

Basic Descriptive Analysis

Numerical Variables

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Guió

0. Basic Principles in descriptive analysis
1. Graphical Descriptive tools for numerical variables
2. Numerical Descriptive tools for numerical variables
3. Descriptive Analysis of Temporal variables
4. Introduction to Space
5. Normality and Exponentiality Evaluation

0. Descriptive analysis

Compact and Informative view of the variable structure

$$\text{DATA} = \text{FIT} + \text{ERROR}$$

General Pattern

Deviations

Characterización

Structural Component

Random Component

Tools

1. Graphical

Visualitze variable's distribution



2. Numerical

Quantify what is observed in he graphs



1. Graphical tools

1. Performing the graph

Mechanical

(software)

2. Reading the graph

Technical

(statistician or data miner)

3. Interpretation

Conceptual

(domain expert)

Contextualization

Graphical tools for numerical variables

1. Histogram

2. Boxplot

3. Others (dotplot, stem and leaf plot....)

Histogram

Visualitzation of frequencies distribution table

Intervalo	Número de Observaciones	Observaciones Acumuladas	Frec Relativas	Frec Acumuladas
45-65	1	1	$1/17 = .06$	0.06
65-75	5	6	0.29	0.35
75-85	5	11	0.29	0.64
85-95	1	12	0.06	0.70
95-105	3	15	0.17	0.87
105-115	0	15	0	0.87
115-125	1	16	0.06	0.93
125-135	0	16	0	0.93
135-145	1	17	0.06	0.99

Frequency
Classes?

Bars' AREAS
PROPORTIONAL
to frequencies

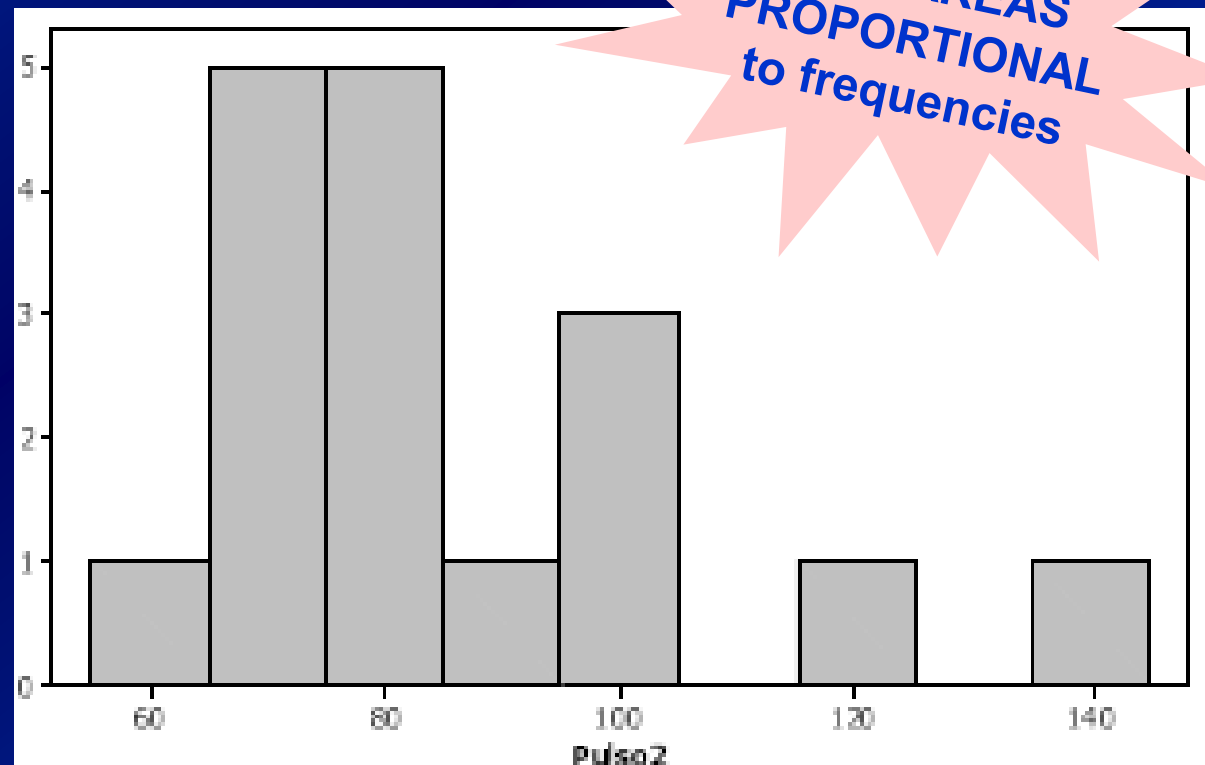
Heuristics: $\begin{cases} 6 \log_{10}(n) & , si \ n < 100 \\ 1,2\sqrt{n} & , si \ n \geq 100 \end{cases}$

$$3,49 \sqrt[3]{n}$$

$$2 \sqrt[3]{d_i n}$$

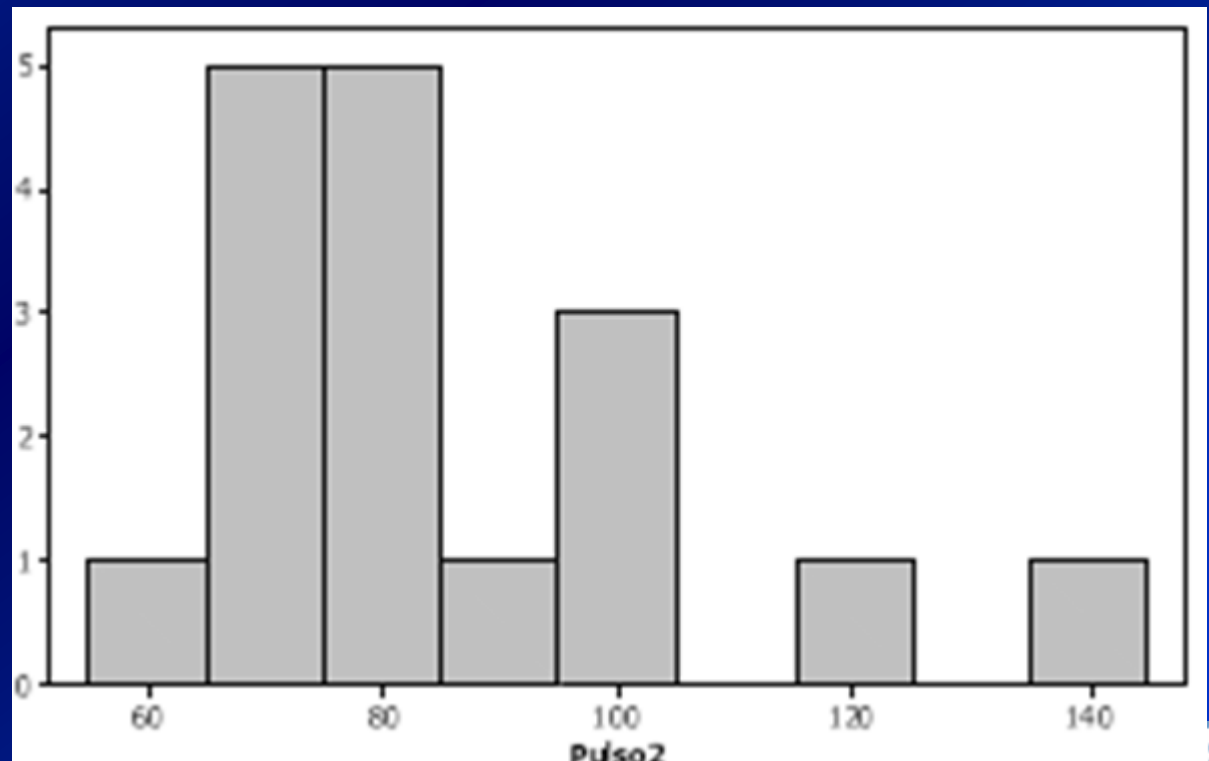
Histogram

Intervalo	Número de Observaciones
45-65	1
65-75	5
75-85	5
85-95	1
95-105	3
105-115	0
115-125	1
125-135	0
135-145	1



READING HISTOGRAMS

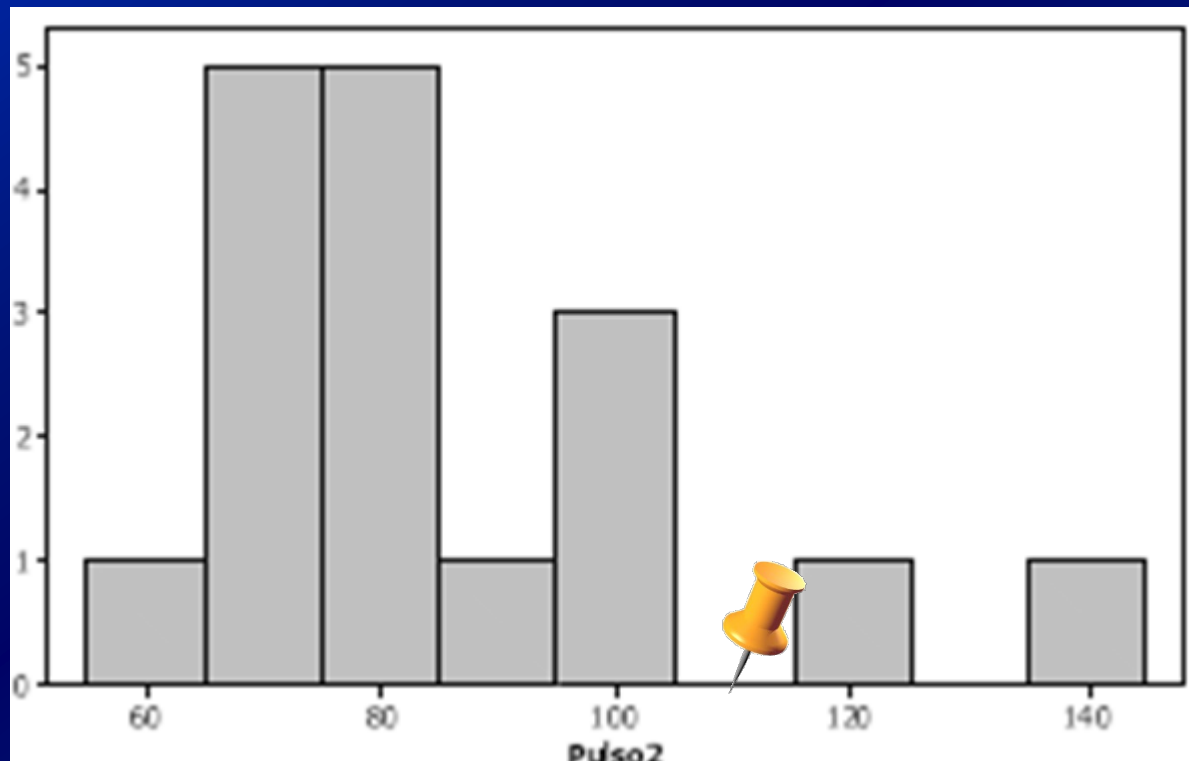
1. Range of variable (max-min)
2. Central trend
3. Dispersion
4. Symmetry
5. Anomalies



