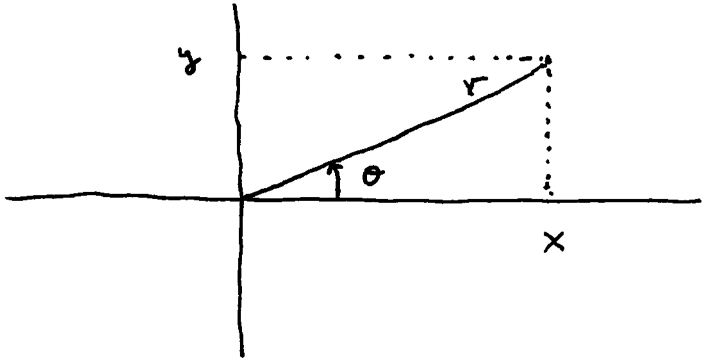
$\boxed{\text{Coordenades polars}}$

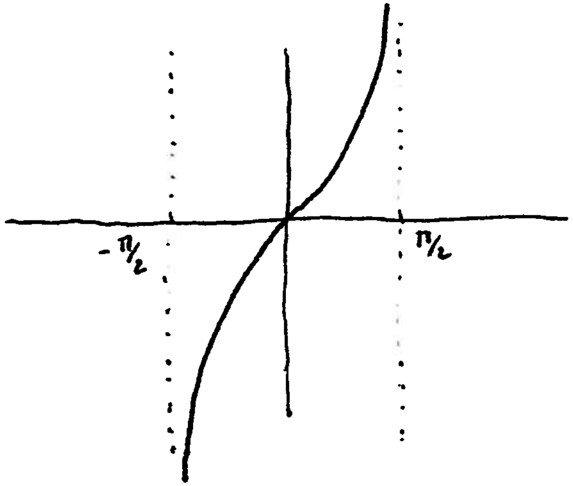
$r \geq 0 \;,\quad 0 \leq \theta < 2\pi$ (algunes vegades també es pren $-\pi < \theta \leq \pi$)



$$\boxed{\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}}$$

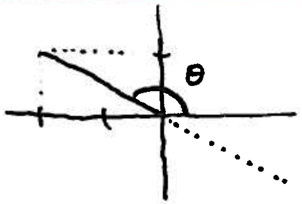
$$\boxed{r = \sqrt{x^2+y^2} \;,\qquad \operatorname{tg}\theta = \frac{y}{x}}$$

$$\theta = \begin{cases} \operatorname{arctg}\frac{y}{x} \,, & x>0,\ y\geq 0 \\ \operatorname{arctg}\frac{y}{x} + \pi \,, & x<0 \\ \operatorname{arctg}\frac{y}{x} + 2\pi \,, & x>0,\ y<0 \\ \pi/2 & x=0,\ y>0 \\ 3\pi/2 & x=0 \quad y<0 \end{cases}$$



gràfica de tg

$\boxed{\text{Ex}}$ El punt $(-2,1)$

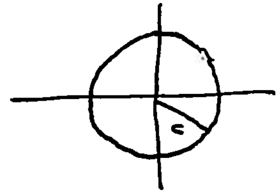


té coordenades polars $r = \sqrt{(-2)^2+1^2} = \sqrt{5}$

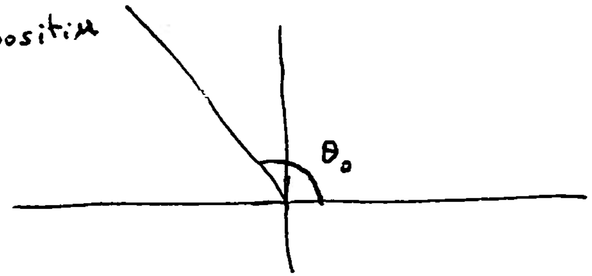
$\operatorname{tg}\theta = \frac{1}{-2} \quad \rightarrow \quad \theta = -0.4636\ldots + \pi = 2.677\ldots$

En coordenades polars r, θ

$r = c$ representa una circumferència de radi c



$\theta = \theta_0$ representa una semirecta que forma un angle θ_0 respecte el semieix x positiu



Eq. de la circumferència en coordenades polars ($c>0$)

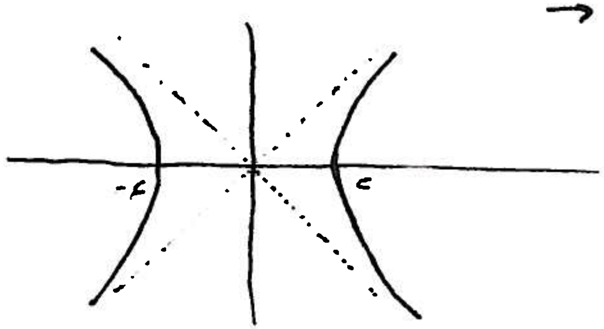
$$x^2+y^2=c^2 \longrightarrow (r\cos\theta)^2+(r\sin\theta)^2=c^2 \longrightarrow r^2(\cos^2\theta+\sin^2\theta)=c^2 \longrightarrow r^2=c^2$$

$$\longrightarrow \boxed{r=c}$$

Eq. d'una hipèrbola

$$x^2-y^2=c^2 \longrightarrow r^2\cos^2\theta - r^2\sin^2\theta = c^2 \longrightarrow r^2(\cos^2\theta-\sin^2\theta)=c^2$$

