Ejercicio práctico de Introducción a la Investigación Operativa

Una empresa fabrica 3 productos, cuyo proceso de fabricación pasa por tres departamentos, en la siguiente tabla se muestran las necesidades en horas de cada departamento para la producción de cada unidad de producto:

PRODUCTO	Departamento 1	Departamento 2	Departamento 3
A	1,50	2,00	0,25
B	3,00	1,00	0,25
C	2,00	2,50	0,25

En un determinado periodo y con la plantilla existente se disponen de 450 horas en el Departamento 1, de 350 en el 2 y de 50 en el 3. Los márgenes brutos por unidad de producto son de 25€ para A, de 28€ para B y de 30€ para el producto C.

a. El modelo de PLE que permite encontrar las producciones (en valores enteros) óptimas maximizando el margen bruto y teniendo en cuenta las limitaciones horarias es:

$$\text{Max } Z = 25X_1+28X_2+30X_3$$
Sujeto a:
$$1{,}50X_1+3{,}00X_2+2{,}00X_3 \leq 450$$
$$2{,}00X_1+1{,}00X_2+2{,}50X_3 \leq 350$$
$$0{,}25X_1+0{,}25X_2+0{,}25X_3 \leq 50$$
$$X_1, X_2, X_3 \geq 0$$
X_1, X_2, X_3 Enteras

Determinad cuál es la solución óptima.

```
libname pr '.';
data pr.pr1;
 input _row_ $13. x1 x2 X3 _type_ $ _rhs_;
      datalines;
Beneficio     25     28     30      MAX       .
Dep1           1.50   3      2      LE      450
Dep2           2      1      2.5    LE      350
Dep3           0.25   0.25   0.25   LE       50
limsup         10000  10000  10000  UPPERBD   .
enteras        1      2      3      INTEGER   .
;
run;

proc print data=pr.pr1;
run;

proc lp data=pr.pr1;
run;
```

Obs	_row_	x1	x2	X3	_type_	_rhs_
1	Beneficio	25.00	28.00	30.00	MAX	.
2	Dep1	1.50	3.00	2.00	LE	450
3	Dep2	2.00	1.00	2.50	LE	350
4	Dep3	0.25	0.25	0.25	LE	50
5	limsup	10000.00	10000.00	10000.00	UPPERBD	.
6	enteras	1.00	2.00	3.00	INTEGER	.

The LP Procedure

Problem Summary

Objective Function	Max Beneficio
Rhs Variable	_rhs_
Type Variable	_type_
Problem Density (%)	66.67

Variables	Number
Integer	3
Slack	3
Total	6

Constraints	Number
LE	3
Objective	1
Total	4

Solution Summary

Integer Optimal Solution

Objective Value	5540
Phase 1 Iterations	0
Phase 2 Iterations	3
Phase 3 Iterations	0
Integer Iterations	0
Integer Solutions	1
Initial Basic Feasible Variables	5
Time Used (seconds)	0
Number of Inversions	4
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	100
Maximum Phase 2 Iterations	100
Maximum Phase 3 Iterations	99999999
Maximum Integer Iterations	100
Time Limit (seconds)	120

Variable Summary

Col	Variable Name	Status	Type	Price	Activity	Reduced Cost
1	x1	BASIC	INTEGER	25	60	0
2	x2	BASIC	INTEGER	28	80	0
3	X3	BASIC	INTEGER	30	60	0
4	Dep1		SLACK	0	0	-5.2
5	Dep2		SLACK	0	0	-4.8
6	Dep3		SLACK	0	0	-30.4

Constraint Summary

Row	Constraint Name	Type	S/S Col	Rhs	Activity	Dual Activity
1	Beneficio	OBJECTVE	.	0	5540	.
2	Dep1	LE	4	450	450	5.2
3	Dep2	LE	5	350	350	4.8
4	Dep3	LE	6	50	50	30.4

b. En el modelo anterior incorporad la existencia de unos costes de preparación a la producción (Costes Fijos) que ascienden a 400€ para A, 550€ para B y 550€ para C. ¿Existen cambios en la producción óptima a causa de la incorporación de los costes fijos? Justificad la respuesta.

