#### EJEMPLOS DE PLANTEAMIENTOS CON BINARIAS

Una empresa fabrica 3 productos, cuyo proceso de fabricación pasa por tres departamentos, en la siguiente tabla se muestran las necesidades en horas de cada departamento para la producción de cada unidad de producto:

PRODUCTO	Departamento 1	Departamento 2	Departamento 3
A	1,50	2,00	0,25
В	3,00	1,00	0,25
C	2,00	2,50	0,25

En un determinado periodo y con la plantilla existente se disponen de 450 horas en el Departamento 1, de 350 en el 2 y de 50 en el 3. Los márgenes brutos por unidad de producto son de 25€ para A, de 28€ para B y de 30€ para el producto C.

a. Plantear el modelo de programación lineal que permita encontrar las producciones óptimas maximizando el margen bruto y teniendo en cuenta las limitaciones horarias.

Max 
$$Z=25X_1+28X_2+30X_3$$
  
Sujeto a:  
 $1,50X_1+3,00X_2+2,00X_3 \le 450$   
 $2,00X_1+1,00X_2+2,50X_3 \le 350$   
 $0,25X_1+0,25X_2+0,25X_3 \le 50$   
 $X_1,X_2,X_3 \ge 0$   
 $X_1,X_2,X_3$  Enteras

b. En el modelo anterior incorporar la existencia de unos costes de preparación a la producción (Costes Fijos) que ascienden a 400€ para A, 550€ para B y 550€ para C.

$$\begin{array}{l} \text{Max } Z = 25X_1 + 28X_2 + 30X_3 - 400Y_1 - 550Y_2 - 550Y_3 \\ \text{Sujeto a:} \\ 1,50X_1 + 3,00X_2 + 2,00X_3 \leq 450 \\ 2,00X_1 + 1,00X_2 + 2,50X_3 \leq 350 \\ 0,25X_1 + 0,25X_2 + 0,25X_3 \leq \ 50 \\ X_1 - M_1Y_1 \leq 0 \\ X_2 - M_2Y_2 \leq 0 \\ X_3 - M_3Y_3 \leq 0 \\ X_1, X_2, X_3 \geq 0, \ Y_1, \ Y_2, \ Y_3 \in \{0,1\} \\ X_1, X_2, X_3 \text{ Enteras} \end{array}$$

- c. Calcular unos valores coherentes para M<sub>1</sub>, M<sub>2</sub> y M<sub>3</sub>.
- d. La empresa se plantea hacer horas extras, por tanto puede eliminar las limitaciones horarias de los tres departamentos. Incorporar este hecho al modelo del apartado b y c.

```
\begin{array}{l} \text{Max } Z = 25X_1 + 28X_2 + 30X_3 - 400Y_1 - 550Y_2 - 550Y_3 \\ \text{Sujeto a:} \\ 1,50X_1 + 3,00X_2 + 2,00X_3 - 450 \leq M_4(1 - Y_4) \\ 2,00X_1 + 1,00X_2 + 2,50X_3 - 350 \leq M_5(1 - Y_5) \\ 0,25X_1 + 0,25X_2 + 0,25X_3 - 50 \leq M_6(1 - Y_6) \\ X_1 - M_1Y_1 \leq 0 \\ X_2 - M_2Y_2 \leq 0 \\ X_3 - M_3Y_3 \leq 0 \\ Y_4 + Y_5 + Y_6 \leq 3 \\ X_1, X_2, X_3 \geq 0; \ Y_1, \ Y_2, \ Y_3, \ Y_4, \ Y_5, \ Y_6 \in \{0,1\} \\ X_1, X_2, X_3 \text{ Enteras} \end{array}
```

