \neg 1)x$	DPF(2.4a)
$12x$	OOA(1.1)

S

Less Steps Solution:

$13x + \neg x$	DOS(2.5a)
$12x$	OOA(1.1)

■

It is possible for a univariate monomial to have more than two terms in its non-canonical form. The associative property of addition will be used to help simplify these expressions.

Definition 2.4.5 — Associative Property of Addition (APA).

$$a+b+c=(a+b)+c \tag{2.7a}$$
$$a+b+c=a+(b+c) \tag{2.7b}$$

Example 2.11 — id:20141121-184652.

Simplify the expression $3x+7x+8x$

S

$(3x+7x)+8x$	APA(2.7a)
$(3+7)x+8x$	DPF(2.4a)
$10x+8x$	OOA(1.1)
$(10+8)x$	DPF(2.4a)
$18x$	OOA(1.1)

S

Less Steps Solution:

$(3x+7x)+8x$	APA(2.7a)
$10x+8x$	OOA(1.1)
$18x$	OOA(1.1)

You might have noticed that this expression could be simplified in one step by adding the coefficient of the three monomials $3x$, $7x$ and $8x$, which have the like term x.

S

Less Steps Solution:

$18x$	OOA(1.1)

Example 2.12 — id:20141106-152020.

Simplify $4x-2x-x$

S

Solution:

$$\begin{array}{rl}
4x - 2x - 1x & \text{MId}(2.1\text{a}) \\
4x + \neg 2x + \neg 1x & \text{DOS}(2.5\text{a}) \\
(4 + \neg 2)x + \neg 1x & \text{DPF}(2.4\text{a}) \\
2x + \neg 1x & \text{OOA}(1.1) \\
(2 + \neg 1)x & \text{DPF}(2.4\text{a}) \\
1x & \text{OOA}(1.1) \\
x & \text{MId}(2.1\text{b})
\end{array}$$

S

Less Steps Solution:

$$\begin{array}{rl}
4x + \neg 2x + \neg x & \text{DOS}(2.5\text{a}) \\
x & \text{OOA}(1.1)
\end{array}$$

■

Example 2.13 — id:20141108-194431.

Simplify $-3 \cdot 7x - 2x \cdot 4$

S

Solution:

$$\begin{array}{rl}
\neg 3 \cdot 7x - 2x \cdot 4 & \text{ONeg}(2.6\text{a}) \\
\neg 3 \cdot 7x + \neg 2x \cdot 4 & \text{DOS}(2.5\text{a}) \\
\neg 3 \cdot 7 \cdot x + \neg 2 \cdot x \cdot 4 & \text{JTC}(1.6) \\
\neg 3 \cdot 7 \cdot x + \neg 2 \cdot 4 \cdot x & \text{CPM}(1.8) \\
(\neg 3 \cdot 7) \cdot x + (\neg 2 \cdot 4) \cdot x & \text{APM}(1.10\text{a}) \\
\neg 21 \cdot x + \neg 8 \cdot x & \text{OOM}(1.4) \\
\neg 21x + \neg 8x & \text{CTJ}(1.7) \\
(\neg 21 + \neg 8)x & \text{DPF}(2.4\text{a}) \\
\neg 29x & \text{OOA}(1.1) \\
-29x & \text{ONeg}(2.6\text{b})
\end{array}$$

S

Less Steps Solution:

$-3 \cdot 7x + \neg 2x \cdot 4$	DOS(2.5a)
$\neg 3 \cdot 7 \cdot x + \neg 2 \cdot 4 \cdot x$	CPM(1.8)
$\neg 21x + \neg 8x$	OOM(1.4)
$-29x$	OOA(1.1)

■

Example 2.14 — id:20141108-194156.

Simplify $3 \cdot 5x + 3x \cdot 4$

S

Solution:

$3 \cdot 5 \cdot x + 3 \cdot x \cdot 4$	JTC(1.6)
$3 \cdot 5 \cdot x + 3 \cdot 4 \cdot x$	CPM(1.8)
$(3 \cdot 5) \cdot x + (3 \cdot 4) \cdot x$	APM(1.10a)
$15 \cdot x + 12 \cdot x$	OOM(1.4)
$15x + 12x$	CTJ(1.7)
$(15 + 12)x$	DPF(2.4a)
$27x$	OOA(1.1)

S

Less Steps Solution:

$3 \cdot 5 \cdot x + 3 \cdot 4 \cdot x$	CPM(1.8)
$15x + 12x$	OOM(1.4)
$27x$	OOA(1.1)

■

Example 2.15 — id:20141108-173613.

Simplify $8x \cdot 5$

S

Solution:

$8 \cdot x \cdot 5$	JTC(1.6)
$8 \cdot 5 \cdot x$	CPM(1.8)
$(8 \cdot 5) \cdot x$	APM(1.10a)
$40 \cdot x$	OOM(1.4)
$40x$	CTJ(1.7)

Binomials

Definition 2.4.6 — Commutative Property of Addition (CPA).

$$ab = ba \tag{2.8}$$

Definition 2.4.7 — Distributive Property Expanding (DPE).

$$a(b+c) = ab+ac \tag{2.9a}$$

$$(b+c)a = ba+ca \tag{2.9b}$$

Example 2.16 — id:20141109-090809.

Simplify by expanding $5(x+4)$

S

Solution:

$5(1x+4)$	MId(2.1a)
$5 \cdot 1x+5 \cdot 5$	DPF(2.4a)
$5 \cdot 1 \cdot x+5 \cdot 5$	JTC(1.6)
$5 \cdot x+25$	OOM(1.4)
$5x+25$	CTJ(1.7)

S

Less Steps Solution:

$5x+20$	DPE(2.9a)

Example 2.17 — id:20141109-091015.

Simplify by expanding $5(3x-9)$

Solution:

$5(3x+\neg 9)$	DOS(2.5a)
$5\cdot 3x+5\cdot\neg 9$	DPE(2.9a)
$5\cdot 3\cdot x+5\cdot\neg 9$	JTC(1.6)
$15\cdot x+\neg 45$	OOM(1.4)
$15x+\neg 45$	CTJ(1.7)
$15x-45$	DOS(2.5b)

Less Steps Solution:

$5(3x+\neg 9)$	DOS(2.5a)
$15x+\neg 40$	DPE(2.9a)
$15x-40$	DOS(2.5b)

■

Example 2.18 — id:20141109-092448.

Simplify by expanding $-(5x+7)$

Solution:

$\neg 1(5x+7)$	MId(2.1a)
$\neg 1\cdot 5x+\neg 1\cdot 7$	DPE(2.9a)
$\neg 1\cdot 5\cdot x+\neg 1\cdot 7$	JTC(1.6)
$\neg 5\cdot x+\neg 7$	OOM(1.4)
$\neg 5x+\neg 7$	CTJ(1.7)
$\neg 5x-7$	DOS(2.5b)
$-5x-7$	ONeg(2.6b)

Less Steps Solution:

$-5x-7$	DPE(2.9a)

■

Example 2.19 — id:20141109-092651.

Simplify by expanding $-13(7x-9)$

Solution:

$\neg 13(7x+\neg 9)$	DOS(2.5a)
$\neg 13\cdot 7x+\neg 13\cdot \neg 9$	DPE(2.9a)
$\neg 13\cdot 7\cdot x+\neg 13\cdot \neg 9$	JTC(1.6)
$\neg 91\cdot x+117$	OOM(1.4)
$\neg 91x+117$	CTJ(1.7)
$-91x+117$	ONeg(2.6b)

Less Steps Solution:

$-13(7x+\neg 9)$	DOS(2.5a)
$-91x+117$	DPE(2.9a)

■

Example 2.20 — id:20141109-092910.

Simplify by expanding $a(x+b)$, where $a,b\in\mathbb{Z}$

Solution:

$a(1x+b)$	MId(2.1a)
$a\cdot 1x+a\cdot b$	DPE(2.9a)
$a\cdot 1\cdot x+a\cdot b$	JTC(1.6)
$1\cdot a\cdot x+a\cdot b$	CPM(1.8)
$1ax+ab$	JTC(1.6)
$ax+ab$	MId(2.1b)

Less Steps Solution:

$ax+ab$	DPE(2.9a)

Example 2.21 — id:20141109-093220.

Simplify by expanding $5(x+2)+4$

Solution:

$5(1x+2)+4$	MId(2.1a)
$5\cdot 1x+5\cdot 2+4$	DPE(2.9a)
$5\cdot 1\cdot x+5\cdot 2+4$	JTC(1.6)
$5\cdot x+10+4$	OOM(1.4)
$5x+10+4$	CTJ(1.7)
$5x+14$	OOA(1.1)

Less Steps Solution:

$5x+10+4$	DPE(2.9a)
$5x+14$	OOA(1.1)

Example 2.22 — id:20141109-093419.

Simplify by expanding $7x+5(4x+8)$

Solution:

$7x+5\cdot 4x+5\cdot 8$	DPE(2.9a)
$7\cdot x+5\cdot 4\cdot x+5\cdot 8$	JTC(1.6)
$7\cdot x+20\cdot x+40$	OOM(1.4)
$7x+20x+40$	CTJ(1.7)
$(7+20)x+40$	DPF(2.4a)
$27x+40$	OOA(1.1)

Example 2.23 — id:20141109-094928.

Simplify by expanding $4(3x+4)+x+6$

S

Solution:

$$
\begin{aligned}
4(3x+4)+1x+6 &\qquad \text{MId}(2.1a)\\
4\cdot 3x+4\cdot 4+1x+6 &\qquad \text{DPE}(2.9a)\\
4\cdot 3\cdot x+4\cdot 4+1\cdot x+6 &\qquad \text{JTC}(1.6)\\
12\cdot x+16+1\cdot x+6 &\qquad \text{OOM}(1.4)\\
12x+16+1x+6 &\qquad \text{CTJ}(1.7)\\
12+1x+16+6 &\qquad \text{CPA}(2.8)\\
(12+1)x+16+6 &\qquad \text{DPF}(2.4a)\\
13x+22 &\qquad \text{OOA}(1.1)
\end{aligned}
$$

S

Less Steps Solution:

$$
\begin{aligned}
12x+16+x+6 &\qquad \text{DPE}(2.9a)\\
12x+x+16+6 &\qquad \text{CPA}(2.8)\\
13x+22 &\qquad \text{OOA}(1.1)
\end{aligned}
$$

Example 2.24 — id:20141109-095151.

