Linear Discriminant Analysis

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LDA

- Find the function that better predicts the class of an object
- Based on Factorial methods

LDA

- Given
  - A data matrix X (nxp) centered
    - n individuals
    - p variables
  - Y categorical variable (m modalities)

- Total Variability of X:
  - $T_{pp} = X'PX$
  - P = diagonal(1/n) $$T_{pp} = V(X) = \frac{(X - \bar{X})^2}{n}$$

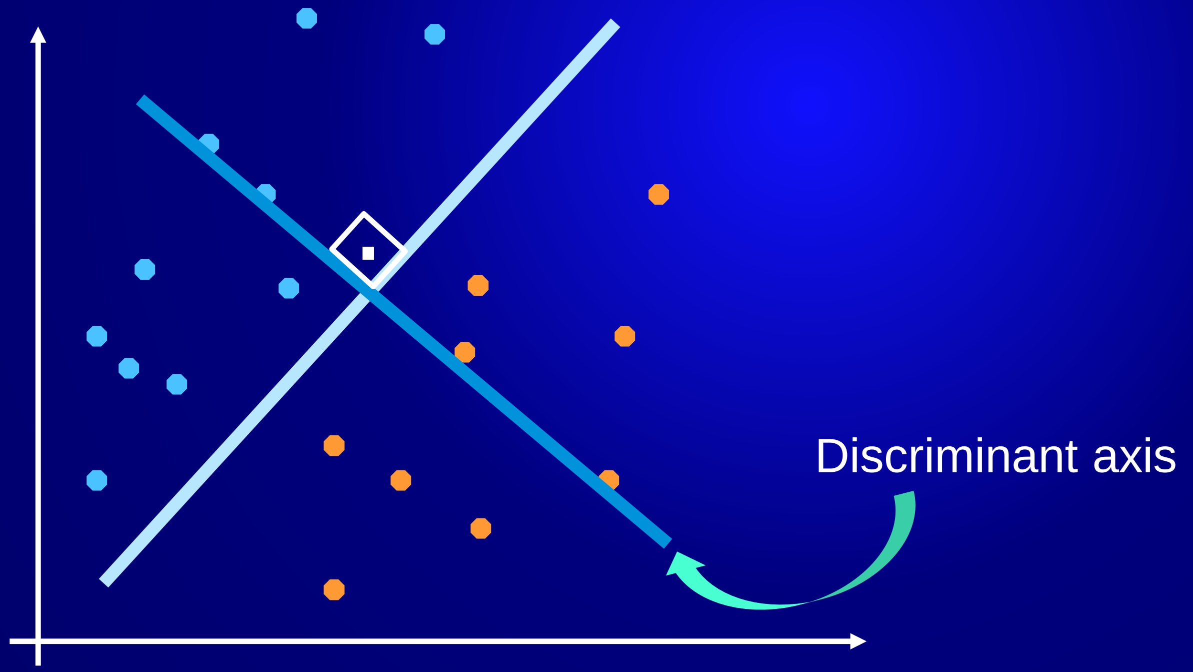
Matricial notation. Care, centered data means $\bar{X}$=0

LDA

- Variability decomposition
  - Total variability $T_{pp} = X^t P X$
  - $T = E + D$
  - $E = G^t H G$
    - G centroids matrix
    - $H = \mathrm{diag}(n_k/n)$
  - $D = X'^t P X$
    - $X' = (x_{ij} - g_{kj})$
    - $P = \mathrm{diag}(1/n)$

Linear Discriminant Analysis

- Find the axis that better discriminates

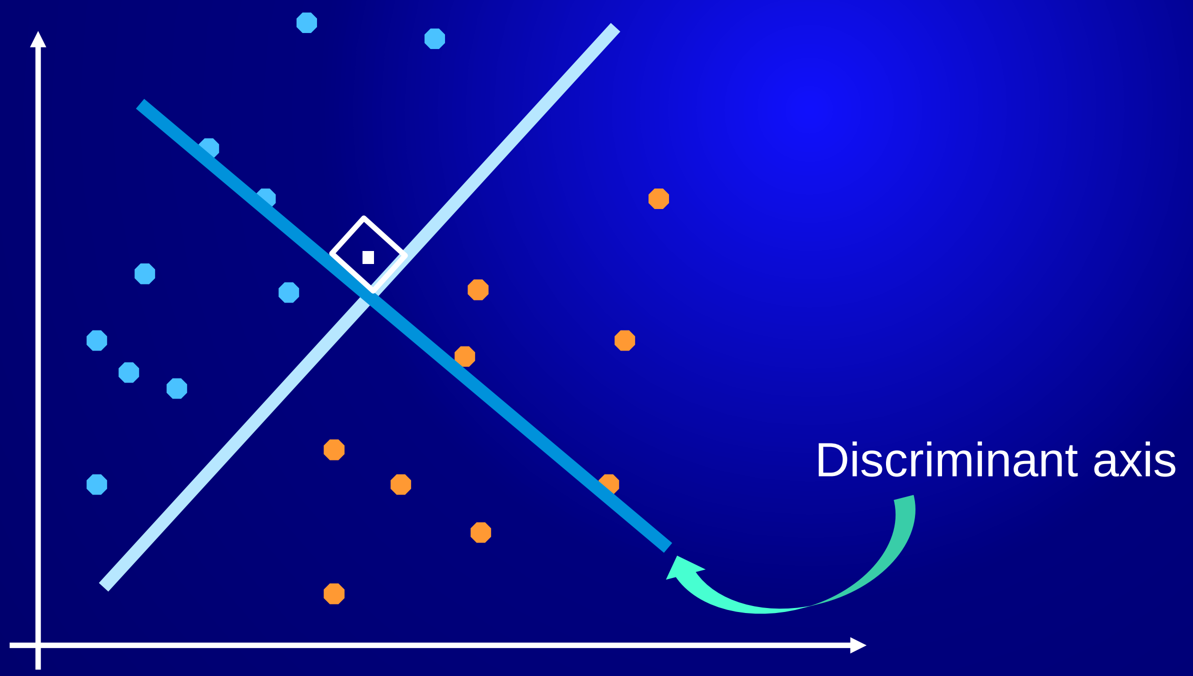


LDA

- Factorial spaces
  - $\langle X_{nxp}, \mathbb{I}_p, P_{nxn} \rangle$
  - $\langle G_{mxp}, \mathbb{I}_p, H_{mxm} \rangle$
  - $\langle G_{mxp}, D_{pxp}, H_{mxm} \rangle$
  - *Projected variabilities on direction* $\vec{w}$
  - *Total variability projected* $\vec{w}^t T \vec{w}$
  - Between Classes variability projected $\vec{w}^t E \vec{w}$
  - Within Classes variability projected $\vec{w}^t D \vec{w}$

Linear Discriminant Analysis

- Find the axis that better discriminates
- Find the axis that maximizes $\vec{w}^t E \vec{w}$



LDA

- <G,D^{-1}, H>
- Search direction $\vec{w}$ such that $\vec{w}^t E \vec{w}$ is maximized
- Diagonalization of

$$\vec{u}^t D^{-1} G^t \; H \, G \, D^{-1} \vec{u}$$

$$\text{subject to } \; \vec{u}^t D \, \vec{u} = 1$$

- The first eigen value points to an eigen vector that gives solution