

# UNIVERSIDADE FEDERAL DE GOIÁS

## TEAM REFERENCE MATERIAL

2018 South America/Brazil Regional Contest

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## Augmenting Path

```
//Augmenting Path for MCBM
#include <bits/stdc++.h>
using namespace std;

const int N = 1000;
vector<int> g[N];

int augment(int v, vector<int> &match, vector<bool> &visited) {
    if (visited[v]) return 0;
    visited[v] = true;

    for (size_t i = 0; i < g[v].size(); i++) {
        int u = g[v][i];

        if (match[u] == -1 || augment(u, match, visited)) {
            match[u] = v;
            return 1;
        }
    }

    return 0;
}
```

## Combinations

```
//Combination Algorithm
#include <bits/stdc++.h>
using namespace std;

const int N = 120;
int comb[N][N];
int combination(int n, int m) {
    return comb[n][m];
}

void pre_compute(int n) {
    assert(n < N-1);
    for (int i = 0; i <= n; i++) {
```

## Debug message

```
//Lib - Debug message for CF
#include <bits/stdc++.h>
using namespace std;

template<typename T>
void debug(T value) {
#ifdef ONLINE_JUDGE
    cout << value << endl;
#endif
}

template <typename T, typename... Args>
```

```
int matching(int left_sz, int right_sz) {
    vector<int> match(left_sz + right_sz, -1);
    vector<bool> visited;

    int mcmb = 0;

    for (int i = 0; i < left_sz; i++) {
        visited.assign(left_sz, false);
        mcmb += augment(i, match, visited);
    }

    return mcmb;
}

int main() {
    ios::sync_with_stdio(false);

    return 0;
}
```

```
    comb[i][0] = comb[i][i] = 1;
    for (int j = 0; j < i; j++) {
        comb[i][j] = comb[i-1][j-1] + comb[i-1][j];
    }
}

int main() {
    ios::sync_with_stdio(false);

    return 0;
}
```

```
void debug(T value, Args... args) {
#ifdef ONLINE_JUDGE
    cout << value;
    debug(args...);
#endif
}

int main() {
    debug("this_message_do_not_will_appear_in_CF.");
    return 0;
}
```

## Dinic (Max Flow)

```
#include <bits/stdc++.h>
using namespace std;

struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(cap) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int v, int u, long long cap) {
        edges.push_back(FlowEdge(v, u, cap));
        edges.push_back(FlowEdge(u, v, 0));
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
    }
};
```

## Euler Tour

```
// Euler Tour
#include <bits/stdc++.h>
using namespace std;

const int N = 100 * 1000 + 10;
vector<int> g[N];
int sz[N], ini[N], cnt = 0;
int64_t path[N];
```

```
        return level[t] != -1;
    }

    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap - edges[id].flow < 1)
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap - edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }

    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }

    int main() {
        return 0;
    }
}
```

```
int dfs(int v, int p) {
    ini[v] = cnt;
    path[cnt] = v;
    cnt++;
    sz[v] = 1;

    for (const auto &u : g[v]) {
        if (u == p) continue;
```

```

        sz[v] += dfs(u, v);
    }

    return sz[v];
}

int main() {
    int n;

    cin >> n;

    for (int i = 0, x, y; i < n-1; i++) {
        cin >> x >> y;
        g[x].push_back(y);
        g[y].push_back(x);
    }

```

```

    dfs(0, -1);

    for (int i = 0; i < cnt; i++) {
        cout << path[i] << "\n";
    }
    cout << endl;

    int v = 1;
    // subarvore de v
    for (int i = ini[v]; i < ini[v]+sz[v]; i++) {
        cout << path[i] << "\n";
    }
    cout << endl;

    return 0;
}

```

## Extended Euclids

```

// Euclids Algorithm and Modular Multiplicative Inverse
#include <bits/stdc++.h>
using namespace std;

pair<int, int> extendedEuclids(int a, int b) {
    if (b == 0) {
        return {1, 0};
    } else {
        auto p = extendedEuclids(b, a % b);
        return {p.second,
                p.first - floor((double)a / b) * p.second};
    }
}

```

```

int inverseMod(int a, int n) {
    auto p = extendedEuclids(a, n);
    return (p.first % n + n) % n;
}

int main() {
    ios::sync_with_stdio(false);

    return 0;
}

```

## Fenwick 2D

```

//Lib - Fenwick Tree 2D
#include <bits/stdc++.h>
using namespace std;

const int N = 1010;
int ft[N][N];

int last(int v) {
    return v & -v;
}

void update(int x, int y, int val) {
    for (int i = x; i < N; i += last(i)) {
        for (int j = y; j < N; j += last(j)) {
            ft[i][j] += val;
        }
    }
}

int query(int x, int y) {

```

```

    int s = 0;
    for (int i = x; i > 0; i -= last(i)) {
        for (int j = y; j > 0; j -= last(j)) {
            s += ft[i][j];
        }
    }
    return s;
}

int sum(int x1, int y1, int x2, int y2) {
    return query(x2, y2) - query(x2, y1 - 1)
        - query(x1 - 1, y2) + query(x1 - 1, y1 - 1);
}

int main() {
    ios::sync_with_stdio(false);
    return 0;
}

```

## Fenwick 2D (Xor)

```
//Lib - Fenwick Tree 2D XOR
#include <bits/stdc++.h>
using namespace std;

const int N = 1010;
int64_t dp[4][N][N];

int parity(int x, int y) {
    int res = 0;
    if (x % 2) res++;
    if (y % 2) res += 2;
    return res;
}

int64_t query(int x, int y) {
    int64_t res = 0;
    x++;
    y++;
    int whichSquare = parity(x, y);
    for (int i = x; i > 0; i -= (i & (-i))) {
        for (int j = y; j > 0; j -= (j & (-j))) {
            res ^= dp[whichSquare][i][j];
        }
    }
    return res;
}

void update(int x, int y, int64_t val) {
    x++;
    y++;
    int whichSquare = parity(x, y);
    for (int i = x; i <= N; i += (i & (-i))) {
        for (int j = y; j <= N; j += (j & (-j))) {
            dp[whichSquare][i][j] ^= val;
        }
    }
}
```

## Fenwick

```
#include <bits/stdc++.h>
using namespace std;

const int N = 2*1e5+10;

int bit[N];

// execute the query [1, x]
int query(int x) {
    int s = 0;

    while (x) {
        s += bit[x];
        x -= (x & -x);
    }

    return s;
}
```

```
int query2d(int x1, int y1, int x2, int y2) {
    int r = query(x2, y2);
    r ^= query(x2, y1 - 1);
    r ^= query(x1 - 1, y2);
    r ^= query(x1 - 1, y1 - 1);
    return r;
}

void update2d(int x1, int y1, int x2, int y2, int val) {
    update(x1, y1, val);
    update(x1, y2 + 1, val);
    update(x2 + 1, y1, val);
    update(x2 + 1, y2 + 1, val);
}

int main() {
    ios::sync_with_stdio(false);

    int n, m, p;
    int a, b, c, d;
    cin >> n >> m;
    for (int i = 0; i < m; i++) {
        cin >> p >> a >> b >> c >> d;

        update2d(a, b, c, d, p);
    }

    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            cout << query2d(i, j, i, j);
            if (j < n)
                cout << " ";
        }
        cout << endl;
    }
    return 0;
}
```

```
}

// execute the update [1,x]
void update(int x, int v) {
    while (x <= N) {
        bit[x] += v;
        x += (x & -x);
    }
}

int main() {
    ios::sync_with_stdio(false);

    return 0;
}
```

## Floyd Warshall

```
// Lib - Floyd Warshall
#include <bits/stdc++.h>
using namespace std;

const int N = 110, inf = 1e9;
int64_t dist[N][N];

void floyd_warshall(int n) {
    for (int v = 1; v <= n; v++) {
        for (int u = 1; u <= n; u++) {
            for (int q = 1; q <= n; q++) {
                dist[u][q] = min(dist[u][q], dist[u][v] + dist[v][q]);
            }
        }
    }
}

int main() {
    int t, n, m;

    cin >> t;

    while (t--) {
        cin >> n >> m;

        for (int v = 1; v <= n; v++) {
            for (int u = 1; u <= n; u++) {
```

```
                if (u == v)
                    dist[v][u] = 0;
                else
                    dist[v][u] = inf;
            }
        }

        int64_t d;
        for (int i = 0, x, y; i < m; i++) {
            cin >> x >> y >> d;
            dist[x][y] = min(dist[x][y], d);
            dist[y][x] = min(dist[y][x], d);
        }

        floyd_warshall(n);

        for (int v = 1; v <= n; v++) {
            for (int u = 1; u <= n; u++) {
                cout << dist[v][u] << " ";
            }
            cout << "\n";
        }
    }
    return 0;
}
```

## Gaussian Elimination

```
// Lib - Gaussian Elimination
// UVA 11319 - Stupid Sequence
#include <bits/stdc++.h>
using namespace std;

const int N = 7, M = 7; // N variables, M equations
const int INF = 1e9;
const double EPS = 1e-9;

struct Matrix {
    double m[M][N+1];
    double* operator[](size_t i) {
        return m[i];
    };
};

struct Vector {
    double v[N];
    double& operator[](size_t i) {
        return v[i];
    };
};

// n variables, m equations
// note: for reducing error, implicit pivoting can be used
int elimination(Matrix &aug, int n, int m, Vector &ans) {
    int where[N];
```

```
    memset(where, -1, sizeof(where));

    for (int col = 0, row = 0, lgst; col < n && row < m; col++) {
        lgst = row;
        for (int i = row+1; i < m; i++) {
            if (fabs(aug.m[i][col]) > fabs(aug.m[lgst][col])) {
                lgst = i;
            }
        }

        if (fabs(aug.m[lgst][col]) < EPS) // independent variable or impossible system
            continue;

        swap(aug.m[row], aug.m[lgst]);

        where[col] = row; // assign a row for variable of column col

        // zero elements in column col (except in row)
        for (int i = 0; i < m; i++) {
            if (i == row) continue;

            double c = aug.m[i][col] / aug.m[row][col];

            for (int j = col; j <= n; j++) {
                aug.m[i][j] -= aug.m[row][j] * c;
            }
        }
    }
```

```

    row++;
}

// assign answer to variables
memset(ans.v, 0, sizeof(ans.v));
for (int j = 0; j < n; j++) {
    if (where[j] != -1) {
        ans.v[j] = aug.m[where[j]][n] / aug.m[where[j]][j];
    }
}

// verify if system is impossible
for (int i = 0; i < m; i++) {
    double sum = 0;
    for (int j = 0; j < n; j++) {
        sum += (ans.v[j] * aug.m[i][j]);
    }

    if (fabs(sum - aug.m[i][n]) > EPS) {
        return 0; // impossible system
    }
}

for (int j = 0; j < n; j++) {
    if (where[j] == -1) // infinite solutions
        return INF;
}

return 1;
}

int main() {
    int t;
    int n = 7;
    const int Z = 1500;
    uint64_t v, ans2[N], y[Z];

    Matrix aug;
    Vector ans;

    cin >> t;
    while (t--) {
        for (int i = 0; i < n; i++) {
            v = 1;
            for (int j = 0; j < n; j++) {
                aug[i][j] = v;
                v *= (i+1);
            }
        }

        for (int i = 0; i < Z; i++) {
            cin >> y[i];
        }
    }
}

```

```

for (int i = 0; i < n; i++) {
    aug[i][n] = y[i];
}

bool has_solution = true;

if (elimination(aug, n, n, ans) != 1) {
    has_solution = false;
} else {
    for (int j = 0; j < n; j++) {
        if (ans[j] < -EPS) {
            // it is negative
            has_solution = false;
            break;
        }

        ans2[j] = static_cast<uint64_t>(ans[j] + EPS);

        if (ans2[j] > 1000) {
            // greater than 1000
            has_solution = false;
            break;
        }
    }

    // verify if ans2 vector generate the results of function of input
    for (int i = 0; i < 1500; i++) {
        uint64_t f = 0;
        v = 1;
        for (int j = 0; j < n; j++) {
            f += ans2[j] * v;
            v *= (i+1);
        }

        if (f != y[i]) {
            has_solution = false;
            break;
        }
    }
}

if (has_solution) {
    for (int j = 0; j < n; j++) {
        cout << ans2[j] << "\n"[j == n-1];
    }
} else {
    cout << "This_is_a_smart_sequence!" << endl;
}

return 0;
}

```

## Generate all combinations

```
#include <bits/stdc++.h>
using namespace std;

void gen_comb(vector<int> a, int r) {
    assert(r <= (int) a.size());

    vector<bool> v(a.size());
    fill(v.begin(), v.begin() + r, true);

    do {
        for (size_t i = 0; i < v.size(); i++) {
            if (v[i]) {
                cout << a[i] << "_";
            }
        }
        cout << "\n";
    } while (prev_permutation(v.begin(), v.end()));
}
```

## Geometry

```
#include <bits/stdc++.h>
using namespace std;

#define INF 1e9
#define EPS 1e-9
#define PI acos(-1.0) // important constant; alternative #define PI (2.0 * acos(0.0))

double DEG_to_RAD(double d) {
    return d * PI / 180.0;
}

double RAD_to_DEG(double r) {
    return r * 180.0 / PI;
}

// struct point_i { int x, y; }; // basic raw form, minimalist mode
struct point_i {
    int x, y; // whenever possible, work with point_i
    point_i() {
        x = y = 0; // default constructor
    }
    point_i(int _x, int _y) : x(_x), y(_y) {}
}; // user-defined

struct point {
    double x, y; // only used if more precision is needed
    point() {
        x = y = 0.0; // default constructor
    }
    point(double _x, double _y) : x(_x), y(_y) {} // user-defined
    bool operator < (point other) const { // override less than operator
        if (fabs(x - other.x) > EPS) // useful for sorting
            return x < other.x; // first criteria, by x-coordinate
        return y < other.y;
    } // second criteria, by y-coordinate
    // use EPS (1e-9) when testing equality of two floating points
    bool operator == (point other) const {
        return (fabs(x - other.x) < EPS && (fabs(y - other.y) < EPS));
    }
};
```

```
    } while (prev_permutation(v.begin(), v.end()));
}

int main() {
    const int N = 50, R = 4;
    vector<int> a(N);

    for (int i = 0; i < N; i++) {
        a[i] = rand();
    }

    gen_comb(a, R);
    return 0;
}
```

```
double dist(point p1, point p2) { // Euclidean distance
    // hypot(dx, dy) returns sqrt(dx * dx + dy * dy)
    return hypot(p1.x - p2.x, p1.y - p2.y);
} // return double

// rotate p by theta degrees CCW w.r.t origin (0, 0)
point rotate(point p, double theta) {
    double rad = DEG_to_RAD(theta); // multiply theta with PI / 180.0
    return point(p.x * cos(rad) - p.y * sin(rad),
                p.x * sin(rad) + p.y * cos(rad));
}

struct line {
    double a, b, c;
}; // a way to represent a line

// the answer is stored in the third parameter (pass by reference)
void pointsToLine(point p1, point p2, line &l) {
    if (fabs(p1.x - p2.x) < EPS) { // vertical line is fine
        l.a = 1.0;
        l.b = 0.0;
        l.c = -p1.x; // default values
    } else {
        l.a = -(double)(p1.y - p2.y) / (p1.x - p2.x);
        l.b = 1.0; // IMPORTANT: we fix the value of b to 1.0
        l.c = -(double)(l.a * p1.x) - p1.y;
    }
}

// not needed since we will use the more robust form: ax + by + c = 0 (see above)
struct line2 {
    double m, c;
}; // another way to represent a line

int pointsToLine2(point p1, point p2, line2 &l) {
    if (abs(p1.x - p2.x) < EPS) { // special case: vertical line
        l.m = INF; // l contains m = INF and c = x_value
        l.c = p1.x; // to denote vertical line x = x_value
        return 0; // we need this return variable to differentiate result
    }
    else {
```



```

    l.m = (double)(p1.y - p2.y) / (p1.x - p2.x);
    l.c = p1.y - l.m * p1.x;
    return l; // l contains m and c of the line equation y = mx + c
}

bool areParallel(line l1, line l2) { // check coefficients a & b
    return (fabs(l1.a-l2.a) < EPS) && (fabs(l1.b-l2.b) < EPS);
}

bool areSame(line l1, line l2) { // also check coefficient c
    return areParallel(l1,l2) && (fabs(l1.c - l2.c) < EPS);
}

// returns true (+ intersection point) if two lines are intersect
bool areIntersect(line l1, line l2, point &p) {
    if (areParallel(l1, l2)) return false; // no intersection
    // solve system of 2 linear algebraic equations with 2 unknowns
    p.x = (l2.b * l1.c - l1.b * l2.c) / (l2.a * l1.b - l1.a * l2.b);
    // special case: test for vertical line to avoid division by zero
    if (fabs(l1.b) > EPS) p.y = -(l1.a * p.x + l1.c);
    else p.y = -(l2.a * p.x + l2.c);
    return true;
}

struct vec {
    double x, y; // name: 'vec' is different from STL vector
    vec(double _x, double _y) : x(_x), y(_y) {}
};

vec toVec(point a, point b) { // convert 2 points to vector a->b
    return vec(b.x - a.x, b.y - a.y);
}

vec scale(vec v, double s) { // nonnegative s = [1 .. 1 .. >1]
    return vec(v.x * s, v.y * s);
} // shorter.same.longer

point translate(point p, vec v) { // translate p according to v
    return point(p.x + v.x, p.y + v.y);
}

// convert point and gradient/slope to line
void pointSlopeToLine(point p, double m, line &l) {
    l.a = -m; // always -m
    l.b = 1; // always 1
    l.c = -((l.a * p.x) + (l.b * p.y));
} // compute this

void closestPoint(line l, point p, point &ans) {
    line perpendicular; // perpendicular to l and pass through p
    if (fabs(l.b) < EPS) { // special case 1: vertical line
        ans.x = -(l.c);
        ans.y = p.y;
        return;
    }

    if (fabs(l.a) < EPS) { // special case 2: horizontal line
        ans.x = p.x;
        ans.y = -(l.c);
        return;
    }

```

```

    pointSlopeToLine(p, 1 / l.a, perpendicular); // normal line
    // intersect line l with this perpendicular line
    // the intersection point is the closest point
    areIntersect(l, perpendicular, ans);
}

```

```

// returns the reflection of point on a line
void reflectionPoint(line l, point p, point &ans) {
    point b;
    closestPoint(l, p, b); // similar to distToLine
    vec v = toVec(p, b); // create a vector
    ans = translate(translate(p, v), v);
} // translate p twice

```

```

double dot(vec a, vec b) {
    return (a.x * b.x + a.y * b.y);
}

```

```

double norm_sq(vec v) {
    return v.x * v.x + v.y * v.y;
}

```

```

// returns the distance from p to the line defined by
// two points a and b (a and b must be different)
// the closest point is stored in the 4th parameter (byref)
double distToLine(point p, point a, point b, point &c) {
    // formula: c = a + u * ab
    vec ap = toVec(a, p), ab = toVec(a, b);
    double u = dot(ap, ab) / norm_sq(ab);
    c = translate(a, scale(ab, u)); // translate a to c
    return dist(p, c);
} // Euclidean distance between p and c

```

```

// returns the distance from p to the line segment ab defined by
// two points a and b (still OK if a == b)
// the closest point is stored in the 4th parameter (byref)
double distToLineSegment(point p, point a, point b, point &c) {
    vec ap = toVec(a, p), ab = toVec(a, b);
    double u = dot(ap, ab) / norm_sq(ab);
    if (u < 0.0) {
        c = point(a.x, a.y); // closer to a
        return dist(p, a);
    } // Euclidean distance between p and a
    if (u > 1.0) {
        c = point(b.x, b.y); // closer to b
        return dist(p, b);
    } // Euclidean distance between p and b
    return distToLine(p, a, b, c);
} // run distToLine as above

```

```

double angle(point a, point o, point b) { // returns angle aob in rad
    vec oa = toVec(o, a), ob = toVec(o, b);
    return acos(dot(oa, ob) / sqrt(norm_sq(oa) * norm_sq(ob)));
}

```

```

double cross(vec a, vec b) {
    return a.x * b.y - a.y * b.x;
}

```

```

//// another variant
//int area2(point p, point q, point r) { // returns 'twice' the area of this triangle A-B-C
// return p.x * q.y - p.y * q.x +
// q.x * r.y - q.y * r.x +

```

```
// r.x * p.y - r.y * p.x;
//}

// note: to accept collinear points, we have to change the '> 0'
// returns true if point r is on the left side of line pq
bool ccw(point p, point q, point r) {
    return cross(toVec(p, q), toVec(p, r)) > 0;
}

// returns true if point r is on the same line as the line pq
bool collinear(point p, point q, point r) {
    return fabs(cross(toVec(p, q), toVec(p, r))) < EPS;
}

// circles

int insideCircle(point_i p, point_i c, int r) { // all integer version
    int dx = p.x - c.x, dy = p.y - c.y;
    int Euc = dx * dx + dy * dy, rSq = r * r; // all integer
    return Euc < rSq ? 0 : Euc == rSq ? 1 : 2;
} //inside/border/outside

bool circle2PtsRad(point p1, point p2, double r, point &c) {
    double d2 = (p1.x - p2.x) * (p1.x - p2.x) +
        (p1.y - p2.y) * (p1.y - p2.y);
    double det = r * r / d2 - 0.25;
    if (det < 0.0) return false;
    double h = sqrt(det);
    c.x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
    c.y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;
    return true;
} // to get the other center, reverse p1 and p2

double perimeter(double ab, double bc, double ca) {
    return ab + bc + ca;
}

double perimeter(point a, point b, point c) {
    return dist(a, b) + dist(b, c) + dist(c, a);
}

double area(double ab, double bc, double ca) {
    // Heron's formula, split sqrt(a * b) into sqrt(a) * sqrt(b); in implementation
    double s = 0.5 * perimeter(ab, bc, ca);
    return sqrt(s) * sqrt(s - ab) * sqrt(s - bc) * sqrt(s - ca);
}

double area(point a, point b, point c) {
    return area(dist(a, b), dist(b, c), dist(c, a));
}

double rInCircle(double ab, double bc, double ca) {
    return area(ab, bc, ca) / (0.5 * perimeter(ab, bc, ca));
}

double rInCircle(point a, point b, point c) {
    return rInCircle(dist(a, b), dist(b, c), dist(c, a));
}

// assumption: the required points/lines functions have been written
// returns 1 if there is an inCircle center, returns 0 otherwise
// if this function returns 1, ctr will be the inCircle center
// and r is the same as rInCircle
```

```
int inCircle(point p1, point p2, point p3, point &ctr, double &r) {
    r = rInCircle(p1, p2, p3);
    if (fabs(r) < EPS) return 0; // no inCircle center

    line l1, l2; // compute these two angle bisectors
    double ratio = dist(p1, p2) / dist(p1, p3);
    point p = translate(p2, scale(toVec(p2, p3), ratio / (1 + ratio)));
    pointsToLine(p1, p, l1);

    ratio = dist(p2, p1) / dist(p2, p3);
    p = translate(p1, scale(toVec(p1, p3), ratio / (1 + ratio)));
    pointsToLine(p2, p, l2);

    areIntersect(l1, l2, ctr); // get their intersection point
    return 1;
}

double rCircumCircle(double ab, double bc, double ca) {
    return ab * bc * ca / (4.0 * area(ab, bc, ca));
}

double rCircumCircle(point a, point b, point c) {
    return rCircumCircle(dist(a, b), dist(b, c), dist(c, a));
}

// assumption: the required points/lines functions have been written
// returns 1 if there is a circumCenter center, returns 0 otherwise
// if this function returns 1, ctr will be the circumCircle center
// and r is the same as rCircumCircle
int circumCircle(point p1, point p2, point p3, point &ctr, double &r) {
    double a = p2.x - p1.x, b = p2.y - p1.y;
    double c = p3.x - p1.x, d = p3.y - p1.y;
    double e = a * (p1.x + p2.x) + b * (p1.y + p2.y);
    double f = c * (p1.x + p3.x) + d * (p1.y + p3.y);
    double g = 2.0 * (a * (p3.y - p2.y) - b * (p3.x - p2.x));
    if (fabs(g) < EPS) return 0;

    ctr.x = (d*e - b*f) / g;
    ctr.y = (a*f - c*e) / g;
    r = dist(p1, ctr); // r = distance from center to 1 of the 3 points
    return 1;
}

// returns true if point d is inside the circumCircle defined by a,b,c
int inCircumCircle(point a, point b, point c, point d) {
    return (a.x - d.x) * (b.y - d.y) * ((c.x - d.x) * (c.x - d.x) +
        (c.y - d.y) * (c.y - d.y)) +
        (a.y - d.y) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) * (b.y - d.y))
        * (c.x - d.x) +
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) * (b.x - d.x)
        * (c.y - d.y) -
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) * (b.y - d.y)
        * (c.x - d.x) -
        (a.y - d.y) * (b.x - d.x) * ((c.x - d.x) * (c.x - d.x) + (c.y - d.y)
        * (c.y - d.y)) -
        (a.x - d.x) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) * (b.y - d.y))
        * (c.y - d.y) > 0 ? 1 : 0;
}

bool canFormTriangle(double a, double b, double c) {
    return (a + b > c) && (a + c > b) && (b + c > a);
}
```

```
// returns the perimeter, which is the sum of Euclidian distances
// of consecutive line segments (polygon edges)
double perimeter(const vector<point> &P) {
    double result = 0.0;
    for (int i = 0; i < (int)P.size()-1; i++) // remember that P[0] = P[n-1]
        result += dist(P[i], P[i+1]);
    return result;
}

// returns the area, which is half the determinant
double area(const vector<point> &P) {
    double result = 0.0, x1, y1, x2, y2;
    for (int i = 0; i < (int)P.size()-1; i++) {
        x1 = P[i].x;
        x2 = P[i+1].x;
        y1 = P[i].y;
        y2 = P[i+1].y;
        result += (x1 * y2 - x2 * y1);
    }
    return fabs(result) / 2.0;
}

// returns true if we always make the same turn while examining
// all the edges of the polygon one by one
bool isConvex(const vector<point> &P) {
    int sz = (int)P.size();
    if (sz <= 3) return false; // a point/sz=2 or a line/sz=3 is not convex
    bool isLeft = ccw(P[0], P[1], P[2]); // remember one result
    for (int i = 1; i < sz-1; i++) // then compare with the others
        if (ccw(P[i], P[i+1], P[(i+2) % sz]) != isLeft)
            return false; // different sign -> this polygon is concave
    return true;
} // this polygon is convex

// returns true if point p is in either convex/concave polygon P
bool inPolygon(point pt, const vector<point> &P) {
    if ((int)P.size() == 0) return false;
    double sum = 0; // assume the first vertex is equal to the last vertex
    for (int i = 0; i < (int)P.size()-1; i++) {
        if (ccw(pt, P[i], P[i+1]))
            sum += angle(P[i], pt, P[i+1]); // left turn/ccw
        else sum -= angle(P[i], pt, P[i+1]);
    } // right turn/cw
    return fabs(sum - 2*PI) < EPS;
}

// line segment p-q intersect with line A-B.
point lineIntersectSeg(point p, point q, point A, point B) {
    double a = B.y - A.y;
    double b = A.x - B.x;
    double c = B.x * A.y - A.x * B.y;
    double u = fabs(a * p.x + b * p.y + c);
    double v = fabs(a * q.x + b * q.y + c);
    return point((p.x * v + q.x * u) / (u+v), (p.y * v + q.y * u) / (u+v));
}

// cuts polygon Q along the line formed by point a -> point b
// (note: the last point must be the same as the first point)
vector<point> cutPolygon(point a, point b, const vector<point> &Q) {
    vector<point> P;
```

```
    for (int i = 0; i < (int)Q.size(); i++) {
        double left1 = cross(toVec(a, b), toVec(a, Q[i])), left2 = 0;
        if (i != (int)Q.size()-1) left2 = cross(toVec(a, b), toVec(a, Q[i+1]));
        if (left1 > -EPS) P.push_back(Q[i]); // Q[i] is on the left of ab
        if (left1 * left2 < -EPS) // edge (Q[i], Q[i+1]) crosses line ab
            P.push_back(lineIntersectSeg(Q[i], Q[i+1], a, b));
    }
    if (!P.empty() && !(P.back() == P.front()))
        P.push_back(P.front()); // make P's first point = P's last point
    return P;
}

point pivot;
bool angleCmp(point a, point b) { // angle-sorting function
    if (collinear(pivot, a, b)) // special case
        return dist(pivot, a) < dist(pivot, b); // check which one is closer
    double dlx = a.x - pivot.x, dly = a.y - pivot.y;
    double d2x = b.x - pivot.x, d2y = b.y - pivot.y;
    return (atan2(dly, dlx) - atan2(d2y, d2x)) < 0;
} // compare two angles

vector<point> CH(vector<point> P) { // the content of P may be reshuffled
    int i, j, n = (int)P.size();
    if (n <= 3) {
        if (!(P[0] == P[n-1])) P.push_back(P[0]); // safeguard from corner case
        return P; // special case, the CH is P itself
    }

    // first, find P0 = point with lowest Y and if tie: rightmost X
    int P0 = 0;
    for (i = 1; i < n; i++)
        if (P[i].y < P[P0].y || (P[i].y == P[P0].y && P[i].x > P[P0].x))
            P0 = i;

    point temp = P[P0];
    P[0] = P[P0];
    P[P0] = temp; // swap P[P0] with P[0]

    // second, sort points by angle w.r.t. pivot P0
    pivot = P[0]; // use this global variable as reference
    sort(++P.begin(), P.end(), angleCmp); // we do not sort P[0]

    // third, the ccw tests
    vector<point> S;
    S.push_back(P[n-1]);
    S.push_back(P[0]);
    S.push_back(P[1]); // initial S
    i = 2; // then, we check the rest
    while (i < n) { // note: N must be >= 3 for this method to work
        j = (int)S.size()-1;
        if (ccw(S[j-1], S[j], P[i])) S.push_back(P[i]); // left turn, accept
        else S.pop_back();
    } // or pop the top of S until we have a left turn
    return S;
} // return the result

int main() {
    return 0;
}
```

## Getchar unlocked

```
#include <bits/stdc++.h>
using namespace std;

bool readChar(char &c) {
    c = getchar_unlocked();
    return c != EOF;
}

inline void writeChar(char c) {
    putchar_unlocked(c);
}

template<typename T>
bool readInt(T &n) {
    n = 0;
    register bool neg = false;
    register char c = getchar_unlocked();
    if( c == EOF) { n = -1; return false; }
    while (!('0' <= c && c <= '9')) {
        if( c == '-' ) neg = true;
        c = getchar_unlocked();
    }
    while ('0' <= c && c <= '9') {
        n = n * 10 + c - '0';
        c = getchar_unlocked();
    }
    n = (neg ? (-n) : (n));
    return true;
}
```

## Grundy Number

```
// Codeforces - Permutation Game - 1003C
// Grundy Number
// Game description:
/* A token is placed in one of the cells. They take
 * alternating turns of moving the token around the board,
 * with Alice moving first. The current player can move from
 * cell i to cell j only if the following two conditions are satisfied:
 *  ** the number in the new cell j must be strictly larger than
 *  ** the number in the old cell i (i.e. a[j] > a[i]), and
 *  ** the distance that the token travels during this turn
 *  ** must be a multiple of the number in the old cell
 *  (i.e. |i - j| mod a[i] == 0).
 * Whoever is unable to make a move, loses.
 */
/* Note: for combining various games, make nim-sum (xor-sum) of
 * grundy numbers of each game. If this value is 0, first player
 * loses, otherwise first player win.
 */

#include <bits/stdc++.h>
using namespace std;

const int N = 100 * 1000 + 10;
int n;
int a[N];
int memo[N];
```

```
}

template<typename T>
inline void writeInt(T n){
    register int idx = 20;
    if( n < 0 ) putchar_unlocked('-');
    n = abs(n);
    char out[21];
    out[20] = '\0';
    do{
        idx--;
        out[idx] = n % 10 + '0';
        n /= 10;
    }while(n);
    do{ putchar_unlocked(out[idx++]); } while (out[idx] != '\0');
}

int main() {
    ios::sync_with_stdio(false);

    int x;
    while (readInt(x)) {
        writeInt(x);
    }
    return 0;
}
```

```
int calculateMex(unordered_set<int> set) {
    int mex = 0;

    while (set.find(mex) != set.end())
        mex++;

    return mex;
}

int calculateGrundy(int i) {
    if (memo[i] != -1)
        return memo[i];

    if (a[i] == n)
        return 0;

    unordered_set<int> set;

    for (int j = i + a[i]; j < n; j += a[i]) {
        if (a[j] > a[i])
            set.insert(calculateGrundy(j));
    }

    for (int j = i - a[i]; j >= 0; j -= a[i]) {
        if (a[j] > a[i]) {
```

```

        set.insert(calculateGrundy(j));
    }
}

return memo[i] = calculateMex(set);
}

int main() {
    cin >> n;

    for (int i = 0; i < n; i++) {