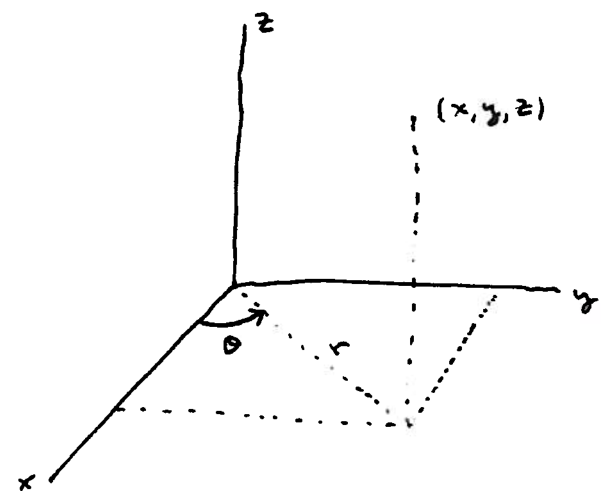
$$\longrightarrow r^2\cos 2\theta = c^2$$



$$r=\sqrt{\frac{c^2}{\cos 2\theta}}$$

Coordenades cilíndriques

(r, θ, z)



$$\boxed{\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}}$$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \operatorname{tg}\theta = \dfrac{y}{x} \end{cases}$$ (en el sentit de les polars)

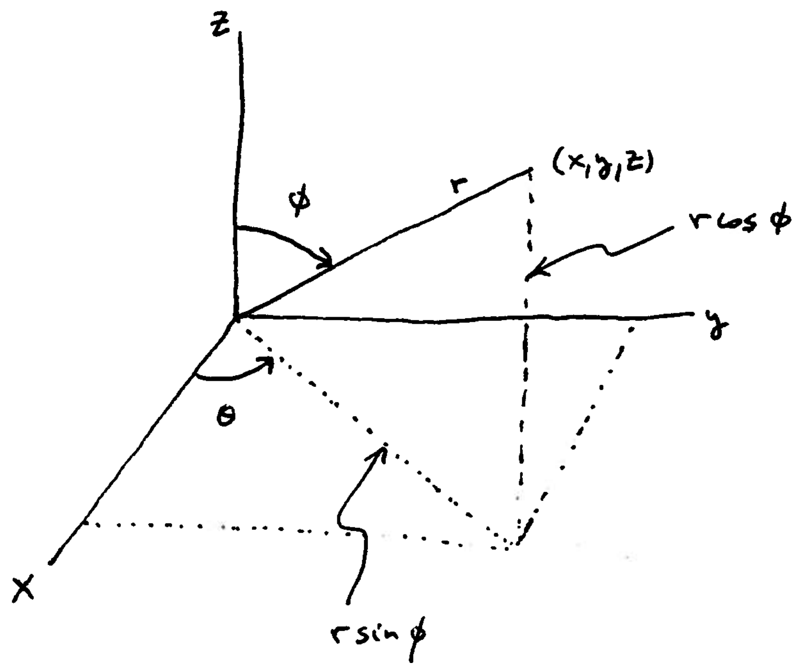
Eq. d'un cilindre : $x^2+y^2 = c^2 \longrightarrow r^2 = c^2 \longrightarrow \boxed{r = c}$

Eq. d'una esfera : $x^2+y^2+z^2 = c^2 \longrightarrow \boxed{r^2+z^2 = c^2}$

(suposem $c > 0$)

Coordenades esfèriques

(r, θ, ϕ) $\qquad r \geq 0, \quad 0 \leq \theta < 2\pi, \quad 0 \leq \phi \leq \pi$



$$\boxed{\begin{aligned} x &= r \sin\phi \cos\theta \\ y &= r \sin\phi \sin\theta \\ z &= r \cos\phi \end{aligned}}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\operatorname{tg}\theta = \frac{y}{x} \longrightarrow \text{"}\,\theta = \operatorname{arctg}\frac{y}{x}\,\text{"}$$

$$\cos\phi = \frac{z}{r} \longrightarrow \phi = \arccos\frac{z}{\sqrt{x^2+y^2+z^2}}$$

Eq d'una esfera: $x^2 + y^2 + z^2 = c^2$, $\quad c > 0$

$$\rightarrow \underbrace{r^2\sin^2\phi\cos^2\theta + r^2\sin^2\phi\sin^2\theta}_{r^2\sin^2\phi\,(\cos^2\theta + \sin^2\theta)} + r^2\cos^2\phi = c^2$$

$$\rightarrow r^2\sin^2\phi + r^2\cos^2\phi = c^2 \longrightarrow r^2 = c^2 \longrightarrow \boxed{r = c}$$

Funcions

Considerarem funcions $f : A \subset \mathbb{R}^n \longrightarrow \mathbb{R}^m$

- si $n > 1$ es diuen funcions de diverses variables (n variables)
- si $m = 1$ es diuen funcions reals o funcions escalars
- si $m > 1$ es diuen funcions vectorials

També escriurem

$$x \longmapsto f(x)$$

Ex:

- $f(x,y) = \left(\sqrt{x^2+y^2}, \frac{xy}{x^2+y^2}\right)$
- temperatura en un forn

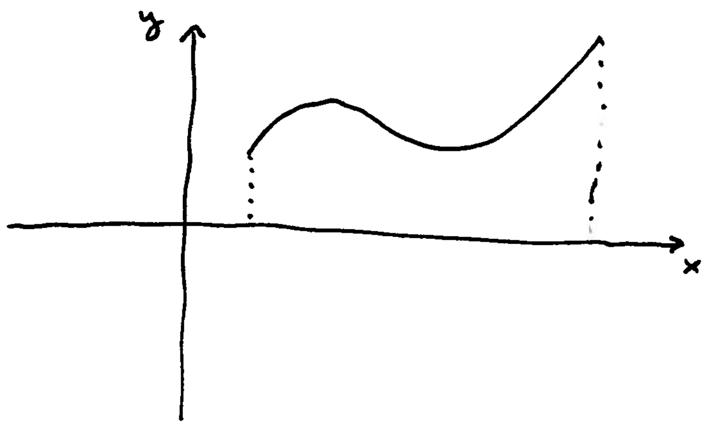
$T(x,y,z)$ $\quad T : U \subset \mathbb{R}^3 \to \mathbb{R}$

- posició i orientació d'un satèl·lit artificial $(x(t), y(t), z(t), \theta(t), \psi(t))$

$$f : I \subset \mathbb{R} \to \mathbb{R}^5$$

Gràfica d'una funció

$f : I \subset \mathbb{R} \longrightarrow \mathbb{R}$



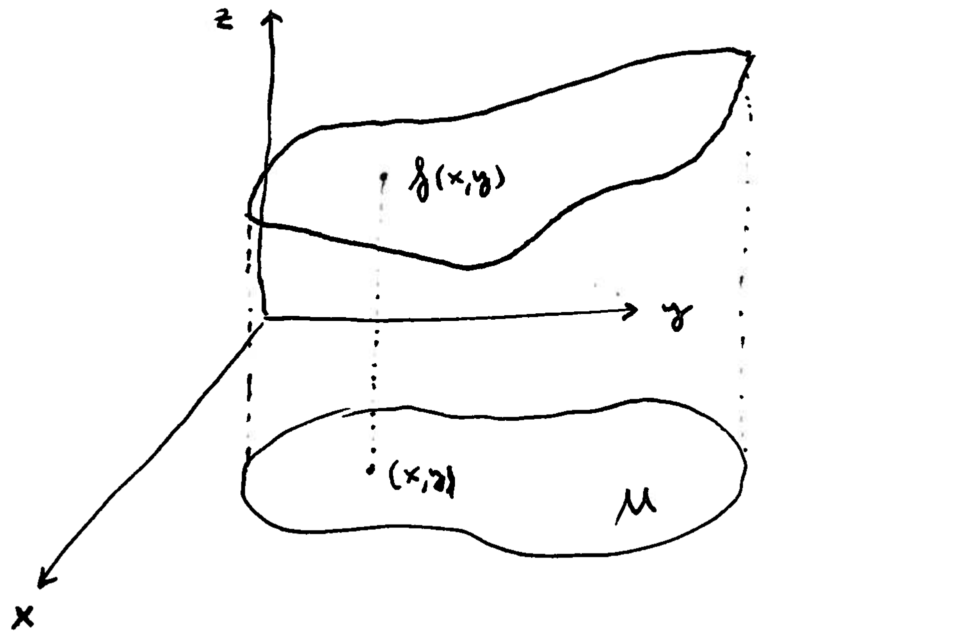
$$\operatorname{graf} f = \{ (x, f(x)) \mid x \in I \} \subset \mathbb{R}^2$$

$f : M \subset \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$$\boxed{\operatorname{graf} f = \{ (x, f(x)) \mid x \in M \} \subset \mathbb{R}^{n+m}}$$

En general és molt difícil de visualitzar

Si $n = 2$ i $m = 1$, $\operatorname{graf} f \subset \mathbb{R}^3$

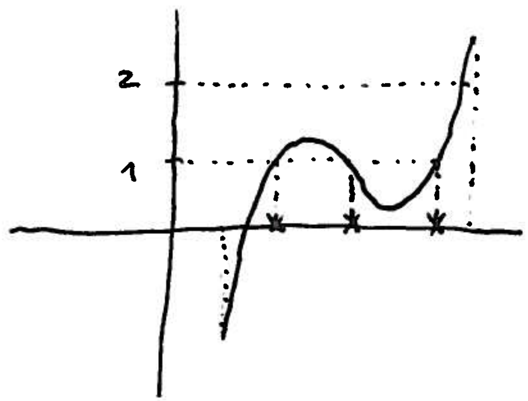


Conjunts de nivell

Donada $f: \mathcal{U} \subset \mathbb{R}^m \longrightarrow \mathbb{R}$ definim conjunt de nivell de valor c al conjunt

$$\boxed{L_c = \{x \in \mathcal{U} \mid f(x) = c\} \subset \mathcal{U} \subset \mathbb{R}^m}$$

$\boxed{m=1}$

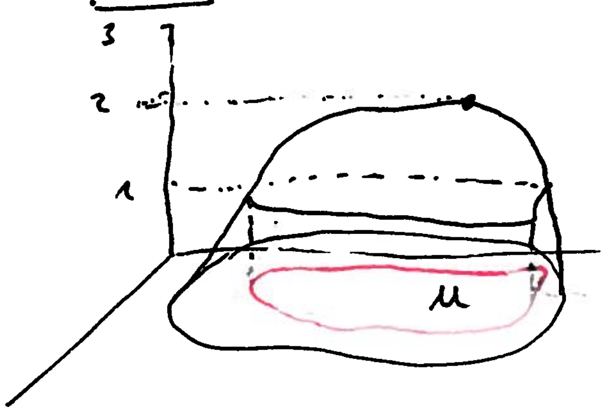


L_2 és un únic punt

L_1 està format per 3 punts

L_0 és un únic punt

$\boxed{m=2}$



$L_3 = \phi$

L_2 és un únic punt

L_1 és una corba

$\boxed{m=3}$

$L_c \subset \mathbb{R}^3$

exemple: $f(x,y,z) = x^2+y^2+z^2$

per a cada $c > 0$

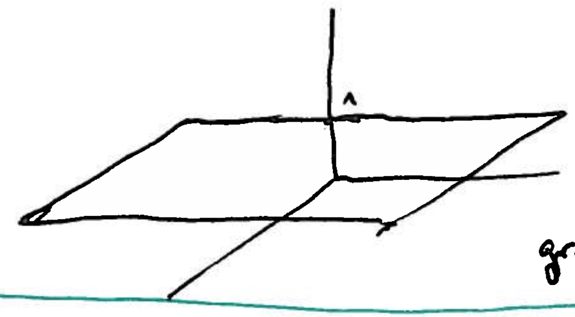
L_c és una esfera de radi $\sqrt{c}$

si $c=0$, $L_c = \{(0,0,0)\}$

si $c<0$, $L_c = \emptyset$

Ex. 1 $\quad f(x,y) = 1 \quad , \quad f: \mathbb{R}^2 \longrightarrow \mathbb{R}$