Simplify by expanding $5(x-4)+3x-5$

S

Solution:

$$
\begin{aligned}
5(1x-4)+3x-5 && \text{MId}(2.1a)\\
5(1x+\neg 4)+3x+\neg 5 && \text{DOS}(2.5a)\\
5\cdot 1x+5\cdot\neg 4+3x+\neg 5 && \text{DPE}(2.9a)\\
5\cdot 1\cdot x+5\cdot\neg 4+3\cdot x+\neg 5 && \text{JTC}(1.6)\\
5\cdot x+\neg 20+3\cdot x+\neg 5 && \text{OOM}(1.4)\\
5x+\neg 20+3x+\neg 5 && \text{JTC}(1.6)\\
5x+3x+\neg 20+\neg 5 && \text{CPA}(2.8)\\
(5+3)x+\neg 20+\neg 5 && \text{DPF}(2.4a)\\
8x+\neg 25 && \text{OOA}(1.1)\\
8x-25 && \text{DOS}(2.5b)
\end{aligned}
$$

S

Less Steps Solution:

$$
\begin{aligned}
5(x+\neg 4)+3x+\neg 5 && \text{DOS}(2.5a)\\
5x+\neg 20+3x+\neg 5 && \text{DPE}(2.9a)\\
5x+3x+\neg 20+\neg 5 && \text{CPA}(2.8)\\
8x+\neg 25 && \text{OOA}(1.1)\\
8x-25 && \text{DOS}(2.5b)
\end{aligned}
$$

■

Example 2.25 — id:20141109-095536.

Simplify by expanding $8x-5-4(x-3)$

S

Solution:

$$
\begin{aligned}
8x-5-4(1x-3) && \text{MId}(2.1a)\\
8x+\neg 5+\neg 4(1x+\neg 3) && \text{DOS}(2.5a)\\
8x+\neg 5+\neg 4\cdot 1x+\neg 4\cdot\neg 3 && \text{DPE}(2.9a)\\
8\cdot x+\neg 5+\neg 4\cdot 1\cdot x+\neg 4\cdot\neg 3 && \text{JTC}(1.6)\\
8\cdot x+\neg 5+\neg 4\cdot x+12 && \text{OOM}(1.4)\\
8x+\neg 5+\neg 4x+12 && \text{CTJ}(1.7)\\
8x+\neg 4x+\neg 5+12 && \text{CPA}(2.8)\\
(8+\neg 4)x+\neg 5+12 && \text{DPF}(2.4a)\\
4x+7 && \text{OOA}(1.1)
\end{aligned}
$$

S

Less Steps Solution:

$$\begin{aligned} 8x+\neg5+\neg4(x+\neg3) &\qquad \text{DOS}(2.5a)\\ 8x+\neg5+\neg4x+12 &\qquad \text{DPE}(2.9a)\\ 8x+\neg4x+\neg5+12 &\qquad \text{CPA}(2.8)\\ 4x+7 &\qquad \text{OOA}(1.1) \end{aligned}$$

■

Example 2.26 — id:20141109-095842.

Simplify by expanding $5(x+3)+3(x+2)$

S

Solution:

$$\begin{aligned} 5\cdot x+5\cdot 3+3\cdot x+3\cdot 2 &\qquad \text{DPE}(2.9a)\\ 5\cdot x+15+3\cdot x+6 &\qquad \text{OOM}(1.4)\\ 5x+15+3x+6 &\qquad \text{CTJ}(1.7)\\ 5x+3x+15+6 &\qquad \text{CPA}(2.8)\\ (3+5)x+15+6 &\qquad \text{DPF}(2.4a)\\ 8x+21 &\qquad \text{OOA}(1.1) \end{aligned}$$

S

Less Steps Solution:

$$\begin{aligned} 5x+15+3x+6 &\qquad \text{DPE}(2.9a)\\ 5x+3x+15+6 &\qquad \text{CPA}(2.8)\\ 8x+21 &\qquad \text{OOA}(1.1) \end{aligned}$$

■

2.5 Degree 2 Univariate Polynomials

Monomials

Example 2.27 — id:20141106-151138.

Simplify $4x^2+12x^2$

S

Solution:

$$
\begin{aligned}
(4+12)x^2 & \quad \text{DPF(2.4a)} \\
16x^2 & \quad \text{OOA(1.1)}
\end{aligned}
$$

Less Steps Solution:

$$
16x^2 \quad \text{OOA(1.1)}
$$

Example 2.28 — id:20141106-154547.

Simplify $x^2 - x + x^2 + x$

Solution:

$$
\begin{aligned}
1x^2 - 1x + 1x^2 + 1x & \quad \text{MId(2.1a)} \\
1x^2 + \neg 1x + 1x^2 + 1x & \quad \text{DOS(2.5a)} \\
1x^2 + 1x^2 + \neg 1x + 1x & \quad \text{CPA(2.8)} \\
(1+1)x^2 + (\neg 1 + 1)x & \quad \text{DPF(2.4a)} \\
2x^2 + 0x & \quad \text{OOA(1.1)} \\
2x^2 & \quad \text{MId(2.1b)}
\end{aligned}
$$

Less Steps Solution:

$$
\begin{aligned}
x^2 + \neg x + x^2 + x & \quad \text{DOS(2.5a)} \\
x^2 + x^2 + \neg x + x & \quad \text{CPA(2.8)} \\
2x^2 & \quad \text{OOA(1.1)}
\end{aligned}
$$

Example 2.29 — id:20141108-194709.

Simplify $-2x4x - x \cdot -x3$

Solution:

$$
\begin{aligned}
-2x4x - 1x \cdot 1x3 &\qquad \text{MId}(2.1a)\\
\neg 2x4x - 1x \cdot 1x3 &\qquad \text{ONeg}(2.6a)\\
\neg 2 \cdot x \cdot 4 \cdot x + \neg 1 \cdot x \cdot 1 \cdot x \cdot 3 &\qquad \text{JTC}(1.6)\\
\neg 2 \cdot 4 \cdot x \cdot x + \neg 1 \cdot \neg 1 \cdot 3 \cdot x \cdot x &\qquad \text{CPM}(1.8)\\
\neg 2 \cdot 4 \cdot x^2 + \neg 1 \cdot \neg 1 \cdot 3 \cdot x^2 &\qquad \text{PrCBPo}(2.10a)\\
(\neg 2 \cdot 4) \cdot x^2 + (\neg 1 \cdot \neg 1 \cdot 3)x^2 &\qquad \text{APM}(1.10a)\\
\neg 8x^2 + 3x^2 &\qquad \text{OOM}(1.4)\\
(\neg 8 + 3)x^2 &\qquad \text{DPF}(2.4a)\\
\neg 5x^2 &\qquad \text{OOA}(1.1)\\
-5x^2 &\qquad \text{ONeg}(2.6b)
\end{aligned}
$$

S

Less Steps Solution:

$$
\begin{aligned}
-2x4x + \neg x \cdot -x3 &\qquad \text{DOS}(2.5a)\\
\neg 2 \cdot 4 \cdot x \cdot x + 3 \cdot \neg x \cdot \neg x &\qquad \text{CPM}(1.8)\\
\neg 2 \cdot 4 \cdot x^2 + 3 \cdot x^2 &\qquad \text{PrCBPo}(2.10a)\\
\neg 8x^2 + 3x^2 &\qquad \text{OOM}(1.4)\\
-5x^2 &\qquad \text{OOA}(1.1)
\end{aligned}
$$

■

Rule 2.5.1 — Product of a Common Base Powers (PrCBPo).

$$b^m \cdot b^n = b^{m+n} \tag{2.10a}$$
$$b^{m+n} = b^m \cdot b^n \tag{2.10b}$$

Rule 2.5.2 — Quotient of a Common Base Powers (QCBPo).

$$\frac{b^m}{b^n} = b^{m-n} \tag{2.11a}$$
$$b^{m-n} = \frac{b^m}{b^n} \tag{2.11b}$$

Rule 2.5.3 — Power of a Power (PoPo).

$$(b^m)^k = b^{m \cdot k} \tag{2.12a}$$
$$b^{m \cdot k} = (b^m)^k \tag{2.12b}$$

Example 2.30 — id:20141108-191616.

Simplify $-5x \cdot 4x$

S

Solution:

$\neg 5x \cdot 4x$	ONeg(2.6a)
$\neg 5 \cdot x \cdot 4 \cdot x$	JTC(1.6)
$\neg 5 \cdot 4 \cdot x \cdot x$	CPM(1.8)
$\neg 5 \cdot 4 \cdot x^2$	PrCBPo(2.10a)
$(\neg 5 \cdot 4) \cdot x^2$	APM(1.10a)
$\neg 20 \cdot x^2$	OOM(1.4)
$\neg 20x^2$	CTJ(1.7)
$-20x^2$	ONeg(2.6b)

S

Less Steps Solution:

$\neg 5 \cdot 4 \cdot x \cdot x$	CPM(1.8)
$\neg 5 \cdot 4 \cdot x^2$	PrCBPo(2.10a)
$-20x^2$	OOM(1.4)

■

Binomials

Example 2.31 — id:20141106-152339.

Simplify $3x^2 + 2x + 5x^2 + 4x$

S

Solution:

$3x^2 + 5x^2 + 2x + 4x$	CPA(2.8)
$(3+5)x^2 + (2+4)x$	DPF(2.4a)
$8x^2 + 6x$	OOA(1.1)

If needed we could continue and express it in the simplified factored form using the distributive property

$(4x+3)2x$ DPF(2.4a)

Less Steps Solution:

$$3x^2+5x^2+2x+4x \quad \text{CPA(2.8)}$$
$$8x^2+6x \quad \text{OOA(1.1)}$$

Example 2.32 — id:20141107-121834.

Simplify $\left(\sqrt{9-x^2}\right)^2$

Solution:

$$\left(\sqrt{9-1x^2}\right)^2 \quad \text{MId(2.1a)}$$
$$\left(\sqrt{9+\neg 1x^2}\right)^2 \quad \text{DOS(2.5a)}$$
$$\left[\left(9+\neg x^2\right)^{\frac{1}{2}}\right]^2 \quad \text{RTPo(1.17)}$$
$$9+\neg 1x^2 \quad \text{PoPo(2.12a)}$$
$$\neg 1x^2+9 \quad \text{CPA(2.8)}$$
$$\neg x^2+9 \quad \text{MId(2.1a)}$$
$$-x^2+9 \quad \text{ONeg(2.6b)}$$

Less Steps Solution:

$$9-x^2 \quad \text{PoPo(2.12a)}$$

It might be easier to view this using a variable substitution for the radicand, $9-x^2$*. Let* $k=9+\neg 1x^2$.

$$\begin{aligned}
&\left(\sqrt{k}\right)^2 && \text{MId(2.1a)}\\
&\left(\sqrt{k}\right)^2 && \text{DOS(2.5a)}\\
&\left[(k)^{\frac{1}{2}}\right]^2 && \text{RTPo(1.17)}\\
&k && \text{PoPo(2.12a)}\\
&9+\neg 1x^2 && \text{CPA(2.8)}\\
&\neg 1x^2+9 && \text{CPA(2.8)}\\
&\neg x^2+9 && \text{MId(2.1a)}\\
&-x^2+9 && \text{ONeg(2.6b)}
\end{aligned}$$

D

Dependencies:example 6.2-20141105-144223

Example 2.33 — id:20141209-145211.

Simplify $2x(2x+4)+x^2\cdot 2\cdot 1+0$

S

Solution:

$$\begin{aligned}
2x\cdot 2x+2x\cdot 4+x^2\cdot 2\cdot 1+0 &&& \text{DPE(2.9a)}\\
2\cdot x\cdot 2\cdot x+2\cdot x\cdot 4+x^2\cdot 2\cdot 1+0 &&& \text{JTC(1.6)}\\
2\cdot 2\cdot x\cdot x+2\cdot 4\cdot x+2\cdot 1\cdot x^2+0 &&& \text{CPM(1.8)}\\
2\cdot 2\cdot x^2+2\cdot 4\cdot x+2\cdot 1\cdot x^2+0 &&& \text{PrCBPo(2.10a)}\\
4\cdot x^2+8\cdot x+2\cdot x^2+0 &&& \text{OOM(1.4)}\\
4x^2+8x+2x^2+0 &&& \text{CTJ(1.7)}\\
4x^2+2x^2+8x &&& \text{APA(2.7a)}\\
(4+2)x^2+8x &&& \text{DPF(2.4b)}\\
6x^2+8x &&& \text{OOA(1.1)}
\end{aligned}$$

D

Dependencies:example 5.6-20141209-144203

Trinomials

Example 2.34 — id:20141109-133008.

Simplify by expanding $(x+5)(x-8)$

Solution:

$$
\begin{aligned}
(1x+5)(1x-8) && \text{MId}(2.1a)\\
(1x+5)(1x+\neg 8) && \text{DOS}(2.5a)\\
1x(1x+\neg 8)+5(1x+\neg 8) && \text{DPE}(2.9b)\\
1x\cdot 1x+1x\cdot \neg 8+5\cdot 1x+5\cdot \neg 8 && \text{DPE}(2.9a)\\
1\cdot x\cdot 1\cdot x+1\cdot x\cdot \neg 8+5\cdot 1\cdot x+5\cdot \neg 8 && \text{JTC}(1.6)\\
1\cdot 1\cdot x\cdot x+\neg 8\cdot 1\cdot x+1\cdot 5\cdot x+\neg 8\cdot 5 && \text{CPM}(1.8)\\
1\cdot 1\cdot x^2+\neg 8\cdot 1\cdot x+1\cdot 5\cdot x+\neg 8\cdot 5 && \text{PrCBPo}(2.10a)\\
1\cdot x^2+\neg 8\cdot x+5\cdot x+\neg 40 && \text{OOM}(1.4)\\
1x^2+\neg 8x+5x+\neg 40 && \text{CTJ}(1.7)\\
1x^2+\neg 3x+\neg 40 && \text{OOA}(1.1)\\
1x^2-3x-40 && \text{DOS}(2.5b)\\
x^2-3x-40 && \text{MId}(2.1a)
\end{aligned}
$$

Less Steps Solution:

$$
\begin{aligned}
(x+5)(x+\neg 8) && \text{DOS}(2.5a)\\
x(x+\neg 8)+5(x+\neg 8) && \text{DPE}(2.9b)\\
x^2+\neg 8x+5x+\neg 40 && \text{DPE}(2.9a)\\
x^2-3x-40 && \text{OOA}(1.1)
\end{aligned}
$$

■

Example 2.35 — id:20141109-133316.

Simplify by expanding $(x+a)(x+b)$, where $a,b\in\mathbb{Z}$

Solution:

$$
\begin{aligned}
(1x+a)(1x+b) &\quad \text{MId(2.1a)}\\
1x(1x+b)+a(1x+b) &\quad \text{DPE(2.9b)}\\
1x\cdot 1x+1x\cdot b+a\cdot 1x+a\cdot b &\quad \text{DPE(2.9a)}\\
1\cdot x\cdot 1\cdot x+1\cdot x\cdot b+a\cdot 1\cdot x+a\cdot b &\quad \text{JTC(1.6)}\\
1\cdot 1\cdot x\cdot x+1\cdot b\cdot x+1\cdot a\cdot x+a\cdot b &\quad \text{CPM(1.8)}\\
1\cdot 1\cdot x^2+1\cdot b\cdot x+1\cdot a\cdot x+a\cdot b &\quad \text{PrCBPo(2.10a)}\\
1\cdot x^2+1\cdot b\cdot x+1\cdot a\cdot x+a\cdot b &\quad \text{OOM(1.4)}\\
1x^2+1bx+1ax+ab &\quad \text{CTJ(1.7)}\\
1x^2+(1b+1a)x+ab &\quad \text{DPF(2.4a)}\\
x^2+(b+a)x+ab &\quad \text{MId(2.1b)}
\end{aligned}
$$

S

Less Steps Solution:

$$
\begin{aligned}
x(x+b)+a(x+b) &\quad \text{DPE(2.9b)}\\
x^2+(b+a)x+ab &\quad \text{DPE(2.9a)}
\end{aligned}
$$

■

Example 2.36 — id:20141109-140659.

Simplify by expanding $(2x+3)(5x+13)$

S

Solution:

$$
\begin{aligned}
2x(5x+13)+3(5x+13) &\quad \text{DPE(2.9b)}\\
2x\cdot 5x+2x\cdot 13+3\cdot 5x+3\cdot 13 &\quad \text{DPE(2.9a)}\\
2\cdot x\cdot 5\cdot x+2\cdot x\cdot 13+3\cdot 5\cdot x+3\cdot 13 &\quad \text{JTC(1.6)}\\
2\cdot 5\cdot x\cdot x+2\cdot 13\cdot x+5\cdot 3\cdot x+3\cdot 13 &\quad \text{CPM(1.8)}\\
2\cdot 5\cdot x^2+2\cdot 13\cdot x+5\cdot 3\cdot x+3\cdot 13 &\quad \text{PrCBPo(2.10a)}\\
10\cdot x^2+26\cdot x+16\cdot x+39 &\quad \text{OOM(1.4)}\\
10x^2+26x+15x+39 &\quad \text{CTJ(1.7)}\\
10x^2+41x+39 &\quad \text{OOA(1.1)}
\end{aligned}
$$

S

Less Steps Solution:

$2x(5x+13)+3(5x+13)$	DPE(2.9b)
$10x^2+26x+15x+39$	DPE(2.9a)
$10x^2+41x+39$	OOA(1.1)

Example 2.37 — id:20141109-141019.

Simplify by expanding $(-3x-5)(7x+8)$

S

Solution:

$(\neg 3x-5)(7x+8)$	ONeg(2.6a)
$(\neg 3x+\neg 5)(7x+8)$	DOS(2.5a)
$\neg 3x(7x+8)+\neg 5(7x+8)$	DPE(2.9b)
$\neg 3x\cdot 7x+\neg 3x\cdot 8+\neg 5\cdot 7x+\neg 5\cdot 8$	DPE(2.9a)
$\neg 3\cdot x\cdot 7\cdot x+\neg 3\cdot x\cdot 8+\neg 5\cdot 7\cdot x+\neg 5\cdot 8$	JTC(1.6)
$\neg 3\cdot 7\cdot x\cdot x+\neg 3\cdot 8\cdot x+\neg 5\cdot 7\cdot x+\neg 5\cdot 8$	CPM(1.8)
$\neg 3\cdot 7\cdot x^2+\neg 3\cdot 8\cdot x+\neg 5\cdot 7\cdot x+\neg 5\cdot 8$	PrCBPo(2.10a)
$\neg 21\cdot x^2+\neg 24\cdot x+\neg 35\cdot x+\neg 40$	OOM(1.4)
$\neg 21x^2+\neg 24x+\neg 35x+\neg 40$	CTJ(1.7)
$\neg 21x^2+\neg 59x+\neg 40$	OOA(1.1)
$\neg 21x^2-59x-40$	DOS(2.5b)
$-21x^2-59x-40$	ONeg(2.6b)

S

Less Steps Solution:

$(-3x+\neg 5)(7x+8)$	DOS(2.5a)
$\neg 3x(7x+8)+\neg 5(7x+8)$	DPE(2.9b)
$\neg 21x^2+\neg 24x+\neg 35x+\neg 40$	CTJ(2.9a)
$-21x^2-59x-40$	OOA(1.1)

Example 2.38 — id:20141109-141347.

Simplify by expanding $(ax+b)(cx+d)$, where $a,b,c,d\in\mathbb{Z}$

S

Solution:

$ax(cx+d)+b(cx+d)$	DPE(2.9b)
$ax\cdot cx+ax\cdot d+b\cdot cx+b\cdot d$	DPE(2.9a)
$a\cdot x\cdot c\cdot x+a\cdot x\cdot d+b\cdot c\cdot x+b\cdot d$	JTC(1.6)
$a\cdot c\cdot x\cdot x+a\cdot d\cdot x+b\cdot c\cdot x+b\cdot d$	CPM(1.8)
$a\cdot c\cdot x^2+a\cdot d\cdot x+b\cdot c\cdot x+b\cdot d$	PrCBPo(2.10a)
$acx^2+adx+bcx+bd$	CTJ(1.7)
$acx^2+(ad+bc)x+bd$	DPF(2.4a)

S

Less Steps Solution:

$ax(cx+d)+b(cx+d)$	DPE(2.9b)
$acx^2+(ad+bc)x+bd$	DPE(2.9a)

■

Example 2.39 — id:20141105-161225.

Simplify $\left(2-\frac{x}{2}\right)^2$ by expanding.

S

Solution:

$$\left(2-\frac{1x}{2}\right)^2 \quad \text{MId(2.1a)}$$
$$\left(2+\neg\frac{1x}{2}\right)^2 \quad \text{DOS(2.5a)}$$
$$\left(2+\neg\frac{1x}{2}\right)\left(2+\neg\frac{1x}{2}\right) \quad \text{PoTF(1.13)}$$
$$2\left(2+\neg\frac{1x}{2}\right)+\neg\frac{1x}{2}\left(2+\neg\frac{1x}{2}\right) \quad \text{DPE(2.9b)}$$
$$2\cdot 2+2\cdot\neg\frac{1x}{2}+\neg\frac{1x}{2}\cdot 2+\neg\frac{1x}{2}\cdot\neg\frac{1x}{2} \quad \text{DPE(2.9a)}$$
$$2\cdot 2+2\cdot\neg\frac{1}{2}\cdot x+\neg\frac{1}{2}\cdot x\cdot 2+\neg\frac{1}{2}\cdot x\cdot\neg\frac{1}{2}\cdot x \quad \text{JTC(1.6)}$$
$$2\cdot 2+2\cdot\neg\frac{1}{2}\cdot x+\neg\frac{1}{2}\cdot 2\cdot x+\neg\frac{1}{2}\cdot\neg\frac{1}{2}\cdot x\cdot x \quad \text{CPM(1.8)}$$
$$2\cdot 2+2\cdot\neg\frac{1}{2}\cdot x+\neg\frac{1}{2}\cdot 2\cdot x+\neg\frac{1}{2}\cdot\neg\frac{1}{2}\cdot x^2 \quad \text{PrCBPo(2.10a)}$$
$$4+\neg 1\cdot x+\neg 1\cdot x+\frac{1}{4}\cdot x^2 \quad \text{OOM(1.4)}$$
$$4+\neg 1x+\neg 1x+\frac{1}{4}x^2 \quad \text{CTJ(1.7)}$$
$$\frac{1}{4}x^2+\neg 1x+\neg 1x+4 \quad \text{CPA(2.8)}$$
$$\frac{1}{4}x^2+\neg 2x+4 \quad \text{OOA(1.1)}$$
$$\frac{1}{4}x^2-2x+4 \quad \text{DOS(2.5b)}$$

S

Less Steps Solution:

$$7x+20x+40 \quad \text{DPE(2.9a)}$$
$$27x+40 \quad \text{OOA(1.1)}$$

2.6 Degree 3 Univariate Polynomials

Monomials

Binomials

Trinomials

Polynomials

2.7 Degree n Univariate Polynomials

Monomials

Binomials

Trinomials

Polynomials

3. Equations

3.1 Equality

Property 3.1.1 — Reflexive Property of Eqaulity (RPE).

$$a = a \tag{3.1a}$$

Property 3.1.2 — Substitution Property of Equality (SPE).

Given $a = b$, then

$$E(a) = E(b) \tag{3.2}$$

$E(x)$ represents any expression.

Property 3.1.3 — Symmetric Property of Equality (SyPE).

$$a = b \quad \text{then} \quad b = a \tag{3.3a}$$

Property 3.1.4 — Transitive Property of Equality (TPE).

$$\text{if} \quad a = b \quad \text{and} \quad b = c \quad \text{then} \quad a = c \tag{3.4a}$$

3.2 Solving Linear Equations

Example 3.1 — id:20141206-102142.

Solve the equation $x + a = b$ for x

S

Solution:

$[x+a] + \neg a = [b] + \neg a$	SPE(3.2) + AI(1.19a)
$x + (a + \neg a) = b + \neg a$	APA(2.7b)
$x + 0 = b + \neg a$	OOA(1.1)
$x = b + \neg a$	AId(2.3b)
$x = b - a$	DOS(2.5b)

■

Example 3.2 — id:20141111-222931.

Solve the equations $x + 8 = 0$

S

Solution:

$[x+8] + \neg 8 = [0] + \neg 8$	SPE(3.2) + AI(1.19a)
$x + (8 + \neg 8) = 0 + \neg 8$	APA(2.7b)
$x + 0 = \neg 8$	OOA(1.1)
$x = \neg 8$	AId(2.3b)
$x = -8$	ONeg(2.6b)

S

Less Steps Solution:

$[x+8] + \neg 8 = [0] + \neg 8$	SPE(3.2) + AI(1.19a)
$x = -8$	OOA(1.1)

D

Dependencies:
example 4.2-20141111-190212

■

Example 3.3 — id:20141206-101632.

Solve the equation $x+4=7$

S

Solution:

$$\begin{aligned} [x]+4+\neg 4 &= [7]+\neg 4 && \text{SPE}(3.2)+\text{AI}(1.19a) \\ x+(4+\neg 4) &= 7+\neg 4 && \text{APA}(2.7b) \\ x+0 &= 3 && \text{OOA}(1.1) \\ x &= 3 && \text{AId}(2.3b) \end{aligned}$$

S

Less Steps Solution:

$$\begin{aligned} [x]+4+\neg 4 &= [7]+\neg 4 && \text{SPE}(3.2)+\text{AI}(1.19a) \\ x &= 3 && \text{OOA}(1.1) \end{aligned}$$

■

Example 3.4 — id:20141206-101107.

Solve the equation $x-8=15$ for x

S

Solution:

$$\begin{aligned} x+\neg 8 &= 15 && \text{DOS}(2.5a) \\ [x+\neg 8]+8 &= [15]+8 && \text{SPE}(3.2)+\text{AI}(1.19a) \\ x+(\neg 8+8) &= 15+8 && \text{APA}(2.7b) \\ x+0 &= 23 && \text{OOA}(1.1) \\ x &= 23 && \text{AId}(2.3b) \end{aligned}$$

S

Less Steps Solution:

$$\begin{aligned} [x+\neg 8]+8 &= [15]+8 && \text{SPE}(3.2)+\text{AI}(1.19a) \\ x &= 23 && \text{OOA}(1.1) \end{aligned}$$

■

Example 3.5 — id:20141206-102404.

Solve the equation $5x = 9$ for x.

Solution:

$$
\begin{aligned}
\frac{1}{5}[5x] &= \frac{1}{5}[9] && \text{SPE(3.2)} + \text{MI(1.9a)} \\
\frac{1}{5}\cdot[5\cdot x] &= \frac{1}{5}\cdot 9 && \text{JTC(1.6)} \\
\left(\frac{1}{5}\cdot 5\right)\cdot x &= \frac{1}{5}\cdot 9 && \text{APM(1.10b)} \\
1\cdot x &= \frac{9}{5} && \text{OOM(1.4)} \\
x &= \frac{9}{5} && \text{MId(2.1b)}
\end{aligned}
$$

Less Steps Solution:

$$
\begin{aligned}
\frac{1}{5}[5x] &= \frac{1}{5}[9] && \text{SPE(3.2)} + \text{MI(1.9a)} \\
x &= \frac{9}{5} && \text{OOM(1.4)}
\end{aligned}
$$

Example 3.6 — id:20141206-104404.

Solve the equation $ax = b$ for x.

Solution:

$$
\begin{aligned}
\frac{1}{a}[ax] &= \frac{1}{a}[b] && \text{SPE(3.2)} + \text{MI(1.9a)} \\
\frac{1}{a}\cdot(a\cdot x) &= \frac{1}{a}\cdot b && \text{JTC(1.6)} \\
\left(\frac{1}{a}\cdot a\right)\cdot x &= \frac{1}{a}\cdot b && \text{APM(1.10b)} \\
1\cdot x &= \frac{b}{a} && \text{OOM(1.4)} \\
x &= \frac{b}{a} && \text{MId(2.1b)}
\end{aligned}
$$

Example 3.7 — id:20141206-102723.

Solve the equation $-2x = 7$ for x

S

Solution:

$$\begin{aligned}
\neg 2x &= 7 && \text{ONeg(2.6a)}\\
\neg\frac{1}{2}[\neg 2x] &= \neg\frac{1}{2}[7] && \text{SPE(3.2)} + \text{MI(1.9b)}\\
\neg\frac{1}{2}\cdot(\neg 2\cdot x) &= \neg\frac{1}{2}\cdot 7 && \text{JTC(1.6)}\\
\left(\neg\frac{1}{2}\cdot\neg 2\right)\cdot x &= \neg\frac{1}{2}\cdot 7 && \text{APM(1.10b)}\\
1\cdot x &= \neg\frac{7}{2} && \text{OOM(1.4)}\\
1\cdot x &= -\frac{7}{2} && \text{ONeg(2.6b)}\\
x &= -\frac{7}{2} && \text{MId(2.1b)}
\end{aligned}$$

S

Less Steps Solution:

$$\begin{aligned}
\neg\frac{1}{2}[\neg 2x] &= \neg\frac{1}{2}[7] && \text{SPE(3.2)} + \text{MI(1.9b)}\\
x &= -\frac{7}{2} && \text{OOM(1.4)}
\end{aligned}$$

■

Example 3.8 — id:20141111-215726.