READING HISTOGRAMS

Central trend

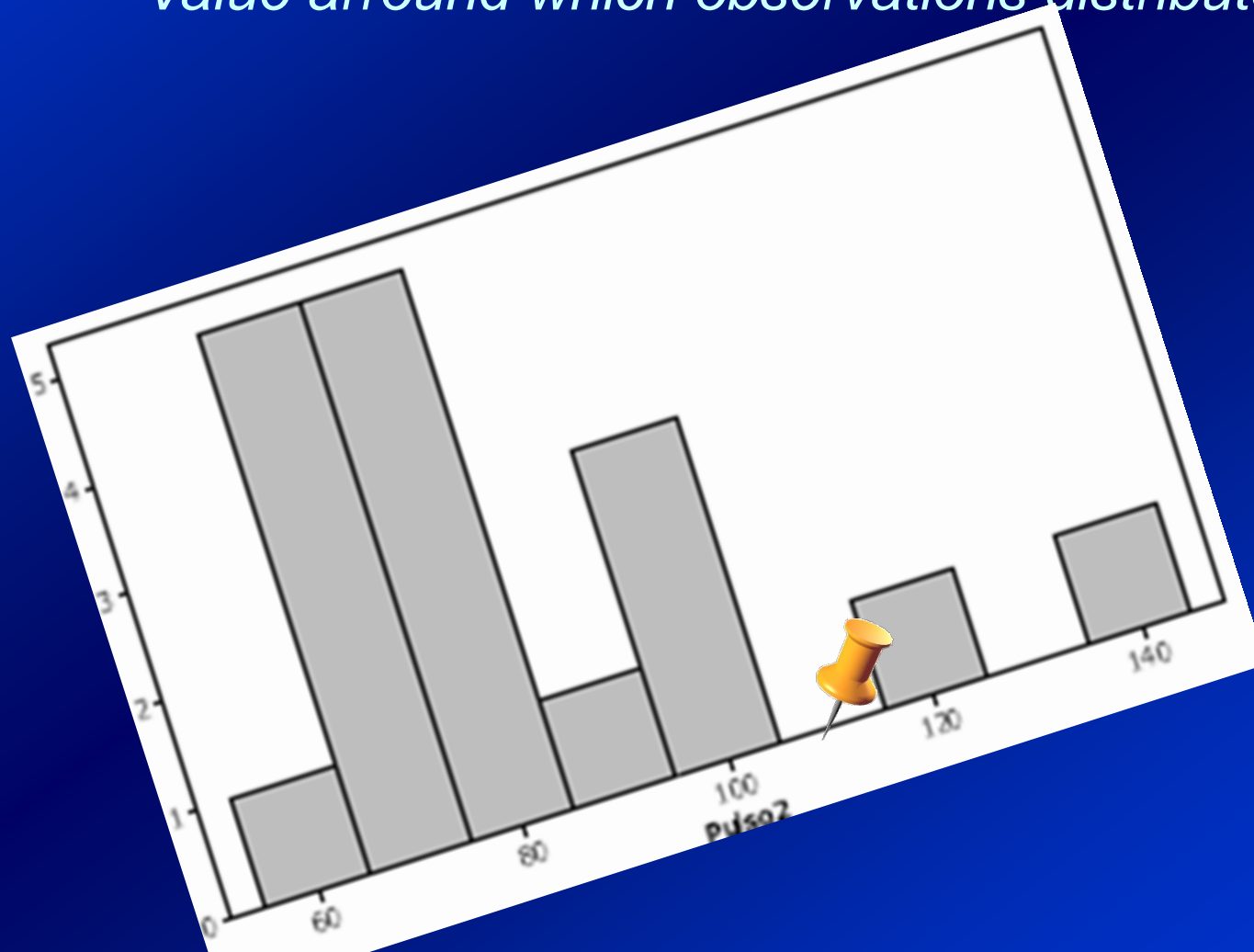
Value around which observations distribute



READING HISTOGRAMS

Central trend

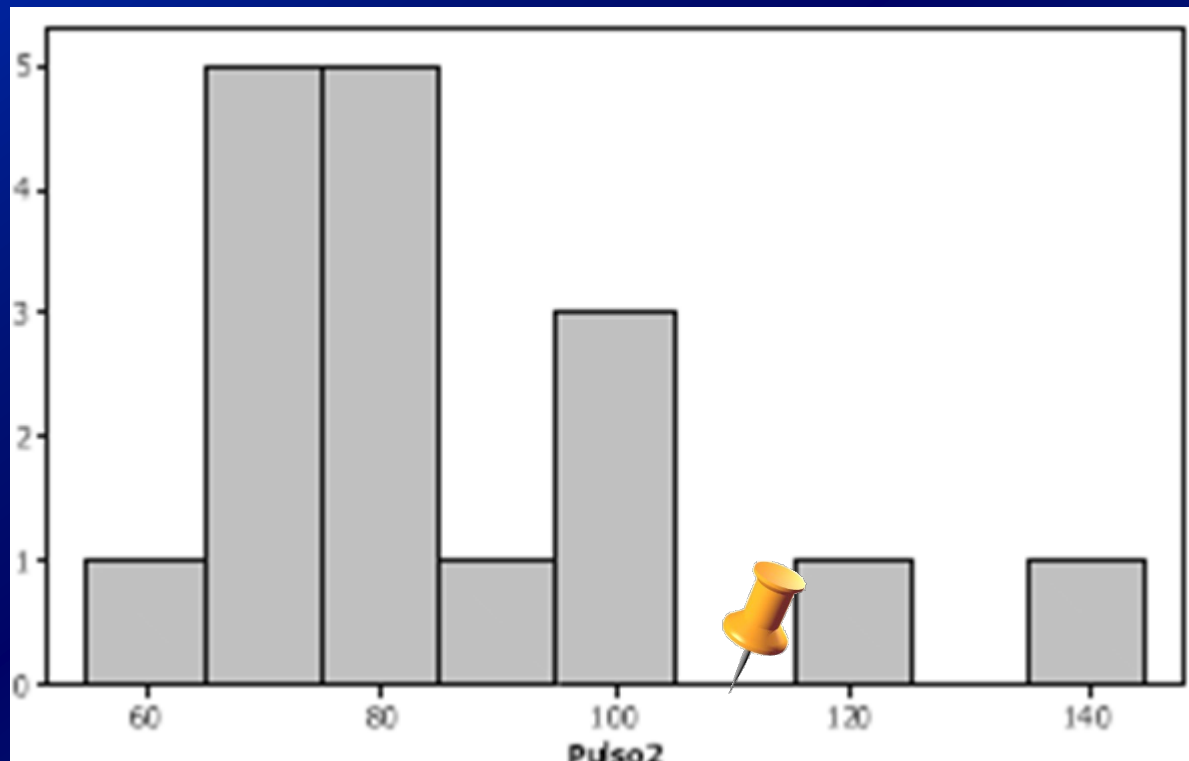
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READING HISTOGRAMS

Central trend

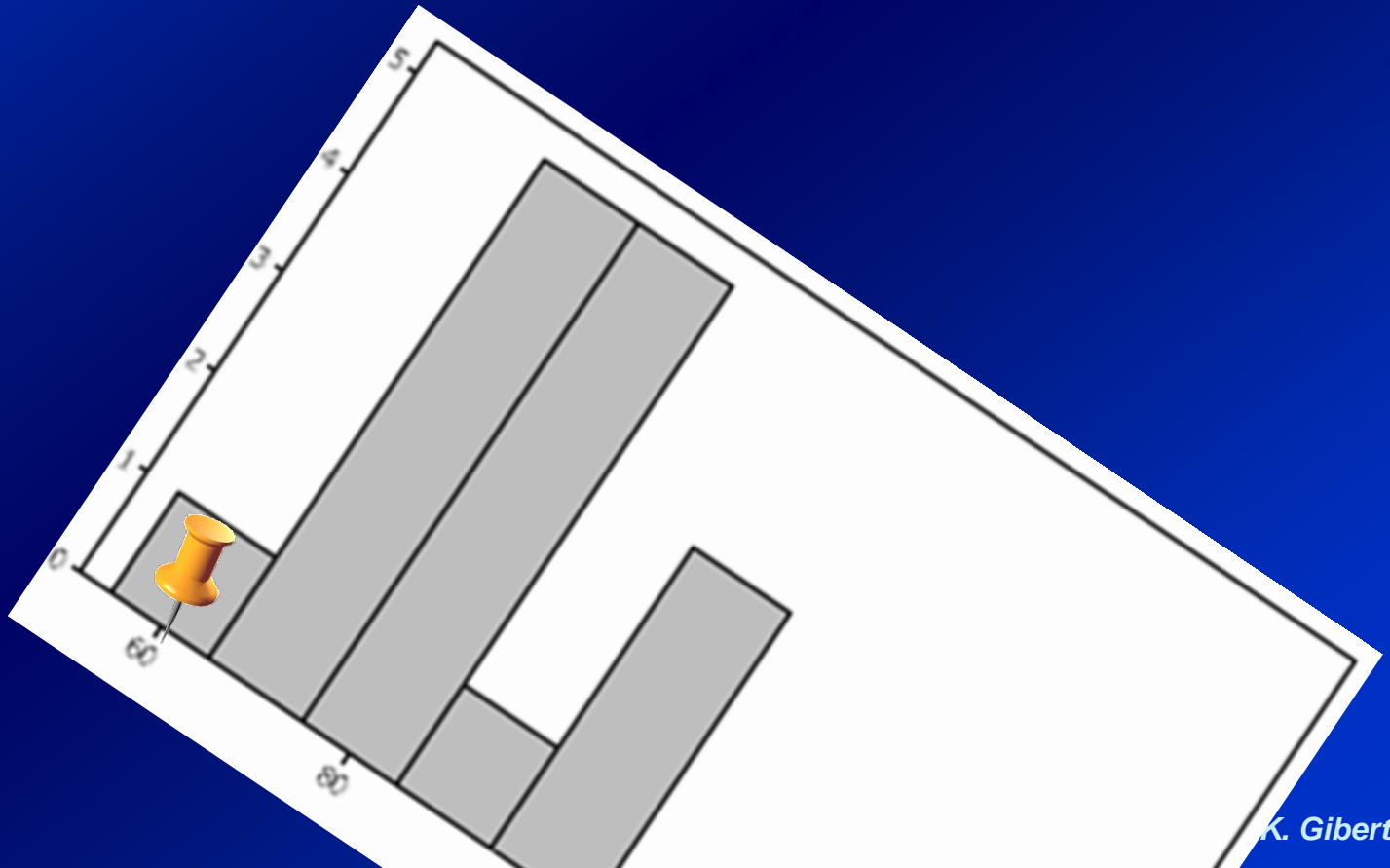
Value around which observations distribute



READING HISTOGRAMS

Central trend

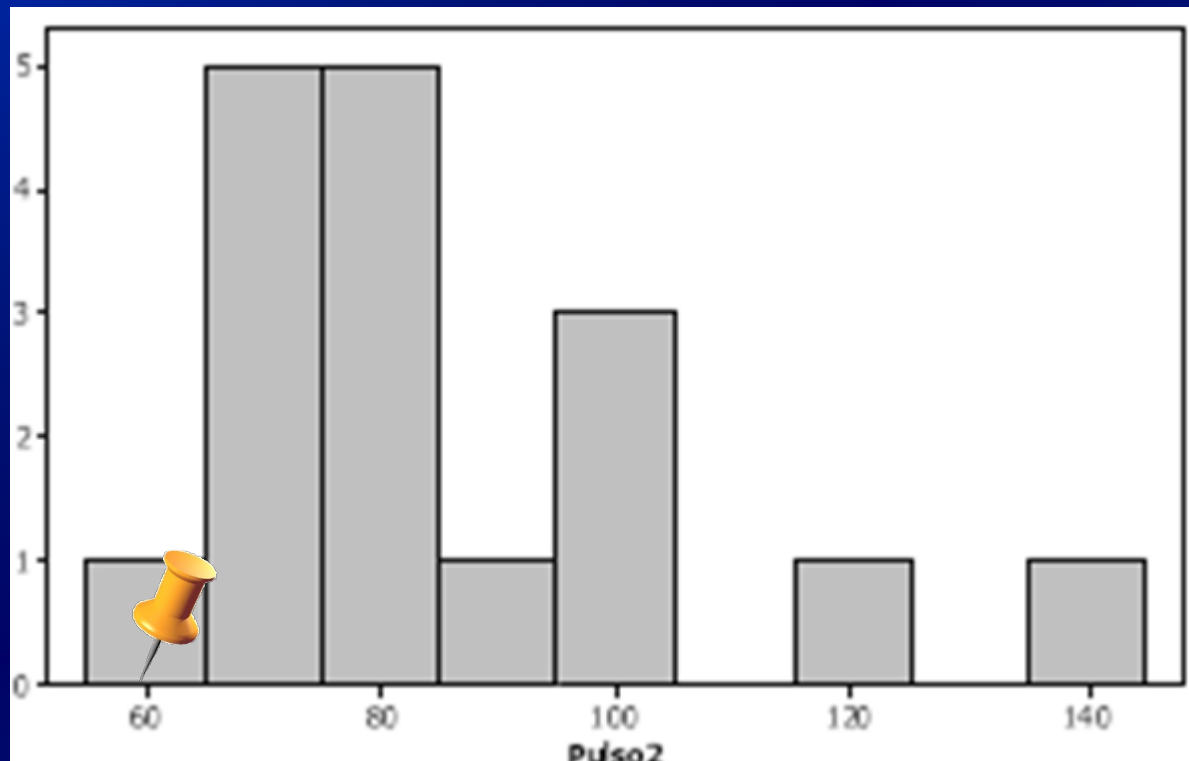
Value around which observations distribute



READING HISTOGRAMS

Central trend

Value around which observations distribute

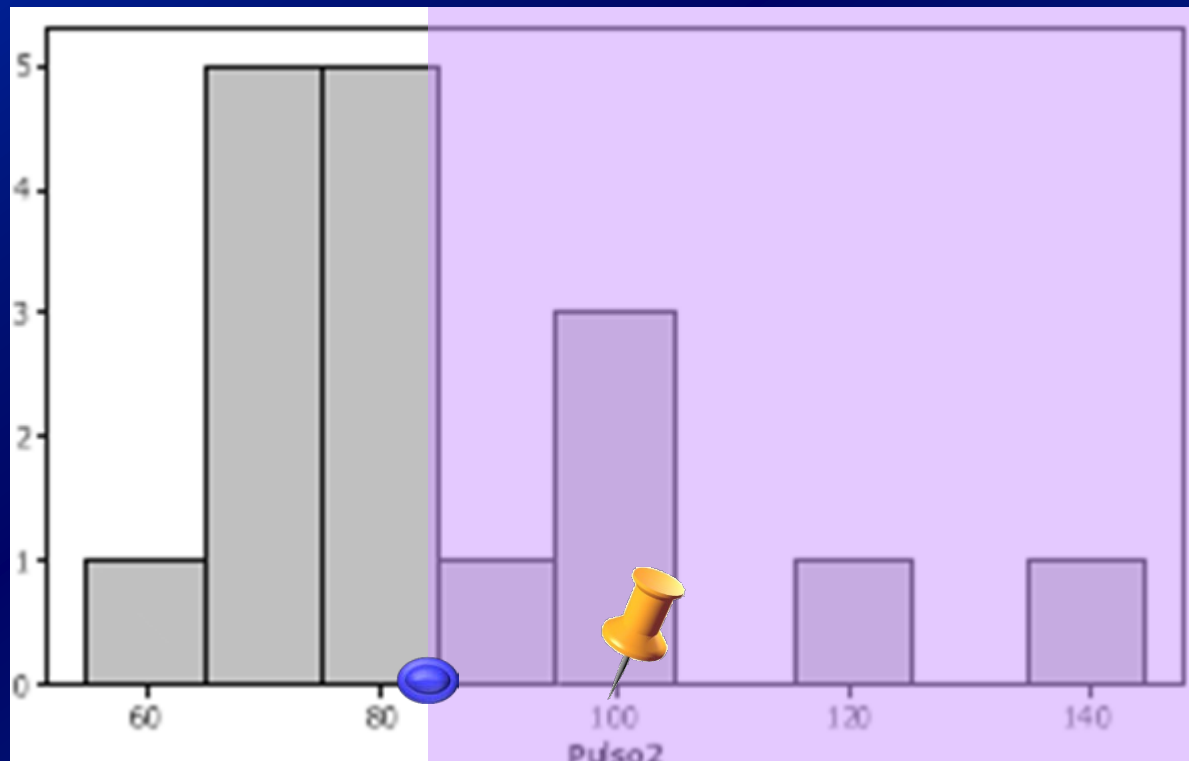


READING HISTOGRAMS

Dispersion/Variability

How observations concentrate around central trend?