Resolver los modelos de los apartados a., b. (c.) y d. utilizando SAS/OR.

```
libname t4 '.';
data t4.exem_int;
 input _row_ $7. x1 x2 x3 _type_ $ _rhs_;
      datalines;
Obj
        25
                28
                       30
                                 MAX
         1.50
                 3.00
                        2.00
                                 LE
                                           452
res1
         2.00
                1.00
                        2.50
                                  _{
m LE}
                                           352
res2
res3
         0.25
                 0.25
                        0.25
                                 LE
                                            53
limsup
        10000
                10000
                        10000
                                 UPPERBD
enteras 1
                                  INTEGER
;
run;
proc lp data=t4.exem_int;
run;
data t4.exem_int_binCF;
 input _row_ $7. x1 x2 x3 y1 y2 y3 _type_ $ _rhs_;
      datalines;
                        30
Obj
                                 -400
                                        -550
                                               -550
         25
                28
                                                         MAX
                3.00
                        2.00
                                   0
                                         0
                                                0
                                                         LE
                                                                   450
         1.50
res1
         2.00
                1.00
                        2.50
                                   0
                                         0
                                                0
                                                         LE
                                                                   350
res2
                                         0
                                                0
        0.25
                0.25
                         0.25
                                   0
                                                                    50
res3
                                                         LE
                         0
                              -10000
                                         0
                                                0
CF1
         1
                 0
                                                         LE
                                                                     0
                                      -10000
         0
                         0
                               0
                                                0
                                                                     0
CF2
                 1
                                                         LE
         0
                                              -10000
                                                                     0
CF3
                 0
                         1
                               0
                                           0
                                                         _{
m LE}
                10000
limsup
        10000
                        10000
                                                         UPPERBD
enteras 1
                 2
                         3
                                                         INTEGER
                                          2
bin
                                   1
                                                         BINARY
run;
proc lp data=t4.exem_int_binCF;
run;
```

```
data t4.exem_int_binCF_RES;
 input _row_ $7. x1 x2 x3 y1 y2 y3 y4 y5 y6 _type_ $ _rhs_;
      datalines;
                      30
                             -400
                                     -550 -550
                                                          0
Obj
        25
               28
                                                    Ω
                                                                0
                                                                      MAX
        1.50
                                                   10000 0
res1
               3.00
                      2.00
                                0
                                      0
                                            0
                                                                0
                                                                      LE
                                                                             10450
res2
        2.00
               1.00
                      2.50
                                0
                                      0
                                            0
                                                     0
                                                         10000 0
                                                                      LE
                                                                             10350
res3
        0.25
               0.25
                      0.25
                                0
                                      0
                                            0
                                                     0
                                                           0
                                                                10000 LE
                                                                             10050
CF1
        1
               0
                      0
                            -10000
                                      0
                                            0
                                                     0
                                                           0
                                                                0
                                                                      LE
                                                                                0
CF2
        0
               1
                      0
                                0 -10000
                                            0
                                                     0
                                                           0
                                                                0
                                                                      LE
                                                                                0
CF3
        0
               0
                      1
                                0
                                      0 -10000
                                                     0
                                                           0
                                                                0
                                                                      LE
                                                                                0
un
        0
               0
                      0
                                0
                                      0
                                            0
                                                     1
                                                           1
                                                                1
                                                                      GE
                                                                                1
limsup 10000
               10000
                      10000
                                                                      UPPERBD
enteras 1
               2
                                                                      INTEGER
bin
                                                           5
                                                                6
                                                                      BINARY
;
run;
proc lp data=t4.exem int binCF RES MAXIT=500;
run; libname t4 '.';
data t4.exem_int;
 input _row_ $7. x1 x2 x3 _type_ $ _rhs_;
      datalines;
Obj
       25
              28
                     30
                              MAX
        1.50
             3.00
                     2.00
                              LE
                                       450
res1
        2.00
             1.00
                      2.50
                              LE
                                       350
res2
               0.25
                                        50
        0.25
                      0.25
                              LE
res3
limsup 10000 10000 10000
                              UPPERBD
enteras 1 2 3 INTEGER
;
run;
proc lp data=t4.exem_int;
run;
data t4.exem_int_binCF;
 input _row_ $7. x1 x2 x3 y1 y2 y3 _type_ $ _rhs_;
      datalines;
Obj
        25
               28
                      30 -400
                                     -550 -550
                                                    MAX
        1.50
               3.00
                      2.00
                                0
                                      0
                                            0
                                                    LE
                                                             450
res1
        2.00
               1.00
                      2.50
                                0
                                      0
                                            0
                                                    LE
                                                             350
res2
        0.25
               0.25
                      0.25
                                0
                                      0
                                            0
                                                              50
res3
                                                    LE
                            -10000
                                      0
                                                               0
CF1
        1
               0
                      0
                                            0
                                                    LE
                            0
                                   -10000
                                            0
CF2
        0
               1
                      0
                                                    LE
                                                               0
                                          -10000
CF3
               0
                      1
                             0
                                       0
                                                    LE
                                                               0
               10000
limsup 10000
                      10000
                                                    UPPERBD
                                            .
               2
                      3
                                                    INTEGER
enteras 1
bin
                                                    BINARY
;
run;
proc lp data=t4.exem_int_binCF;
run;
```

```
data t4.exem_int_binCF_RES;
 input _row_ $7. x1 x2 x3 y1 y2 y3 y4 y5 y6 _type_ $ _rhs_;
     datalines;
              28
                    30 -400
       25
                                   -550 -550
                                                  0 0
Obj
                                                              0
                                                                    MAX
                                                  10000 0
              3.00
res1
       1.50
                     2.00
                              0
                                    0
                                           0
                                                              0
                                                                    _{
m LE}
                                                                           10450
res2
       2.00
              1.00
                      2.50
                               0
                                     0
                                           0
                                                   0
                                                        10000 0
                                                                    LE
                                                                           10350
res3
       0.25
             0.25
                      0.25
                              0
                                     0
                                           0
                                                    0
                                                         0
                                                              10000 LE
                                                                           10050
CF1
       1
              0
                      0
                          -10000
                                     0
                                           0
                                                    0
                                                          0
                                                              0
                                                                    LE
                                                                              0
CF2
       0
              1
                      0
                              0 -10000
                                           0
                                                    0
                                                          0
                                                              0
                                                                    _{
m LE}
                                                                              0
CF3
       0
              0
                      1
                               0
                                    0 -10000
                                                    0
                                                          0
                                                                    LE
                                                                              0
un
       0
              0
                      0
                               0
                                     0
                                           0
                                                    1
                                                          1
                                                              1
                                                                    LE
                                                                              3
limsup 10000 10000
                     10000
                                                                    UPPERBD
enteras 1
              2
                                                                    INTEGER
bin
                                                          5
                                                              6
                                                                    BINARY
;
run;
proc lp data=t4.exem int binCF RES MAXIT=500;
                                     The LP Procedure
                                     Problem Summary
                             Objective Function
                                                    Max Obj
                             Rhs Variable
                                                      _rhs_
                              Type Variable
```