Solve the equation $2x + 5 = 0$ for x

S

Solution:

$$\begin{aligned}
[2x+5]+\neg 5 &= [0]+\neg 5 && \text{SPE(3.2)}+\text{AI(1.19a)}\\
2x+(5+\neg 5) &= 0+\neg 5 && \text{APA(2.7a)}\\
2x+0 &= \neg 5 && \text{OOA(1.1)}\\
2x &= \neg 5 && \text{AId(2.3a)}\\
\frac{1}{2}[2x] &= \frac{1}{2}[\neg 5] && \text{SPE(3.2)}+\text{MI(1.9a)}\\
\frac{1}{2}\cdot 2\cdot x &= \frac{1}{2}\cdot \neg 5 && \text{JTC(1.6)}\\
\left(\frac{1}{2}\cdot 2\right)\cdot x &= \frac{1}{2}\cdot \neg 5 && \text{APM(1.10a)}\\
1\cdot x &= \frac{\neg 5}{2} && \text{OOM(1.4)}\\
1x &= -\frac{5}{2} && \text{ONeg(2.6b)}\\
x &= -\frac{5}{2} && \text{MId(2.1b)}
\end{aligned}$$

S

Less Steps Solution:

$$[2x+5]+\neg 5 = [0]+\neg 5 \qquad \text{SPE(3.2)}+\text{AI(1.19a)} \tag{3.5}$$
$$2x = \neg 5 \qquad \text{OOA(1.1)} \tag{3.6}$$
$$\frac{1}{2}[2x] = \frac{1}{2}[\neg 5] \qquad \text{SPE(3.2)}+\text{MI(1.9a)} \tag{3.7}$$
$$x = -\frac{5}{2} \qquad \text{OOM(1.4)} \tag{3.8}$$

D

Dependencies:
example 4.3-20141111-192213

3.3 Solving Quadratic Equations

Example 3.9 — id:20141107-131748.

Solve the equation $2-x^2=0$ for x

S

Solution:

$$\begin{aligned}
2 - 1x^2 &= 0 && \text{MId(2.1a)}\\
2 + \neg 1x^2 &= 0 && \text{DOS(2.5a)}\\
\left[2 + \neg 1x^2\right] + 1x^2 &= [0] + 1x^2 && \text{SPE(3.2)} + \text{AI(1.19a)}\\
2 + \left(\neg 1x^2 + 1x^2\right) &= 0 + 1x^2 && \text{APA(2.7a)}\\
2 + 0 &= 0 + 1x^2 && \text{OOA(1.1)}\\
2 &= 1x^2 && \text{AId(2.3a)}\\
2 &= x^2 && \text{MId(2.1b)}\\
\pm[2]^{\frac{1}{2}} &= \left[x^2\right]^{\frac{1}{2}} && \\
\pm 2^{\frac{1}{2}} &= x && \text{PoPo(2.12a)}\\
\pm\sqrt{2} &= x && \text{PoTR(1.18)}\\
x &= \pm\sqrt{2} && \text{SyPE(3.3a)}
\end{aligned}$$

■

3.3.1 Completing the Square

Completing the square is an algebraic algorithm used to find the solutions of quadratic equations of the form, $ax^2 + bx + c = 0$. Essentially, we want to manipulate this equation such that $x =$ some values. To gain some understanding of how this algorithm works, we will consider each step individually. Let's begin with a quadratic equation in the general form: $ax^2 + bx + c = 0$.

Since we are trying to manipulate the equation $ax^2 + bx + c = 0$ such that $x =$ some value, we first want the coefficient factor a to be equal to 1. This is done by multiplying both expressions by the multiplicative inverse of a (step 1) followed by simplifying both expressions.

$$\begin{aligned}
\frac{1}{a}\left[ax^2 + bx + c\right] &= \frac{1}{a}[0] && \text{SPE(3.2)} + \text{MI(1.9a)}\\
\frac{1}{a}\cdot ax^2 + \frac{1}{a}\cdot bx + \frac{1}{a}\cdot c &= \frac{1}{a}[0] && \text{DPE(2.9a)}\\
\frac{1}{a}\cdot a\cdot x^2 + \frac{1}{a}\cdot b\cdot x + \frac{1}{a}\cdot c &= \frac{1}{a}[0] && \text{JTC(1.6)}\\
x^2 + \frac{b}{a}\cdot x + \frac{c}{a} &= 0 && \text{OOM(1.4)}\\
x^2 + \frac{b}{a}x + \frac{c}{a} &= 0 && \text{CTJ(1.7)}
\end{aligned}$$

We now have three terms in the left hand expression where the first two terms have at least one variable factor, x. The problem is that the third term is a constant and we want $x =$ some value. This text step is to get rid of the $\frac{c}{a}$ term, which can be done by using the additive inverse followed by simplifying both expressions.

$$\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]+\neg\frac{c}{a}=[0]+\neg\frac{c}{a} \qquad \text{SPE(3.2)}+\text{AI(1.19a)}$$
$$x^2+\frac{b}{a}x+\frac{c}{a}+\neg\frac{c}{a}=0+\neg\frac{c}{a} \qquad \text{APA(2.7a)}$$
$$x^2+\frac{b}{a}x=\neg\frac{c}{a} \qquad \text{OOA(1.1)}$$

The next step is called completing the square. The idea is to add a *NeW* constant, k, to the left-hand expression, $x^2+\frac{b}{a}x+k$, such that the quadratic expression can then be factored as two identical factors, $(x+m)(x+m)=(x+m)^2$, where $k=m\cdot m$. Notice that since we are adding a constant term, k, to the left-hand expression, then we must also add this constant, k, to the right-hand expression, $x^2+\frac{b}{a}x+k=\neg\frac{c}{a}+k$. To determine the values of both m and k we should refer to the the organization of the two factors that make up the product of a quadratic expression, $x^2+\frac{b}{a}x+k=(x+m)^2$.

Since both factors of this new quadratic expression are the same, both terms that make up the middle term, must also be the same. We know that $mx+mx=\frac{b}{a}x$, so we should be able to determine the value of m from this equation. If we can determine the value of m, then we can determine the value of k.

$$\frac{b}{a}x=mx+mx$$
$$=2mx \qquad \text{OOA(1.1)}$$

Solving for m,

$$2mx=\frac{b}{a}x$$
$$2\cdot m\cdot x=\frac{b}{a}\cdot x \qquad \text{JTC(1.6)}$$
$$\frac{1}{2}\left[2\cdot m\cdot x\right]=\frac{1}{2}\left[\frac{b}{a}\cdot x\right] \qquad \text{SPE(3.2)}+\text{MI(1.9a)}$$
$$m\cdot x=\frac{b}{2a}\cdot x \qquad \text{OOM(1.4)}$$
$$\left[m\cdot x\right]\frac{1}{x}=\left[\frac{b}{2a}\cdot x\right]\frac{1}{x} \qquad \text{SPE(3.2)}+\text{MI(1.9a)}$$
$$m=\frac{b}{2a} \qquad \text{OOM(1.4)}$$

$$
\begin{aligned}
\left[x^2+\frac{b}{a}x\right]+\left(\frac{b}{2a}\right)^2 &= \left[\neg\frac{c}{a}\right]+\left(\frac{b}{2a}\right)^2 && \text{SPE(3.2)}\\
x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}+\left(\frac{b}{2a}\right)^2 && \text{APA(2.7b)}\\
\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right) &= \neg\frac{c}{a}+\left(\frac{b}{2a}\right)^2 && \text{DPF(2.4b)}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}+\left(\frac{b}{2a}\right)^2 && \text{PoTF(1.13)}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}+\frac{(b)^2}{(2a)^2} && \text{PoQPo(1.11a)}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}+\frac{b^2}{2^2a^2} && \text{PoPrPo}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}+\frac{b^2}{4a^2} && \text{OOE}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c}{a}\cdot\frac{4a}{4a}+\frac{b^2}{4a^2} && \text{MId}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{c\cdot 4\cdot a}{a\cdot 4\cdot a}+\frac{b^2}{4a^2} && \text{JTC}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{4\cdot a\cdot c}{4\cdot a\cdot a}+\frac{b^2}{4a^2} && \text{CPM}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{4\cdot a\cdot c}{4\cdot a^2}+\frac{b^2}{4a^2} && \text{PrCBPo}\\
\left(x+\frac{b}{2a}\right)^2 &= \neg\frac{4ac}{4a^2}+\frac{b^2}{4a^2} && \text{CTJ}\\
\left(x+\frac{b}{2a}\right)^2 &= \frac{\neg 4ac+b^2}{4a^2} && \text{CD}\\
\left(x+\frac{b}{2a}\right)^2 &= \frac{b^2+\neg 4ac}{4a^2} && \text{CPA}\\
\left[\left(x+\frac{b}{2a}\right)^2\right]^{\frac{1}{2}} &= \pm\left[\frac{b^2+\neg 4ac}{4a^2}\right]^{\frac{1}{2}} && \text{PoI}
\end{aligned}
$$

$$x + \frac{b}{2a} = \pm\left[\frac{b^2 + \neg 4ac}{4a^2}\right]^{\frac{1}{2}} \qquad \text{PoPrPo}$$

$$x + \frac{b}{2a} = \pm\frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{\left[4a^2\right]^{\frac{1}{2}}} \qquad \text{PoQPo}$$

$$x + \frac{b}{2a} = \pm\frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{4^{\frac{1}{2}}a} \qquad \text{PoPrPo}$$

$$x + \frac{b}{2a} = \pm\frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{OOE}$$

$$\left[x + \frac{b}{2a}\right] + \neg\frac{b}{2a} = \left[\pm\frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a}\right] + \neg\frac{b}{2a} \qquad \text{AI}$$

$$x + \frac{b}{2a} + \neg\frac{b}{2a} = \pm\frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} + \neg\frac{b}{2a} \qquad \text{APA}$$

$$x + \frac{b}{2a} + \neg\frac{b}{2a} = \neg\frac{b}{2a} \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{CPA}$$

$$x = \neg\frac{b}{2a} \pm \frac{\left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{OOA}$$

$$x = \frac{\neg b \pm \left[b^2 + \neg 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{CD}$$

$$x = \frac{\neg b \pm \left[b^2 - 4ac\right]^{\frac{1}{2}}}{2a} \qquad \text{DOS}$$

$$x = \frac{\neg b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{ETR}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{ONeg}$$

II

DP-SL

4. Functions

4.1 Evaluating Functions

Example 4.1 — id:20141106-083703.

Given $f(x) = \frac{1}{12}x^3 - x^2 + 4x$, evaluate $f(3) - f(2)$.