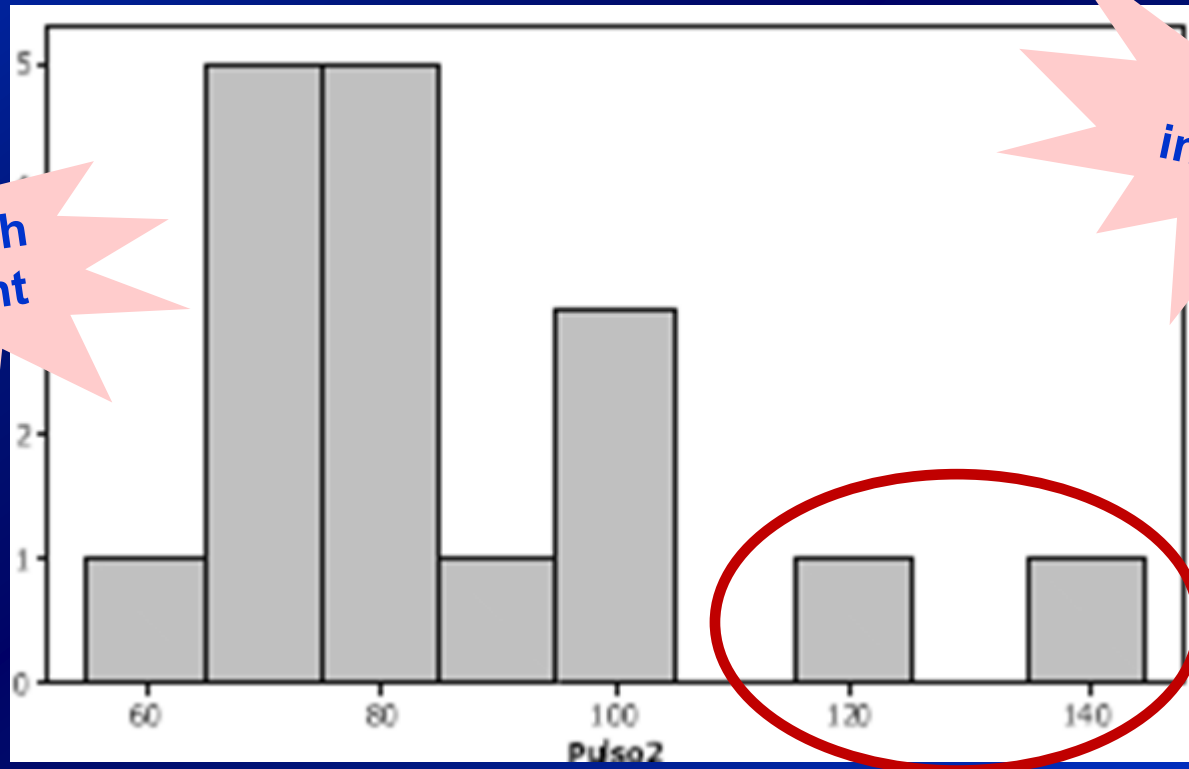
Mean distance to central trend



READING HISTOGRAMS

Anomalies

Outliers: Observations anomaly far from rest



CARE with
Treatment

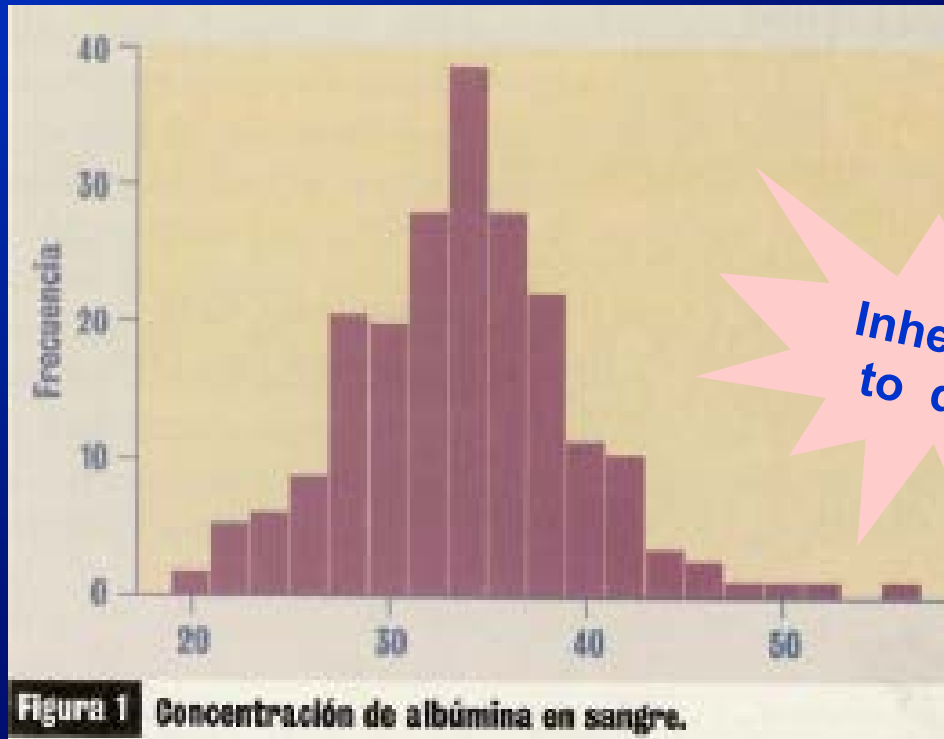
Identify
individual

READING HISTOGRAMS

Main Patterns

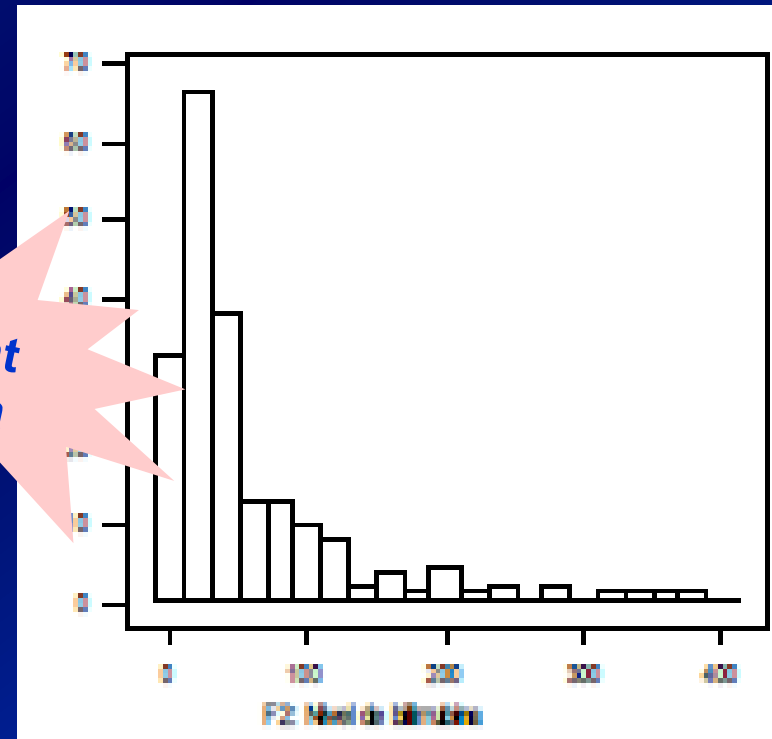
[Gibert, JANO1996]

Albumine



*Inherent
to data*

Bilirrubine



Symmetric

Asymmetric

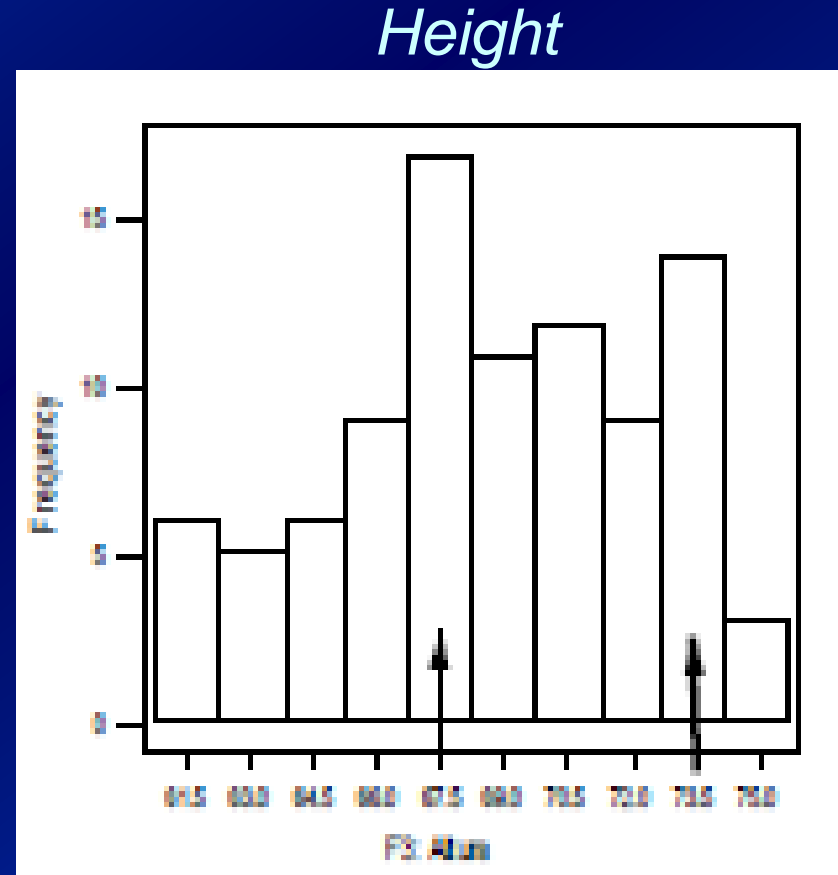
READING HISTOGRAMS

Main Patterns

Multimodality

Several central trends!!

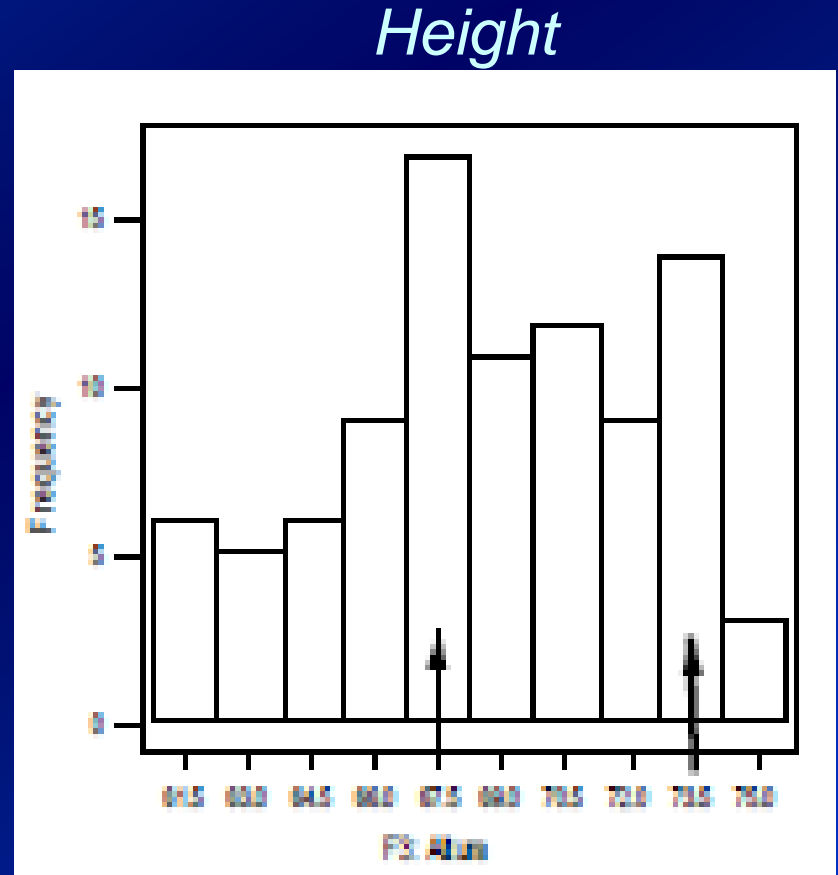
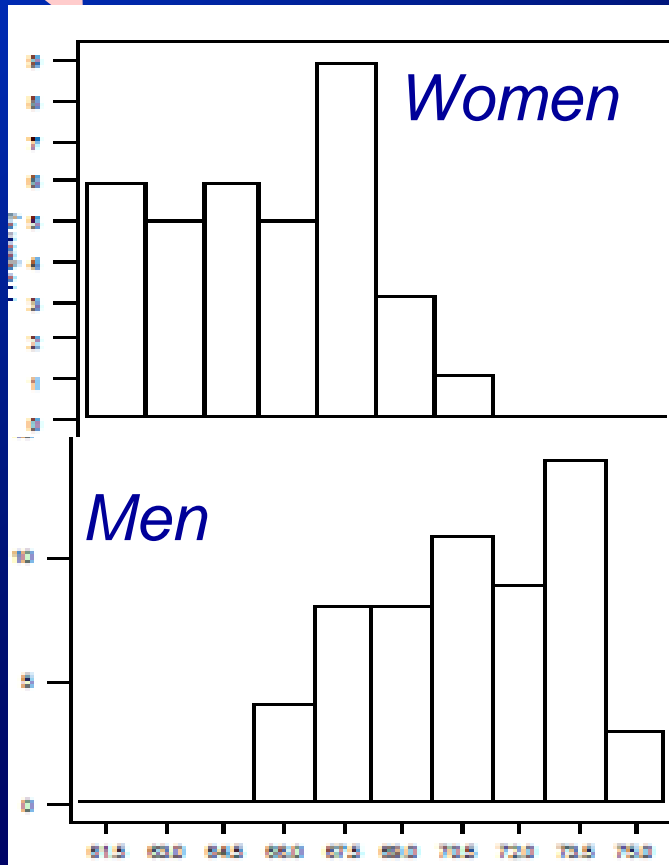
Find
discriminant
factor



READING HISTOGRAMS

Main Patterns

Find
discriminant
factor



READING HISTOGRAMS

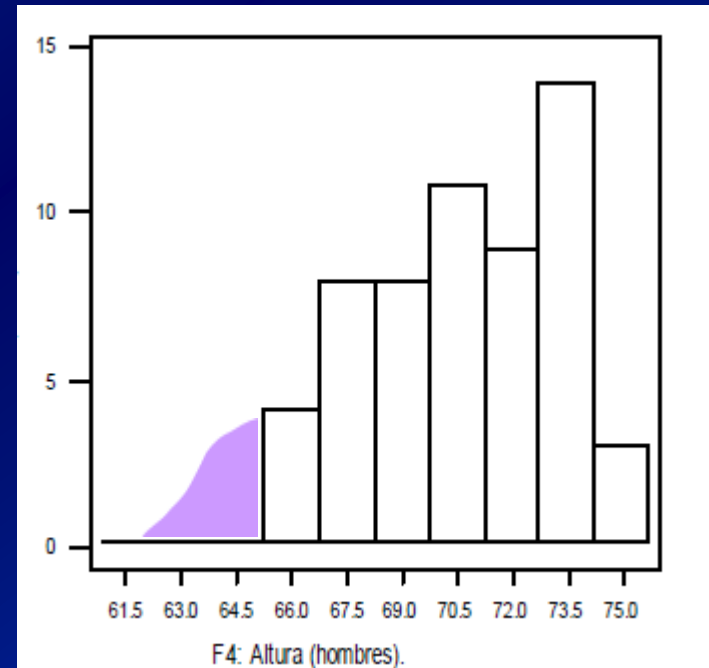
Main Patterns

Height of Men

Scarped

Part of distribution trunked!