```

## Hashing

```

// String Hash - Polynomial rolling hash
// Codeforces - Equivalent Strings - 559B
#include <bits/stdc++.h>
using namespace std;

const int N = 200100;
const uint64_t E = 33;
const uint64_t C = 5381ULL;
const uint64_t MOD = int(1e9) + 7;

struct Hash {
    vector<int> v;
    uint64_t h[N];
    uint64_t poww[N];

    Hash() {
        poww[0] = 1ULL;
        for (int i = 1; i < N; i++) {
            poww[i] = poww[i-1] * E;
            poww[i] %= MOD;
        }

        for (int i = 0; i < 26; i++) {
            v.push_back(i);
        }

        srand(time(nullptr));
        random_shuffle(v.begin(), v.end());
    }

    void init(string s) {
        int n = s.size();
        h[0] = C;
        for (int i = 0; i < n; i++) {
            h[i+1] = h[i] * E + v[s[i] - 'a'];
            h[i+1] %= MOD;
        }
    }
}

```

## Kruskal

```

    cin >> a[i];
}

memset(memo, -1, sizeof(memo));
for (int i = 0; i < n; i++) {
    cout << (calculateGrundy(i) > 0 ? "A" : "B");
}
cout << "\n";
return 0;
}

uint64_t get_hash(int i, int j) {
    return ((h[j+1] - (h[i] * poww[j-i+1]) % MOD) + MOD) % MOD;
}

};

Hash h1, h2;
string s1, s2;

bool solve(int l, int r, int x, int y) {
    if (h1.get_hash(l, r) == h2.get_hash(x, y)) {
        return true;
    } else if ((r - l + 1) % 2 == 0) {
        int mid1 = (l + r) >> 1, mid2 = (x + y) >> 1;

        return (solve(l, mid1, x, mid2) && solve(mid1+1, r, mid2+1, y))
            || (solve(mid1+1, r, x, mid2) && solve(l, mid1, mid2+1, y));
    }

    return false;
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    cin >> s1 >> s2;

    h1.init(s1);
    h2.init(s2);

    if (solve(0, s1.size() - 1, 0, s2.size() - 1)) {
        cout << "YES\n";
    } else {
        cout << "NO\n";
    }

    return 0;
}

```

```
// UVA - Bond - 11354
// similar to problem "Caminhoes"
// Kruskal and queries response inside kruskal
#include <bits/stdc++.h>
using namespace std;

namespace UF {
    const int N = 100 * 1000 + 10;
    int parent[N];
    int sz[N];
    void init(int size) {
        assert(size < N);
        for (int i = 0; i < size; i++) {
            parent[i] = i;
            sz[i] = 1;
        }
    }

    int find(int x) {
        if (parent[x] == x) return x;
        else return parent[x] = find(parent[x]);
    }

    void join(int x, int y) {
        x = find(x);
        y = find(y);

        if (x == y) return;

        if (sz[x] < sz[y]) {
            parent[x] = y;
            sz[y] += sz[x];
        } else {
            parent[y] = x;
            sz[x] += sz[y];
        }
    }
}

struct edge {
    int x, y;
    int64_t w;
    bool operator< (const edge &other) {
        return this->w < other.w || (this->w == other.w && this->x < other.x);
    }
};

struct query {
    int x, y, ans = -1;
};

const int N = 100 * 1000 + 10;
edge edges[N];
query queries[N];
vector<int> comp_queries[N];

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    int n, m, x, y, q;
    int64_t w;
    bool first = true;

```

```
while (cin >> n >> m) {
    if (!first) {
        cout << "\n";
    }
    first = false;

    for (int i = 0; i < m; i++) {
        cin >> edges[i].x >> edges[i].y >> edges[i].w;
    }

    for (int i = 1; i <= n; i++) {
        comp_queries[i].clear();
    }

    cin >> q;

    for (int i = 0; i < q; i++) {
        cin >> x >> y;
        queries[i].x = x;
        queries[i].y = y;
        queries[i].ans = -1;

        comp_queries[x].push_back(i);
        comp_queries[y].push_back(i);
    }

    sort(edges, edges+m);

    UF::init(n+1);

    for (int i = 0; i < m; i++) {
        x = edges[i].x;
        y = edges[i].y;
        w = edges[i].w;

        if (UF::find(x) != UF::find(y)) {
            int small = UF::find(x),
                large = UF::find(y);

            if (UF::sz[small] >= UF::sz[large]) {
                swap(small, large);
            }

            // note: should choose small and large in the same
            // manner that union-find do

            UF::join(x, y);

            for (const auto &id : comp_queries[small]) {
                if (queries[id].ans == -1 &&
                    UF::find(queries[id].x) == UF::find(queries[id].y)) {
                    queries[id].ans = w;
                } else {
                    comp_queries[large].push_back(id);
                }
            }

            // store queries in large will maintain
            // access to queries, because UF::find return the large

            comp_queries[small].clear();
        }
    }
}
```

```

    }

    for (int i = 0; i < q; i++) {
        cout << queries[i].ans << "\n";
    }
}

```

## Lowest Common Ancestor (LCA)

```

//Lib - LCA
#include <bits/stdc++.h>
using namespace std;

const int N = 100100, L = 20;
vector<pair<int, int64_t>> g[N];
int64_t dist[N];
int parent[N][L], lvl[N];

void dfs(int v, int p) {
    parent[v][0] = p;
    lvl[v] = lvl[p] + 1;

    int64_t w;
    for (int j = 0, u; j < (int)g[v].size(); j++) {
        u = g[v][j].first;
        w = g[v][j].second;

        if (u == p) continue;

        dist[u] = dist[v] + w;
        dfs(u, v);
    }
}

void pre_lca(int n) {
    for (int i = 1; i < L; i++) {
        for (int v = 0; v < n; v++) {
            parent[v][i] = parent[parent[v][i-1]][i-1];
        }
    }
}

int lca(int a, int b) {
    if (lvl[a] < lvl[b]) {

```

## Longest Increasing Sequence (LIS)

```

#include <bits/stdc++.h>
using namespace std;

int main() {
    ios::sync_with_stdio(false);
    const int N = 1000;
    int n;
    int64_t a[N];

    cin >> n;
    for (int i = 0; i < n; i++) {

```

```

    }

    return 0;
}

```

```

        swap(a, b);
    }

    int d = lvl[a] - lvl[b];

    for (int i = L-1; i >= 0; i--) {
        if (d & (1 << i)) {
            a = parent[a][i];
        }
    }

    if (a == b) return a;

    for (int i = L-1; i >= 0; i--) {
        if (parent[a][i] != parent[b][i]) {
            a = parent[a][i];
            b = parent[b][i];
        }
    }

    return parent[a][0];
}

int main() {
    ios::sync_with_stdio(false);

    int n;
    cin >> n;
    dfs(0, 0);
    pre_lca(n);

    return 0;
}

