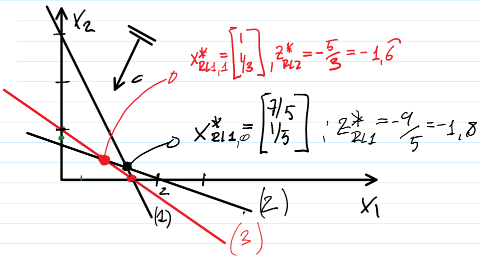
$$(PE)\begin{cases}\min & -x_1 \quad -2x_2 \\ \text{s.a.:} & 2x_1 \quad +x_2 \quad \leq 3 \ (1) \\ & x_1 \quad +3x_2 \quad \leq 2 \ (2) \\ & x_1, \quad x_2 \quad \geq 0 \ , enteres\end{cases}$$



$\boxed{\text{B\&C amb 1 tall}}$

$\boxed{\text{It 1.}}$ $L = \{(PE1)\}$, $z^*_{PE1} = -\infty$, $z^* = +\infty$

- Selecció: (PE1)
- Relaxació reforçada:

▶ $(RL1,\emptyset)$: $x^*_{RL1,\emptyset} = [7/5, 1/5]'$, $z^*_{RL1,\emptyset} = -9/5 = -1.8$; Tall Gomory: $\boxed{2x_1 + 3x_2 \leq 3 \ (3)}$

▶ $(RL1,1)$: $(PE1,1)$
$$\begin{cases}\min & -x_1 - 2x_2 \\ \text{s.a.:} & 2x_1 + x_2 \leq 3 \ (1) \\ & x_1 + 3x_2 \leq 2 \ (2) \\ & 2x_1 + 3x_2 \leq 3 \ (3) \\ & x \geq \emptyset, \ x \in \mathbb{Z}^2\end{cases}$$

$\boxed{x^*_{RL1,\emptyset}:}$ $\mathcal{B} = \{1,2\}$, $B = \begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$, $B^{-1} = \begin{bmatrix}3/5 & -1/5\\-1/5 & 2/5\end{bmatrix}$ ✓

▪ Reoptimització amb l'ASD

1.– $\tilde{B}^{-1} = \begin{bmatrix}B^{-1} & 0\\ -a'_{B,m+1}B^{-1} & 1\end{bmatrix}$

$$a'_{m+1} = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ [2, 3, \emptyset, \emptyset] \end{array}$$
$a'_{B,m+1} = [2, 3]$

$$\tilde{B}^{-1} = \left[\begin{array}{cc|c} 3/5 & -1/5 & \emptyset \\ -1/5 & 2/5 & \emptyset \\ \hline -3/5 & -4/5 & 1\end{array}\right]$$

$$-a'_{B,m+1} \cdot B^{-1} = -[2, 3]\begin{bmatrix}3/5 & -1/5\\-1/5 & 2/5\end{bmatrix} = -[3/5, 4/5]$$

$\tilde{\mathcal{B}} = \{1,2,5\}$, $x_5 = 3 - 2\cdot\frac{7}{5} - 3\cdot\frac{1}{5} = -2/5$; $\tilde{x}_B = [7/5, 1/5, -2/5]' \not\geq \emptyset$

$\tilde{r}' = r' = c_N - c_B \cdot B^{-1} \cdot A_N = [\emptyset] - [-1, -2]\cdot\begin{bmatrix}3/5 & -1/5\\-1/5 & 2/5\end{bmatrix}\cdot I = [1/5, 3/5] \geq \emptyset$

2.– $\boxed{p = 3}$; $B(3) = 5$

3.– $dr_N = (\beta_3 \cdot A_N)' = [-\frac{3}{5} \ -\frac{4}{5} \ 1] \cdot \begin{array}{c}\begin{matrix} x_3 & x_4\end{matrix}\\ \begin{bmatrix}1 & 0\\0 & 1\\0 & 0\end{bmatrix}\end{array} = [-3/5 \ -4/5] \not\geq \emptyset$

