Efficient learning of sparse image representations using homeostatic regulation

Victor Boutin
Aix Marseille Univ, CNRS,
INT, Inst Neurosciences Timone
Marseille, France

Franck Ruffier
Aix Marseille Univ, CNRS,
ISM, Inst Movement Sci,
Marseille, France

Laurent U Perrinet
Aix Marseille Univ, CNRS,
INT, Inst Neurosciences Timone
Marseille, France

Abstract— **One core advantage of sparse representations is the efficient coding of complex signals using compact codes. For instance, it allows for the representation of any sample as a combination of few elements drawn from a large dictionary of basis functions. In the context of the efficient processing of natural images, we propose here that sparse coding can be optimised by designing a proper homeostatic rule regulating the competition between the elements of the dictionary. Indeed, a common design for unsupervised learning rules relies on a gradient descent over a cost measuring representation quality with respect to sparseness. The sparseness constraint introduces a competition which can be optimised by ensuring that each item in the dictionary is selected as often as others. We implemented this rule by introducing a gain normalisation similar to what is observed in biological neural networks. We validated this theoretical insight by challenging the matching pursuit sparse coding algorithm with the same learning rule but with or without homeostasis. Simulations show that for a given homeostasis rule, gradient descent performed similarly the learning of a dataset of image patches. While the coding accuracy did not vary much, including homeostasis changed qualitatively the learned features. In particular, homeostasis results in a more homogeneous set of orientation selective filters, which is closer to what is found in the visual cortex of mammals. To further validate these results, we applied this algorithm to the optimisation of a visual system to be embedded in an aerial robot. In summary, this biologically-inspired learning rule demonstrates that principles observed in neural computations can help improve real-life machine learning algorithms.**

It is observed that simple cell neurones in mammalian primary visual cortex are selective to orientation, spatial localisation, and frequencies [1]. It is demonstrated that developing a coding strategy that maximises sparseness is sufficient to form receptive fields that account for all three of the above properties [2]. Visual items composing natural images are often sparse, such that the brain may use this sparseness to reconstruct images with only a few set of these items [3]. This is supporting the idea that an unsupervised learning algorithm based on sparse coding could be use to describe efficiently image processing in the primary visual cortex.

Most of existing models of unsupervised learning aim at optimising a cost defined on prior assumptions on representation's sparseness. For instance, learning is accomplished in SparseNet [4] on patches taken from natural images as a sequence of coding and learning steps. First, knowing a dictionary of receptive fields Φ_i, the sparse coding is achieved using a gradient descent over a convex cost derived from a sparse prior probability distribution function of the coefficients a_i. Then, knowing this sparse solution, learning is defined as slowly changing the dictionary using Hebbian learning. In general, the parameterisation of the prior has major impacts on results of the sparse coding and thus on the emergence of edge-like receptive fields and requires proper tuning. In fact, the definition of the prior corresponds to an objective sparseness and does not always fit to the observed probability distribution function of the coefficients. In particular, this could be a problem *during* learning if we use the cost to measure representation efficiency for this learning step. An alternative is to use a more generic ℓ_0 norm sparseness, by simply counting the number of non-zero coefficients:

$$\mathscr{C}_0(\mathbf{a}|\mathbf{I},\Phi) = \frac{1}{2\sigma_n^2}\|\mathbf{I}-\Phi\mathbf{a}\|^2 + \lambda\|\mathbf{a}\|_0$$

It was found that by using an algorithm like Matching Pursuit, the learning algorithm could provide results similar to SparseNet, but without the need of parametric assumptions on the prior [5]. However, we observed that this class of algorithms could lead to solutions corresponding to a local minimum of the objective function: Some solutions seem as efficient as others for representing the signal but do not represent edge-like features homogeneously. In particular, during the early learning phase, some cells may learn "faster" than others. There is the need for a homeostasis mechanism that will ensure convergence of learning. The goal of this work is to study the specific role of homeostasis in learning sparse representations and to propose a homeostasis mechanism which optimises the learning of an efficient neural representation.

To achieve this, we first formulate analytically the problem of representation efficiency in a population of sensory neurones. For the particular ℓ_0 norm sparseness, we show that sparseness is optimal, in term of Shannon entropy, when average activity within the neural population is uniformly balanced (i.e. each neurone is selected with the same probability when encoding a large set of data). To achieve this uniformity, we define an homeostatic gain control mechanism based on histogram equalisation, that is in transforming coefficients in terms of z-scores $z_i(a_i) = P(\cdot > a_i)$. The cumulative distribution z_i for each coefficient of the sparse vector is calculated using Hebbian learning to smooth its evolution during learning. At the coding level, this z-score function is incorporated in the matching step of the matching pursuit algorithm, to modulate the choice of the most as that with the maximal z-score: $i^* = \text{Argmax}_i z_i(a_i)$. The rest of the algorithm is left unchanged.

We compared qualitatively the set Φ of receptive filters generated by the proposed algorithm when the homeostasis is first turned-off and then enabled (see Fig. 1). A more quantitative study of the coding is shown by comparing selection distribution of sparse coefficients when the homeostasis mechanism is turned on (see Fig. 2). We demonstrate that forcing the learning activity to be uniformly spread among all receptive fields results in a faster convergence of the representation error, and in an increase of the Shanon entropy. Finally, an interesting perspective is to apply the homeostatic regulation algorithm in a classical fully connected deep-learning neural network and applied on the MNIST recognition task. By using the sparse coefficients as the input layer of the network, we can compare the performance obtained with and without the homeostatic mechanism. Preliminary results show that the improvement in efficiency is more acute when using sparse representations (5 out of 324 coefficients).

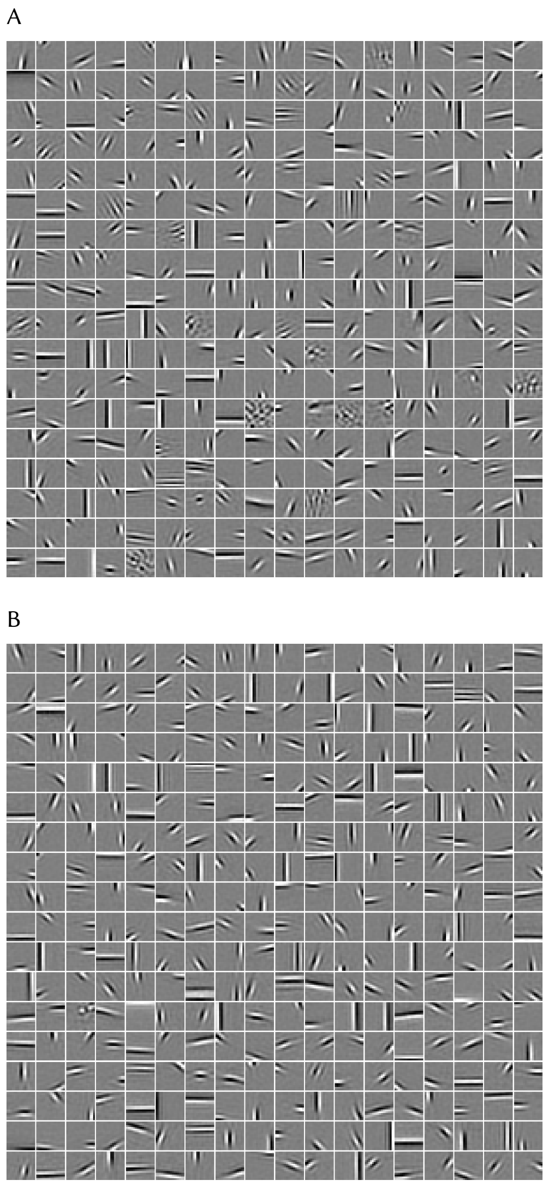

Figure 1. **Role of homeostasis in learning sparse representations**: We show the results of Sparse Hebbian Learning using two different homeostasis algorithms at convergence (20000 learning steps). 324 filters of the same size as the image patches (16×16) are presented in a matrix (separated by a white border). Note that their position in the matrix is arbitrary as in ICA. (A) When switching off the cooperative homeostasis during learning, the corresponding Sparse Hebbian Learning algorithm converges to a set of filters that contains some less localized filters and some high-frequency Gabor functions that correspond to more "textural" features. One may wonder if these filters are inefficient and capturing noise or if they rather correspond to independent features of natural images in the LGM model. (B) Results with the same coding and learning algorithm but by enabling homeostasis.