Type Variable Problem Density (%)	_type_ 66.67
Variables	Number
Integer Slack	3
Total	6
Constraints	Number
LE Objective	3 1
Total	4

#### Solution Summary

#### Integer Optimal Solution

Objective Value	5540
Phase 1 Iterations	0
Phase 2 Iterations	3
Phase 3 Iterations	0
Integer Iterations	0
Integer Solutions	1

Initial Basic Feasible Variables	5
Time Used (seconds)	0
Number of Inversions	4
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	100
Maximum Phase 2 Iterations	100
Maximum Phase 3 Iterations	99999999
Maximum Integer Iterations	100
Time Limit (seconds)	120

## Variable Summary

	Variable					Reduced
Col	Name	Status	Type	Price	Activity	Cost
1	x1	BASIC	INTEGER	25	60	0
2	x2	BASIC	INTEGER	28	80	0
3	х3	BASIC	INTEGER	30	60	0
4	res1		SLACK	0	0	-5.2
5	res2		SLACK	0	0	-4.8
6	res3		SLACK	0	0	-30.4

## The LP Procedure

## Constraint Summary

	Constraint		S/S			Dual
Row	Name	Type	Col	Rhs	Activity	Activity
1	Obj	OBJECTVE		0	5540	
2	res1	LE	4	450	450	5.2
3	res2	LE	5	350	350	4.8
4	res3	LE	6	50	50	30.4

## The LP Procedure

## Problem Summary

Objective Function	Max Obj
Rhs Variable	_rhs_
Type Variable	_type_
Problem Density (%)	29.17
Variables	Number
Integer	3
Binary	3
Slack	6
Total	12

Constraints	Number
LE	6
Objective	1
Total	7

The LP Procedure

## Integer Iteration Log

Iter Problem Condition Objective Branched Value Sinfeas Active Proximity

1	O ACTIVE	5529.9 y2	0.008	0.02	2	
2	-1 ACTIVE	4984.3 y3	0.006	0.012	3	
3	2 ACTIVE	4746 y1	0.01	0.01	4	
4	-3 SUBOPTIMAL	4350 .			2	143.5
5	-2 FATHOMED	4437.6 .			1	143.5
6	1 FATHOMED	4368 .			0	

