Solucions problemes tema 5: Tests d'Hipòtesis

1. (a)

$$P(x \in \{1,2\}|H_0) = P(X=1|H_0) + P(X=2|H_0) = \frac{1}{50} + \frac{1}{50} = \frac{1}{25} = 0.04 \leq 0.05$$

$$P(x \in \{3\}|H_0) = P(X=3|H_0) = \frac{1}{25} = 0.04 \leq 0.05$$

   (b) Si $x = 3 \notin W_1 \Rightarrow$ acceptaríem H_0.
   Si $x = 3 \in W_2 \Rightarrow$ rebutjaríem H_0

   (c) Prenent $a = 1$, per exemple, tenim

$$\text{si } x \in W_1 \quad f_1(x) = \frac{1}{4} \geq af_0(x) = \frac{1}{50}$$

$$\text{si } x \notin W_1 \quad f_1(x)\frac{1}{48} \leq af_0(x) = \frac{1}{25}$$

   Així doncs, W_1 és regió crítica d'un test UMP. Omservem, doncs, que no hi ha unicitat.

2.

3.

4. Regió crítica: $W = \{(x_1, \cdots, x_n) \,:\, \sum_{i=1}^{n} (x_i - \mu)^2 > k_\alpha\}$.

5.

6. $W = \{x :\, -2\ln\Lambda > \chi_1^2(\alpha)\}$

$$\Lambda = \frac{\hat{p}^{(n+m)}(1-\hat{p})^{(n+m)(1-\hat{p})}}{\hat{p}_1^{n\hat{p}_1}(1-\hat{p}_1)^{n(1-\hat{p}_1)}\,\hat{p}_2^{m\hat{p}_2}(1-\hat{p}_2)^{m(1-\hat{p}_2)}}$$

7. $W = \{x :\, n\frac{S^2}{\sigma_0^2} < \chi_{n-1}^2(1-\alpha/2)\} \cup \{x :\, n\frac{S^2}{\sigma_0^2} > \chi_{n-1}^2(\alpha/2)\}$

8.

9.