

PRÁCTICA EN EL AULA DE INFORMÁTICA

2 de mayo de 2016

Sea una empresa dedicada a la fabricación de 3 productos. Los inputs necesarios en el proceso de producción son, por unidad fabricada, los siguientes:

| PRODUCTO | MATERIA PRIMA | MANO DE OBRA |
|----------|---------------|--------------|
| 1 | 4 | 3 |
| 2 | 2 | 2 |
| 3 | 1 | 2 |

La materia prima se adquiere a un proveedor que es capaz de suministrar hasta 245 unidades físicas a un precio de 10 u.m./u.f.. La plantilla de la empresa supone 315 horas de trabajo efectivo a un coste de 5 u.m./hora. Por razones de demanda la fabricación del primer producto debe ser como mínimo de 20 unidades. Los beneficios brutos unitarios son 20 u.m./u.f., 30 u.m./u.f. y 10 u.m./u.f., respectivamente.

El modelo de programación lineal que permita determinar cuál es la producción que maximiza el beneficio bruto mensual es el siguiente.

$$\text{MAX } Z = 20X_1 + 30X_2 + 10X_3$$

Sujeto a:

$$4X_1 + 2X_2 + 1X_3 \leq 245$$

$$3X_1 + 2X_2 + 2X_3 \leq 315$$

$$1X_1 \geq 20$$

$$X_1, X_2, X_3 \geq 0$$

1. Con la información anterior, vuelve a plantear y resuelve el modelo de modo que el objetivo sea el cumplimiento, en la medida de lo posible, de las siguientes metas:

- I. Alcanzar un beneficio bruto total de 2500 u.m.
- II. Evitar que sobre materia prima mensualmente.
- III. No subutilizar la capacidad productiva contratada.
- IV. Servir el pedido mensual de 20 unidades de producto 1.
- V. Si fuera necesario realizar horas extras, que estas no superen las 10 horas.

Interpreta los resultados obtenidos.

2. Resuelve el modelo de programación lineal del inicio del enunciado considerando que las variables de decisión toman valores enteros. Utiliza el PROC LP de SAS/OR. Interpreta la solución. Describe las iteraciones realizadas por el algoritmo Branch and Bound.

SOLUCIÓN:

1.

$$\text{Mín } Z = (1/2500)d_1^- + (1/245)d_2^- + (1/315)d_3^- + (1/20)d_4^- + (1/10)d_5^+$$

Sujeto a:

$$20X_1 + 30X_2 + 10X_3 + d_1^- - d_1^+ = 2500$$

$$4X_1 + 2X_2 + 1X_3 + d_2^- = 245$$

$$3X_1 + 2X_2 + 2X_3 + d_3^- - d_3^+ = 315$$

$$X_3 + d_4^- - d_4^+ = 20$$

$$d_3^+ + d_5^- - d_5^+ = 10$$

$$X_1, X_2, X_3 \geq 0$$

$$d_i^-, d_i^+ \geq 0 \text{ y } d_i^- \times d_i^+ = 0, \forall i=1, \dots, 5$$

```
*METAS;
data pr.pr3_METAS;
  input _row_ $13. x1 x2 x3 d1me d1ma d2me d3me d3ma d4me d4ma d5me d5ma _type_ $ _rhs_;
  datalines;
Objetivo      0      0      0 4.00E-04  0 4.08E-03  3.17E-03  0      5E-02  0 0  1E-01 MIN .
Beneficio      20     30     10 1      -1 0      0      0      0      0 0 0  EQ  2500
Materia_Prima  4      2      1 0      0 1      0      0      0      0 0 0  EQ  245
Mano_de_Obra   3      2      2 0      0 0      1      -1     0      0 0 0  EQ  315
Pedido         1      0      0 0      0 0      0      0      1     -1 0 0  EQ  20
Horas_Extras   0      0      0 0      0 0      0      1      0      0 1 -1  EQ  10
;
run;

proc print data=pr.pr3_METAS;
run;

proc lp data=pr.pr3_METAS;
run;
```

| Obs | _row_ | x1 | x2 | x3 | d1me | d1ma | d2me | d3me | d3ma | d4me | d4ma | d5me | d5ma | _type_ | _rhs_ |
|-----|---------------|----|----|----|--------|------|---------|---------|------|------|------|------|------|--------|-------|
| 1 | Objetivo | 0 | 0 | 0 | 0.0004 | 0 | 0.00408 | 0.00317 | 0 | 0.05 | 0 | 0 | 0.1 | MIN | . |
| 2 | Beneficio | 20 | 30 | 10 | 1.0000 | -1 | 0.00000 | 0.00000 | 0 | 0.00 | 0 | 0 | 0.0 | EQ | 2500 |
| 3 | Materia_Prima | 4 | 2 | 1 | 0.0000 | 0 | 1.00000 | 0.00000 | 0 | 0.00 | 0 | 0 | 0.0 | EQ | 245 |
| 4 | Mano_de_Obra | 3 | 2 | 2 | 0.0000 | 0 | 0.00000 | 1.00000 | -1 | 0.00 | 0 | 0 | 0.0 | EQ | 315 |
| 5 | Pedido | 1 | 0 | 0 | 0.0000 | 0 | 0.00000 | 0.00000 | 0 | 1.00 | -1 | 0 | 0.0 | EQ | 20 |
| 6 | Horas Extras | 0 | 0 | 0 | 0.0000 | 0 | 0.00000 | 0.00000 | 1 | 0.00 | 0 | 1 | -1.0 | EQ | 10 |