S

Solution:

$$
\begin{aligned}
f(3)-f(2) &= \left(\frac{[3]^3}{12}-[3]^2+4[3]\right)-\left(\frac{[2]^3}{12}-[2]^2+4[2]\right) \\
&= \left(\frac{(3)^3}{12}-(3)^2+4(3)\right)-1\left(\frac{(2)^3}{12}-(2)^2+4(2)\right) && \text{MId(2.1a)} \\
&= \left(\frac{(3)^3}{12}+\neg(3)^2+4(3)\right)+\neg 1\left(\frac{(2)^3}{12}+\neg(2)^2+4(2)\right) && \text{DOS(2.5a)} \\
&= \left(\frac{27}{12}+\neg 9+4(3)\right)+\neg 1\left(\frac{8}{12}+\neg 4+4(2)\right) && \text{OOE(1.5)} \\
&= \left(\frac{27}{12}+\neg 9+12\right)+\neg 1\left(\frac{8}{12}+\neg 4+8\right) && \text{OOM(1.4)} \\
&= \left(\frac{27}{12}+3\right)+\neg 1\left(\frac{8}{12}+4\right) && \text{OOA(1.1)} \\
&= \left(\frac{27+36}{12}\right)+\neg 1\left(\frac{8+48}{12}\right) && \text{FOOA(1.3a)} \\
&= \left(\frac{63}{12}\right)+\neg 1\left(\frac{56}{12}\right) && \text{OOA(1.1)} \\
&= \left(\frac{63}{12}\right)+\neg\frac{56}{12} && \text{OOM(1.4)} \\
&= \frac{7}{12} && \text{OOA(1.1)}
\end{aligned}
$$

D

Dependencies:
example 6.2-20141105-144223

4.2 Quadratic Functions

4.2.1 Completing the Square

Definition 4.2.1 — Completing The Square. Completing the square is the process used to convert a quadratic polynomial function $f(x) = ax^2 + bx + c$ to the form

$$f(x) = a\left(x+\frac{b}{2a}\right)^2 + \left(c-\frac{b^2}{4a}\right)$$

We can simplify this form by defining $B = \dfrac{b}{2a}$ and $C = c - \dfrac{b^2}{4a}$, which gives us

$$f(x) = a(x+B)+C$$

[mathworld:completethesquare]

$$
\begin{aligned}
f(x) &= ax^2+bx+c \\
&= a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right] && \text{DPF(2.4b)} \\
&= a\left[x^2+\frac{b}{a}x+k+\neg k+\frac{c}{a}\right] && \text{AId(2.3b)} \\
&= a\left[\left(x^2+\frac{b}{a}x+k\right)+\left(-k+\frac{c}{a}\right)\right] && \text{APA(2.7b)}
\end{aligned}
$$

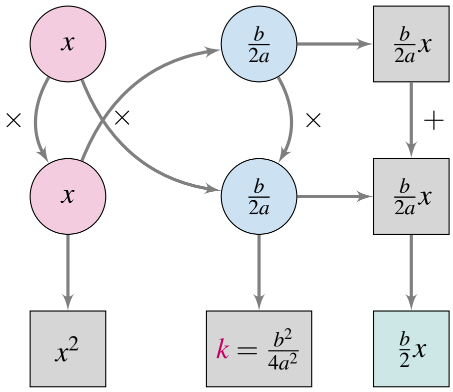


Figure 4.1: Factoring Organizer used to find the value of k

$$
\begin{aligned}
f(x) &= a\left[\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+\left(\frac{-b^2}{4a^2}+\frac{c}{a}\right)\right] \\
&= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{-b^2}{4a^2}+\frac{c}{a}\right] && \text{DPF(2.4b)} \\
&= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{-b^2}{4a^2}+\frac{4ac}{4a^2}\right] \\
&= a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right] && \text{OOA(1.1)} \\
&= a\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a} && \text{DPE(2.9a)} \\
&= a\left(x+\frac{b}{2a}\right)^2+\left(c-\frac{b^2}{4a}\right)
\end{aligned}
$$

4.3 Rational Functions

Definition 4.3.1 — Asympote. An asymptote is a line or curve that apporoaches a given curve arbitrarily close.[**mathworld:asymptote**]

A vertical asymptote is a vertical line $x_{va} = c$, that is used to visualize the values of x for which the function is not defined.

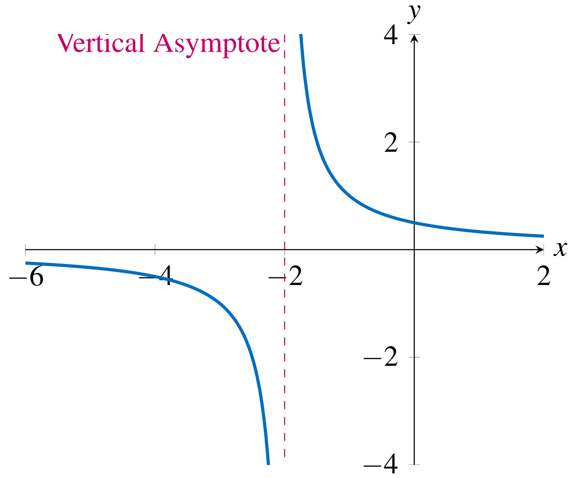


Figure 4.2: Vertical Asymptote

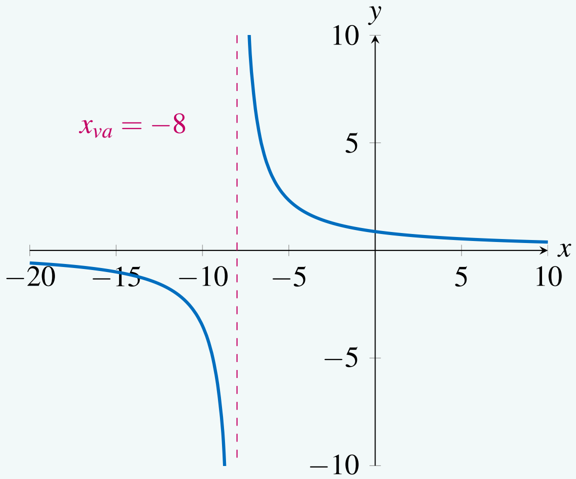
Example 4.2 — id:20141111-190212.

Find the vertical asymptote of the function $R(x) = \dfrac{7}{x+8}$

S

Solution:
We are interested in the values of x for which the denominator of $R(x)$ has a value of zero.

$$\begin{aligned} x_{va} + 8 &= 0 \\ x_{va} &= -8 \qquad \text{solving} \end{aligned}$$



Example 4.3 — id:20141111-192213.

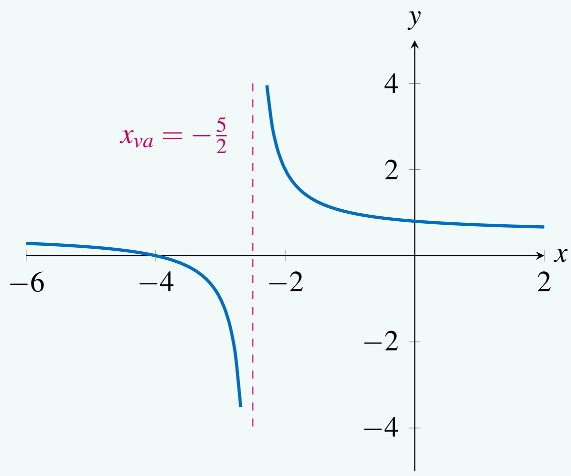
Find the vertical asymptote of the function $R(x) = \dfrac{x+4}{2x+5}$

S

Solution:

We are interested in the values of x for which the denominator of $R(x)$ has a value of zero.

$$
\begin{aligned}
2x_{va} + 5 &= 0 \\
x_{va} &= -\frac{5}{2} && \text{solving 3.8}
\end{aligned}
$$



5. Differentiation

5.1 Limit of the Difference Quotient

Definition 5.1.1 — Derivative. The derivative of a function $f(x)$ with respect to the variable x is defined as

$$f'(x) \equiv \lim_{\Delta x \to 0} \underbrace{\frac{f(x+\Delta x) - f(x)}{\Delta x}}_{\text{Difference Quotient}} \tag{5.1}$$

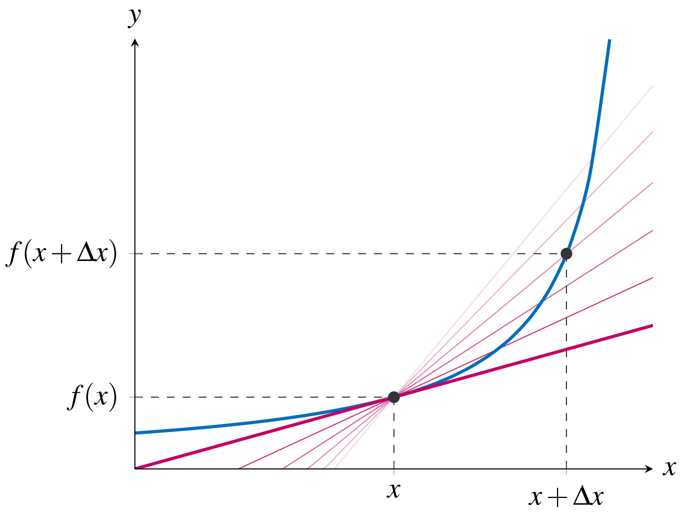


Figure 5.1: [**mooculus:textbook**]

Example 5.1 — id:20141219-212546.

Differentiate the function $f(x) = 5$

S

Solution:

$$
\begin{aligned}
f(x) &= 5x^0 && \text{PoID(1.16a)} \\
f'(x) &= \lim_{\Delta x \to 0} \frac{5[x+\Delta x]^0 - 5[x]^0}{\Delta x} && \text{SPE(3.2)\&DBFP(5.1)} \\
f'(x) &= \lim_{\Delta x \to 0} \frac{5(1) - 5(1)}{\Delta x} && \text{PoID(1.16b)} \\
f'(x) &= \lim_{\Delta x \to 0} 0 && \text{OOM(1.4)} \\
f'(x) &= 0
\end{aligned}
$$

■

5.2 Derivative of a Monomial Functions

Rule 5.2.1 — Derivative of a Constant (DC).

$$[c]' = 0 \tag{5.2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[c] = 0 \tag{5.3}$$

Rule 5.2.2 — Derivative of a Constant Multiple (DCM).

$$[cf(x)]' = c\,[f(x)]' \tag{5.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[cf(x)] = c\frac{\mathrm{d}}{\mathrm{d}x}[f(x)] \tag{5.5}$$

Rule 5.2.3 — Derivative of a Power (DPo).

$$[x^n]' = nx^{n-1} \tag{5.6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^n] = nx^{n-1} \tag{5.7}$$

Example 5.2 — id:20141124-153017.

