Lliste de problemes

4.1 Expressen la funció polinòmica $f(x,y) = x^3 + y^3 + xy^2$ en potències de $(x-1)$ i $(y-2)$

4.2 Desenvolopen $g(x,y) = \log(x+y)$ al voltant de $(1,1)$ fins a ordre 3.

4.3 Escriviu la fórmula de Taylor fins a ordre 2 per a $f(x,y,z) = e^{a(x+y+z)}$ al voltant de $(0,0,0)$

4.4 Escriviu la fórmula de Taylor fins a ordre 2 per a

$$f(x,y) = \sin(xy) + \cos(xy) \qquad \text{al voltant de } (0,0)$$

4.5 Desenvolopen $f(x,y) = x^y$ en un entorn de $(1,1)$ fins a ordre 3 i calculen aproximadament $(1.1)^{1.02}$.

4.1 $f(x,y) = x^3 + y^3 + x\,y^2$ en potències de $(x-1)$ i $(y-2)$

$$\frac{\partial f}{\partial x} = 3x^2 + y^2, \qquad \frac{\partial f}{\partial y} = 3y^2 + 2xy$$

$$\frac{\partial^2 f}{\partial x^2} = 6x, \qquad \frac{\partial^2 f}{\partial y \partial x} = 2y, \qquad \frac{\partial^2 f}{\partial y^2} = 6y + 2x$$

$$\frac{\partial^3 f}{\partial x^3} = 6, \qquad \frac{\partial^3 f}{\partial y \partial x^2} = 0, \qquad \frac{\partial^3 f}{\partial y^2 \partial x} = 2, \qquad \frac{\partial^3 f}{\partial y^3} = 6$$

$$f(x,y) = f(1,2) + \frac{\partial f}{\partial x}(1,2)\,(x-1) + \frac{\partial f}{\partial y}(1,2)\,(y-2)$$

$$+ \frac{1}{2!}\left[ \frac{\partial^2 f}{\partial x^2}(1,2)\,(x-1)^2 + 2\frac{\partial^2 f}{\partial x \partial y}(1,2)\,(x-1)(y-2) + \frac{\partial^2 f}{\partial y^2}(1,2)\,(y-2)^2 \right]$$

$$+ \frac{1}{3!}\left[ \frac{\partial^3 f}{\partial x^3}(1,2)\,(x-1)^3 + 3\frac{\partial^3 f}{\partial y \partial x^2}(1,2)\,(x-1)^2(y-2) + 3\frac{\partial^3 f}{\partial y^2 \partial x}(1,2)\,(x-1)(y-2)^2 + \frac{\partial^3 f}{\partial y^3}(1,2)\,(y-2)^3 \right]$$

$$= 13 + 7(x-1) + 16(y-2) + 3(x-1)^2 + 4(x-1)(y-2) + 7(y-2)^2$$

$$+ (x-1)^3 + (x-1)(y-2)^2 + (y-2)^3$$

4.2 $\quad g(x,y) = \log(x+y) \qquad (x_0, y_0) = (1,1)$

$$\frac{\partial g}{\partial x} = \frac{1}{x+y}, \qquad \frac{\partial g}{\partial y} = \frac{1}{x+y}$$

$$\frac{\partial^2 g}{\partial x^2} = \frac{-1}{(x+y)^2}, \qquad \frac{\partial^2 g}{\partial x \partial y} = \frac{-1}{(x+y)^2}, \qquad \frac{\partial^2 g}{\partial y^2} = \frac{-1}{(x+y)^2}$$

$$\frac{\partial^3 g}{\partial x^3} = \frac{2}{(x+y)^3}, \qquad \frac{\partial^3 g}{\partial y \partial x^2} = \frac{2}{(x+y)^3}, \qquad \frac{\partial^3 g}{\partial y^2 \partial x} = \frac{2}{(x+y)^3}, \qquad \frac{\partial^3 g}{\partial y^3} = \frac{2}{(x+y)^3}$$

$$g(x,y) = g(1,1) + \frac{\partial g}{\partial x}(1,1)\,(x-1) + \frac{\partial g}{\partial y}(1,1)\,(y-1)$$
$$+ \frac{1}{2!}\left[\frac{\partial^2 g}{\partial x^2}(1,1)\,(x-1)^2 + 2\,\frac{\partial^2 g}{\partial y \partial x}(1,1)\,(x-1)(y-1) + \frac{\partial^2 g}{\partial y^2}(1,1)\,(y-1)^2\right]$$
$$+ \frac{1}{3!}\left[\frac{\partial^3 g}{\partial x^3}(1,1)\,(x-1)^3 + 3\,\frac{\partial^3 g}{\partial y \partial x^2}(1,1)\,(x-1)^2(y-1) + 3\,\frac{\partial^3 g}{\partial y^2 \partial x}(1,1)\,(x-1)(y-1)^2 + \frac{\partial^3 g}{\partial y^3}(1,1)\,(y-1)^3\right]$$