The LP Procedure

Problem Summary

| | |
|---------------------|--------------|
| Objective Function | Min Objetivo |
| Rhs Variable | _rhs_ |
| Type Variable | _type_ |
| Problem Density (%) | 33.33 |

| | |
|-----------|--------|
| Variables | Number |
|-----------|--------|

| | |
|--------------|----|
| Non-negative | 12 |
|--------------|----|

| | |
|-------|----|
| Total | 12 |
|-------|----|

| | |
|-------------|--------|
| Constraints | Number |
|-------------|--------|

| | |
|----|---|
| EQ | 5 |
|----|---|

| | |
|-----------|---|
| Objective | 1 |
|-----------|---|

| | |
|-------|---|
| Total | 6 |
|-------|---|

Solution Summary

Terminated Successfully

| | |
|----------------------------------|--------------|
| Objective Value | 0.03 |
| Phase 1 Iterations | 2 |
| Phase 2 Iterations | 4 |
| Phase 3 Iterations | 0 |
| Integer Iterations | 0 |
| Integer Solutions | 0 |
| Initial Basic Feasible Variables | 7 |
| Time Used (seconds) | 0 |
| Number of Inversions | 3 |
| Epsilon | 1E-8 |
| Infinity | 1.797693E308 |
| Maximum Phase 1 Iterations | 100 |
| Maximum Phase 2 Iterations | 100 |
| Maximum Phase 3 Iterations | 99999999 |
| Maximum Integer Iterations | 100 |
| Time Limit (seconds) | 120 |

Variable Summary

| Col | Variable Name | Status | Type | Price | Activity | Reduced Cost |
|-----|---------------|--------|---------|---------|----------|--------------|
| 1 | x1 | BASIC | NON-NEG | 0 | 20 | 0 |
| 2 | x2 | BASIC | NON-NEG | 0 | 37.5 | 0 |
| 3 | x3 | BASIC | NON-NEG | 0 | 90 | 0 |
| 4 | d1me | BASIC | NON-NEG | 0.0004 | 75 | 0 |
| 5 | d1ma | | NON-NEG | 0 | 0 | 0.0004 |
| 6 | d2me | | NON-NEG | 0.00408 | 0 | 0.01208 |
| 7 | d3me | | NON-NEG | 0.00317 | 0 | 0.00117 |
| 8 | d3ma | | NON-NEG | 0 | 0 | 0.002 |
| 9 | d4me | | NON-NEG | 0.05 | 0 | 0.032 |
| 10 | d4ma | | NON-NEG | 0 | 0 | 0.018 |
| 11 | d5me | BASIC | NON-NEG | 0 | 10 | 0 |
| 12 | d5ma | | NON-NEG | 0.1 | 0 | 0.1 |

Constraint Summary

| Row | Constraint Name | Type | S/S Col | Rhs | Activity | Dual Activity |
|-----|-----------------|-----------|---------|------|----------|---------------|
| 1 | Objetivo | OBJECTIVE | . | 0 | 0.03 | . |
| 2 | Beneficio | EQ | . | 2500 | 2500 | 0.0004 |
| 3 | Materia_Prima | EQ | . | 245 | 245 | -0.008 |
| 4 | Mano_de_Obra | EQ | . | 315 | 315 | 0.002 |
| 5 | Pedido | EQ | . | 20 | 20 | 0.018 |
| 6 | Horas Extras | EQ | . | 10 | 10 | 0 |

2.

```
data pr.pr3;
  input _row_ $13. x1 x2 X3 _type_ $ _rhs_;
  datalines;
Beneficio      20      30      10  MAX      .
Materia_Prima  4       2       1  LE      245
Mano_de_Obra   3       2       2  LE      315
Pedido         1       0       0  GE      20
limsup         10000   10000  10000  UPPERBD .
enteras        1       2       3  INTEGER .
;
```

```
run;
```

```
proc print data=pr.pr3;
```

```
run;
```

/*To help monitor the growth of the branch-and-bound tree, the LP procedure reports on the status of each problem that is solved. The report, displayed in the Integer Iteration Log, can be used to reconstruct the branch-and-bound tree. Each row in the report describes the results of the attempted solution of the linear program at a node in the tree. In the following discussion, a problem on a given line in the log is called the current problem. The following columns are displayed in the report: Iter identifies the number of the branch-and-bound iteration. Problem identifies how the current problem fits in the branch-and-bound tree. Condition reports the result of the attempted solution of the current problem. Values for Condition are:

ACTIVE: The current problem was solved successfully.

