

# Méthodes Bayésiennes

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**1. Yield of Potatoes.** A researcher is investigating the relationship between yield of potatoes ( $y$ ) and level of fertilizer ( $x$ ). She divides a field into eight plots of equal size and applied fertilizer at a different level to each plot. The level of fertilizer and yield for each plot is recorded below:

| Fertilizer level<br>$x$ | Yield<br>$y$  |
|-------------------------|---------------|
| 1                       | 25            |
| 1.5                     | 31            |
| 2                       | 27            |
| 2.5                     | 28            |
| 3                       | 36            |
| 3.5                     | 35            |
| 4                       | Not available |
| 4.5                     | 34            |

Suppose that we know that yield given the fertilizer level is  $Normal(\beta_0 + \beta_1 x, \sigma)$  and no prior information is available.

- Write the Bayesian Model.
- Find a 95% Bayesian credible interval for  $\beta_1$ .
- Find a 95% credible interval for  $y$  given  $x=4$ .

**2. Leukemia: Time to Event Data.** Feigl and Zelen (1965) present data on the survival times in weeks of patients who were diagnosed with leukemia. The patients were classified according to one characteristic of white cells referred to as AG+ and AG-. The  $n_1=17$  times from diagnosis to death for the AG+ group are: 65, 156, 100, 134, 16, 108, 121, 4, 39, 143, 56, 26, 22, 1, 1, 5, 65, and the  $n_2=16$  observations for the AG- group are: 56, 65, 17, 7, 16, 22, 3, 4, 2, 3, 8, 4, 3, 30, 4, 43. No prior information is available. Suppose a two-sample exponential model for handling the leukemia data.

- Write the Bayesian Model.
- Calculate a 95% credible interval for the weeks to death from diagnosis for the AG+ and AG- group.
- Calculate a 95% credible interval for the ratio of the 24-week probabilities of survival for the two groups, defined as:

$$\frac{S_{AG+}(24)}{S_{AG-}(24)}.$$

Note: The probability of survival at time  $t$  is defined as:  $S(t) = 1 - \int_0^t f(x)dx$ , where in this exercise  $f(x)$  is the exponential distribution.