```

```

    cin >> a[i];
}

set<int64_t> st;
auto it = st.begin();
for (int i = 0; i < n; i++) {
    if (st.find(a[i]) != st.end()) continue;

    st.insert(a[i]);
    it = st.find(a[i]);
    if (next(it) != st.end())

```

```

        st.erase(next(it));
    }

    cout << st.size() << endl;

```

## Longest Increasing Sequence (Print elements)

```

// LIS algorithm with function to print the sequence
// UVA - What Goes Up - 481
#include <bits/stdc++.h>
using namespace std;

namespace LIS {
    const int N = 100 * 1000 + 10;
    int parent[N], l[N], l_id[N], l_end;
    void lis(int a[], const int n) {
        l_end = 0;
        for (int i = 0; i < n; i++) {
            int pos = lower_bound(l, l+l_end, a[i]) - l;

            l[pos] = a[i];
            l_id[pos] = i;
            parent[i] = pos ? l_id[pos-1] : -1;

            if (pos == l_end) {
                l_end++;
            }
        }
    }

    vector<int> get_lis(int a[]) {
        stack<int> st;
        for (int x = l_id[l_end-1]; x != -1; x = parent[x]) {
            st.push(a[x]);
        }
    }
}

```

## Longest Increasing Sequence with Fenwick

```

// LIS using Fenwick Tree
#include <bits/stdc++.h>
using namespace std;

const int M = 310;
int ft[M];

void update(int x, int v) {
    x += 3;
    while (x < M) {
        ft[x] = max(ft[x], v);
        x += (x & -x);
    }
}

int query(int x) {
    x += 3;
    int ans = 0;

```

```

    return 0;
}

```

```

    vector<int> ans;
    while (!st.empty()) {
        ans.push_back(st.top());
        st.pop();
    }
    return ans;
}

int main() {
    const int N = 100 * 1000 + 10;
    int x, n = 0;
    int a[N];

    while (cin >> x) {
        a[n++] = x;
    }

    LIS::lis(a, n);

    vector<int> ans = LIS::get_lis(a);
    cout << ans.size() << "\n\n";
    for (const auto &x : ans) {
        cout << x << "\n";
    }

    return 0;
}

```

```

    while (x > 0) {
        ans = max(ans, ft[x]);
        x -= (x & -x);
    }
    return ans;
}

int dp[1000100];

int main() {
    const int N = 110;
    int n;
    int64_t a[N];

    cin >> n;
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }

```



```

int lis = 1;

for (int i = 0; i < n; i++) {
    dp[i] = query(a[i%n]-1) + 1;
    update(a[i%n], dp[i]);
    lis = max(lis, dp[i]);
}

```

## Matrix Power

```

// Codeforces - Gym 101845 - Univ. Nacional de Colombia PC
// Apple Trees
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 5;

struct matrix {
    int64_t m[MAXN][MAXN];
};

matrix mult(matrix a, matrix b, int64_t mod) {
    matrix ans;
    for (int i = 0; i < MAXN; i++) {
        for (int j = 0; j < MAXN; j++) {
            ans.m[i][j] = 0;
            for (int k = 0; k < MAXN; k++) {
                ans.m[i][j] += (a.m[i][k] * b.m[k][j]) % mod;
            }
            ans.m[i][j] %= mod;
        }
    }

    return ans;
}

matrix power(matrix base, int64_t exp, int64_t mod) {
    matrix ans;
    for (int i = 0; i < MAXN; i++) {
        for (int j = 0; j < MAXN; j++) {
            ans.m[i][j] = (i == j) ? 1 : 0;
            base.m[i][j] %= mod;
        }
    }

    while (exp) {
        if (exp & 1) {
            ans = mult(ans, base, mod);
        }

        base = mult(base, base, mod);
        exp >>= 1;
    }

    return ans;
}

```

## Max Flow

```

}

cout << lis << endl;
return 0;
}

exp >>= 1;
}

return ans;
}

int main() {
    const int64_t mod = 1000000007;
    int64_t n;

    cin >> n;

    if (n < 10) {
        cout << "1\n";
        return 0;
    }

    matrix a = {{{{16, 9, 4, 1, 0},
                  {1, 0, 0, 0, 0},
                  {0, 1, 0, 0, 0},
                  {0, 0, 1, 0, 0},
                  {0, 0, 0, 1, 0}}}};

    a = power(a, n/10, mod);

    int64_t ans = 0;
    for (int i = 0; i < 4; i++) {
        ans += a.m[i][0];
        ans %= mod;
    }

    if (n % 10 < 5) {
        ans += a.m[4][0];
        ans %= mod;
    }

    cout << ans << "\n";
    return 0;
}

```

```

// Lib - Edmonds-Karp Algorithm
// UVA - 820 - Network Bandwidth
#include <bits/stdc++.h>
using namespace std;

struct Edmonds {
    static const int64_t inf = 1e18;
    static const int V = 150;
    int64_t adj[V][V];
    vector<int> g[V];
    int p[V]; // parent vector
    int64_t dist[V];
    int n, s, t;

    Edmonds(int n, int s, int t) : n(n), s(s), t(t) {
        clear();
    }

    void clear() {
        for (int i = 0; i < n; i++)
            g[i].clear();
        memset(adj, 0, sizeof(adj));
    }

    int64_t augmented() {
        int u = t;
        int64_t minimum = inf;

        // find minimum flow in augmented path
        while (p[u] != -1) {
            minimum = min(adj[p[u]][u], minimum);
            u = p[u];
        }

        // walk in augment path updating flow
        u = t;
        while (p[u] != -1) {
            adj[p[u]][u] -= minimum;
            adj[u][p[u]] += minimum;
            u = p[u];
        }

        return minimum; // return minimum flow in augmented path
    }

    int64_t flow() {
        int64_t mf = 0, // max flow answer
            f = 1;
        int u;

        while (f > 0) {
            f = 0;
            for (int i = 0; i < n; i++)
                dist[i] = inf, p[i] = -1;
            dist[s] = 0;

            queue<int> q;
            q.push(s);

            while (!q.empty()) {
                u = q.front();
                q.pop();