4.– $\theta^*_D = \min\left\{\frac{-1/5}{-3/5}, \frac{-3/5}{-4/5}\right\} = \min\{1/3, 3/4\} = 1/3$; $\boxed{q = 3}$

5.– $r' := r' + \theta^*_D \cdot dr'_N = [1/5, 3/5] + \frac{1}{3}[-\frac{3}{5}, -\frac{4}{5}] = [\frac{1}{5} - \frac{1}{5} = \emptyset, \frac{3}{5} - \frac{4}{15} = \frac{5}{15}] = [\emptyset, 1/3]$

$d_B = -B^{-1} \cdot A_3 = \begin{bmatrix}-3/5\\1/5\\3/5\end{bmatrix}$, $\theta^* = -\frac{-2/5}{3/5} = \frac{2}{3}$,

$$x_B := \begin{bmatrix}7/5\\1/5\\-2/5\end{bmatrix} + \frac{2}{3}\cdot\begin{bmatrix}-3/5\\1/5\\3/5\end{bmatrix} = \begin{bmatrix}\frac{7}{5} - \frac{2}{5} = 1 \\ 1/5 + 2/15 = \frac{5}{15} = 1/3 \\ -2/5 + \frac{2}{5} = \emptyset\end{bmatrix}, \quad x_q = x_3 = \theta^* = 2/3$$

Coincideix

$B = \{1,2,3\}$; $B = \begin{bmatrix}2 & 1 & 1\\1 & 3 & 0\\2 & 3 & 0\end{bmatrix}$; $\det(B) = 3 - (6) = -3$

$B^T = \begin{bmatrix}2 & 1 & 2\\1 & 3 & 3\\1 & \emptyset & \emptyset\end{bmatrix}$; $B^{-1} = \begin{bmatrix}\emptyset & -1 & 1\\ \emptyset & 2/3 & -1/3\\ 1 & 4/3 & -5/3\end{bmatrix}$; $x_B = \begin{bmatrix}\emptyset & -1 & 1\\ \emptyset & 2/3 & -1/3\\ 1 & 4/3 & -5/3\end{bmatrix}\begin{bmatrix}3\\2\\3\end{bmatrix} = \begin{bmatrix}1\\1/3\\2/3\end{bmatrix} \geq \emptyset$

2.– $x_B \geq \emptyset \rightarrow$ òptim.

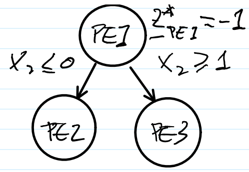
▪ $x^*_{RL1,1} = [1, 1/3]'$; $z^*_{RL1,1} = -5/3 = -1.\hat{6}$; $\underline{z}^*_{PE1} = \lceil z^*_{RL1,1} \rceil = -1$

- Eliminació : no es pot
- Separació : $[x^* \ ] = 1/\ldots \rightarrow \{(PE2): (PE1) \wedge x_2 \leq \emptyset$

(PE1) $\underline{z}^*_{PE1} = -1$

$x_2 \leq 0$ $\quad$ $x_2 \geq 1$

- Eliminació : no es pot
- Separació : $[x^*_{RL1,1}]_2 = 1/3 \to \begin{cases} (PE2): (PE1) \wedge x_2 \le 0 \\ (PE3): (PE1) \wedge x_2 \ge 0 \end{cases}$



$\boxed{IV.2}$ $L = \{(PE2),(PE3)\}$, $\underline{z}^*_{PE1} = -1$, $z^* = +\infty$

- Selecció : (PE2)
- Relaxació reforçada:

B(RL2,0) :

▪ Reoptimització a partir de $x^*_{RL1,1}$:

1.- $\mathcal{B} = \{1,2,3\}$; $B^{-1} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 2/3 & -1/3 \\ 1 & 4/3 & -5/3 \end{bmatrix}$; $a'_{m+1} x \le b_{m+1}$: $x_2 \le 0$ (4)