Differentiate $f(x) = -3$

S

Solution:

$f'(x) = [-3]'$ SPE(3.2)
$f'(x) = 0$ DC(5.2)

D

Dependencies:
example 5.4-20141124-152503

Example 5.3 — id:20141124-141850.

Differentiate $f(x) = x^2$

S

Solution:

$f'(x) = [x^2]'$ SPE(3.2)
$f'(x) = 2x^{2-1}$ DPo(5.6)
$f'(x) = 2x^1$ OOA(1.1)
$f'(x) = 2x$ MId(2.1b)

S

Less Steps Solution:

$f'(x) = 2x$ DPo(5.6)

D

Dependencies:
example 5.4-20141124-152503

5.3 Derivative of Polynomial Functions

Rule 5.3.1 — Derivative of a Sum (DS).

$$[f(x)+g(x)]' = f'(x)+g'(x) \tag{5.8}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}[f(x)]+\frac{\mathrm{d}}{\mathrm{d}x}[g(x)] \tag{5.9}$$

Example 5.4 — id:20141124-152503.

Differentiate $f(x) = x^2 - 3$

S

Solution:

$$
\begin{aligned}
f(x) &= x^2 + \neg 3 && \text{DOS(2.5a)} \\
f'(x) &= [x^2 + \neg 3]' && \text{SPE(3.2)} \\
f'(x) &= [x^2]' + [\neg 3]' && \text{DS(5.8)} \\
f'(x) &= [x^2]' + 0 && \text{DC(5.2)} \\
f'(x) &= [x^2]' && \text{AId(2.3a)} \\
f'(x) &= 2x && \text{DPo(5.6) goto 5.3}
\end{aligned}
$$

S

Less Steps Solution:

$$f(x) = 2x^2 \qquad \text{DPo(5.6)\&DC(5.6)}$$

D

Dependencies:
example 5.12-20141124-205219

Example 5.5 — id:20141128-151834.

Differentiate $f(x) = 3x^2 - 6x + 4$

S

Solution:

$$
\begin{aligned}
f(x) &= 3x^2 + \neg 6x + 4 && \text{DOS(2.5a)} \\
f'(x) &= [3x^2 + \neg 6x + 4]' && \text{SPE(3.2)} \\
f'(x) &= [3x^2]' + [6x]' + [4]' && \text{DS(5.8)} \\
f'(x) &= [3x^2]' + [6x]' + 0 && \text{DC(5.2)} \\
f'(x) &= [3x^2]' + [6x]' && \text{AId(2.3a)} \\
f'(x) &= 3[x^2]' + 6[x]' && \text{DCM(5.4)} \\
f'(x) &= 3(2x) + 6(1) && \text{DPo(5.6)} \\
f'(x) &= 6x + 6 && \text{OOM(1.4)}
\end{aligned}
$$

S

$$
f'(x) = 6x + 6 \qquad \text{DS(5.8)}
$$

■

Rule 5.3.2 — Derivative of a Product (DPr).

$$
[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x) \tag{5.10}
$$

$$
\frac{\mathrm{d}}{\mathrm{d}x}[f(x)g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}[f(x)]g(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}[g(x)] \tag{5.11}
$$

Example 5.6 — id:20141209-144203.

Differentiate $f(x) = x^2(2x+4)$

S

Solution:

$$
\begin{aligned}
f'(x) &= [x^2(2x+4)]' && \text{SPE(3.2)} \\
f'(x) &= [x^2]'(2x+4) + x^2[2x+4]' && \text{DPr(5.10)} \\
f'(x) &= [x^2]'(2x+4) + x^2[2x] + [4]' && \text{DS(5.8)} \\
f'(x) &= [x^2]'(2x+4) + x^2 \cdot 2[x] + [4]' && \text{DCM(5.4)} \\
f'(x) &= 2x(2x+4) + x^2 \cdot 2 \cdot 1 + [4]' && \text{DPo(5.6)} \\
f'(x) &= 2x(2x+4) + x^2 \cdot 2 \cdot 1 + 0 && \text{DC(5.2)} \\
f'(x) &= 6x^2 + 8x && \text{simplify goto 2.33}
\end{aligned}
$$

■

Example 5.7 — id:20141209-142321.

Differentiate $f(x) = x^2 \cos(x)$

S

Solution:

$$\begin{aligned}
f'(x) &= [x^2 \cos(x)]' && \text{SPE(3.2)}\\
f'(x) &= [x^2]' \cos(x) + x^2 [\cos(x)]' && \text{DPr(5.10)}\\
f'(x) &= 2x \cos(x) + x^2 [\cos(x)]' && \text{DPo(5.6)}\\
f'(x) &= 2x \cos(x) + x^2 (-1 \sin(x)) && \text{DCos(5.16)}\\
f'(x) &= 2x \cos(x) - x^2 \sin x && \text{OOM(1.4)}
\end{aligned}$$

■

Example 5.8 — id:20141209-151354.

Differentiate $f(x) = \sin(x) \sin(x)$

S

Solution:

$$\begin{aligned}
f'(x) &= [\sin(x) \sin(x)] && \text{SPE(3.2)}\\
f'(x) &= [\sin(x)]' \sin(x) + \sin(x) [\sin(x)] && \text{DPr(5.10)}\\
f'(x) &= \cos(x) \sin(x) + \sin(x) \cos(x) && \text{DSin(5.14)}\\
f'(x) &= \cos(x) \sin(x) + \cos(x) \sin(x) && \text{CPM(1.8)}\\
f'(x) &= 2 \cos(x) \sin(x) && \text{OOA(1.1)}
\end{aligned}$$

■

5.4 Derivative of Rational Functions

Rule 5.4.1 — Derivative of a Quotient (DQ).

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2} \tag{5.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{\mathrm{d}}{\mathrm{d}x}[f(x)]\, g(x) - f(x) \frac{\mathrm{d}}{\mathrm{d}x}[g(x)]}{[g(x)]^2} \tag{5.13}$$

5.5 Derivative of Trigonometric Functions

Rule 5.5.1 — Derivative of Sine (DSin).

$$[\sin(x)]' = \cos(x) \tag{5.14}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\sin(x)] = \cos(x) \tag{5.15}$$

Rule 5.5.2 — Derivative of Cosine (DCos).

$$[\cos(x)]' = -\sin(x) \tag{5.16}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\cos(x)] = -\sin(x) \tag{5.17}$$

5.6 Derivative of Exponential Functions

5.7 Derivative of Logarithmic Functions

Rule 5.7.1 — Derivative of a Natural Logarithm (DNL).

$$[\ln x]' = \frac{1}{x} \tag{5.18}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln x] = \frac{1}{x} \tag{5.19}$$

5.8 Derivative of Composite Functions

Rule 5.8.1 — Derivative of a Composite Function (DComp).

$$[f(g(x))]' = [g(x)]'[f(g(x))]' \tag{5.20}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] = \frac{\mathrm{d}}{\mathrm{d}x}[g(x)]\frac{\mathrm{d}}{\mathrm{d}x}[f(g(x))] \tag{5.21}$$

Example 5.9 — id:20141124-203850.

Differentiate $y = \ln(3x)$

S

Solution:

After identifying that $y = \ln(3x)$ is a composite function, we let $u = 3x$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x} \qquad \text{DComp(5.21)}$$

We need to find the factors $\frac{\mathrm{d}y}{\mathrm{d}u}$ and $\frac{\mathrm{d}u}{\mathrm{d}x}$.

$$y = \ln(u) \qquad u = 3x$$
$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 3$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(5.21)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} \qquad \text{DNL(5.18)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot 3 \qquad \text{DPo(5.6)}$$
$$\frac{dy}{dx} = \frac{1}{3x} \cdot 3$$
$$\frac{dy}{dx} = \frac{3}{3x} \qquad \text{OOM(1.4)}$$
$$\frac{dy}{dx} = \frac{1}{x}$$

D

Dependencies:
example 5.12-20141124-205219

Example 5.10 — id:20141128-160248.

Differentiate $y = \ln(3x^2 - 6x + 4)$

S

Solution: After identifying that $y = \ln(3x^2 - 6x + 4)$ is a composite function, we let $u = 3x^2 - 6x + 4$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(5.21)}$$

We need to find the factors $\frac{dy}{du}$ and $\frac{du}{dx}$.

$$y = \ln(u) \qquad u = 3x^2 - 6x + 4$$
$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = 6x - 6 \qquad \text{goto } 5.5$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(5.21)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} \qquad \text{DNL(5.18)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot (6x-6) \qquad \text{DPo(5.6)}$$
$$\frac{dy}{dx} = \frac{1}{3x^2-6x+4} \cdot (6x-6)$$
$$\frac{dy}{dx} = \frac{6x-6}{3x^2-6x+4} \qquad \text{OOM(1.4)}$$

■

Example 5.11 — id:20141128-155506.

Differentiate $y = \ln(\cos x)$

S

Solution: After identifying that $y = \ln(\cos x)$ is a composite function, we let $u = \cos x$ and thus we get a new function $y = \ln(u)$.

Using chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(5.21)}$$

We need to find the factors $\frac{dy}{du}$ and $\frac{du}{dx}$.

$$y = \ln(u) \qquad u = \cos x$$
$$\frac{dy}{du} = \frac{1}{u} \qquad \frac{du}{dx} = -\sin x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{DComp(5.21)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx} \qquad \text{DNL(5.18)}$$
$$\frac{dy}{dx} = \frac{1}{u} \cdot -\sin x \qquad \text{DPo(5.6)}$$
$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot -\sin x$$
$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} \qquad \text{OOM(1.4)}$$
$$\frac{dy}{dx} = -\tan x$$

■

Example 5.12 — id:20141124-205219.