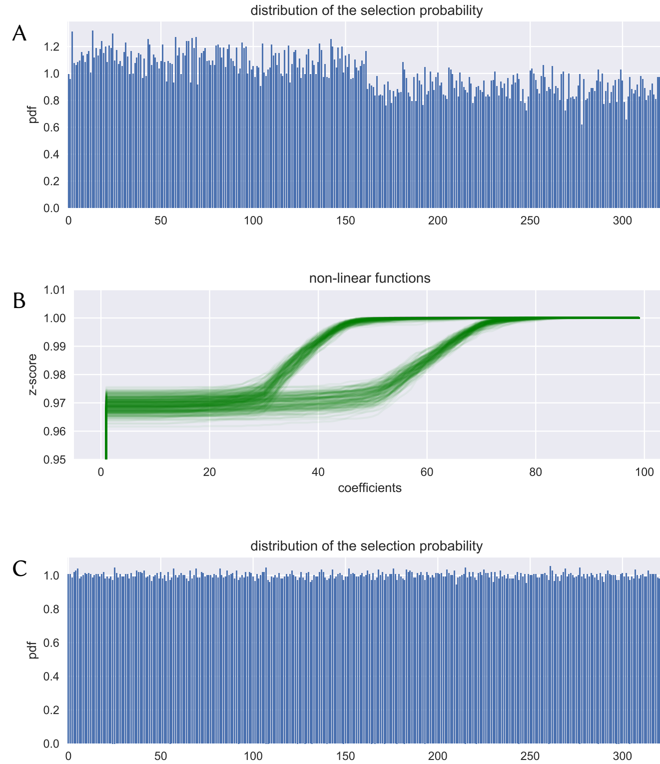

Figure 2. **Quantitative role of homeostasis in sparse coding**: We show the results of Sparse Coding using the two different homeostasis algorithms using surrogate data where each filter was equiprobable but for which we manipulated the first half of the coefficients to be artificially twice as big. (A) Such a situation replicates a situation arising during learning when a sub-group of filters is more active, e. g. because it learned more salient features. Here, we show the probability of the selection of the different filters (normalised to an average of 1) which shows a bias of the standard Matching Pursuit to select more often filters whose activity is higher. (B) Non-linear homeostatic functions learned using Hebbian learning. These functions were initialised as the cumulative distribution function of uniform random variables. Then they are used to modify choices in the Matching step of the Matching Pursuit algorithm. Progressively, the non-linear functions converge to the (hidden) cumulative distributions of the coefficients of the surrogate, clearly showing the group of filters with twice a big coefficients. (C) At convergence, the probability of choosing any filter is uniform. As a result, entropy is maximal, a property which is essential for the optimal representation of signals in distributed networks such as the brain.

Acknowledgment

This work was supported by the Doc2Amu project which received funding from a co-fund with the European Union's Horizon 2020 research and innovation programme and the region Provence Alpes Cote d'Azur. Correspondence and requests for materials should be addressed to LUP (email:laurent.perrinet@univ-amu.fr). Code and supplementary material available at http://invibe.net/LaurentPerrinet/Publications/BoutinRuffierPerrinet17spars.

References

[1] D. H. Hubel and T. N. Wiesel, "Receptive fields and functional architecture of monkey striate cortex", *The Journal of Physiology*, vol. 195, no. 1, pp. 215–243, 1968.

[2] B. A. Olshausen and D. J. Field, "Emergence of simple-cell receptive field properties by learning a sparse code for natural images", *Nature*, vol. 381, no. 6583, p. 607, 1996.

[3] L. U. Perrinet, "Sparse models for computer vision", in *Biologically Inspired Computer Vision*, G. Cristóbal, L. Perrinet, and M. S. Keil, Eds., Wiley-VCH Verlag, 2015, ch. 13.

[4] B. A. Olshausen and D. J. Field, "Sparse coding with an overcomplete basis set: A strategy employed by V1?", *Vision Research*, vol. 37, no. 23, pp. 3311–3325, 1997.

[5] L. U. Perrinet, "Role of homeostasis in learning sparse representations", *Neural Computation*, vol. 22, no. 7, pp. 1812–36, 2010.