

Llista de problemes (2)

- 1.13 Esbossen corbes de nivell i la gràfica de $f: \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x,y) = -xy$.
- 1.14 Idem $f(x,y) = x^2 + 4y^2$
- 1.15 Descriu les corbes de nivell de $f(x,y) = x^3 - x$
- 1.16 Descriu les corbes de nivell i la gràfica de $f(x,y) = (100 - x^2 - y^2)^{1/2}$
- 1.17 Descriu les corbes de nivell de $f(x,y) = x^2 + xy$
- 1.18 Descriu les corbes de nivell i la gràfica de $f(x,y) = \max(|x|, |y|)$
- 1.19 Determinen les superfícies de nivell de $f(x,y,z) = 4x^2 + y^2 + 9z^2$
- 1.20 Esbossen les superfícies $y^2 + z^2 = 4$, $y^2 = x^2 + z^2$
- 1.21 Determinen les corbes de nivell de $f(x,y) = \frac{2xy}{x^2 + y^2}$ (usen coord. polars)
- 1.22 Donada $f: \mathbb{R}^2 - \{(0,0)\} \rightarrow \mathbb{R}$ en coordenades polars per $f(r,\theta) = \frac{\cos 2\theta}{r^2}$, troben les corbes de nivell respecte els eixos x i y .

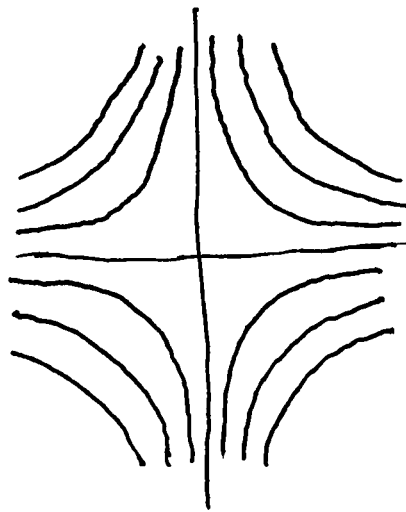
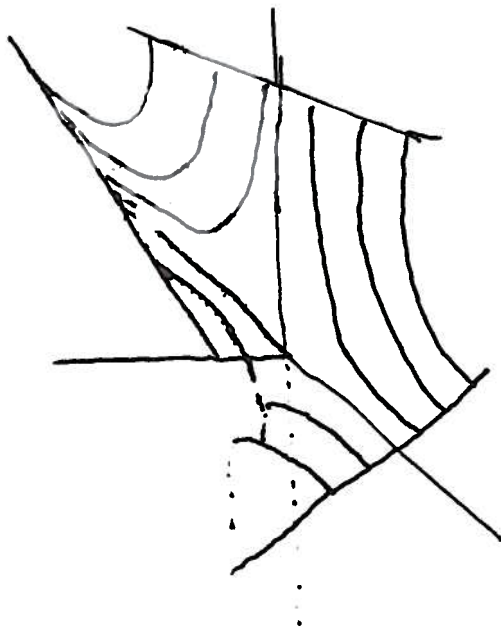
1.13

Corbes de nivell : gràfica de
 $f(x,y) = -xy$, $f: \mathbb{R}^2 \rightarrow \mathbb{R}$

corbes de nivell

$$\begin{array}{lcl}
 L_0 = \{ -xy = 0 \} & \xrightarrow{\substack{x=0 \\ y=0}} & \\
 L_1 = \{ -xy = 1 \} & \xrightarrow{\quad} & y = -\frac{1}{x} \\
 L_2 = \{ -xy = 2 \} & \xrightarrow{\quad} & y = -\frac{2}{x} \\
 L_{-1} = \{ -xy = -1 \} & \xrightarrow{\quad} & y = \frac{1}{x}
 \end{array}$$

si $x=1$, $f(1,y) = -y$ | si $y=1$, $f(x,1) = -x$



1.14

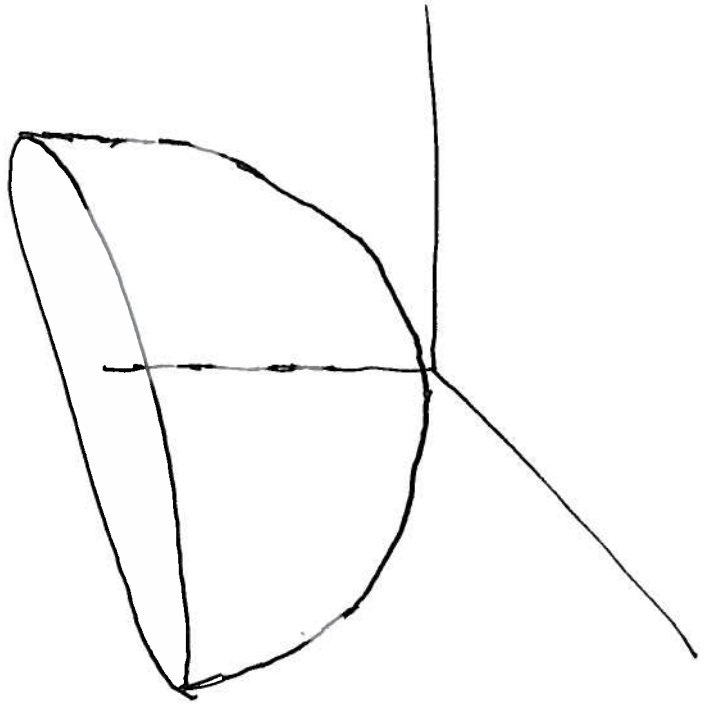
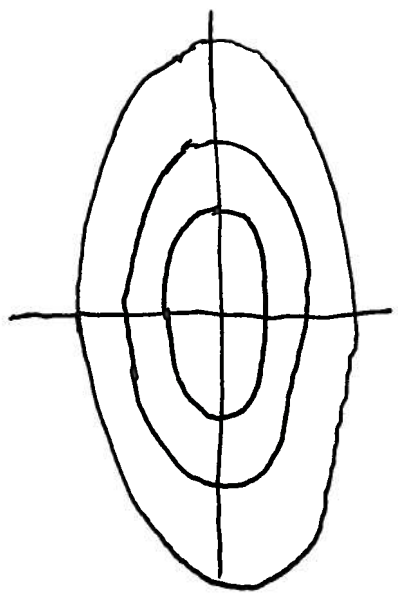
Corbes de nivell : gràfica de
 $f: \mathbb{R}^2 \rightarrow \mathbb{R}, f(x,y) = x^2 + 4y^2$

corbes de nivell

$$L_c = \emptyset \quad \text{si } c < 0$$

$$L_0 = \{x^2 + 4y^2 = 0\} \rightarrow L_0 = \{(0,0)\}$$

$$L_c = \{x^2 + 4y^2 = c\} \rightarrow \frac{x^2}{c} + \frac{y^2}{\frac{c}{4}} = 1 \quad \text{el·lipse de semieixos } \sqrt{c}, \sqrt{\frac{c}{4}}$$



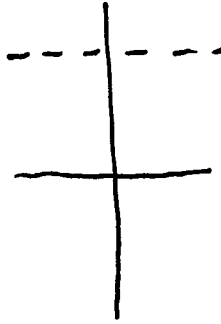
1.15

Corbes de nivell de

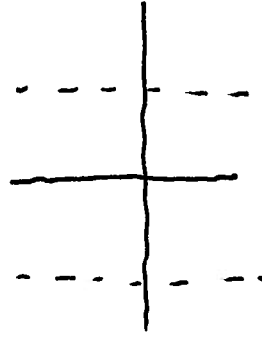
$$f: \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x, y) = x^3 - x$$

$$L_c = \{(x, y) \mid x^3 - x = c\} \subset \mathbb{R}^2$$

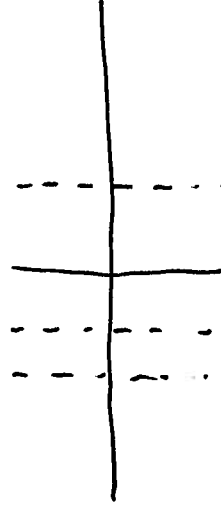
si: $c > \frac{2\sqrt{3}}{9}$, $L_c =$ una recta



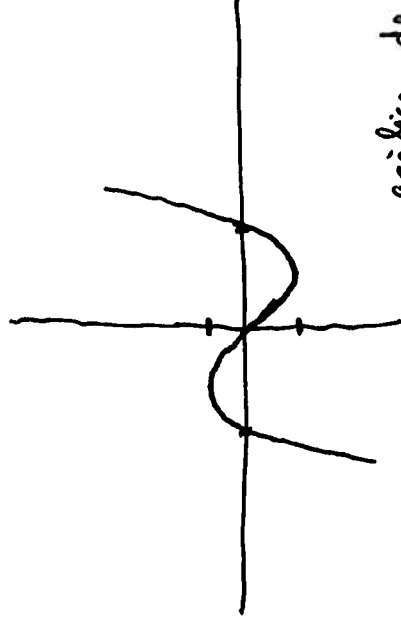
si: $c = \frac{2\sqrt{3}}{9}$, $L_c =$ dues rectes



si: $-\frac{2\sqrt{3}}{9} < c < \frac{2\sqrt{3}}{9}$ $L_c =$ tres rectes



etc.



gràfica de
 $g(x) = x^3 - x$

Extrems de g : $g'(x) = 3x^2 - 1 = 0$

$$x^2 = \frac{1}{3}, \quad x = \pm \frac{1}{\sqrt{3}}$$

$$g\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{1}{\sqrt{3}}\right)^3 - \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\left(\frac{1}{3} - 1\right) = \frac{-2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

$$g\left(-\frac{1}{\sqrt{3}}\right) = \frac{2\sqrt{3}}{9}$$

1.16

Corbes de nivell i gràfica de

$$f(x, y) = (100 - x^2 - y^2)^{1/2}$$

$$\text{domini} = \{100 - x^2 - y^2 \geq 0\}$$

$$x^2 + y^2 \leq 100 = 10^2$$

disc de radi 10

$$L_0 = \{(100 - x^2 - y^2)^{1/2} = 0\}$$

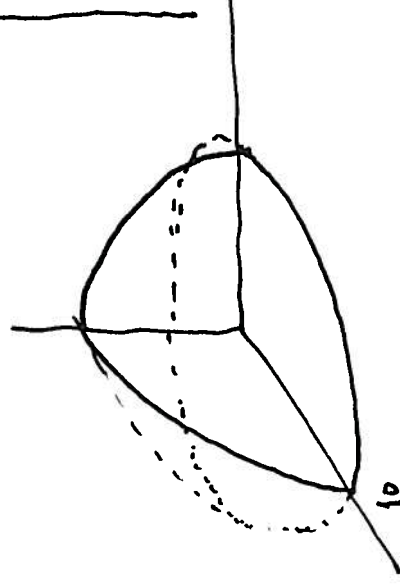
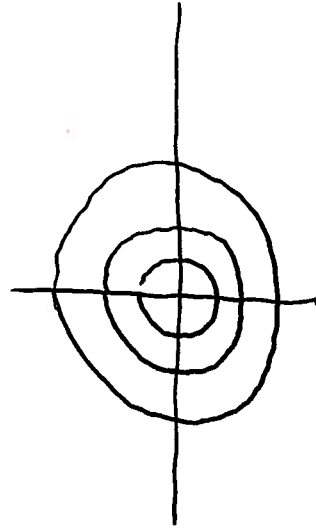
$$= \{100 - x^2 - y^2 = 0\} \quad \text{circumferència de radi 10}$$

$$L_4 = \{(100 - x^2 - y^2)^{1/2} = 4\} = \{100 - x^2 - y^2 = 16\} = \{x^2 + y^2 = 84\}$$

$$L_6 = \dots = \{x^2 + y^2 = 64\}$$

$$L_{10} = \{(100 - x^2 - y^2)^{1/2} = 10\} = \{100 - x^2 - y^2 = 100\} = \{(0, 0)\}$$

$$\begin{aligned} (100 - x^2 - y^2)^{1/2} &= z \\ 100 - x^2 - y^2 &= z^2 \\ 100 &= x^2 + y^2 + z^2 \end{aligned}$$



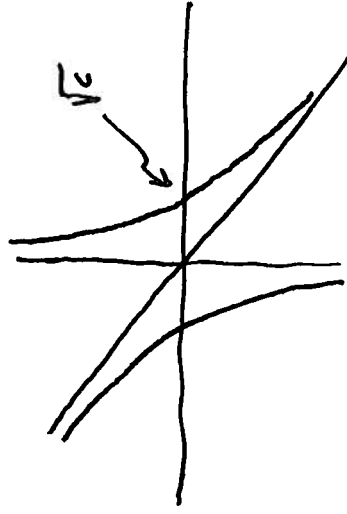
1.17

corres al nivell de

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x, y) = x^2 + xy$$

$$L_0 = \{x^2 + xy = 0\} \rightarrow x(x+y) = 0 \rightarrow \begin{matrix} x=0 \\ y=-x \end{matrix}$$

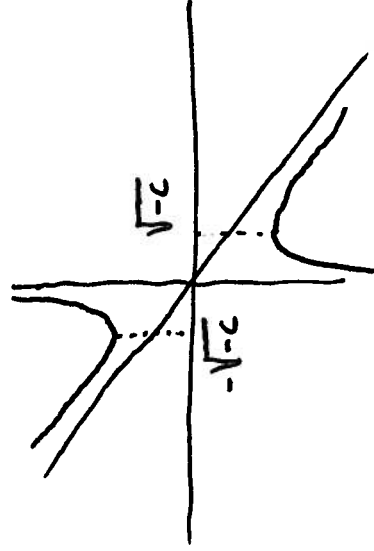
$$c > 0, \quad L_c = \{x^2 + xy = c\} \rightarrow xy = c - x^2 \rightarrow$$



$$y = \frac{c-x^2}{x} = -x + \frac{c}{x}$$

$$y' = -1 - \frac{c}{x^2} < 0$$

$$c < 0, \quad L_c \text{ ve donada per } y = -x + \frac{c}{x} = \frac{c-x^2}{x}.$$



Ara la funció no s'anul·la

$$y' = -1 - \frac{c}{x^2} = 0 \Leftrightarrow \frac{c}{x^2} = -1$$

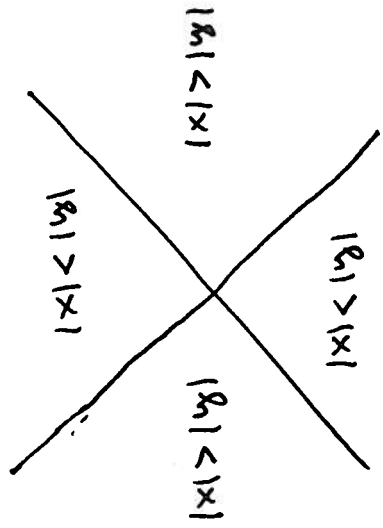
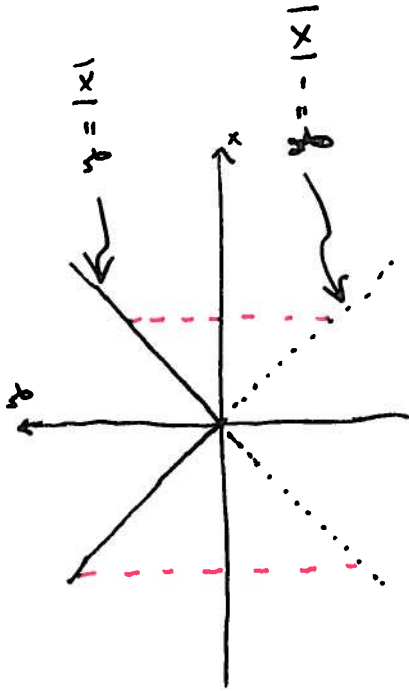
$$\Leftrightarrow x^2 = -c \Leftrightarrow x = \pm \sqrt{-c}$$

(1.18)

Corbes de nivell i gràfica de

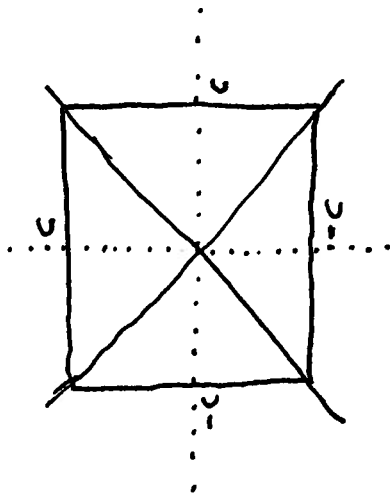
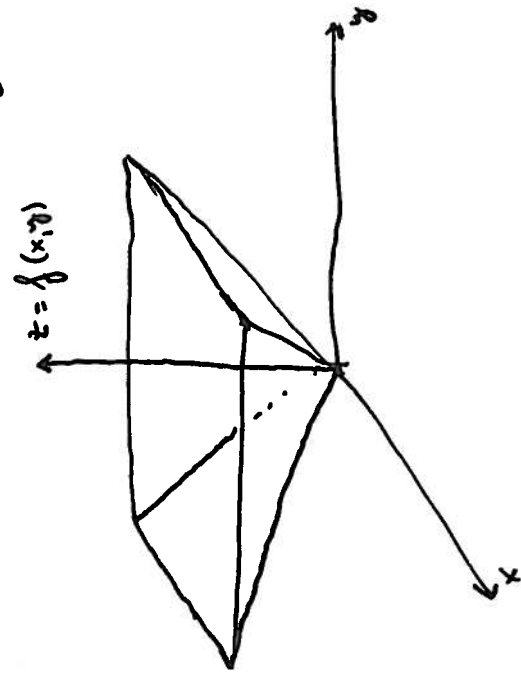
$$f: \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x, y) = \max(|x|, |y|)$$

Primer regim com són les regions $|x| > |y|$, $|x| < |y|$



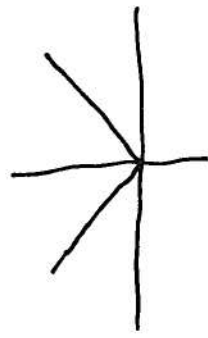
Prenem $c > 0$

$$c = \max(|x|, |y|) = \begin{cases} |x| & \text{si } |x| \geq |y| \\ |y| & \text{si } |x| < |y| \end{cases}$$



Notem que la recerca $y = 0$ dóna

$$g(x) = f(x, 0) = \max(|x|, 0) = |x|$$



1.19

Superfícies de nivell de
 $f(x, y, z) = 4x^2 + y^2 + 9z^2$

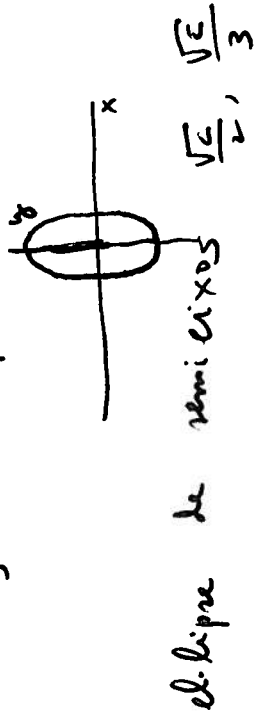
superfícies de nivell:

$$L_c = \phi \quad \text{si } c < 0$$

$$L_0 = \{(0, 0, 0)\}$$

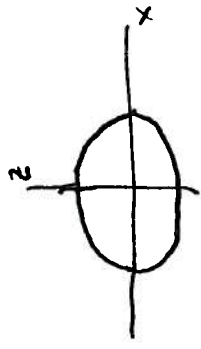
$$\text{si } c > 0 \quad 4x^2 + y^2 + 9z^2 = c \quad \longrightarrow \quad \frac{x^2}{\frac{c}{4}} + \frac{y^2}{c} + \frac{z^2}{\frac{c}{9}} = 1$$

secció $L_c \cap \{z=0\} = \left\{ \frac{x^2}{\frac{c}{4}} + \frac{y^2}{c} = 1 \right\}$ el·lipse de semieixos $\frac{\sqrt{c}}{2}, \sqrt{c}$



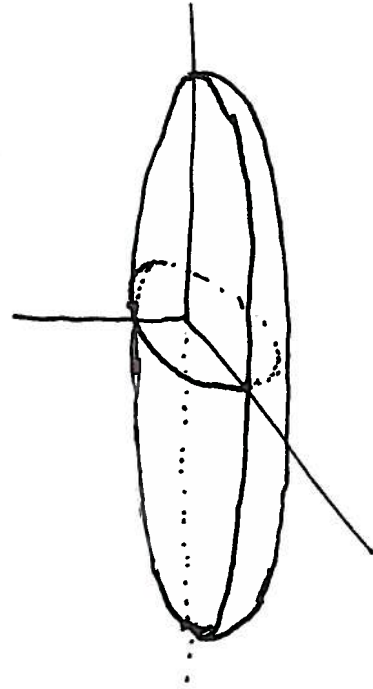
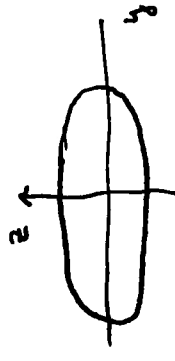
$$L_c \cap \{y=0\} = \left\{ \frac{x^2}{\frac{c}{4}} + \frac{z^2}{\frac{c}{9}} = 1 \right\}$$

el·lipse de semieixos $\frac{\sqrt{c}}{2}, \frac{\sqrt{c}}{3}$



$$L_c \cap \{x=0\} = \left\{ \frac{y^2}{c} + \frac{z^2}{\frac{c}{9}} = 1 \right\}$$

el·lipse de semieixos $\sqrt{c}, \frac{\sqrt{c}}{3}$

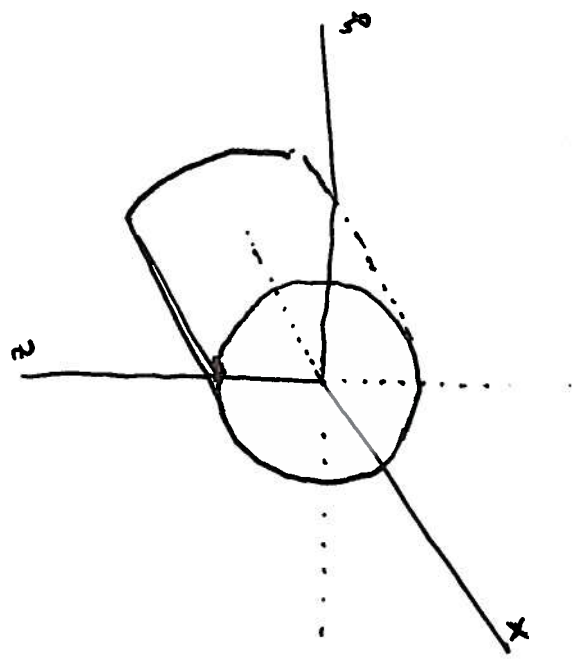


el·lipsoide

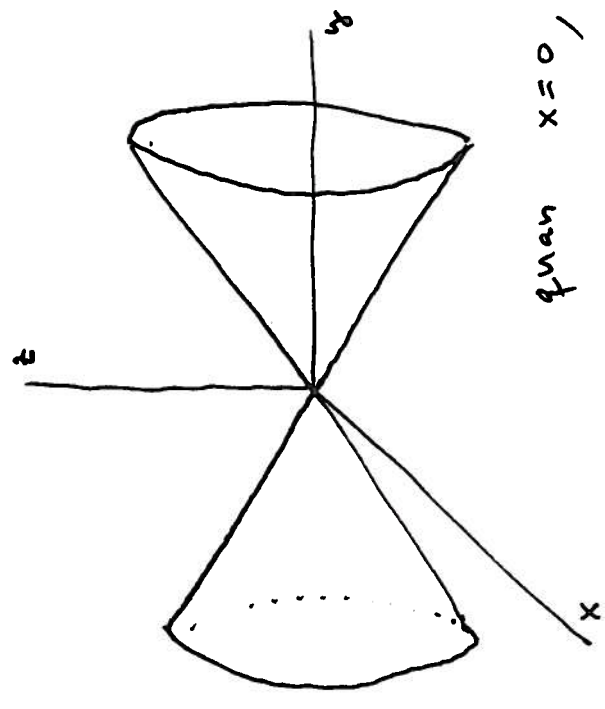
1.20

Esbozen las siguientes superficies

$$y^2 + z^2 = 4$$

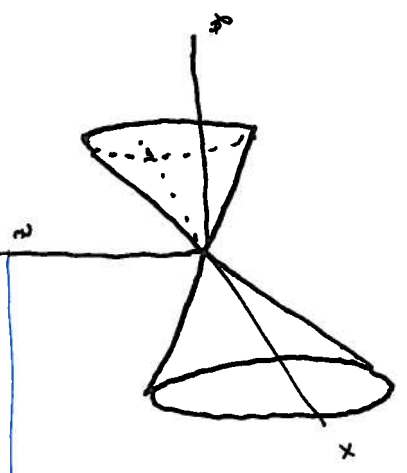


$$y^2 = x^2 + z^2$$

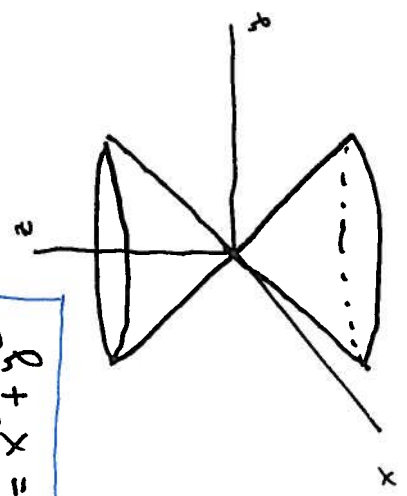


$\text{quan } x=0, y^2=z^2 \Leftrightarrow |y|=|z|$
 $z=0, y^2=x^2 \Leftrightarrow |y|=|x|$
 $y=a, x^2+z^2=a^2$
 circunferencia

$$x^2 = y^2 + z^2$$



$$z^2 = x^2 + y^2$$



(1.2.1)

Corbes de nivell de

$$f: \mathbb{R}^2 \longrightarrow \mathbb{R}, \quad f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & \text{si } (x, y) \neq (0, 0) \\ 0, & \text{si } (x, y) = (0, 0) \end{cases}$$

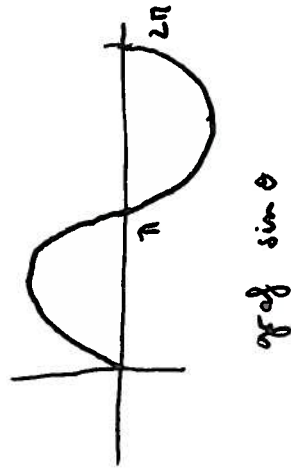
corbes de nivell (en polars)

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\frac{2r \cos \theta r \sin \theta}{r^2 \cos^2 \theta + r^2 \sin^2 \theta} = 2 \sin \theta \cos \theta = \sin 2\theta$$

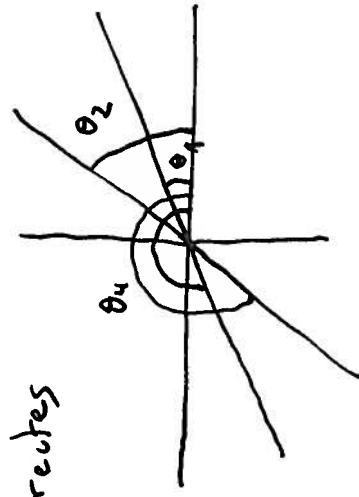
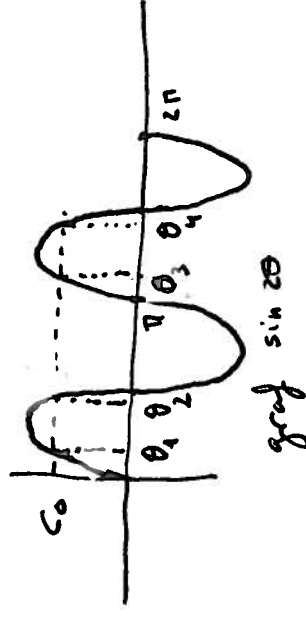
$$\text{Veiem que si } |c| > 1, \quad L_c = \emptyset, \quad L_c = \phi. \quad L_c = \{(r, \theta) \mid \sin 2\theta = c\}$$



$$\text{Si } c = 1, \quad L_c = \left\{ \theta = \frac{\pi}{4}, \frac{5\pi}{4} \right\} \quad \text{dues semirectes}$$

$$\text{Si } -1 < c < 1, \quad L_c \text{ té quatre valors per } \theta \longrightarrow \text{quatre semirectes}$$

$$\text{Si } c = -1, \quad L_c = \left\{ \theta = \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$$

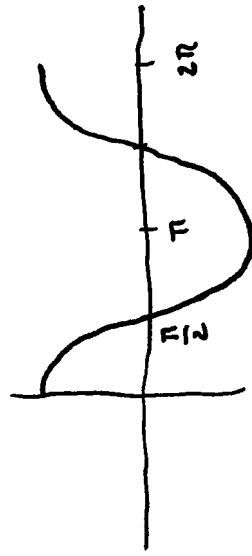


1.2.2

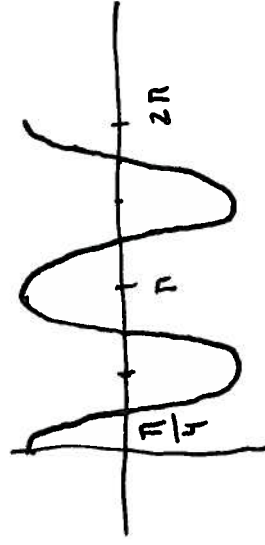
Corbes de nivell respecte als eixos x i y de
 $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ donada en coordenades polars per

$$f(r, \theta) = \frac{\cos 2\theta}{r^2}$$

$$\frac{\cos 2\theta}{r^2} = c \quad \longrightarrow \quad r = \sqrt{\frac{\cos 2\theta}{c}}$$

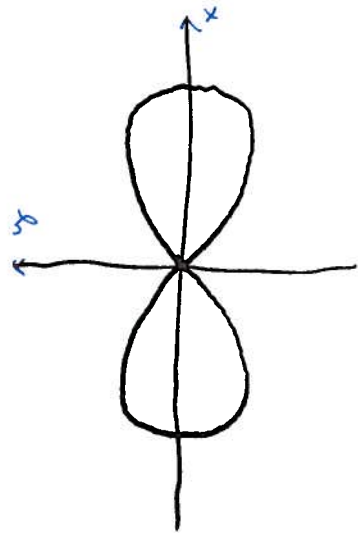


graf $\cos 2\theta$

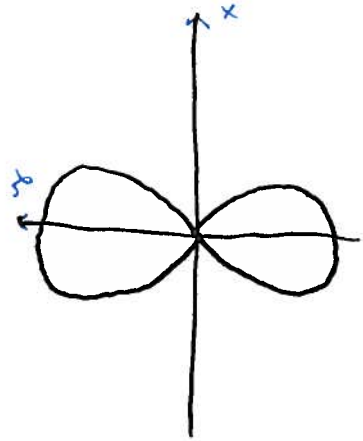


graf $\sin 2\theta$

$c > 0$



$c > 0$



$c < 0$