$x_2 + x_6 = 0$; $-a_{B,4} B^{-1} = -[0,1,0] \cdot \begin{bmatrix} 0 & -1 & 1 \\ 0 & 2/3 & -1/3 \\ 1 & 4/3 & -5/3 \end{bmatrix} = [0, -2/3, 1/3]$

$\tilde{B}^{-1} = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 0 & 2/3 & -1/3 & 0 \\ 1 & 4/3 & -5/3 & 0 \\ 0 & -2/3 & 1/3 & 1 \end{bmatrix}$, $\tilde{x}_B = \begin{bmatrix} 1 \\ 1/3 \\ 2/3 \\ x_6 = -x_2 = -1/3 \end{bmatrix}$; $\tilde{r}' = r' = [0, 1/3]$

2.- $x_{B(4)} = x_6 = -1/3 \to p = 4$, $B(p) = 6$

3.- $d'_{r_N} = \beta_4 \cdot A_N = [0, -2/3, 1/3, 1] \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{array}{cc} x_4 & x_5 \\ [-2/3, & 1/3] \end{array} \not\ge 0$

4.- $\theta^*_D = \min\left\{-\frac{0}{-2/3}\right\} = 0$, $q = 4$

5.- $r'_N := r'_N + \theta^*_D \cdot d'_{r_N} = [0, 1/3] + 0 \cdot d_{r_N} = [0, 1/3]$

$d_B = -\tilde{B}^{-1} A_4 = \begin{bmatrix} 1 \\ -2/3 \\ -4/3 \\ 2/3 \end{bmatrix}$, $\theta^* = -\frac{x_{B(q)}}{d_{B(q)}} = -\frac{-1/3}{2/3} = 1/2 \to \boxed{x_q = x_4 = 1/2}$

$x_B := \begin{bmatrix} 1 \\ 1/3 \\ 2/3 \\ -1/3 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ -2/3 \\ -4/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\mathcal{B} \leftarrow \{1,2,3,4\}$, $B = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \\ 2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \to B^{-1} = \begin{bmatrix} 0 & 0 & 1/2 & -3/2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & -1/2 & -3/2 \end{bmatrix}$

$\left\{ \begin{array}{l} 2x_1 + x_2 + x_3 = 3 \\ x_1 + 3x_2 + x_4 = 2 \\ 2x_1 + 3x_2 = 3 \\ \boxed{x_2 = 0} \end{array} \right. \quad \begin{array}{l} 2x_1 + x_3 = 3 \to x_3 = 3 - 3 = 0 \\ x_1 + x_4 = 2 \to x_4 = 2 - \frac{3}{2} = 1/2 \\ 2x_1 = 3 \to x_1 = 3/2 \end{array}$

2.- $x_B := \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix} \ge 0 \to x^*_{RL2,0}$

▪ Tall de Gomory sobre $x^*_{RL2,0}$: $\boxed{x_1 \le 1}$ (5)

▶ (RL2,1) ; (PE2.1) $\left\{\begin{array}{lllll} \min & -x_1 - 2x_2 & & & \\ \text{s.a.:} & 2x_1 + x_2 + x_3 & = 3 & (1) & \\ & x_1 + 3x_2 + x_4 & = 2 & (2) & \\ & 2x_1 + 3x_2 + x_5 & = 3 & (3) & \leftarrow \text{Gomory} \\ & x_2 + x_6 & = 0 & (4) & \leftarrow \text{Ramificació} \\ & x_1 + x_7 & = 1 & (5) & \leftarrow \text{Gomory} \\ & x \geq 0,\ x \in \mathbb{Z}^2 & & & \end{array}\right.$

▪ Reoptimit. ASD a partir $x^*_{RL2,0}$

1.- $\mathcal{B} = \{1,2,3,4\}$, $B = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \\ 2 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$; $B^{-1} = \begin{bmatrix} 0 & 0 & 1/2 & -3/2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 2 \\ 0 & 1 & -1/2 & -3/2 \end{bmatrix}$