Differentiate $y = \left(x^2 - 1\right) \ln(3x)$

S

Solution:

$$
\begin{aligned}
y' &= \left[x^2 - 3\right]' \cdot \ln x + \left(x^2 - 3\right) \cdot [\ln 3x]' && \text{DPr(5.10)} \\
y' &= 2x \cdot \ln x + \left(x^2 - 3\right) \cdot [\ln 3x]' && \text{differentiate goto 5.4} \\
y' &= 2x \cdot \ln x + \left(x^2 - 3\right) \cdot \frac{1}{x} && \text{differentiate goto 5.9} \\
y' &= 2x \cdot \ln x + \frac{x^2 - 3}{x} && \text{OOM(1.4)} \\
y' &= 2x^2 \ln x + \frac{x^2 - 3}{x} && \text{JTC(1.6)} \\
y' &= \frac{2x^2 \ln x + \left(x^2 - 3\right)}{x} && \text{OOA(1.1)}
\end{aligned}
$$

■

6. Integration

6.1 Integration Rules

Rule 6.1.1 — Integral of a Constant (IC).

$$\int c\,\mathrm{d}x = cx + k \tag{6.1}$$

Rule 6.1.2 — Integral of a Constant Multiple (ICM).

$$\int cx\,\mathrm{d}x = c\int x\,\mathrm{d}x \tag{6.2}$$

Rule 6.1.3 — Integral of a Power (IPo).

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} \qquad \text{for} x \neq -1 \tag{6.3}$$

Rule 6.1.4 — Integral of a Sum (IS).

$$\int f(x) + g(x)\,\mathrm{d}x = \int f(x)\,\mathrm{d}x + \int g(x)\,\mathrm{d}x \tag{6.4}$$

Example 6.1 — id:20141105-172103.

Integrate $\int \frac{1}{4}x^2 - 2x + 4\,\mathrm{d}x$

S

Solution:

$$
\begin{aligned}
\int \frac{1}{4}x^2 - 2x + 4\,dx &= \int \frac{1}{4}x^2 + \neg 2x + 4\,dx && \text{DOS(2.5a)}\\
&= \int \frac{1}{4}x^2\,dx + \int \neg 2x\,dx + \int 4\,dx && \text{IS(6.4)}\\
&= \frac{1}{4}\int x^2\,dx + \neg 2\int x\,dx + \int 4\,dx && \text{ICM(6.2)}\\
&= \frac{1}{4}\int x^2\,dx + \neg 2\int x\,dx + 4(x) + c_1 && \text{IC(6.1)}\\
&= \frac{1}{4}\left(\frac{x^3}{3}\right) + c_2 + \neg 2\left(\frac{x^2}{2}\right) + c_3 + 4(x) + c_1 && \text{IPo(6.3)}\\
&= \frac{x^3}{12} + c_2 - x^2 + c_3 + 4x + c_1 && \text{OOM(1.4)}\\
&= \frac{x^3}{12} - x^2 + 4x + \underbrace{c_1 + c_2 + c_3}_{\text{Just a constant}} && \text{CPA(2.8)}\\
&= \frac{x^3}{12} - x^2 + 4x + C && \text{OOA(1.1)}
\end{aligned}
$$

D

Dependencies:
example 6.2-20141105-144223

■

Theorem 6.1.5 — Fundamental Theorem of Calculus (FThmC). If f is continuous on the closed interval $[a,b]$ and F is the indefinite integral of f on $[a,b]$, then

$$
\begin{aligned}
\int_a^b f(x)\,dx &= F(b) - F(a) && (6.5a)\\
&= [F(x)]_a^b && (6.5b)
\end{aligned}
$$

6.2 Volumes of Revolution

Definition 6.2.1 — Volume of Revolution: Method of Disks. Let f be a nonnegative and continuous function on the closed interval $[a,b]$, then the solid of revolution obtained by rotating the curve $f(x)$ about the x-axis from $x = a$ to $x = b$ has volume given by

$$V = \pi \int_a^b [f(x)]^2\,dx \tag{6.6}$$

Example 6.2 — id:20141105-144223.

Find the volume of the solid formed by rotating the region enclosed by $y = 2 - \dfrac{x}{2}$ and $y = 0$ through 2π about the x-axis on the closed interval $[2,3]$.

S

Solution:

$$\begin{aligned}
V &= \pi \int_a^b [f(x)]^2 \,\mathrm{d}x && \text{VRD(6.6)}\\
&= \pi \int_2^3 \left[2 - \frac{x}{2}\right]^2 \mathrm{d}x && \text{SPE(3.2)}\\
&= \pi \int_2^3 \frac{1}{4}x^2 + \neg 2x + 4\,\mathrm{d}x && \text{simplify: goto 2.39}\\
&= \pi \left[\frac{x^3}{12} - x^2 + 4x\right]_2^3 && \text{integrate goto 6.1}\\
&= \pi \left[\frac{[3]^3}{12} - [3]^2 + 4[3] - \left(\frac{[2]^3}{12} - [2]^2 + 4[2]\right)\right] && \text{FThmC(6.5a)}\\
&= \pi \left(\frac{7}{12}\right) && \text{evaluate: example 4.1}\\
&= \frac{7}{12}\pi && \text{CPM(1.8)}
\end{aligned}$$

■

Example 6.3 — id:20141106-114907.

Find the volume of the solid formed by rotating the region enclosed by $y = \sqrt{9 - x^2}$ and $y = 0$ through 2π about the x-axis on the closed interval $[1, 3]$.

S

Solution:

$$\begin{aligned}
V &= \pi \int_a^b [f(x)]^2 \,\mathrm{d}x && \text{VRD(6.6)}\\
&= \pi \int_1^3 \left[\sqrt{9 - x^2}\right]^2 \mathrm{d}x && \text{SPE(3.2)}\\
&= \pi \int_1^3 9 - x^2 \,\mathrm{d}x && \text{simplify: goto 2.32}\\
&= \pi \left[9x - \frac{x^3}{3}\right]_1^3 && \text{integrate}\\
&= \pi \left[9[3] - \frac{[3]^3}{3} - \left(9[1] - \frac{[1]^3}{3}\right)\right] && \text{FThmC(6.5a)}\\
&= \pi \left(\frac{28}{3}\right) && \text{evaluate}\\
&= \frac{28}{3}\pi && \text{CPM(1.8)}
\end{aligned}$$

■

Example 6.4 — id:20141106-122528.

Find the volume of the solid formed by rotating the region enclosed by $y = \sqrt{\sin x}$ and $y = 0$ through 2π about the x-axis on the closed interval $\left[\frac{\pi}{2}, \pi\right]$.

S

Solution:

$$
\begin{aligned}
V &= \pi \int_a^b [f(x)]^2 \, \mathrm{d}x && \text{VRD(6.6)} \\
&= \pi \int_{\pi/2}^{\pi} \left[\sqrt{\sin\theta}\right]^2 \mathrm{d}\theta && \text{SPE(3.2)} \\
&= \pi \int_{\pi/2}^{\pi} \sin\theta \, \mathrm{d}\theta && \text{PoPo(2.12a)} \\
&= \pi \left[-1\cos\theta\right]_{\pi/2}^{\pi} && \text{integrate} \\
&= \pi \left[-1\cos[\pi] - \left(-1\cos\left[\frac{\pi}{2}\right]\right)\right] && \text{FThmC(6.5a)} \\
&= \pi(1) && \text{evaluate} \\
&= \pi && \text{MId(2.1b)}
\end{aligned}
$$

■

Example 6.5 — id:20141108-083108.

Find the volume of the solid formed by rotating the region enclosed by $y = \sqrt{2 - x^2}$ and $y = 0$ through 2π about the x-axis.

S

Since no closed interval is stated, we will need to find where the curves $y = \sqrt{2 - x^2}$ and $y = 0$ intersect to determine the closed interval. Solving this system of equations (see example 3.9), we find that $x = \pm\sqrt{2}$.

$$
\begin{aligned}
V &= \pi \int_a^b [f(x)]^2 \, \mathrm{d}x && \text{VRD(6.6)} \\
&= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[\sqrt{2-x^2}\right]^2 \mathrm{d}x \\
&= \pi \int_{-\sqrt{2}}^{\sqrt{2}} 2 - x^2 \, \mathrm{d}x && \text{PoPo(2.12a)} \\
&= \pi \left[2x - \frac{x^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}} && \text{integrate} \\
&= \pi \left[2\left[\sqrt{2}\right] - \frac{\left[\sqrt{2}\right]^3}{3} - \left(2\left[-\sqrt{2}\right] - \frac{\left[-\sqrt{2}\right]^3}{3}\right)\right] && \text{FThmC(6.5a)} \\
&= \pi \left(\frac{8}{2}\sqrt{2}\right) && \text{simplify goto 2.32} \\
&= \frac{8}{2}\pi\sqrt{2} && \text{CPM(1.8)}
\end{aligned}
$$

■

Example 6.6 — id:20141125-084541.

Find the volume of the solid formed by rotating the region enclosed by $y = \sqrt{2-x^2}$ and $y = 0$ through 2π about the x-axis.

S

Since no closed interval is stated, we will need to find where the curves $y = 4 - x^2$ and $y = 0$ intersect to determine the closed interval. Solving this system of equations (see the similar example 3.9), we find that $x = \pm 2$.

Since the parabolar $y = 4 - x^2$ is symmetric about the y-axis, we can find the volume on the closed interval [0,2] and multiply this by a factor of two to include the volume on the closed interval [-2,0].

S

Solution:

$$
\begin{aligned}
V &= 2\cdot\pi\int_a^b [f(x)]^2\,\mathrm{d}x && \text{VRD(6.6)}\\
&= 2\cdot\pi\int_0^2 \left[4-x^2\right]^2\mathrm{d}x\\
&= 2\cdot\pi\int_0^2 x^4-8x^2+16\,\mathrm{d}x\\
&= 2\cdot\pi\left[\frac{x^5}{5}-\frac{8x^3}{3}+16x\right]_0^2\\
&= 2\cdot\pi\left(\frac{[2]^5}{5}-\frac{8[2]^3}{3}+16[2]\right) && \text{FThmC(6.5a)}\\
&= \frac{512\pi}{15}
\end{aligned}
$$

■

Bibliography

Books

Website

Articles

[Smi13] James Smith. “Article title”. In: 14.6 (Mar. 2013), pages 1–8.

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