$$\text{Max } Z= 25X_1+28X_2+30X_3-400Y_1-550Y_2-550Y_3$$

Sujeto a:

$$1{,}50X_1+3{,}00X_2+2{,}00X_3\leq 450$$
$$2{,}00X_1+1{,}00X_2+2{,}50X_3\leq 350$$
$$0{,}25X_1+0{,}25X_2+0{,}25X_3\leq 50$$
$$X_1-M_1Y_1\leq 0$$
$$X_2-M_2Y_2\leq 0$$
$$X_3-M_3Y_3\leq 0$$
$$X_1,X_2,X_3\geq 0,\ Y_1,\ Y_2,\ Y_3\in\{0,1\}$$
$$X_1,X_2,X_3 \text{ Enteras}$$

```
data pr.pr1;
 input _row_ $13. x1 x2 x3 y1 y2 y3 _type_ $ _rhs_;
      datalines;
Beneficio     25     28     30    -400    -550    -550          MAX    .
Dep1           1.50   3      2       0       0       0           LE   450
Dep2           2      1      2.5     0       0       0           LE   350
Dep3           0.25   0.25   0.25    0       0       0           LE    50
CF1            1      0      0    -10000     0       0           LE     0
CF2            0      1      0       0    -10000     0           LE     0
CF3            0      0      1       0       0    -10000         LE     0
limsup     10000  10000  10000       .       .       .      UPPERBD     .
enteras        1      2      3       .       .       .      INTEGER     .
bin            .      .      .       1       2       3      BINARY      .
;
run;

proc print data=pr.pr1;
run;

proc lp data=pr.pr1 IMAXIT=200;
run;
```

Obs	_row_	x1	x2	x3	y1	y2	y3	_type_	_rhs_
1	Beneficio	25.00	28.00	30.00	-400	-550	-550	MAX	.
2	Dep1	1.50	3.00	2.00	0	0	0	LE	450
3	Dep2	2.00	1.00	2.50	0	0	0	LE	350
4	Dep3	0.25	0.25	0.25	0	0	0	LE	50
5	CF1	1.00	0.00	0.00	-10000	0	0	LE	0
6	CF2	0.00	1.00	0.00	0	-10000	0	LE	0
7	CF3	0.00	0.00	1.00	0	0	-10000	LE	0
8	limsup	10000.00	10000.00	10000.00	.	.	.	UPPERBD	.
9	enteras	1.00	2.00	3.00	.	.	.	INTEGER	.
10	bin	.	.	.	1	2	3	BINARY	.

The LP Procedure

Problem Summary

Objective Function	Max Beneficio
Rhs Variable	_rhs_
Type Variable	_type_
Problem Density (%)	29.17

Variables	Number
Integer	3
Binary	3
Slack	6
Total	12

Constraints	Number
LE	6
Objective	1
Total	7

Integer Iteration Log

Iter	Problem	Condition	Objective	Branched	Value	Sinfeas	Active	Proximity
1	0	ACTIVE	5529.9	y2	0.008	0.02	2	.
2	-1	ACTIVE	4984.3	y3	0.006	0.012	3	.
3	2	ACTIVE	4746	y1	0.01	0.01	4	.
4	-3	SUBOPTIMAL	4350	.	.	.	2	143.5
5	-2	FATHOMED	4437.6	.	.	.	1	143.5
6	1	FATHOMED	4368	.	.	.	0	.

Solution Summary

Integer Optimal Solution

Objective Value	4350
Phase 1 Iterations	0
Phase 2 Iterations	6
Phase 3 Iterations	6
Integer Iterations	6
Integer Solutions	1
Initial Basic Feasible Variables	8
Time Used (seconds)	0
Number of Inversions	6
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	100
Maximum Phase 2 Iterations	100
Maximum Phase 3 Iterations	99999999
Maximum Integer Iterations	200
Time Limit (seconds)	120

Variable Summary

Col	Variable Name	Status	Type	Price	Activity	Reduced Cost
1	x1	BASIC	INTEGER	25	100	0
2	x2	BASIC	INTEGER	28	100	0
3	x3	DEGEN	INTEGER	30	0	0
4	y1		BINARY	-400	1	-400
5	y2		BINARY	-550	1	-550
6	y3		BINARY	-550	0	39450
7	Dep1		SLACK	0	0	-2
8	Dep2	BASIC	SLACK	0	50	0
9	Dep3		SLACK	0	0	-88
10	CF1	BASIC	SLACK	0	9900	0
11	CF2	BASIC	SLACK	0	9900	0
12	CF3		SLACK	0	0	-4