(only adult men)



READING HISTOGRAMS

Main Patterns

Dentat

Measurement approximations!

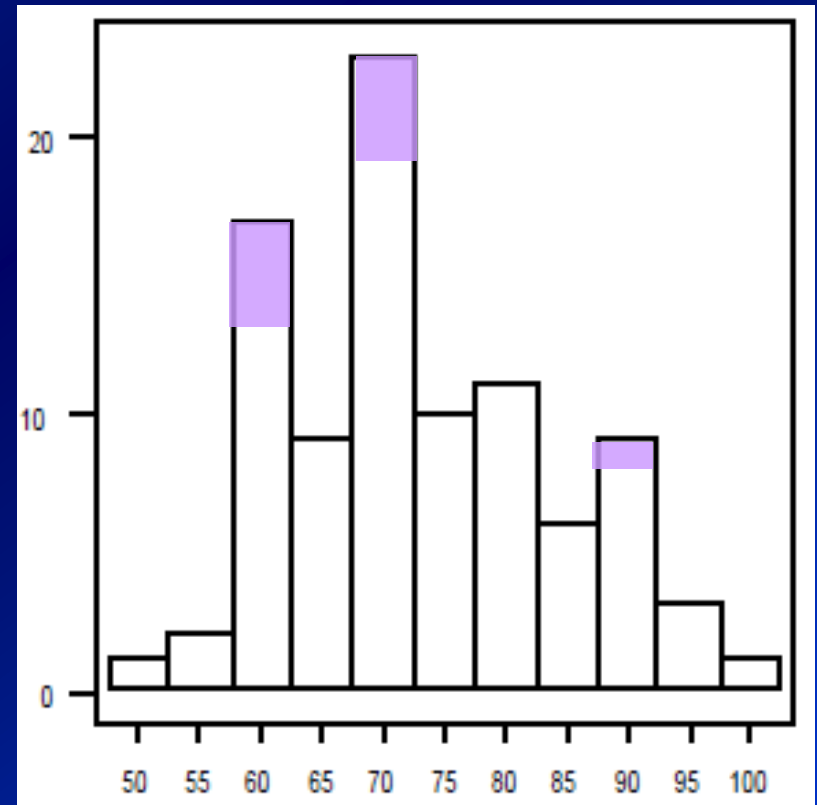
Count one minute

Count 10 sec x 6

Count 25 sec x 4

...

Pulse per minute



Tools

1. Graphical

Visualitze variable's distribution



2. Numerical

Quantify what is observed in he graphs



2. Numerical tools

Quantify and synthetize characteristics of a distribution

1. According to the information provided

1. Central trend statistics
2. Variability statistics

2. According to the stability

1. Classic
2. Robust

Numerical tools

for numerical variables

	Robusto	Clásico
Posición	Mediana Cuartiles Percentiles Moda	Media
Dispersión	Distancia entre cuartiles	S Desviación estándar S^2 Varianza Coef. variación Amplitud

Numerical tools

Mean

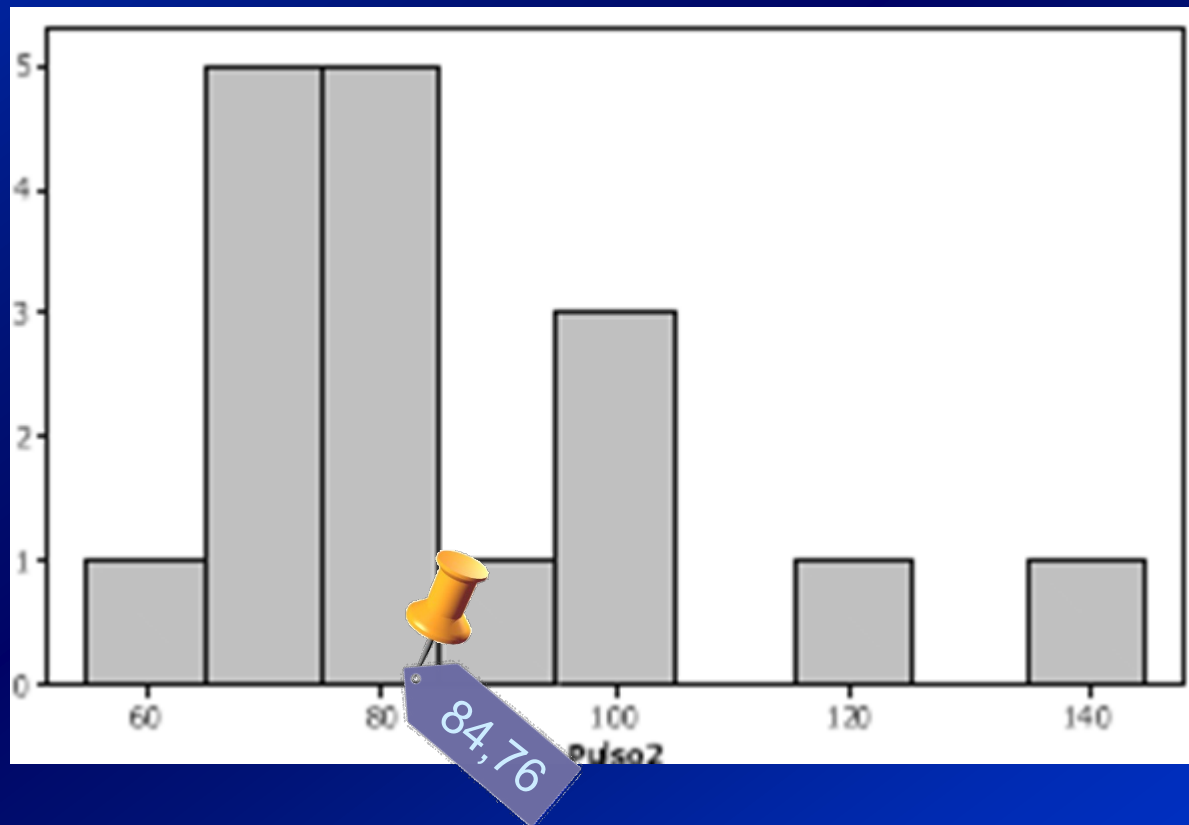
x_1, x_2, \dots, x_n are n observations of a variable X

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$$

READING HISTOGRAMS

Central trend

Value around which observations distribute



Numerical tools

Mean

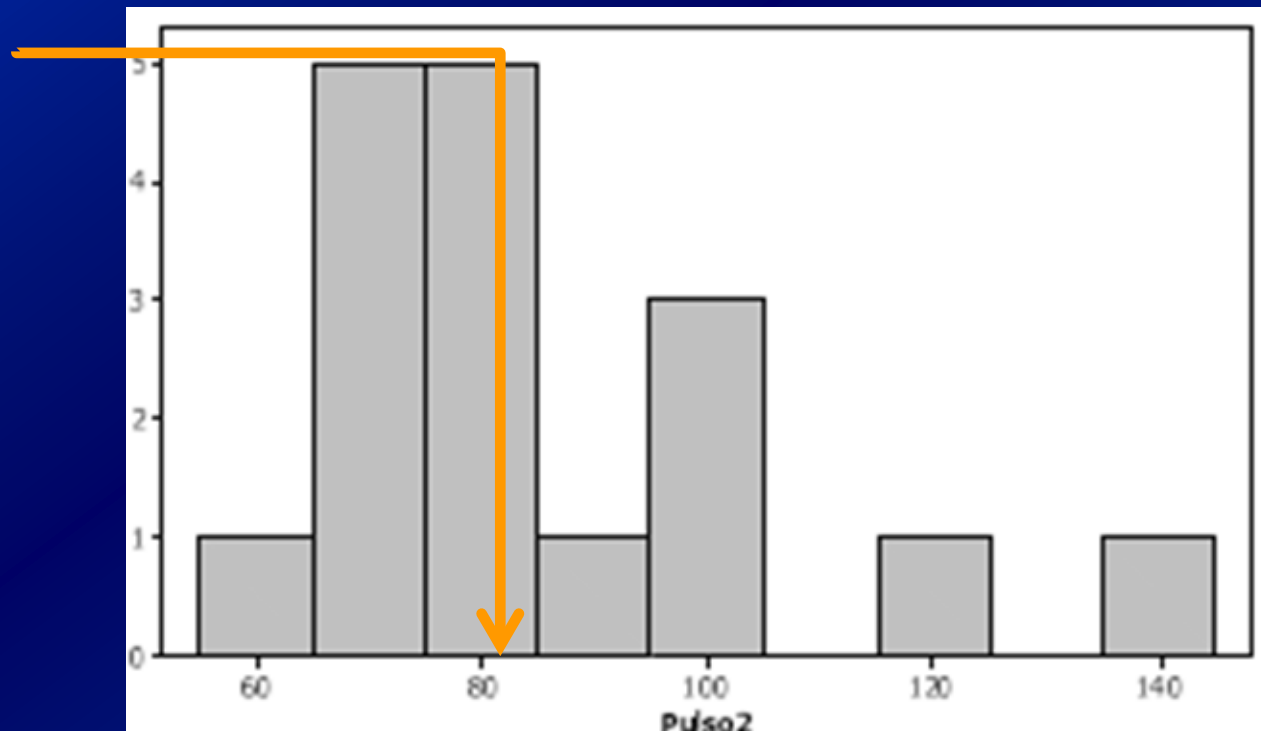


$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

Numerical tools

Mode

The most frequent observation



Numerical tools

Dispersion Measures

$$V(X) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} \quad s_X = \sqrt{V(X)} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

$$qV(X) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \quad q \cdot s_X = \sqrt{qV(X)} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