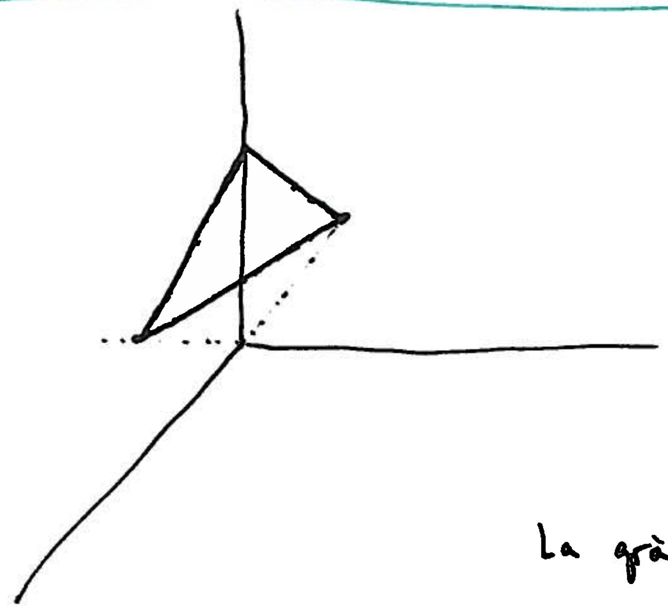
$L_c = \emptyset$ si $c \neq 1$, $\quad L_1 = \mathbb{R}^2$

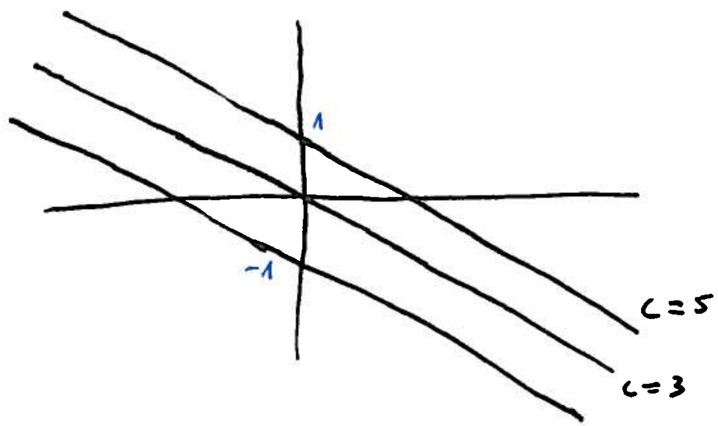


graf f

Ex. 2 $\quad f: \mathbb{R}^2 \longrightarrow \mathbb{R} \quad , \quad f(x,y) = x + 2y + 3$

$L_c = \{ (x,y) \mid x + 2y + 3 = c \}$ $\qquad y = \frac{1}{2}(-x + c - 3)$ $\quad$ rectes de pendent $-\frac{1}{2}$





corbes de nivell

La gràfica té eq. $\quad z = x + 2y + 3$

$x + 2y - z + 3 = 0$

$\boxed{\text{Ex. 3}}$ $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$, $f(x,y) = x^2 + y^2$

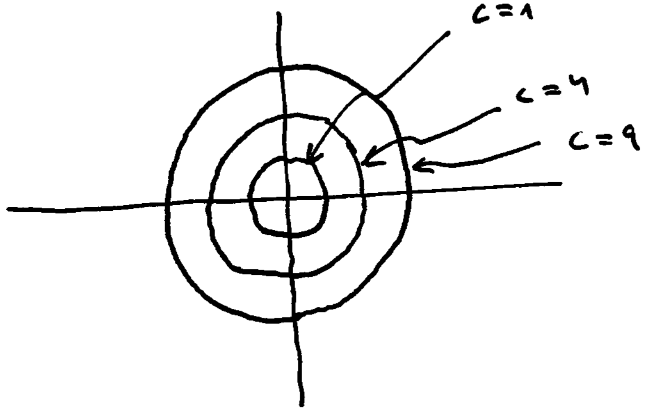
<u>corbes de nivell</u>

$L_c = \{(x,y) \mid x^2+y^2 = c\}$

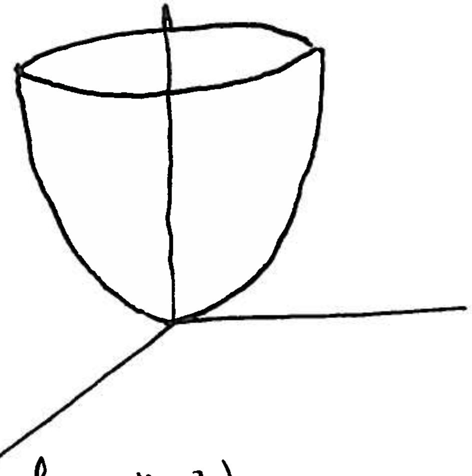
$\to$ circumferència radi $\sqrt{c}$ si $c>0$

$\to$ $(0,0)$ si $c=0$

$\to$ $\emptyset$ si $c<0$



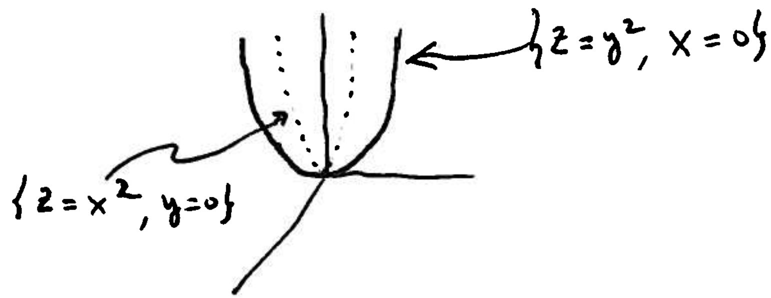
La <u>gràfica</u> és $z = x^2+y^2$



<u>seccions</u> : interseccions de la gràfica amb plans

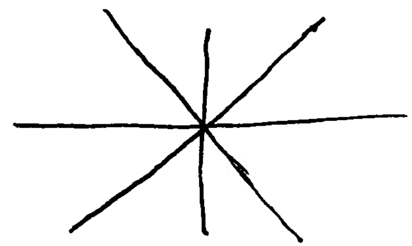
$\text{graf } f \cap \{x=0\} \to z = y^2$ (paràbola en el pla y-z)