## The LP Procedure

## Solution Summary

## Integer Optimal Solution

Objective Value	4350
Phase 1 Iterations	0
Phase 2 Iterations	6
Phase 3 Iterations	6
Integer Iterations	6
Integer Solutions	1
Initial Basic Feasible Variables	8
Time Used (seconds)	0
Number of Inversions	6
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	100
Maximum Phase 2 Iterations	100
Maximum Phase 3 Iterations	99999999
Maximum Integer Iterations	100
Time Limit (seconds)	120

#### The LP Procedure

## Variable Summary

	Variable					Reduced
Col	Name	Status	Type	Price	Activity	Cost
					•	
1	x1	BASIC	INTEGER	25	100	0
2	x2	BASIC	INTEGER	28	100	0
3	х3	DEGEN	INTEGER	30	0	0

4 y1		BINARY	-400	1	-400
5 y2		BINARY	-550	1	-550
6 y3		BINARY	-550	0	39450
7 res	1	SLACK	0	0	-2
8 res	2 BASIC	SLACK	0	50	0
9 res	3	SLACK	0	0	-88
10 CF1	BASIC	SLACK	0	9900	0
11 CF2	BASIC	SLACK	0	9900	0
12 CF3		SLACK	0	0	- 4

## Constraint Summary

	Constraint		S/S			Dual
Row	Name	Туре	Col	Rhs	Activity	Activity
1	0bj	OBJECTVE		0	4350	
2	res1	LE	7	450	450	2
3	res2	LE	8	350	300	0
4	res3	LE	9	50	50	88
5	CF1	LE	10	0	- 9900	0
6	CF2	LE	11	0	- 9900	0
7	CF3	LE	12	0	0	4

## The LP Procedure

#### Problem Summary

Objective Function Rhs Variable Type Variable Problem Density (%)	Max Obj _rhs_ _type_ 25.00
Variables	Number
Integer Binary Slack	3 6 7
Total	16
Constraints	Number
LE Objective	7 1
Total	8