$\tilde{\mathcal{B}} \leftarrow \{1,2,3,4,7\}$

```
B =
 2 1 1 0
 1 3 0 1
 2 3 0 0
 0 1 0 0

--> B^-1

ans =
        0   -0.0000    0.5000   -1.5000
        0         0         0    1.0000
   1.0000         0   -1.0000    2.0000
        0    1.0000   -0.5000   -1.5000
```

$a_5 = [1,0,0,0,0,0]$, $b_5 = 1$

$a_{B,5} = [1,0,0,0]$

```
--> -aB5*B^-1

ans =
        0    0.0000   -0.5000    1.5000
```

```
B2 =
 2 1 1 0 0
 1 3 0 1 0
 2 3 0 0 0
 0 1 0 0 0
 1 0 0 0 1

--> B2^-1

ans =
        0   -0.0000    0.5000   -1.5000         0
        0         0         0    1.0000         0
   1.0000         0   -1.0000    2.0000         0
        0    1.0000   -0.5000   -1.5000         0
  -0.0000    0.0000   -0.5000    1.5000    1.0000
```

$\tilde{B}^{-1} = \begin{bmatrix} 0 & 0 & 1/2 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 2 & 0 \\ 0 & 1 & -1/2 & -3/2 & 0 \\ 0 & 0 & -1/2 & 3/2 & 1 \end{bmatrix}$

$\tilde{x}_B = \begin{bmatrix} x_B \\ \hline x_7 = 1 - x_1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} \ngeq 0$, $\tilde{r}' = r' = [0, 8/5] \geq 0$

2.- $x_B \ngeq 0$: $p = 5$, $B(p) = 7$

3.- $d_{r_N}' = \beta_5 \cdot A_N = [0, 0, -1/2, 3/2, 1] \cdot \begin{array}{c} \begin{array}{cc} x_5 & x_6 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \end{array} = [-1/2, 3/2] \ngeq 0$

4.- $\theta^*_D = -\dfrac{0}{-1/2} = 0 \rightarrow \boxed{q = 5}$

5.- $r_N' := r_N' + 0 \cdot d_{r_N}' = [0, 8/5]$

$d_B = -B^{-1} A_5 = -B^{-1} e_3 = \begin{bmatrix} -1/2 \\ 0 \\ +1 \\ +1/2 \\ +1/2 \end{bmatrix}$; $\theta^* = -\dfrac{x_{B(5)}}{d_{B(5)}} = -\dfrac{-1/2}{1/2} = 1$

$x_B := \begin{bmatrix} 3/2 \\ 0 \\ 0 \\ 1/2 \\ -1/2 \end{bmatrix} + 1 \cdot \begin{bmatrix} -1/2 \\ 0 \\ +1 \\ +1/2 \\ +1/2 \end{bmatrix} = \begin{bmatrix} 3/2 - 1/2 = 1 \\ 0 + 0 = 0 \\ 0 + 1 = 1 \\ 1/2 + 1/2 = 1 \\ -1/2 + 1/2 = 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \geq 0$, $x_5 = \theta^* = 1$

1.- $x_B \geq 0 \rightarrow$ òptim

▶ $x^*_{RL2,1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $z^*_{RL2,1} = 2 \rightarrow z^*_{PE2} = -1$

- Eliminació: $x^*_{RLZ,1} \equiv x^*_{PE2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; z^*_{PE2} = -1$
  - $x^* := [1, 0]'$; $z^* := -1$; $\mathcal{L} \leftarrow \mathcal{L} \setminus \{(PE2)\} = \{(PE3)\}$
  - $z^* = z^*_{PE1} \therefore \mathcal{L} \leftarrow \mathcal{L} \setminus \{(PE3)\} = \emptyset$

$\boxed{\text{It. 3}}$ $\mathcal{L} = \emptyset \therefore \boxed{x^*_{PE1} = [1, 0], \; z^*_{PE1} = -1}$