$\text{graf } f \cap \{y=0\} \to z = x^2$ (paràbola " x-z)



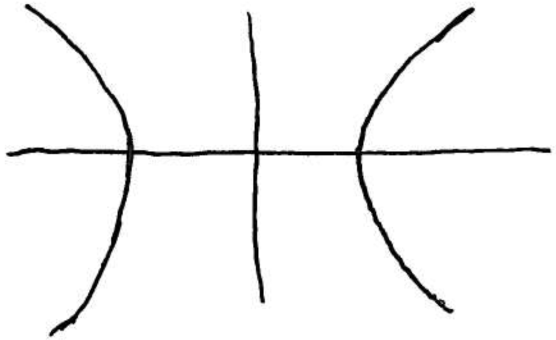
$\boxed{\text{Ex. 4}}$ $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$, $f(x,y) = x^2 - y^2$

<u>corbes de nivell</u>:

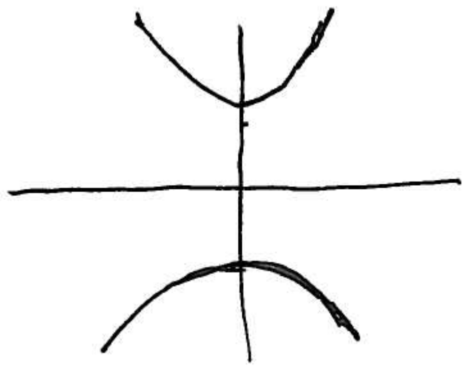


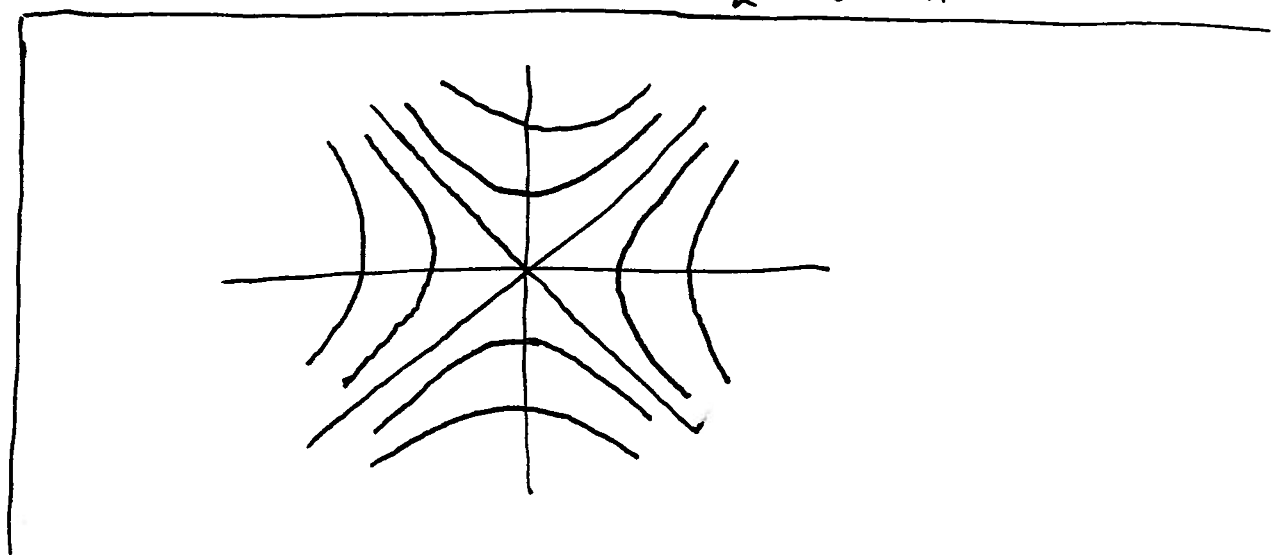
$L_0 = \{x^2 - y^2 = 0\} \rightarrow x^2 = y^2 \longrightarrow x = \pm y$ (dues rectes)

$L_1 = \{x^2 - y^2 = 1\} \longrightarrow x^2 - 1 = y^2 \longrightarrow y = \pm\sqrt{x^2-1}$ (corbes definides per a $x \geq 1$ o $x \leq -1$)



$L_{-1} = \{x^2 - y^2 = -1\} \longrightarrow y^2 = x^2 + 1 \rightarrow y = \pm\sqrt{x^2+1}$ (corbes definides per a tot x)