## Integer Iteration Log

Iter Problem Condition Objective Branched Value Sinfeas Active Proximity

7 -4 ACTIVE 138860 y4 3E-4 0.0003 8 7 SUBOPTIMAL 138860 9 2 ACTIVE 139716.43 y3 0.375 0.80658 10 9 FATHOMED 97341.75 11 -9 ACTIVE 139372.4 x2 986.7 0.43208 12 -11 ACTIVE 139366.72 x3 3745 0.59888 13 -12 INFEASIBLE 139365.05 14 12 ACTIVE 139361.03 x2 987.3 0.43233 15 14 FATHOMED 139351.72 16 -14 FATHOMED 139349.66	3 3 4 4 5 3 4 4 3 3 4 4 3 2 1 2 3	23773.194 23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
3	3 4 5 3 4 2 3 3 4 4 3 2 1 2 3	23773.194 23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
4 -3 ACTIVE 139355.83 y2 0.099 0.0988 5 4 ACTIVE 123650 y4 0.217 0.217 6 5 SUBOPTIMAL 123650	4 5 3 4 2 3 4 3 4 3 2 1 2 3	. 23773.194 23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
4 -3 ACTIVE 139355.83 y2 0.099 0.0988 5 4 ACTIVE 123650 y4 0.217 0.217 6 5 SUBOPTIMAL 123650	4 5 3 4 2 3 4 3 4 3 2 1 2 3	. 23773.194 23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
5	3 4 2 3 3 4 4 3 2 1 2 3 3	23773.194 23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
6 5 SUBOPTIMAL 123650	4 2 3 2 3 4 3 4 3 2 1 2 3	23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
7 -4 ACTIVE 138860 y4 3E-4 0.0003 8 7 SUBOPTIMAL 138860	4 2 3 2 3 4 3 4 3 2 1 2 3	23773.194 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
8 7 SUBOPTIMAL 138860	2 3 4 3 4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
9	3 2 3 4 3 4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
11       -9 ACTIVE       139372.4 x2       986.7 0.43208         12       -11 ACTIVE       139366.72 x3       3745 0.59888         13       -12 INFEASIBLE 139365.05 .       .       .         14       12 ACTIVE       139361.03 x2       987.3 0.43233         15       14 FATHOMED       139351.72 .       .       .         16       -14 FATHOMED       139349.66 .       .       .       .         17       11 FATHOMED       139353.77 .       .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611       .       .       .         19       18 ACTIVE       147416.33 y2       0.119 0.1195       .	3 4 3 4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
12 -11 ACTIVE 139366.72 x3 3745 0.59888 13 -12 INFEASIBLE 139365.05 14 12 ACTIVE 139361.03 x2 987.3 0.43233 15 14 FATHOMED 139351.72 16 -14 FATHOMED 139349.66 17 11 FATHOMED 139353.77 18 -1 ACTIVE 147423.19 x2 1194 0.78611 19 18 ACTIVE 147416.33 y2 0.119 0.1195 20 19 FATHOMED 128975 21 -19 ACTIVE 146932 y4 1E-4 0.0001 22 21 SUBOPTIMAL 146932 23 -18 ACTIVE 147410.94 x1 4577 0.453 24 23 ACTIVE 14740.925 x3 0.5 0.61973 25 -24 ACTIVE 147407.55 x1 4575 0.45312 26 25 ACTIVE 147406.71 x3 1.25 0.36981 27 -26 FATHOMED 147404.17 28 26 ACTIVE 147403.88 x2 1195 0.28652 29 28 FATHOMED 147399.22 30 -28 ACTIVE 147385.5 x1 4573 0.45352 31 30 ACTIVE 147385.5 x1 4573 0.45392 32 31 FATHOMED 147377.17	4 3 4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944 8563.1944
13	3 4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944
14       12 ACTIVE       139361.03 x2       987.3 0.43233         15       14 FATHOMED       139351.72 .       .       .         16       -14 FATHOMED       139349.66 .       .       .         17       11 FATHOMED       139353.77 .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611         19       18 ACTIVE       147416.33 y2       0.119 0.1195         20       19 FATHOMED       128975 .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001       .         22       21 SUBOPTIMAL       146932 .       .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453       .       .       .         24       23 ACTIVE       147409.25 x3       0.5 0.61973       .	4 3 2 1 2 3	8563.1944 8563.1944 8563.1944 8563.1944
15       14 FATHOMED       139351.72 .       .       .       .         16       -14 FATHOMED       139349.66 .       .       .       .         17       11 FATHOMED       139353.77 .       .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611       .       .       .         19       18 ACTIVE       147416.33 y2       0.119 0.1195       .	3 2 1 2 3	8563.1944 8563.1944 8563.1944
17       11 FATHOMED       139353.77 .       .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611         19       18 ACTIVE       147416.33 y2       0.119 0.1195         20       19 FATHOMED       128975 .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       457	2 1 2 3	8563.1944 8563.1944
17       11 FATHOMED       139353.77 .       .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611         19       18 ACTIVE       147416.33 y2       0.119 0.1195         20       19 FATHOMED       128975 .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       457	2 1 2 3	8563.1944 8563.1944
17       11 FATHOMED       139353.77 .       .       .       .         18       -1 ACTIVE       147423.19 x2       1194 0.78611         19       18 ACTIVE       147416.33 y2       0.119 0.1195         20       19 FATHOMED       128975 .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       457	1 2 3	8563.1944
18       -1 ACTIVE       147423.19 x2       1194 0.78611         19       18 ACTIVE       147416.33 y2       0.119 0.1195         20       19 FATHOMED       128975 .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       4571 0.45392         34       33 ACTIVE       147359.77 x2       1197 0.28732	2 3	8561 0314
20       19 FATHOMED       128975 .       .       .       .         21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       4571 0.45392         34       33 ACTIVE       147359.77 x2       1197 0.28732         35       34 FATHOMED       147355.11 .       .       .         36       -34 ACTIVE       147341.39 x1       45		000110011
21       -19 ACTIVE       146932 y4       1E-4 0.0001         22       21 SUBOPTIMAL       146932 .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       4571 0.45392         34       33 ACTIVE       147359.77 x2       1197 0.28732         35       34 FATHOMED       147355.11 .       .       .         36       -34 ACTIVE       147341.39 x1       4569 0.45432	2	8550.9417
22       21 SUBOPTIMAL       146932 .       .       .       .         23       -18 ACTIVE       147410.94 x1       4577 0.453         24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       4571 0.45392         34       33 ACTIVE       147359.77 x2       1197 0.28732         35       34 FATHOMED       147355.11 .       .       .         36       -34 ACTIVE       147341.39 x1       4569 0.45432		
23 -18 ACTIVE 147410.94 x1 4577 0.453 24 23 ACTIVE 147409.25 x3 0.5 0.61973 25 -24 ACTIVE 147407.55 x1 4575 0.45312 26 25 ACTIVE 147406.71 x3 1.25 0.36981 27 -26 FATHOMED 147404.17 28 26 ACTIVE 147403.88 x2 1195 0.28652 29 28 FATHOMED 147399.22 30 -28 ACTIVE 147385.5 x1 4573 0.45352 31 30 ACTIVE 147381.82 x2 1196 0.28692 32 31 FATHOMED 147377.17 33 -31 ACTIVE 147363.44 x1 4571 0.45392 34 33 ACTIVE 147359.77 x2 1197 0.28732 35 34 FATHOMED 147355.11 36 -34 ACTIVE 147341.39 x1 4569 0.45432	3	8550.9417
24       23 ACTIVE       147409.25 x3       0.5 0.61973         25       -24 ACTIVE       147407.55 x1       4575 0.45312         26       25 ACTIVE       147406.71 x3       1.25 0.36981         27       -26 FATHOMED       147404.17 .       .       .         28       26 ACTIVE       147403.88 x2       1195 0.28652         29       28 FATHOMED       147399.22 .       .       .         30       -28 ACTIVE       147385.5 x1       4573 0.45352         31       30 ACTIVE       147381.82 x2       1196 0.28692         32       31 FATHOMED       147377.17 .       .       .         33       -31 ACTIVE       147363.44 x1       4571 0.45392         34       33 ACTIVE       147359.77 x2       1197 0.28732         35       34 FATHOMED       147355.11 .       .       .         36       -34 ACTIVE       147341.39 x1       4569 0.45432		
26		
26	2	475.55333
26	2	474.70625
27 -26 FATHOMED 147404.17	3	472.165
28	2	471.8775
30 -28 ACTIVE 147385.5 x1 4573 0.45352 31 30 ACTIVE 147381.82 x2 1196 0.28692 32 31 FATHOMED 147377.17 33 -31 ACTIVE 147363.44 x1 4571 0.45392 34 33 ACTIVE 147359.77 x2 1197 0.28732 35 34 FATHOMED 147355.11 36 -34 ACTIVE 147341.39 x1 4569 0.45432	3	471.59
31		
32		
33 -31 ACTIVE 147363.44 x1 4571 0.45392 34 33 ACTIVE 147359.77 x2 1197 0.28732 35 34 FATHOMED 147355.11 36 -34 ACTIVE 147341.39 x1 4569 0.45432		
35	2	471.59
35	2	471.59
36 -34 ACTIVE 147341.39 x1 4569 0.45432	3	471.59
	2	
37 36 ACTIVE 147337 71 v2 1108 0 28772		
38		
39 -37 ACTIVE 147319.33 x1 4567 0.45472		
40 39 ACTIVE 147315.66 x2 1199 0.28812		
41 40 FATHOMED 147311		
42 -40 ACTIVE 147297.28 x1 4565 0.45512		
43 42 ACTIVE 147293.6 x2 1200 0.28852	2	
44 43 FATHOMED 147288.95	2 3	471.59
45 -43 ACTIVE 147275.22 x1 4563 0.45552	2 3	471.59
46 45 ACTIVE 147271.55 x2 1201 0.28892	2 3 2 2	
47 46 FATHOMED 147266.89	2 3 2 2 3	471.59
48 -46 ACTIVE 147253.17 x1 4561 0.45592	2 3 2 2 3 2	
49 48 ACTIVE 147249.49 x2 1202 0.28932	2 3 2 2 3 2 2	