Constraint Summary

Row	Constraint Name	Type	S/S Col	Rhs	Activity	Dual Activity
1	Beneficio	OBJECTVE	.	0	4350	.
2	Dep1	LE	7	450	450	2
3	Dep2	LE	8	350	300	0
4	Dep3	LE	9	50	50	88
5	CF1	LE	10	0	-9900	0
6	CF2	LE	11	0	-9900	0
7	CF3	LE	12	0	0	4

c. La empresa se plantea hacer horas extras (CUYO NÚMERO NO ESTÁ LIMITADO), pero las limita al hecho de que al menos deben cumplirse las limitaciones horarias disponibles para uno de los tres departamentos. Incorporar este hecho al modelo del apartado b y calculad la solución óptima en este caso. ¿En qué departamento o departamentos es más rentable realizar horas extras? Justificad la respuesta.

$$\text{Max } Z= 25X_1+28X_2+30X_3-400Y_1-550Y_2-550Y_3$$

Sujeto a:

$$1{,}50X_1+3{,}00X_2+2{,}00X_3-450\leq M_4(1-Y_4)$$
$$2{,}00X_1+1{,}00X_2+2{,}50X_3-350\leq M_5(1-Y_5)$$
$$0{,}25X_1+0{,}25X_2+0{,}25X_3-\ 50\leq M_6(1-Y_6)$$
$$X_1-M_1Y_1\leq 0$$
$$X_2-M_2Y_2\leq 0$$
$$X_3-M_3Y_3\leq 0$$
$$Y_4+Y_5+Y_6=1$$
$$X_1,X_2,X_3\geq 0;\ Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5,\ Y_6\in\{0,1\}$$
$$X_1,X_2,X_3 \text{ Enteras}$$

```
data pr.pr1;
 input _row_ $13. x1 x2 x3 y1 y2 y3 y4 y5 y6 _type_ $ _rhs_;
      datalines;
Beneficio     25     28     30    -400    -550    -550     0     0       0        MAX    .
Dep1          1.50   3      2       0       0       0    10000  0       0        LE   10450
Dep2          2      1      2.5     0       0       0     0    10000    0        LE   10350
Dep3          0.25   0.25   0.25    0       0       0     0     0     10000      LE   10050
CF1           1      0      0    -10000     0       0     0     0       0        LE     0
CF2           0      1      0       0    -10000     0     0     0       0        LE     0
CF3           0      0      1       0       0    -10000   0     0       0        LE     0
Limit         0      0      0       0       0       0     1     1       1        GE     1
limsup      10000  10000  10000     .       .       .     .     .       .     UPPERBD   .
enteras       1      2      3       .       .       .     .     .       .     INTEGER   .
bin           .      .      .       1       2       3     4     5       6     BINARY    .
```

```
;
run;

proc print data=pr.pr1;
run;

proc lp data=pr.pr1 IMAXIT=200;
run;
```

Obs	_row_	x1	x2	x3	y1	y2	y3	y4	y5	y6	_type_	_rhs_
1	Beneficio	25.00	28.00	30.00	-400	-550	-550	0	0	0	MAX	.
2	Dep1	1.50	3.00	2.00	0	0	0	10000	0	0	LE	10450
3	Dep2	2.00	1.00	2.50	0	0	0	0	10000	0	LE	10350
4	Dep3	0.25	0.25	0.25	0	0	0	0	0	10000	LE	10050
5	CF1	1.00	0.00	0.00	-10000	0	0	0	0	0	LE	0
6	CF2	0.00	1.00	0.00	0	-10000	0	0	0	0	LE	0
7	CF3	0.00	0.00	1.00	0	0	-10000	0	0	0	LE	0
8	Limit	0.00	0.00	0.00	0	0	0	1	1	1	GE	1
9	limsup	10000.00	10000.00	10000.00	.	.	.	.	.	.	UPPERBD	.
10	enteras	1.00	2.00	3.00	.	.	.	.	.	.	INTEGER	.
11	bin	.	.	.	1	2	3	4	5	6	BINARY	.