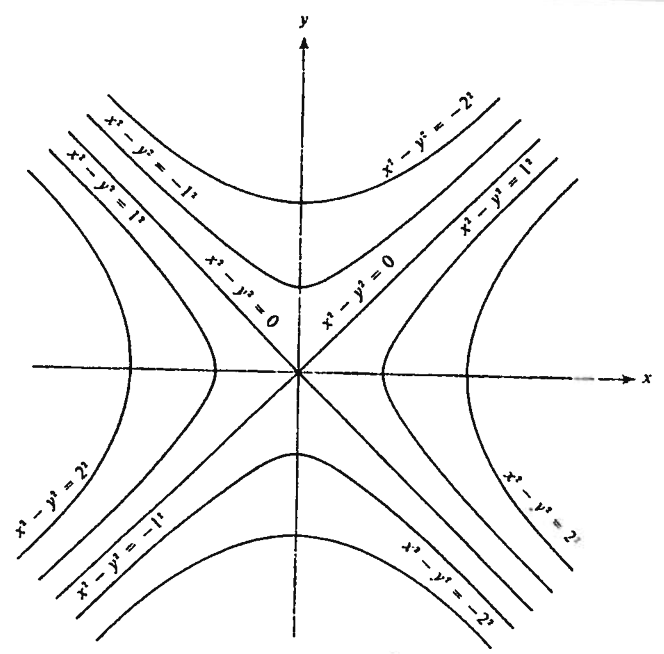


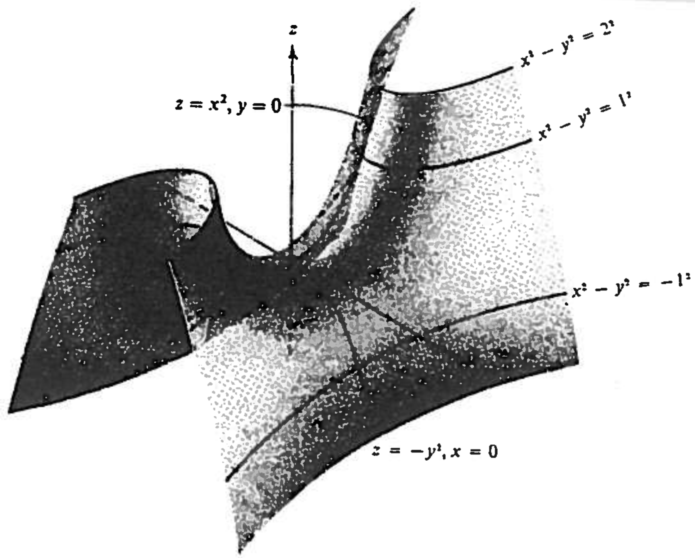


__Gràfica__ de $f(x,y) = x^2 - y^2$ (continuació)

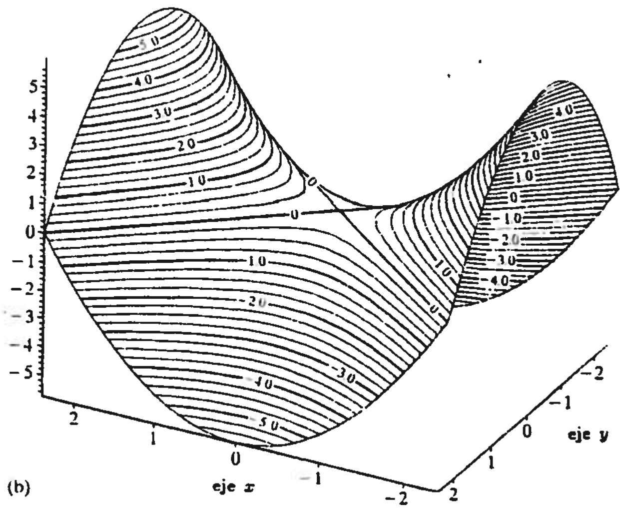
seccions: $\text{graf } f \cap \{x=0\} = \{z = x^2 - y^2\} \cap \{x=0\} = \{z = -y^2\}$ (paràbola)

$\text{graf } f \cap \{y=0\} = \{z = x^2\}$ (paràbola)

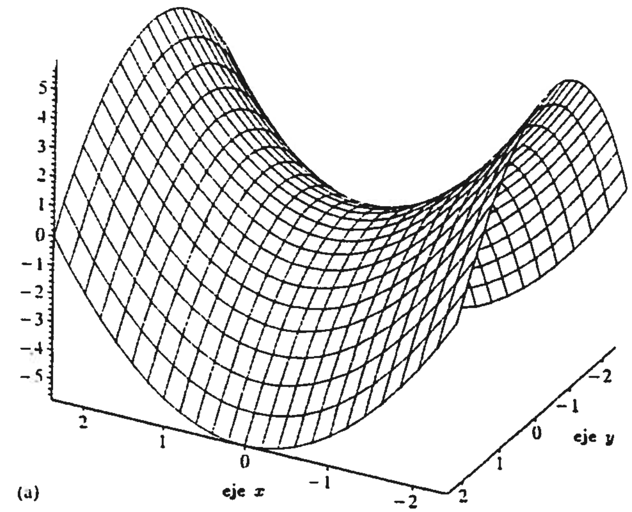




Corbes de nivell i gràfica de $f(x,y) = x^2 - y^2$



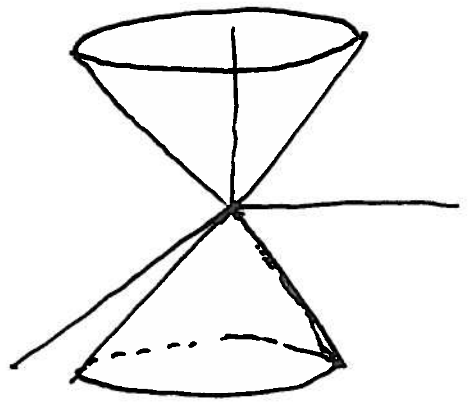
(b)



(a)

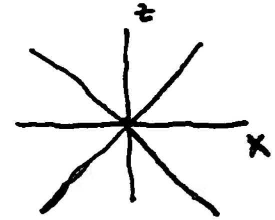
$\boxed{\underline{\text{Ex}}.5}$ Superfícies de nivell de $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$, $f(x,y,z) = x^2+y^2-z^2$

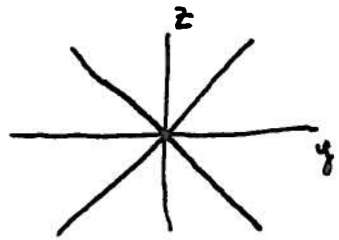
$L_0 = \{x^2+y^2-z^2=0\} \longrightarrow z^2 = x^2+y^2 \longrightarrow z = \pm\sqrt{x^2+y^2}$



quan $y=0$, $z = \pm\sqrt{x^2} = \pm|x|$

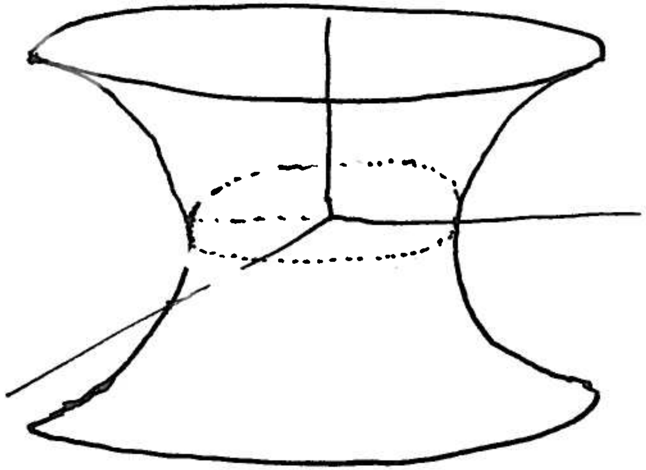
quan $x=0$, $z = \pm\sqrt{y^2} = \pm|y|$