			147403.59		1195	0.45313		462.275
51			147394.28					460.5625
52			147388.89			0.4534		449.535
53			147381.54		1196	0.45353		440.22
54		FATHOMED	147372.22			•		438.5075
55		ACTIVE	147366.83			0.4538	3	427.48
56			147359.48		1197	0.45393		418.165
57			147350.17			•		416.4525
58			147344.78			0.4542	3	405.425
59	58	ACTIVE	147337.43	х2	1198	0.45433	4	396.11
60	59	FATHOMED	147328.11		•			394.3975
61			147322.72			0.4546		383.37
62			147315.37		1199	0.45473	4	374.055
63	62	FATHOMED	147306.06					372.3425
64	-62		147300.67			0.455	3	361.315
65	64	ACTIVE	147293.32	х2	1200	0.45513	4	352
66	65	FATHOMED	147284				3	350.2875
67	-65	ACTIVE	147278.61	х1	4565	0.4554	3	339.26
68	67	ACTIVE	147271.26	х2	1201	0.45553	4	329.945
69	68	FATHOMED	147261.95				3	328.2325
70			147256.56			0.4558	3	317.205
71	70	ACTIVE	147249.21	х2	1202	0.45593	4	312.835
72	71	FATHOMED	147239.89				3	312.835
73	-71	ACTIVE	147234.5	x1	4561	0.4562	3	312.835
74	73	ACTIVE	147227.15	х2	1203	0.45633	4	312.835
75	74	FATHOMED	147217.84				3	312.835
76	-74	ACTIVE	147212.45	x1	4559	0.4566	3	312.835
77	76	ACTIVE	147205.1	х2	1204	0.45673	4	312.835
78	77	FATHOMED	147195.78				3	312.835
79	-77	ACTIVE	147190.39	x1	4557	0.457	3	312.835
80	79	ACTIVE	147183.04	х2	1205	0.45713	4	312.835
81	80	FATHOMED	147173.73				3	312.835
82	-80	ACTIVE	147168.34	x1	4555	0.4574	3	312.835
83	82	ACTIVE	147160.99	х2	1206	0.45753	4	312.835
84	83	FATHOMED	147151.67				3	312.835
85	-83	ACTIVE	147146.28	x1	4553	0.4578	3	312.835
86	85	ACTIVE	147138.93	х2	1207	0.45793	4	312.835
87	86	FATHOMED	147129.62				3	312.835
88	-86	ACTIVE	147124.23	x1	4551	0.4582	3	312.835
89	88	ACTIVE	147116.88	х2	1208	0.45833	4	312.835
90	89	FATHOMED	147107.56				3	312.835
91	-89	ACTIVE	147102.17	x1	4549	0.4586	3	312.835
92	91	ACTIVE	147094.82	х2	1209	0.45873	4	312.835
93	92	FATHOMED	147085.51				3	312.835
94	-92	ACTIVE	147080.12	х1	4547	0.459	3	312.835
95	94	ACTIVE	147072.77	х2	1210	0.45913	4	312.835
96	95	FATHOMED	147063.45				3	312.835
97	-95	ACTIVE	147058.06	x1	4545	0.4594	3	312.835
98	97	ACTIVE	147050.71	х2	1211	0.45953	4	312.835
99	98	FATHOMED	147041.4				3	312.835
100	-98	ACTIVE	147036.01	x1	4543	0.4598	3	312.835
101	100	ACTIVE	147028.66	х2	1212	0.45993	4	312.835
102	101	FATHOMED	147019.34				3	312.835
103	- 101	ACTIVE	147013.95	x1		0.4602		312.835
104		ACTIVE	147006.6			0.46033		312.835