The LP Procedure

Problem Summary

Objective Function	Max Beneficio
Rhs Variable	_rhs_
Type Variable	_type_
Problem Density (%)	25.00

Variables	Number
Integer	3
Binary	6
Slack	6
Surplus	1
Total	16

Constraints	Number
LE	6
GE	1
Objective	1
Total	8

Integer Iteration Log

Iter	Problem	Condition	Objective	Branched	Value	Sinfeas	Active	Proximity
1	0	ACTIVE	138569.09	y1	0.487	0.96974	2	.
2	1	ACTIVE	133033.2	x2	1380	1.45283	3	.
3	2	ACTIVE	133032.69	y3	0.315	0.93337	4	.
159	-116	FATHOMED	5262.0833	.	.	.	11	3899.86
160	158	FATHOMED	3975	.	.	.	10	3800
161	117	SUBOPTIMAL	9250	.	.	.	0	.

Solution Summary

Integer Optimal Solution

Objective Value	9250
Phase 1 Iterations	1
Phase 2 Iterations	10
Phase 3 Iterations	253
Integer Iterations	161
Integer Solutions	2
Initial Basic Feasible Variables	9
Time Used (seconds)	0
Number of Inversions	82
Epsilon	1E-8
Infinity	1.797693E308
Maximum Phase 1 Iterations	100
Maximum Phase 2 Iterations	100
Maximum Phase 3 Iterations	99999999
Maximum Integer Iterations	200
Time Limit (seconds)	120

Variable Summary

Col	Variable Name	Status	Type	Price	Activity	Reduced Cost
1	x1		INTEGER	25	0	-31
2	x2	BASIC	INTEGER	28	350	0
3	x3		INTEGER	30	0	-40
4	y1		BINARY	-400	0	-400
5	y2		BINARY	-550	1	-550
6	y3		BINARY	-550	0	-550
7	y4	ALTER	BINARY	0	0	0
8	y5		BINARY	0	1	-280000
9	y6	ALTER	BINARY	0	0	0
10	Dep1	BASIC	SLACK	0	9400	0
11	Dep2		SLACK	0	0	-28
12	Dep3	BASIC	SLACK	0	9962.5	0
13	CF1	DEGEN	SLACK	0	0	0
14	CF2	BASIC	SLACK	0	9650	0
15	CF3	DEGEN	SLACK	0	0	0
16	Limit	DEGEN	SURPLUS	0	0	0

Constraint Summary

Row	Constraint Name	Type	S/S Col	Rhs	Activity	Dual Activity
1	Beneficio	OBJECTVE	.	0	9250	.
2	Dep1	LE	10	10450	1050	0
3	Dep2	LE	11	10350	10350	28
4	Dep3	LE	12	10050	87.5	0
5	CF1	LE	13	0	0	0
6	CF2	LE	14	0	-9650	0
7	CF3	LE	15	0	0	0
8	Limit	GE	16	1	1	0

NOTAS

/*To help monitor the growth of the branch-and-bound tree, the LP procedure reports on the status of each problem that is solved. The report, displayed in the Integer Iteration Log, can be used to reconstruct the branch-and-bound tree. Each row in the report describes the results of the attempted solution of the linear program at a node in the tree. In the following discussion, a problem on a given line in the log is called the current problem. The following columns are displayed in the report:

Iter
 identifies the number of the branch-and-bound iteration.

Problem
 identifies how the current problem fits in the branch-and-bound tree.

Condition
 reports the result of the attempted solution of the current problem. Values for Condition are:

ACTIVE: The current problem was solved successfully.

INFEASIBLE: The current problem is infeasible.

FATHOMED: The current problem cannot lead to an improved integer solution and therefore it is dropped.

SINGULAR: A singular basis was encountered in attempting to solve the current problem. Solution of this relaxed problem is suspended and will be attempted later if necessary.

SUBOPTIMAL: The current problem has an integer feasible solution.

Objective
 reports the objective value of the current problem.

Branched
 names the variable that is branched in subtrees defined by the descendants of this problem.

Value
 gives the current value of the variable named in the column labeled Branched.

Sinfeas
 gives the sum of the integer infeasibilities in the optimal solution to the current problem.

Active
 reports the total number of nodes currently active in the branch-and-bound tree.

Proximity
 reports the gap between the best integer solution and the current lower (upper for maximizations) bound of all active nodes. */