$L_1 = \{x^2+y^2-z^2=1\} \longrightarrow z^2 = x^2+y^2-1 \longrightarrow z = \pm\sqrt{x^2+y^2-1}$

cal que $x^2+y^2 \geq 1$ (en polars en x-y $r \geq 1$)

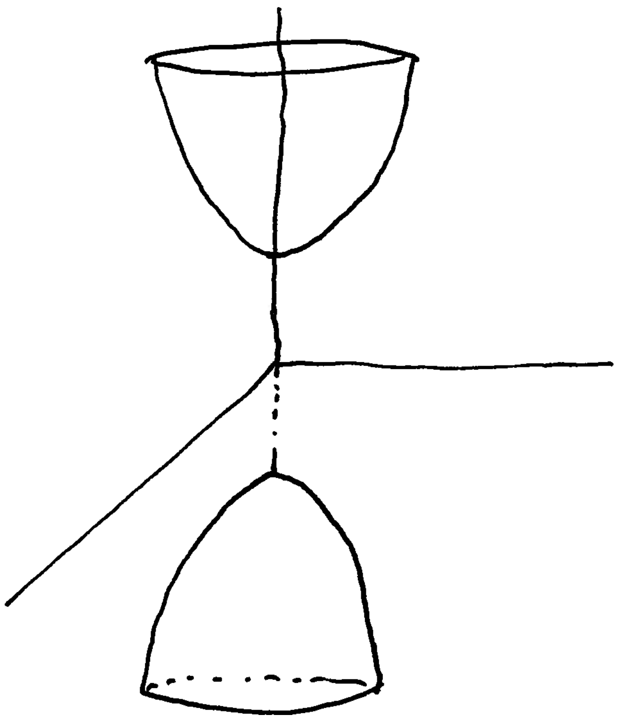


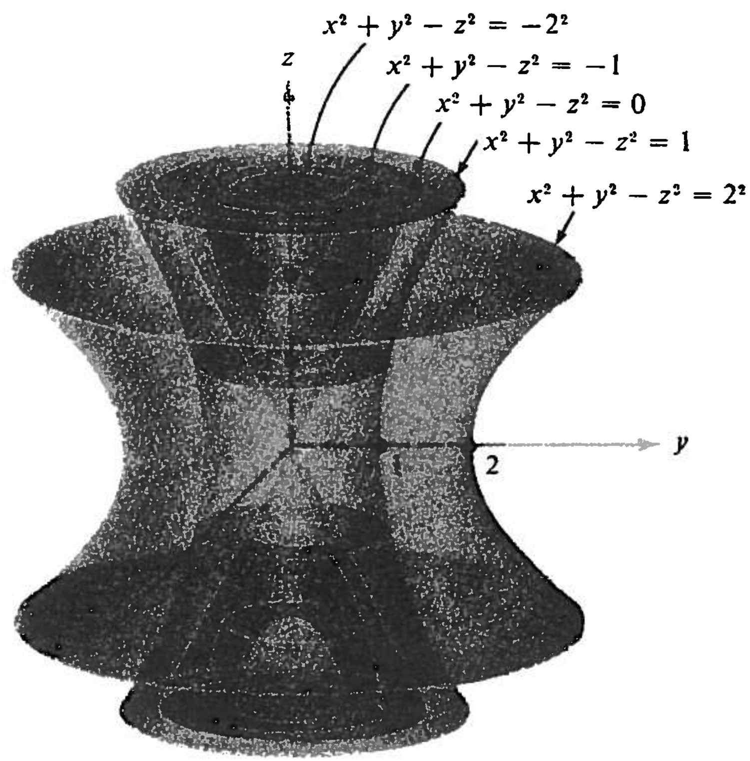
$$L_{-1} = \{ x^2 + y^2 - z^2 = -1 \} \longrightarrow z^2 = x^2 + y^2 + 1 \longrightarrow z = \pm\sqrt{x^2 + y^2 + 1}$$