$(+R_3$

$$= \log 2 + \frac{1}{2}(x-1+y-1) + \frac{1}{2}\left(\frac{-1}{4}\right)\left[(x-1)^2 + 2(x-1)(y-1) + (y-1)^2\right] + \frac{1}{6}\,\frac{2}{8}\left[(x-1)^3 + 3(x-1)^2(y-1) + 3(x-1)(y-1)^2 + (y-1)^3\right]$$

$$+ R_3$$

4.3 $f(x,y,z) = e^{a(x+y+z)}$ , $(x_0,y_0,z_0) = (0,0,0)$

$$\frac{\partial f}{\partial x} = e^{a(x+y+z)}\, a \,, \qquad \frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = e^{a(x+y+z)} \cdot a$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial z \partial x} = \frac{\partial^2 f}{\partial z \partial y} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial z^2} = e^{a(x+y+z)}\, a^2$$

$$f(x,y,z) = f(0,0,0) + \frac{\partial f}{\partial x}(0,0,0)\, x + \frac{\partial f}{\partial y}(0,0,0)\, y + \frac{\partial f}{\partial z}(0,0,0)\, z$$

$$+ \frac{1}{2!}\Big[ \frac{\partial^2 f}{\partial x^2}(0,0,0)\, x^2 + 2\frac{\partial^2 f}{\partial y \partial x}(0,0,0)\, xy + 2\frac{\partial^2 f}{\partial z \partial x}(0,0,0)\, xz$$

$$+ \frac{\partial^2 f}{\partial y^2}(0,0,0)\, y^2 + 2\frac{\partial^2 f}{\partial z \partial y}(0,0,0)\, yz + \frac{\partial^2 f}{\partial z^2}(0,0,0)\, z^2 \Big] + R_3$$

$$= 1 + ax + ay + az + \frac{1}{2}\left[ a^2x^2 + 2a^2xy + 2a^2xz + a^2y^2 + 2a^2yz + a^2z^2 \right] + R_2$$

$$= 1 + a(x+y+z) + \frac{a^2}{2}\left[ x^2 + y^2 + z^2 + 2xy + 2xz + 2yz \right] + R_2$$

4.4

$$f(x,y) = \sin(xy) + \cos(xy), \qquad (x_0,y_0) = (0,0)$$

$$\frac{\partial f}{\partial x} = \cos(xy)\, y - \sin(xy)\, y, \qquad \frac{\partial f}{\partial y} = \cos(xy)\, x - \sin(xy)\, x$$

$$\frac{\partial^2 f}{\partial x^2} = -\sin(xy)\, y^2 - \cos(xy)\, y^2, \qquad \frac{\partial f}{\partial y \partial x} = -\sin(xy)\, xy + \cos(xy)$$

$$- \cos(xy)\, xy - \sin(xy)$$

$$\frac{\partial^2 f}{\partial y^2} = -\sin(xy)\, x^2 - \cos(xy)\, x^2$$

$$f(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)\, x + \frac{\partial f}{\partial y}(0,0)\, y + \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x^2}(0,0)\, x^2 + 2\frac{\partial^2 f}{\partial x \partial y}(0,0)\, xy + \frac{\partial^2 f}{\partial y^2}(0,0)\, y^2\right]$$

$$+ R_2$$

$$= 1 + \frac{1}{2!}\left[2 \cdot 1 \cdot xy\right] + R_2 = 1 + xy + R_2$$

4.5 $\quad f(x,y) = x^y \qquad (1.1)^{1.02} \simeq 1.1020988$

$$\frac{\partial f}{\partial x} = y\, x^{y-1}, \qquad \frac{\partial f}{\partial y} = x^y \log x$$

$$\frac{\partial^2 f}{\partial x^2} = y(y-1)\, x^{y-2}, \qquad \frac{\partial^2 f}{\partial y \partial x} = x^{y-1} + y\, x^{y-1} \log x, \qquad \frac{\partial^2 f}{\partial y^2} = x^y \log^2 x$$

$$\frac{\partial^3 f}{\partial x^3} = y(y-1)(y-2)\, x^{y-3}, \qquad \frac{\partial^3 f}{\partial y \partial x^2} = (2y-1)\, x^{y-2} + (y^2-y)\, x^{y-2} \log x,$$

$$\frac{\partial^3 f}{\partial y^2 \partial x} = y\, x^{y-1} \log^2 x + x^y\, 2 \log x \cdot \frac{1}{x}, \qquad \frac{\partial^3 f}{\partial y^3} = x^y \log^3 x$$

$$f(x,y) = f(1,1) + \frac{\partial f}{\partial x}(1,1)\,(x-1) + \frac{\partial f}{\partial y}(1,1)\,(y-1)$$

$$+ \frac{1}{2!}\left[ \frac{\partial^2 f}{\partial x^2}(1,1)\,(x-1)^2 + 2\, \frac{\partial^2 f}{\partial x \partial y}(1,1)\,(x-1)(y-1) + \frac{\partial^2 f}{\partial y^2}(1,1)\,(y-1)^2 \right]$$

$$+ \frac{1}{3!}\left[ \frac{\partial^3 f}{\partial x^3}(1,1)\,(x-1)^3 + 3\, \frac{\partial^3 f}{\partial y \partial x^2}(1,1)\,(x-1)^2 (y-1) + 3\, \frac{\partial^3 f}{\partial y^2 \partial x}(1,1)\,(x-1)(y-1)^2 + \frac{\partial^3 f}{\partial y^3}(1,1)\,(y-1)^3 \right] + \dots$$

$$= 1 + (x-1) + \frac{1}{2}\big(2(x-1)(y-1)\big) + \frac{1}{6}\, 3\, (x-1)^2 (y-1) + \dots$$

$$f(1.1,\, 1.02) \simeq 1 + 0.1 + 0.1 \times 0.02 + \frac{1}{2}\, 0.1^2 \times 0.02 = 1.1021$$