

Notes on Controllability of Linear Systems

Controllability

Definition:

A system described by

$$\dot{x} = Ax + Bu$$

with state variables $x \in \mathbb{R}^n$ and control variables $u \in \mathbb{R}^r$ is said to be *controllable* if, for any initial condition $a \in \mathbb{R}^n$ and any final condition $b \in \mathbb{R}^n$, and for some $T > 0$, it is possible to find a function

$$u : [0, T] \rightarrow \mathbb{R}^r$$

such that

$$x(0) = a \quad \text{and} \quad x(T) = b.$$

Remarks:

The quantities a and b are characteristics of the system (the plant).

Examples:

a) If $B = 0$, the system

$$\dot{x} = Ax$$

is not controllable. This can be seen by taking

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0 \text{ and } b \neq 0.$$

b) If $B = I$ (where $r = n$), the system

$$\dot{x} = Ax + u$$

has the following general solution: