Formulari de Teoria de Cues

Variable exponencial de paràmetre a ,

$$F_T(t) = P(\{T \le t\}) = \int_{-\infty}^{t} f_T(t)\cdot dt = \begin{cases} 1 - e^{-at} & t \ge 0 \\ 0 & altrament \end{cases} \quad E[T] = \frac{1}{a} \quad V[T] = \frac{1}{a}$$

Distribució de Poisson de paràmetre a ,

$$P(\{X = n\}) = \frac{(a)^n}{n!} e^{-a} \quad n = 0,1,2,... \quad E[X] = a \quad V[X] = a$$

La llei de probabilitats **k-Erlang** (o Erlang de paràmetres *k*, *m*),

$$F_T(t) = P(\{T \le t\}) = 1 - e^{-kmt} \sum_{i=0}^{k-1} \frac{(kmt)^i}{i!} \quad \text{per } t \ge 0 \quad m,k > 0$$

$$E[T] = \frac{1}{m} \qquad E[T] = \frac{1}{km^2}$$

Fórmules de Little:

$$L = `l\ W, \quad L_s = `l\ W_s,\ L_q = `l\ W_q\ ,\ L = L_s + L_q\ ,\ W = W_s + W_q$$

Processos de naixement i mort:

$$C_n = \frac{l_0 l_1 \dots l_{n-1}}{m_1 m_2 \dots m_n} \quad n = 1,2,\dots \quad P_n = \frac{l_0 l_1 \dots l_{n-1}}{m_1 m_2 \dots m_n} \cdot P_0 = C_n \cdot P_0 \quad n = 1,2,\dots \quad i \quad C_0 = 1$$

$$\sum_{n=0}^{\infty} P_n = \sum_{n=0}^{\infty} C_n \cdot P_0 = 1 \ \rightarrow P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n}$$

$$L = \sum_{n=0}^{\infty} n \cdot P_n \qquad L_q = \sum_{n=s}^{\infty} (n-s)\cdot P_n \qquad \overline{l} = \sum_{n=0}^{\infty} l_n \cdot P_n$$

M/M/1:

$$C_n = \frac{l_0 l_1 \dots l_{n-1}}{m_1 m_2 \dots m_n} = \left(\frac{l}{m}\right)^n = r^n \quad n = 1,2,\dots \quad C_0 = 1$$

$$P_n = C_n \cdot P_0 = r^n \cdot P_0 \quad n = 0,1,2,\dots$$

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{\sum_{n=0}^{\infty} r^n} = \frac{1}{\frac{1}{1-r}} = 1 - r$$

$$L = \sum_{n=0}^{\infty} n \cdot P_n = \frac{r}{(1-r)} = \frac{l}{m - l} \qquad L_q = \frac{r^2}{1-r} = \frac{l^2}{m\cdot(m - l)}$$

M/M/s:

$$C_n = \frac{l_0 l_1 \dots l_{n-1}}{m_1 m_2 \dots m_n} = \begin{cases} \frac{1}{n!}\left(\frac{l}{m}\right)^n & n = 1,2,\dots,s-1 \\ \frac{1}{s!}\left(\frac{l}{m}\right)^s \left(\frac{l}{sm}\right)^{n-s} & n = s, s+1,\dots \end{cases} \qquad P_n = C_n \cdot P_0 = \begin{cases} \frac{1}{n!}\left(\frac{l}{m}\right)^n \cdot P_0 & n = 1,2,\dots,s-1 \\ \frac{1}{s!}\left(\frac{l}{m}\right)^s \left(\frac{l}{sm}\right)^{n-s} \cdot P_0 & n = s, s+1,\dots \end{cases}$$

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{l}{m}\right)^n + \sum_{n=s}^{\infty}\frac{1}{s!}\left(\frac{l}{m}\right)^s\left(\frac{l}{sm}\right)^{n-s}} \quad \text{o bé} \quad P_0 = \frac{1}{\sum_{n=0}^{s-1}\frac{\left(l/m\right)^n}{n!} + \frac{\left(l/m\right)^s}{s!}\frac{1}{1-\left(l/sm\right)}}$$

$$L = \sum_{n=0}^{\infty} n\cdot P_n = l\cdot W = l\cdot\left(W_q + \frac{1}{m}\right) = L_q + \frac{l}{m}$$

$$L_q = \sum_{n=s}^{\infty}(n-s)\cdot P_n = \sum_{n=0}^{\infty} n\cdot P_{s+n} = \frac{1}{s!}\left(\frac{l}{m}\right)^s P_0\frac{r}{(1-r)^2}$$

M/M/1/K:

$$C_n = \frac{l_0\, l_1 \ldots l_{n-1}}{m_1 m_2 \ldots m_n} = \begin{cases} \left(\frac{l}{m}\right)^n = r^n & n = 1, 2, \ldots, K \\ 0 & n = K+1, K+2, \ldots \end{cases} \qquad C_0 = 1$$

$$P_n = C_n\cdot P_0 = \begin{cases} r^n\cdot P_0 & n = 0, 1, 2, \ldots, K \\ 0 & n > K \end{cases} \qquad P_0 = \frac{1}{\sum_{n=0}^{K} C_n} = \frac{1}{\sum_{n=0}^{K} r^n} = \frac{1}{\frac{1-r^{K+1}}{1-r}} = \frac{1-r}{1-r^{K+1}}$$

$$L = \frac{r}{1-r} - \frac{(K+1)r^{K+1}}{1-r^{K+1}} \qquad L_q = (L) - (1-P_0) \qquad \bar{l} = l(1-P_K)$$

M/M/s/K:

$$C_n = \frac{l_0\, l_1 \ldots l_{n-1}}{m_1 m_2 \ldots m_n} = \begin{cases} \frac{1}{n!}\left(\frac{l}{m}\right)^n & n = 1, 2, \ldots, s-1 \\ \frac{1}{s!}\left(\frac{l}{m}\right)^s\left(\frac{l}{sm}\right)^{n-s} & n = s, s+1, \ldots, K \qquad i \;\; C_0 = 1 \\ 0 & n = K+1, K+2, \ldots \end{cases}$$

$$P_n = C_n\cdot P_0 = \begin{cases} \frac{1}{n!}\left(\frac{l}{m}\right)^n\cdot P_0 & n = 1, 2, \ldots, s-1 \\ \frac{1}{s!}\left(\frac{l}{m}\right)^s\left(\frac{l}{sm}\right)^{n-s}\cdot P_0 & n = s, s+1, \ldots, K \\ 0 & n = K+1, K+2, \ldots \end{cases} \qquad P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{l}{m}\right)^n + \sum_{n=s}^{K}\frac{1}{s!}\left(\frac{l}{m}\right)^s\left(\frac{l}{sm}\right)^{n-s}}$$

$$L_q = \frac{1}{s!}\left(\frac{l}{m}\right)^s P_0\frac{r}{(1-r)^2}\left(1 - r^{K-s} - (K-s)\cdot r^{K-s}\cdot(1-r)\right). \qquad \bar{l} = l\cdot(1-P_K) \qquad L = L_q + \frac{\bar{l}}{m}$$

M/M/1//N:

$$r = l/m \qquad C_n = \frac{l_0\, l_1 \ldots l_{n-1}}{m_1 m_2 \ldots m_n} = \begin{cases} N(N-1)(N-2)\ldots(N-n+1)\left(\frac{l}{m}\right)^n = \frac{N!}{(N-n)!}\, r^n & n = 1, 2, \ldots, N \\ 0 & n = N+1, N+2, \ldots \end{cases} \qquad C_0 = 1$$

M/M/1//N:

$$\rho = \lambda / \mu$$

$$C_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} = \begin{cases} N(N-1)(N-2)\ldots(N-n+1)\left(\frac{\lambda}{\mu}\right)^n = \frac{N!}{(N-n)!}\rho^n & n = 1,2,\ldots,N \\ 0 & n = N+1, N+2, \ldots \end{cases} \qquad C_0 = 1$$

$$P_n = C_n \cdot P_0 = \begin{cases} \frac{N!}{(N-n)!}\rho^n \cdot P_0 & n = 0,1,2,\ldots,N \\ 0 & n = N+1, N+2, \ldots \end{cases} \qquad P_0 = \frac{1}{\sum_{n=0}^{N} C_n} = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!}\rho^n}$$

$$L_q = N - \frac{\lambda + \mu}{\lambda}(1 - P_0) \qquad \bar{\lambda} = (N - L) \cdot \lambda$$

M/M/s//N:

$$C_n = \frac{\lambda_0 \lambda_1 \ldots \lambda_{n-1}}{\mu_1 \mu_2 \ldots \mu_n} = \begin{cases} \frac{N!}{(N-n)!\,n!}\left(\frac{\lambda}{\mu}\right)^n & n = 1,2,\ldots,s-1 \\ \frac{N!}{(N-n)!\,s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{\lambda}{s\mu}\right)^{n-s} & n = s, s+1, \ldots, N \\ 0 & n = N+1, N+2, \ldots \end{cases} \qquad i \;\; C_0 = 1$$

$$P_n = C_n \cdot P_0 = \begin{cases} \frac{N!}{(N-n)!\,n!}\left(\frac{\lambda}{\mu}\right)^n \cdot P_0 & n = 1,2,\ldots,s-1 \\ \frac{N!}{(N-n)!\,s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{\lambda}{s\mu}\right)^{n-s} \cdot P_0 & n = s, s+1, \ldots, N \\ 0 & n = N+1, N+2, \ldots \end{cases} \qquad \bar{\lambda} = (N - L) \cdot \lambda$$

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{\sum_{n=0}^{s-1} \frac{N!}{(N-n)!\,n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{N} \frac{N!}{(N-n)!\,s!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{\lambda}{s\mu}\right)^{n-s}} \qquad L_q = \sum_{n=s}^{\infty}(n-s) \cdot P_n = \sum_{n=s}^{N-s}(n-s) \cdot P_n$$

M/G/1: $\rho = \frac{\lambda}{\mu} < 1 \qquad L_q = \frac{(\mu^2 \sigma_x{}^2 + 1)\rho^2}{2(1-\rho)} \qquad P_0 = 1 - \rho$

Fòrmula d'aproximació d'Allen-Cuneen per a GI/G/s

$$\mathrm{E}[w_q] = W_q \approx \frac{C(s,\theta)(\lambda^2 \sigma_\tau^2 + \mu^2 \sigma_x^2)}{2s\mu(1-\rho)} \qquad \theta = \frac{\lambda}{\mu}$$

$$C(s,\theta) = P_{M/M/s}(N \geq s) = \frac{\frac{\theta^s}{s!(1-\rho)}}{\sum_{\ell=0}^{s-1} \frac{\theta^\ell}{\ell!} + \frac{\theta^s}{s!(1-\rho)}}$$

Xarxes de Cues

$$\lambda_j = r_j + \sum_{i=1}^{k} \lambda_i p_{ij} \qquad \lambda_j \le s_j \mu_j \qquad j = 1,2,\ldots k \qquad L_{Total} = \sum_{j=1}^{k} L_j \qquad W = \frac{L_{Total}}{\lambda}$$

$$\lambda = \sum_{i=1}^{k} r_j$$