INFEASIBLE: The current problem is infeasible.

FATHOMED: The current problem cannot lead to an improved integer solution and therefore it is dropped.

SINGULAR: A singular basis was encountered in attempting to solve the current problem. Solution of this relaxed problem is suspended and will be attempted later if necessary.

SUBOPTIMAL: The current problem has an integer feasible solution. Objective reports the objective value of the current problem. Branched names the variable that is branched in subtrees defined by the descendants of this problem. Value gives the current value of the variable named in the column labeled Branched. Sinfeas gives the sum of the integer infeasibilities in the optimal solution to the current problem Active reports the total number of nodes currently active in the branch-and-bound tree Proximity reports the gap between the best integer solution and the current lower (upper for maximizations) bound of all active nodes. */

```
proc lp data=pr.pr3;
run;
```

| Obs | _row_ | x1 | x2 | X3 | _type_ | _rhs_ |
|-----|---------------|-------|-------|-------|---------|-------|
| 1 | Beneficio | 20 | 30 | 10 | MAX | . |
| 2 | Materia_Prima | 4 | 2 | 1 | LE | 245 |
| 3 | Mano_de_Obra | 3 | 2 | 2 | LE | 315 |
| 4 | Pedido | 1 | 0 | 0 | GE | 20 |
| 5 | limsup | 10000 | 10000 | 10000 | UPPERBD | . |
| 6 | enteras | 1 | 2 | 3 | INTEGER | . |

The LP Procedure

Problem Summary

| | |
|---------------------|---------------|
| Objective Function | Max Beneficio |
| Rhs Variable | _rhs_ |
| Type Variable | _type_ |
| Problem Density (%) | 55.56 |

| | |
|-----------|--------|
| Variables | Number |
| Integer | 3 |
| Slack | 2 |
| Surplus | 1 |

| | |
|-------|---|
| Total | 6 |
|-------|---|

| | |
|-------------|--------|
| Constraints | Number |
| LE | 2 |
| GE | 1 |
| Objective | 1 |

Total 4

The LP Procedure

Integer Iteration Log

| Iter | Problem | Condition | Objective | Branched | Value | Sinfeas | Active | Proximity |
|------|---------|------------|-----------|----------|-------|---------|--------|-----------|
| 1 | 0 | ACTIVE | 2875 | x2 | 82.5 | 0.5 | 1 | . |
| 2 | 1 | SUBOPTIMAL | 2870 | . | . | . | 0 | . |

The LP Procedure

Solution Summary

Integer Optimal Solution

| | |
|----------------------------------|--------------|
| Objective Value | 2870 |
| Phase 1 Iterations | 1 |
| Phase 2 Iterations | 2 |
| Phase 3 Iterations | 1 |
| Integer Iterations | 2 |
| Integer Solutions | 1 |
| Initial Basic Feasible Variables | 5 |
| Time Used (seconds) | 0 |
| Number of Inversions | 3 |
| Epsilon | 1E-8 |
| Infinity | 1.797693E308 |
| Maximum Phase 1 Iterations | 100 |
| Maximum Phase 2 Iterations | 100 |
| Maximum Phase 3 Iterations | 9999999 |
| Maximum Integer Iterations | 100 |
| Time Limit (seconds) | 120 |

The LP Procedure

Variable Summary

| Col | Variable Name | Status | Type | Price | Activity | Reduced Cost |
|-----|---------------|--------|------|-------|----------|--------------|
|-----|---------------|--------|------|-------|----------|--------------|

| | | | | | | |
|---|---------------|-------|---------|----|----|-----|
| 1 | x1 | BASIC | INTEGER | 20 | 20 | 0 |
| 2 | x2 | | INTEGER | 30 | 82 | 10 |
| 3 | X3 | BASIC | INTEGER | 10 | 1 | 0 |
| 4 | Materia_Prima | | SLACK | 0 | 0 | -10 |
| 5 | Mano_de_Obra | BASIC | SLACK | 0 | 89 | 0 |
| 6 | Pedido | | SURPLUS | 0 | 0 | -20 |

The LP Procedure

Constraint Summary

| Row | Constraint Name | Type | S/S Col | Rhs | Activity | Dual Activity |
|-----|-----------------|-----------|---------|-----|----------|---------------|
| 1 | Beneficio | OBJECTIVE | . | 0 | 2870 | . |
| 2 | Materia_Prima | LE | 4 | 245 | 245 | 10 |
| 3 | Mano_de_Obra | LE | 5 | 315 | 226 | 0 |
| 4 | Pedido | GE | 6 | 20 | 20 | -20 |