105	104	FATHOMED	146997.29			_	3	312.835
106		ACTIVE	146991.9		4539	0.4606	3	312.835
107		ACTIVE				0.46073	4	312.835
108		FATHOMED					3	312.835
109		ACTIVE	146969.84			0.461	3	312.835
110		ACTIVE				0.46113	4	312.835
111		FATHOMED					3	306.465
		ACTIVE	147231.11		4550	0.45632	3	295.4375
112		ACTIVE				0.43032	4	
113					1203	0.20972		290.78
114		FATHOMED	147222.78 147209.06		4557	0 45670	3	284.41
115		ACTIVE				0.45672	3	273.3825
116		ACTIVE	147205.38			0.29012	4	268.725
117		FATHOMED	147200.73			. 45740	3	262.355
118		ACTIVE	147187			0.45712	3	251.3275
119		ACTIVE	147183.33		1205	0.29052	4	246.67
120		FATHOMED			•		3	240.3
121		ACTIVE	147164.95			0.45752	3	229.2725
	121				1206	0.29092		224.615
		FATHOMED				•		218.245
124		ACTIVE	147142.89			0.45792	3	207.2175
125	124	ACTIVE	147139.22	х2	1207	0.29132	4	202.56
126	125	FATHOMED					3	196.19
127	- 125	ACTIVE	147120.84	x1	4549	0.45832	3	185.1625
128	127	ACTIVE	147117.16	х2	1208	0.29172	4	180.505
129	128	FATHOMED	147112.51				3	174.135
130	- 128	ACTIVE	147098.78	x1	4547	0.45872	3	163.1075
131	130	ACTIVE	147095.11	х2	1209	0.29212	4	158.45
132	131	FATHOMED	147090.45				3	152.08
133	- 131	ACTIVE	147076.73	x1	4545	0.45912	3	141.0525
134	133	ACTIVE	147073.05	х2	1210	0.29252	4	136.395
135	134	FATHOMED	147068.4				3	130.025
136	- 134	ACTIVE	147054.67	x1	4543	0.45952	3	118.9975
137	136	ACTIVE	147051	х2	1211	0.29292	4	114.34
138	137	FATHOMED	147046.34				3	107.97
139	- 137	ACTIVE	147032.62	x1	4541	0.45992	3	96.9425
140	139	ACTIVE	147028.94	x2	1212	0.29332	4	92.285
141	140	FATHOMED	147024.29				3	85.915
142	- 140	ACTIVE	147010.56	x1	4539	0.46032	3	74.8875
143	142	ACTIVE	147006.89	x2	1213	0.29372	4	70.23
			147002.23					63.86
		ACTIVE			4537			
			146984.83					48.175
147		FATHOMED						41.805
148		ACTIVE				0.46112		30.7775
149					1215			
			146958.12				_	21.175
			146953.18					19.75
152					4533			
			146947.79					8.7225
					1216	0 20402		4.065
			146936.07				0	
100	104	LATIONED	170300.07	•	•	•	U	•