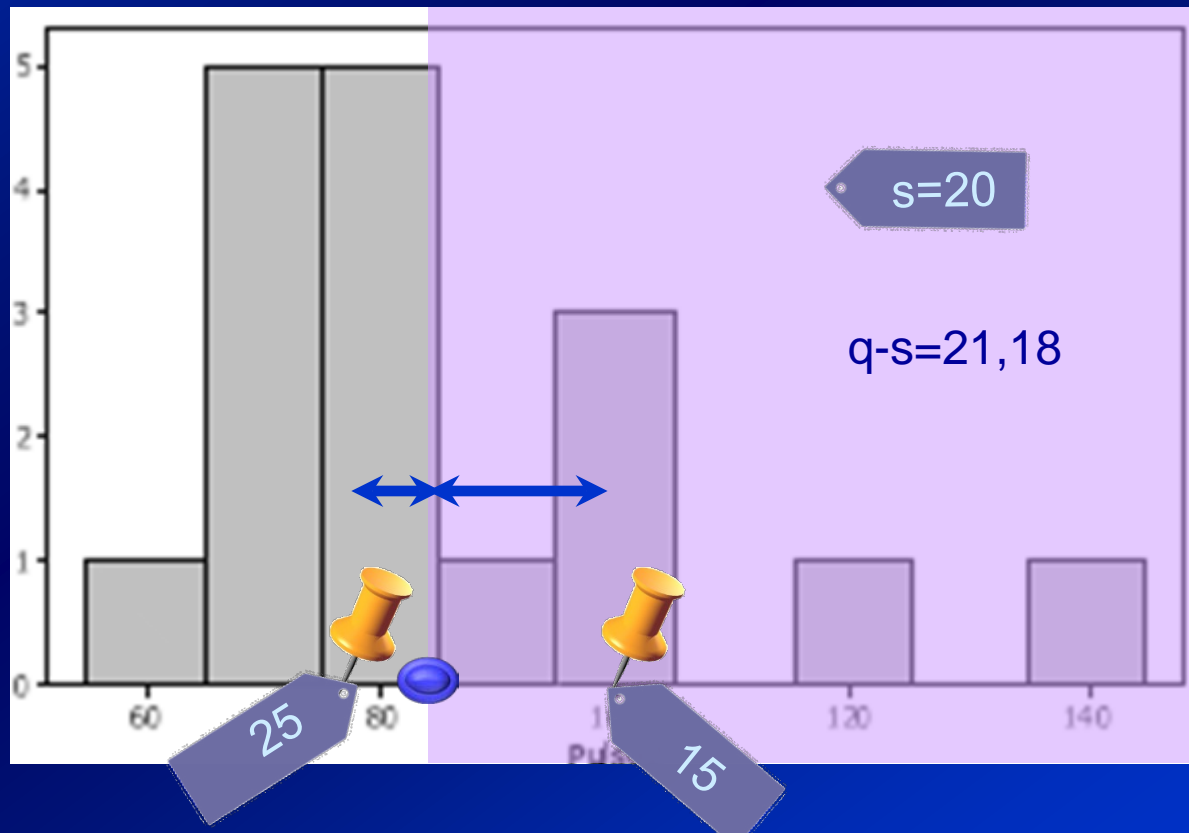
Mean distance to
central trend

READING HISTOGRAMS

Dispersion/Variability

How observations concentrate around central trend?

Mean distance to central trend



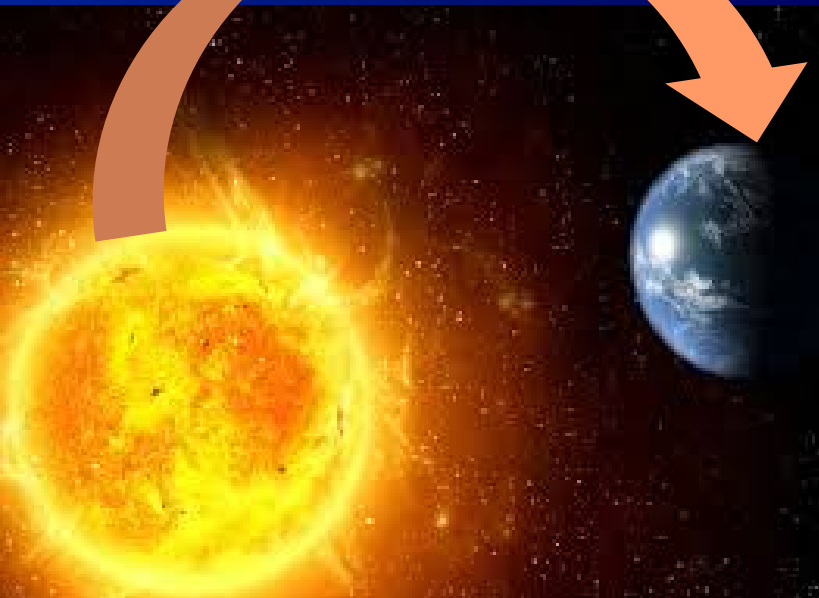
Numerical tools

Dispersion Measures

Variation Coefficient: $\frac{s}{\bar{x}} \times 100$

d./range

d(Sun, Earth): 150×10^6 Km, $s = 10^3$ Km



d(BCN, Moscow): 3000 Km, $s = 10^3$ Km



Numerical tools

5-Number Summary

Robust

$\langle \min, Q1, Me, Q3, \max \rangle$

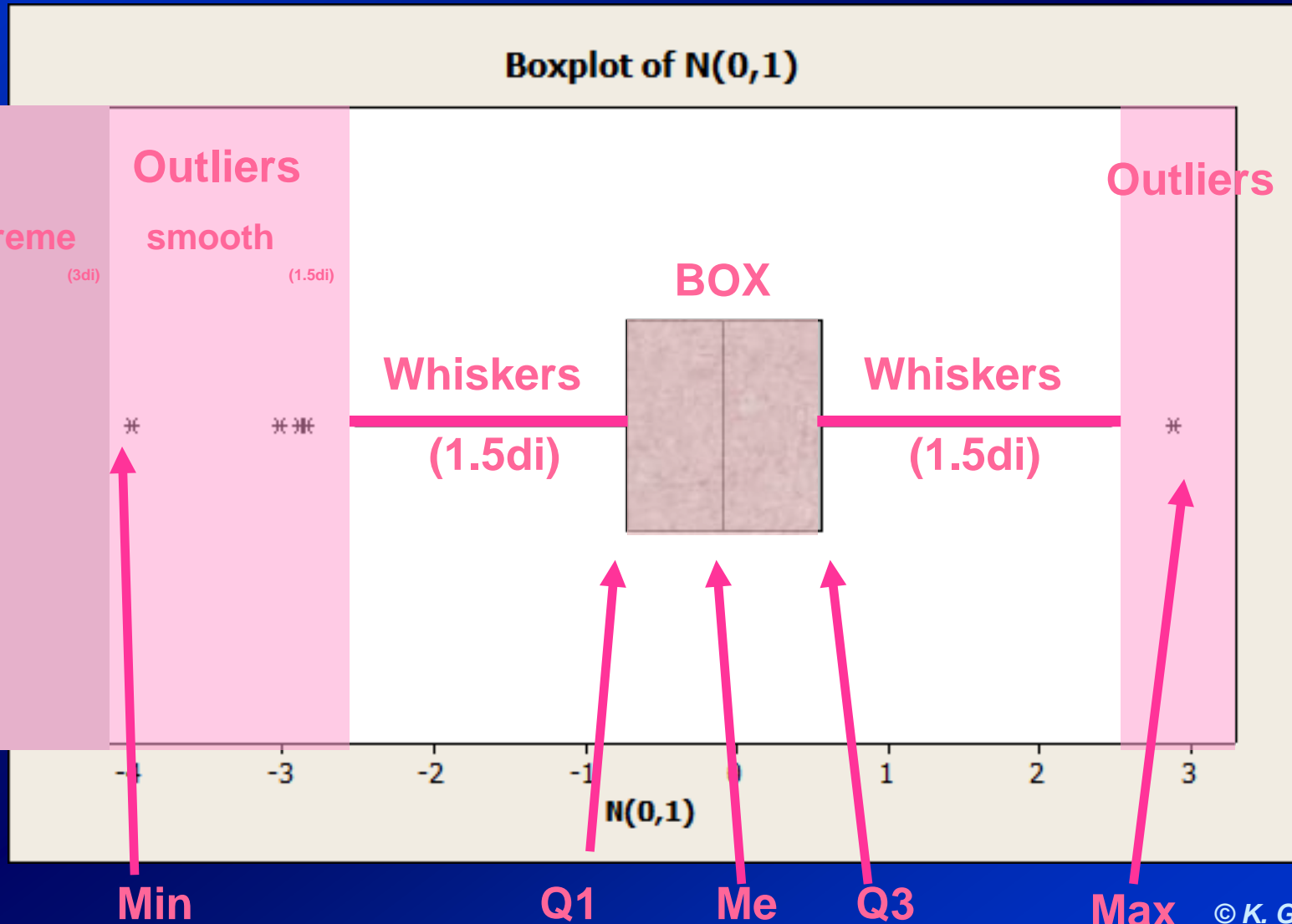


Requires
sorting

Boxplot

[Tukey 1956]