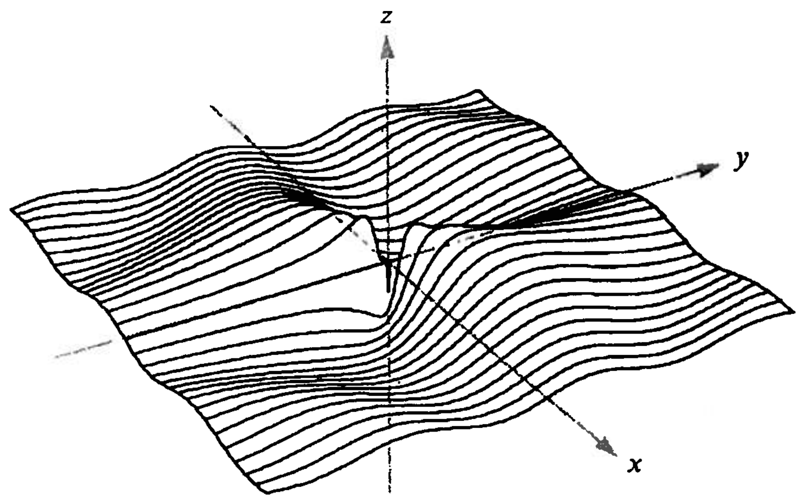
definit $\forall (x,y)$

El valor de z només depèn del radi r de polars en x-y

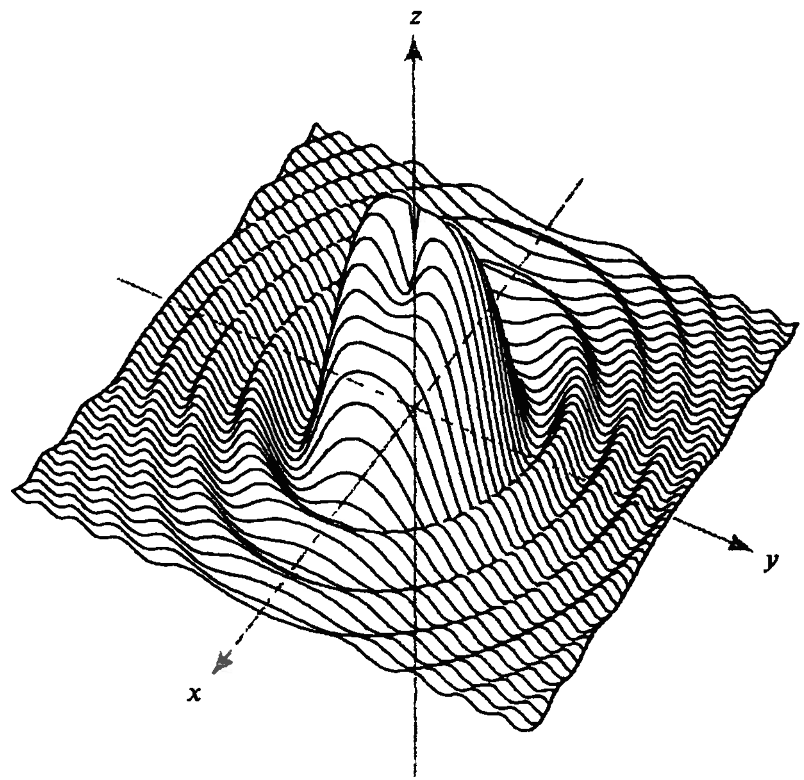
$$z = \pm\sqrt{r^2 + 1}$$



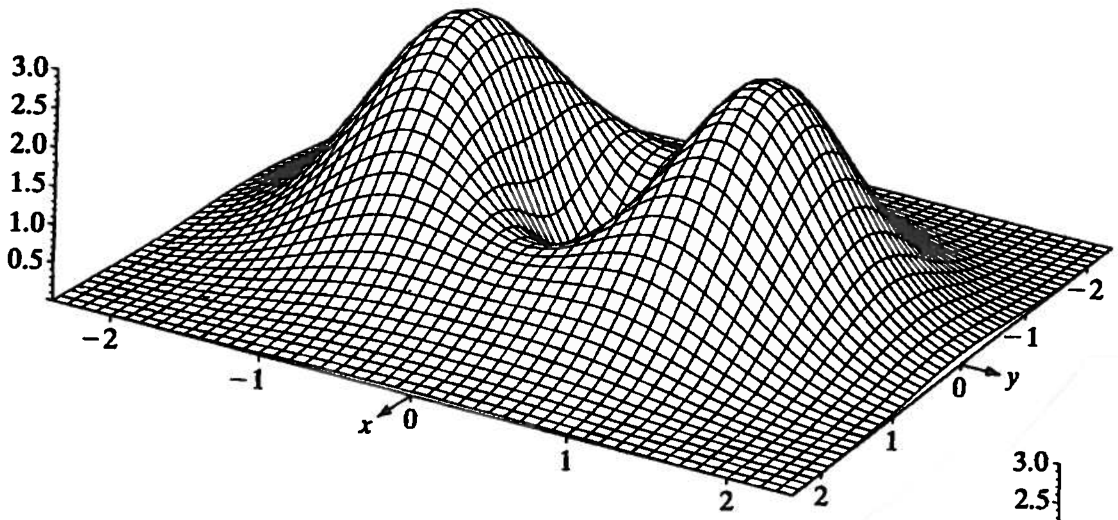


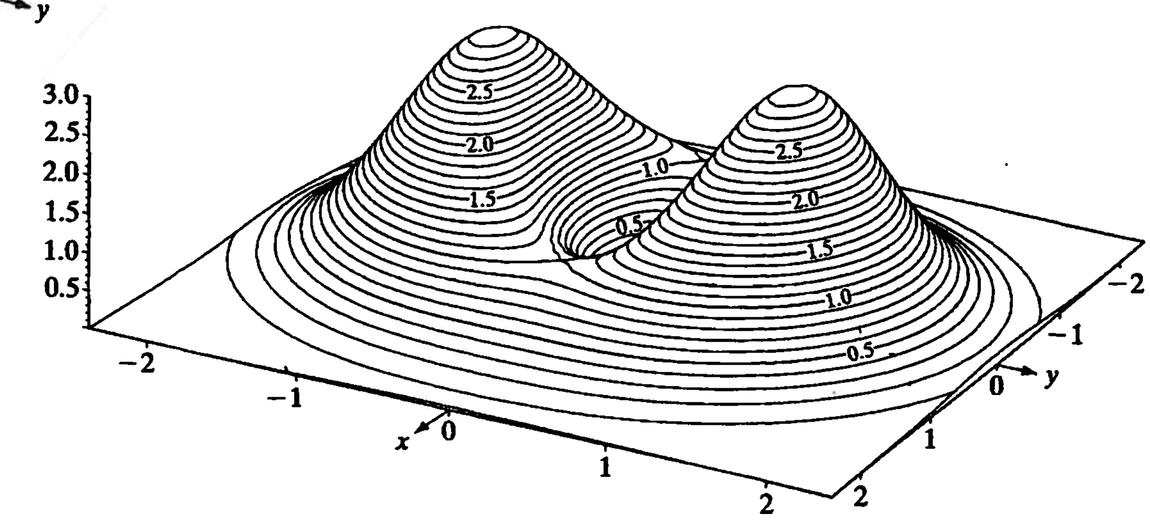


$$z = \frac{(\text{sen } xy)}{(x^2 + y^2)}, 0 < |x| \leq 3, 0 < |y| \leq 3.$$



$$z = \frac{[\text{sen}(2x^2 + 3y^2)]}{[x^2 + y^2]}, 0 < |x| \leq 3, 0 < |y| \leq 3.$$





Funció:

$$f(x,y) = (x^2+3y^2)\,e^{1-x^2-y^2}$$