## Solution Summary

## Integer Optimal Solution

Objective Value	146932
Phase 1 Iterations	0
Phase 2 Iterations	6
Phase 3 Iterations	156
Integer Iterations	155
Integer Solutions	3
Initial Basic Feasible Variables	9
Time Used (seconds)	0
Number of Inversions	60
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	500
Maximum Phase 2 Iterations	500
Maximum Phase 3 Iterations	500
Maximum Integer Iterations	500
Time Limit (seconds)	120

## The LP Procedure

# Variable Summary

	Variable					Reduced
Col	Name	Status	Туре	Price	Activity	Cost
1	x1	BASIC	INTEGER	25	4578	0
2	x2		INTEGER	28	1194	15.5
3	х3		INTEGER	30	0	-1.305
4	y1		BINARY	-400	1	-400
5	y2		BINARY	-550	1	-550
6	у3	DEGEN	BINARY	-550	0	0
7	y4	ALTER	BINARY	0	0	0
8	y5		BINARY	0	0	-125000
9	y6	ALTER	BINARY	0	0	0
10	res1	BASIC	SLACK	0	1	0
11	res2		SLACK	0	0	-12.5
12	res3	BASIC	SLACK	0	8607	0
13	CF1	BASIC	SLACK	0	5422	0
14	CF2	BASIC	SLACK	0	8806	0
15	CF3		SLACK	0	0	-0.055
16	un	BASIC	SLACK	0	3	0

The LP Procedure

## Constraint Summary

	Constraint		S/S			Dual
Row	Name	Type	Col	Rhs	Activity	Activity
1	0bj	OBJECTVE		0	146932	
2	res1	LE	10	10450	10449	0
3	res2	LE	11	10350	10350	12.5
4	res3	LE	12	10050	1443	0
5	CF1	LE	13	0	-5422	0
6	CF2	LE	14	0	-8806	0
7	CF3	LE	15	0	0	0.055
8	un	LE	16	3	0	0