Symbolic representation of 5-Number Summary

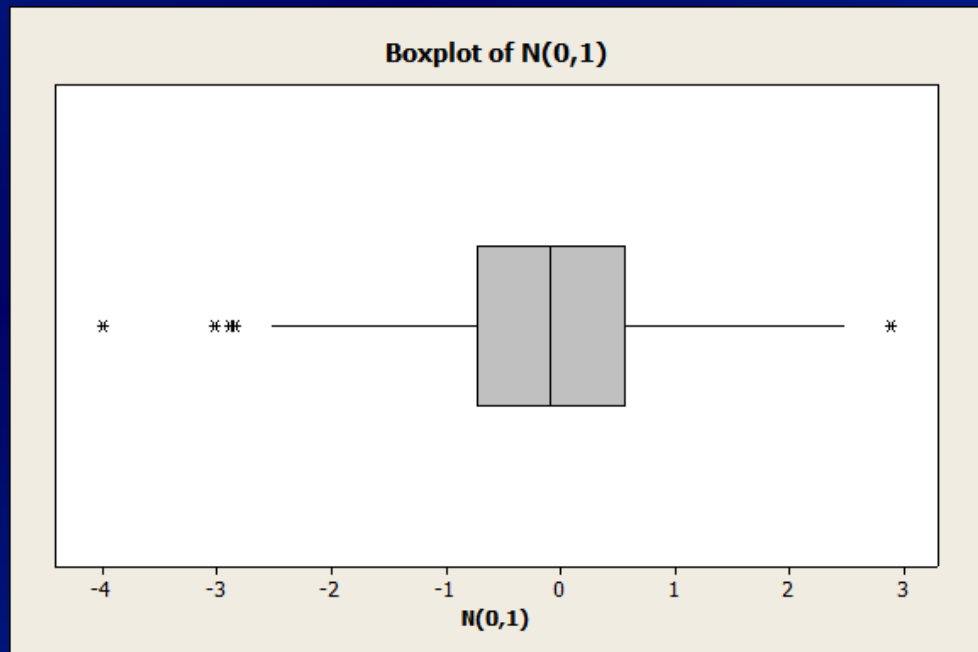


READING BOXPLOTS

1. Range of variable (max-min)
2. Central trend
3. Dispersion
4. Symmetry
5. Anomalies

READING BOXPLOTS

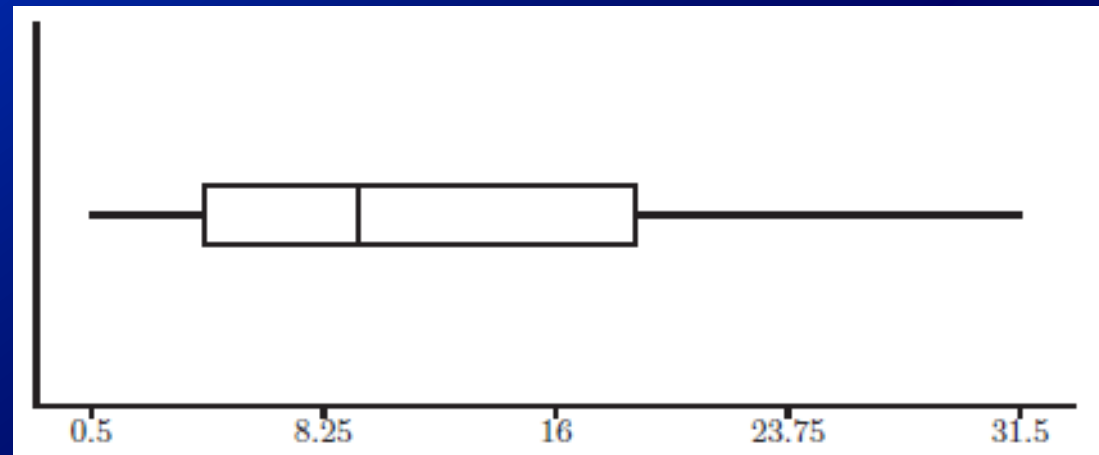
1. Range of variable (max-min)
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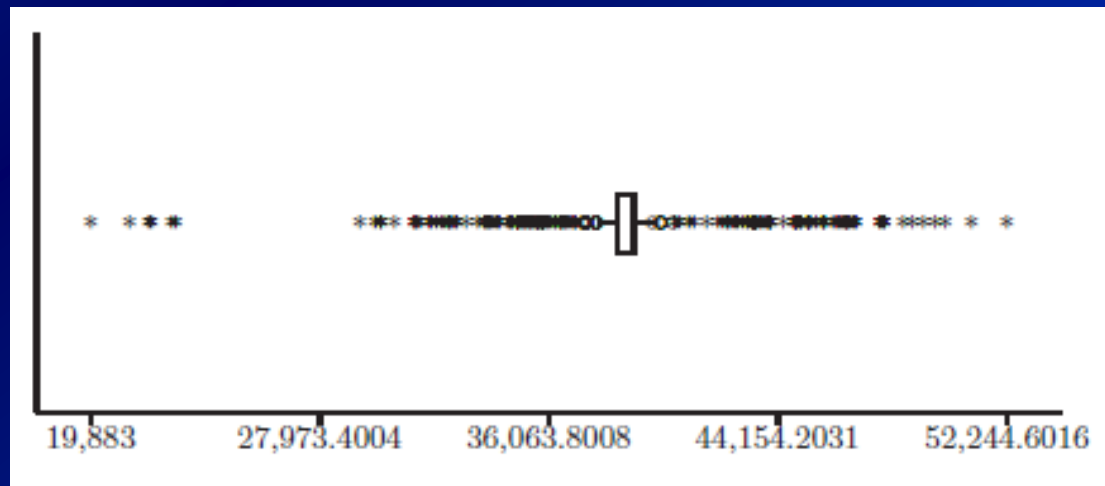
READING BOXPLOTS

Dispersion

Ammonium



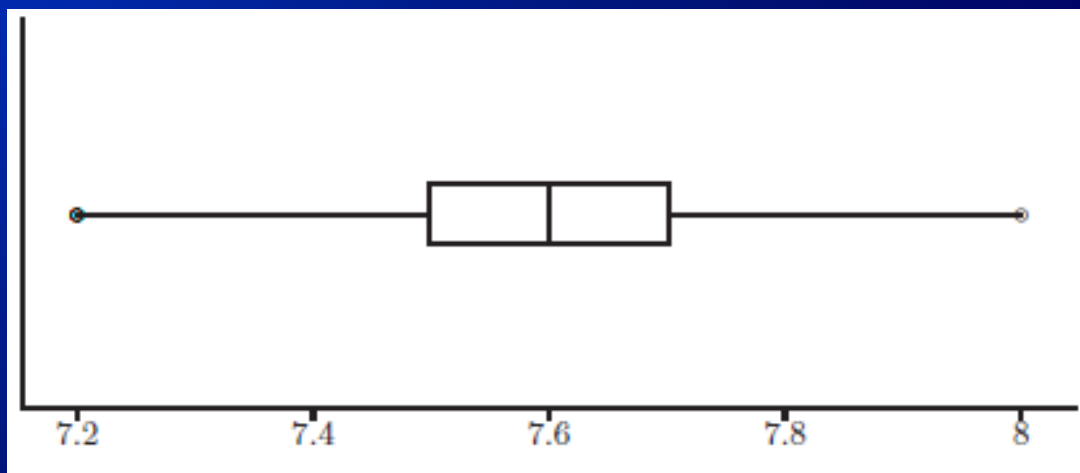
QB-B



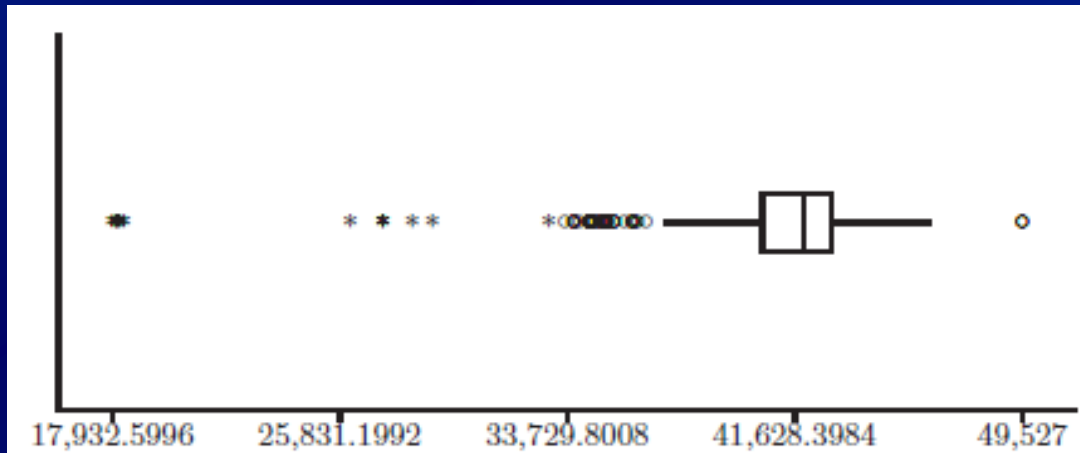
READING BOXPLOTS

Simmetry

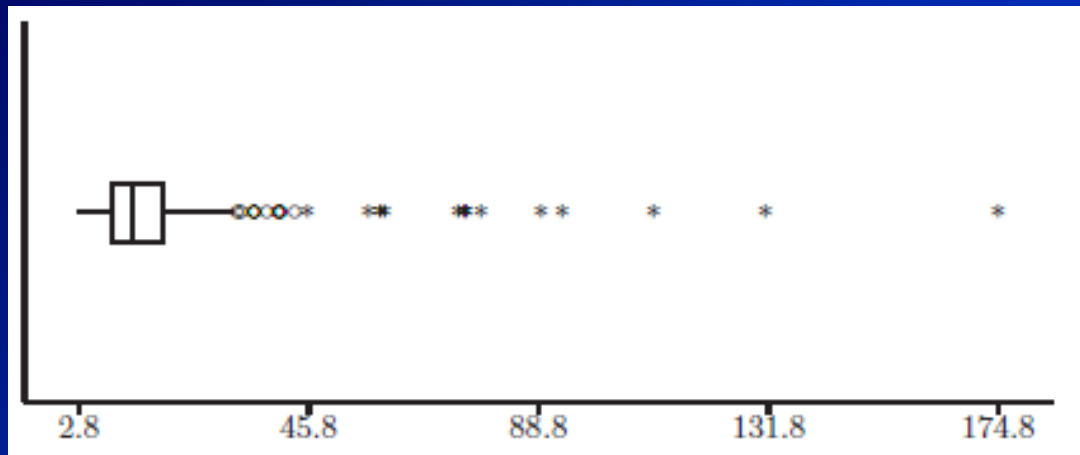
PH



QR-G

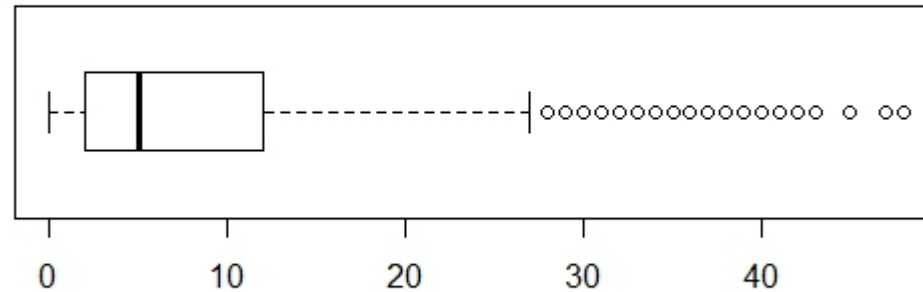


SS-S



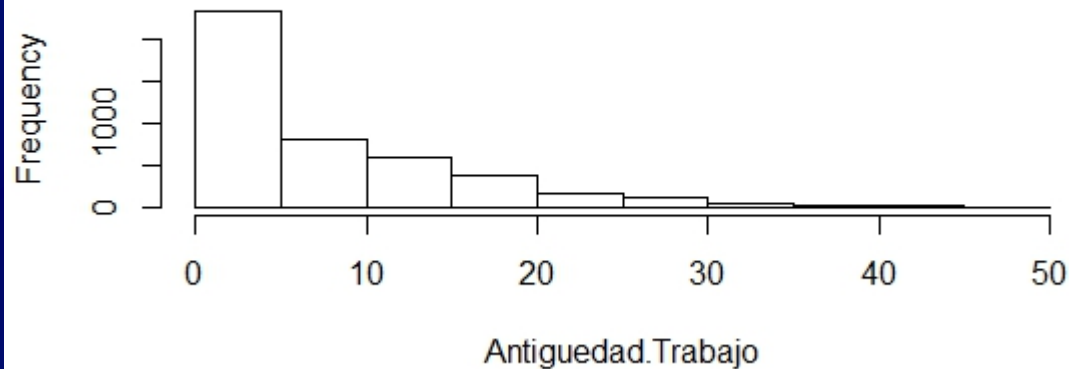
READING BOXPLOTS

Boxplot of Antigüedad.Trabajo



CARE
With *

Histogram of Antigüedad.Trabajo



Symmetry

if Mean \neq Median then

if $Me - Q1 < Q3 - Me$ then asymmetry

else outliers

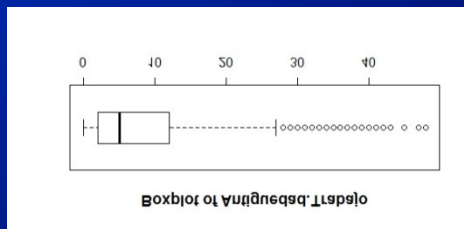
else symmetry without outliers

Synthesis

1. Descriptive analysis of numerical variable

1. Central trend and variability (classical/robust)

2. Graphical and Numerical tools

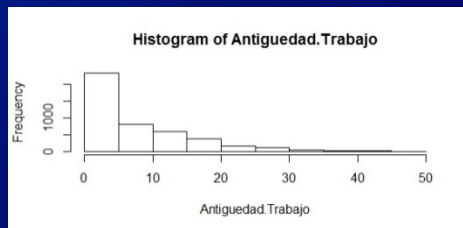


5-Number-Summary:

<min, Q1, Me, Q3, max>

+

mean, q-stdev, variation coefficient



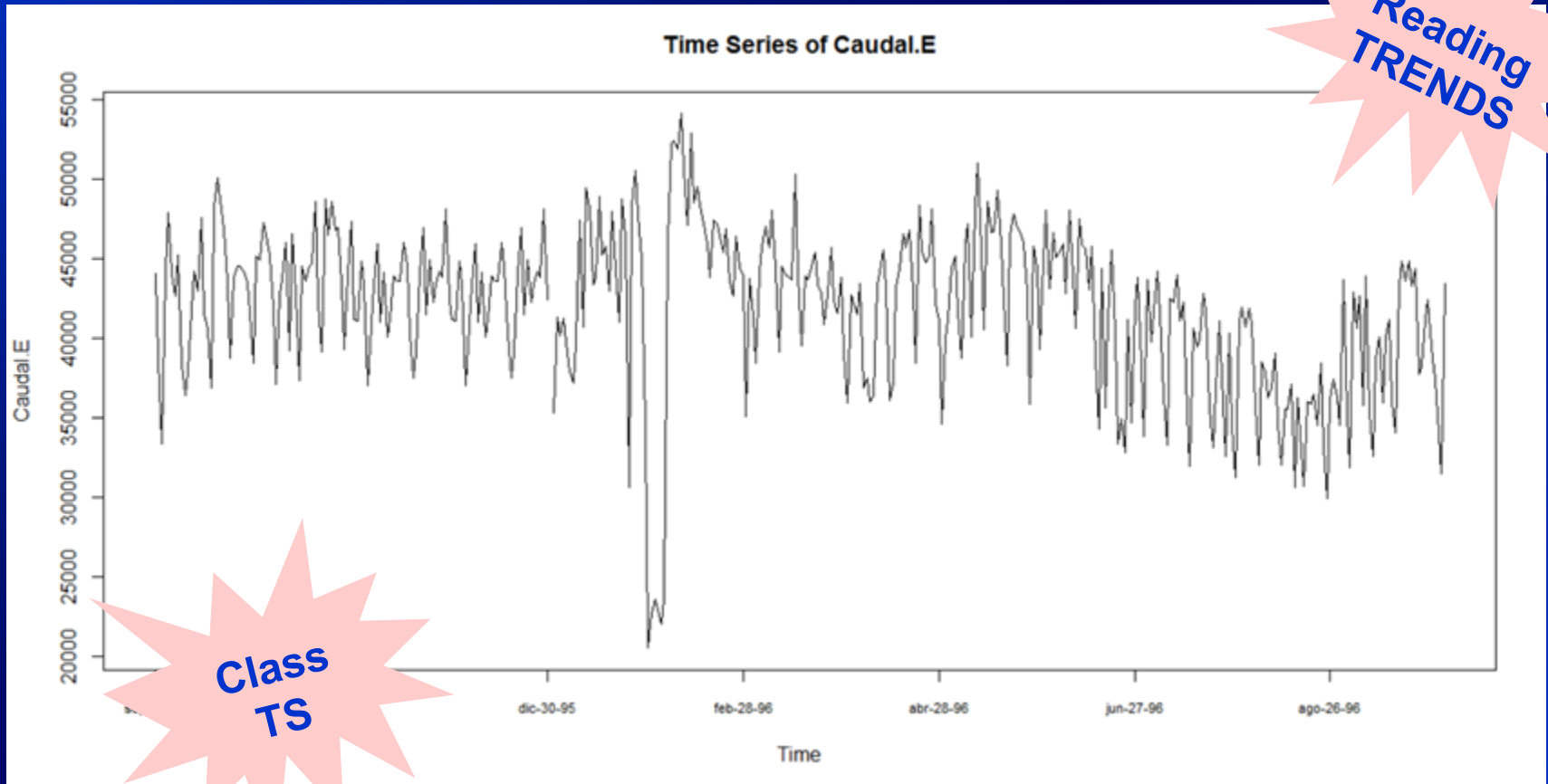
3. Characterize the variable

Central trend, variability, symmetry, n-modality....

3. Cronological data

Observations are sequentially sorted in the dataset

Time series plot



Cronological data

Observations are sequentially sorted in the dataset

Eventually the Date is Available



Cronological data

Date Objects in R

Symbo	Meaning	Example
%d	day as a number (0-31)	01-31
%D	Date format	
%a	abbreviated weekday	Mon
%A	unabbreviated weekday	Monday
%m	month (00-12)	00-12
%b	abbreviated month	Jan
%B	unabbreviated month	January
%y	2-digit year	07
%Y	4-digit year	2007

31/12/2014 : %d/%m/%Y

31-Dic-07: %d-%b-%y

Cronological data

Date Objects in R

Symbo	Meaning	Example
%c	Date and time	
%C	Century	
%H	Hours (00-23)	15
%I	Hours (1-12)	3
%j	Day of the year (0-365)	250
%M	minute (00-59)	January
%S	Second as integer(0-61)	07

23:12:59 = %H:%M:%S

11 12 59 = %I %M %S

Cronological data

Observations are sequentially sorted in the dataset

Eventually the Date is Available



Class
Date

To consider time



Class
POSIXCT

4. Assessing Normality

1. 68-97-99.5 Rule

$$[x \pm s], [x \pm 2s] [x \pm 3s]$$

2. Normality plot (qq-plot, Henri line)

3. Normality assessment test: Shapiro Wilk

$$W = \frac{\left(\sum_{i=1}^N a_i y_i \right)^2}{\sum_{i=1}^N (y_i - m_1)^2}$$

y_i = i th order statistic m_1 = sample mean

a_i = computed as linear regression to the expected value of standard normal order statistics

5. Assessing Exponentiality

The rule of 70

Time of doubling : $70/R$, R growing factor [Moore, McCabe 93]

*X has exponential growth with constant factor R
if needs $70/R$ time to pass from X to $2X$*

Basic Descriptive Analysis Numerical Variables

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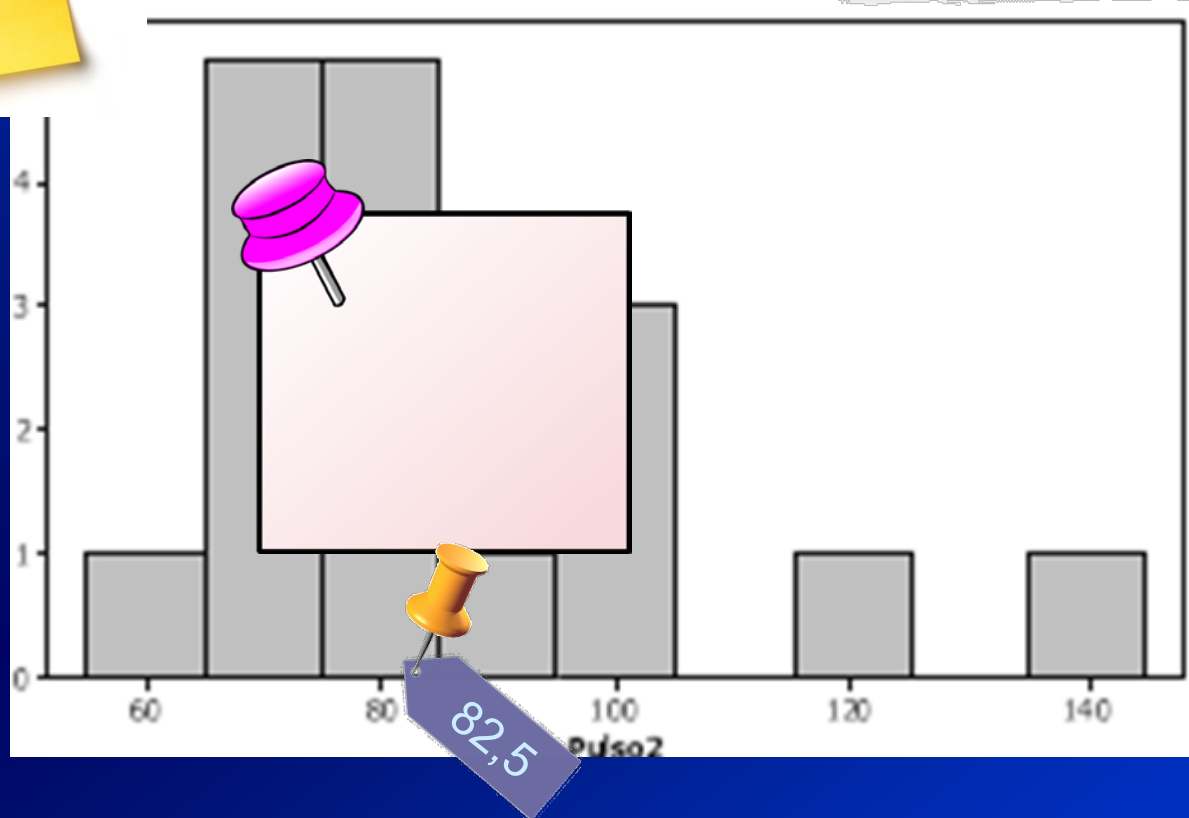


Are there any questions?...

READING HISTOGRAM

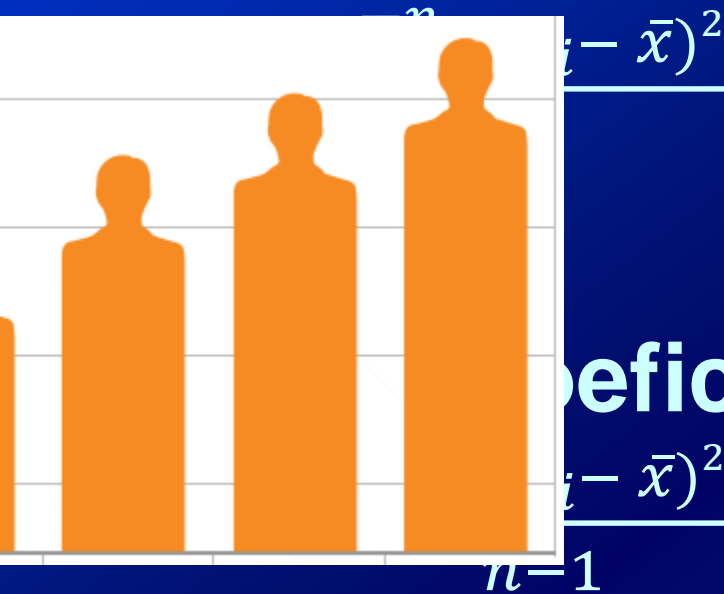
Central tendency

Value around which observations



Numerical tools

Dispersion



Efficient

$$(i - \bar{x})^2$$

$n=1$

Mean dist
central