```

```

                // stop if reach sink t
                if (u == t) break;

                for (const auto &v : g[u]) {
                    if (adj[u][v] > 0 && dist[v] == inf) {
                        dist[v] = dist[u] + 1;
                        q.push(v);
                        p[v] = u;
                    }
                }
            }

            // verify if bfs stop when reach sink t
            if (u == t) {
                // find minimum flow in augmented path
                f = augmented();

                // update max flow of network
                mf += f;
            }
        }

        return mf;
    }

    void add_edge(int v, int u, int64_t cap) {
        adj[v][u] = cap;
        adj[u][v] = 0;
        g[v].push_back(u);
        g[u].push_back(v);
    }

    void add_bi_edge(int v, int u, int64_t cap) {
        adj[v][u] = cap;
        adj[u][v] = cap;
        g[v].push_back(u);
        g[u].push_back(v);
    }

    void inc_edge(int v, int u, int64_t cap) {
        if (adj[v][u] == 0)
            g[v].push_back(u);
        adj[v][u] += cap;
    }
};

int main() {
    ios::sync_with_stdio(false);

    int n, m, k;
    int a, b, source, sink;
    int64_t c;

    k = 1;
    while (cin >> n, n) {
        cin >> source >> sink >> m;
        source--;
        sink--;

        Edmonds edmonds(n, source, sink);

        for (int i = 0; i < m; i++) {
            cin >> a >> b >> c;

```

```

        a--;
        b--;
        edmonds.inc_edge(a, b, c);
        edmonds.inc_edge(b, a, c);
    }

    c = edmonds.flow();

```

## Min Cost Max Flow

```

// Min cost max flow
// UVA 10594 - Data Flow

#include <bits/stdc++.h>
using namespace std;

const int INF = numeric_limits<int>::max();
const int64_t LINF = 1000LL * 1000 * 1000 * 1000 * 1000 * 1000LL;
const int N = 150; // number of vertex
const int M = 4*5002; // number of edges

struct edge {
    // v - from vertex
    // u - to vertex
    int v, u, next;
    int64_t cap, cost;
} edges[M];

int first[N], edgenum = 0;

// initialize algorithm structs
void init(int sz = N) {
    for (int i = 0; i < sz; i++) {
        first[i] = -1;
    }
    edgenum = 0;
}

// add a directed edge v -> u and residual edge v <- u
void add_edge(int v, int u, int64_t cap, int64_t cost) {
    edge &e = edges[edgenum];
    e.v = v;
    e.u = u;
    e.cap = cap;
    e.cost = cost;
    e.next = first[v];
    first[v] = edgenum;
    edgenum++;

    // residual edge
    edge &e2 = edges[edgenum];
    e2.v = u;
    e2.u = v;
    e2.cap = 0;
    e2.cost = -cost;
    e2.next = first[u];
    first[u] = edgenum;
    edgenum++;
}

```

```

        cout << "Network_" << k << "\nThe_bandwidth_is_" << c << ".\n\n";
        k++;
    }

    return 0;
}

```

```

int64_t dist[N];
int in_queue[N], p[N];

// augment - walk in augmented path and update flow
// u - sink or final of path
int64_t augment(int u) {
    int64_t minimum = LINF;

    // find minimum flow in augmented path
    for (int k = p[u]; k != -1; k = p[edges[k].v]) {
        minimum = min(edges[k].cap, minimum);
    }

    // walk in augment path updating flow
    for (int k = p[u]; k != -1; k = p[edges[k].v]) {
        // forward edge
        edges[k].cap -= minimum;
        // residual edge
        edges[k^1].cap += minimum;
    }

    return minimum; // return minimum flow in augmented path
}

// s - source, t - sink, n - number of vertex [0, ..., n-1]
pair<int64_t, int64_t> mcmf(const int s, const int t, const int n) {
    int64_t mf = 0, // max flow answer
        min_cost = 0, // min cost answer
        f = 1; // current min cost

    while (f > 0) {
        f = 0;
        for (int i = 0; i < n; i++) {
            dist[i] = LINF;
            p[i] = -1;
            in_queue[i] = 0;
        }

        dist[s] = 0;

        queue<int> q;
        q.push(s);
        in_queue[s] = 1;
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            in_queue[u] = 0;

            for (int k = first[u], v; k != -1; k = edges[k].next) {
                v = edges[k].u;

```

```

        if (edges[k].cap > 0 && dist[v] > dist[u] + edges[k].cost) {
            dist[v] = dist[u] + edges[k].cost;
            p[v] = k;
            if (in_queue[v] == 0) {
                q.push(v);
                in_queue[v] = 1;
            }
        }
    }

    // verify if bfs stop when reach sink t
    if (dist[t] != LINF) {
        // find minimum flow in augmented path
        f = augment(t);

        // update max flow of network
        min_cost += f * dist[t];
        mf += f;
    }
}

return {mf, min_cost};
}

int main() {
    int a[M], b[M];
    int64_t c[M];
    int n, m;
    int64_t d, k;
    int source, sink;

```

```

while (cin >> n >> m) {
    init(n+10);

    for (int i = 0; i < m; i++) {
        cin >> a[i] >> b[i] >> c[i];
    }

    cin >> d >> k;

    for (int i = 0; i < m; i++) {
        add_edge(a[i], b[i], k, c[i]);
        add_edge(b[i], a[i], k, c[i]);
    }

    source = 0;
    sink = n;

    add_edge(source, 1, d, 0);

    auto p = mcmf(source, sink, n+1);

    if (p.first != d) {
        printf("Impossible.\n");
    } else {
        printf("%jd\n", p.second);
    }
}

return 0;
}

```

## Mo's Algorithm

```

// Mo's Algorithm
// Problem H - Wine Production
#include <bits/stdc++.h>
using namespace std;

// Variables, that hold current "state" of computation
long long current_answer;

// Array to store answers (because the order we achieve them is messed up)
const int N = 100500;
long long answers[N];
int BLOCK_SIZE;
int arr[N];

// We will represent each query as three numbers: L, R, idx. Idx is
// the position (in original order) of this query.
pair< pair<int, int>, int> queries[N];

unordered_map<int, int> cnt, caras;
map<int, int> ok;

// Essential part of Mo's algorithm: comparator, which we will
// use with std::sort. It is a function, which must return True
// if query x must come earlier than query y, and False otherwise.

```

```

inline bool mo_cmp(const pair< pair<int, int>, int> &x,
    const pair< pair<int, int>, int> &y) {
    int block_x = x.first.first / BLOCK_SIZE;
    int block_y = y.first.first / BLOCK_SIZE;
    if (block_x != block_y)
        return block_x < block_y;
    return x.first.second < y.first.second;
}

// When adding a number, we first nullify it's effect on current
// answer, then update cnt array, then account for it's effect again.
inline void add(int pos) {
    cnt[arr[pos]]++;
    int q = cnt[arr[pos]];
    caras[q]++;

    if (caras[q] >= q) {
        ok[q]++;
    }
    if (!ok.empty()) {
        current_answer = (*ok.end()--).first;
    }
    else current_answer = 1;
}

```

```
// Removing is much like adding.
inline void remove(int pos) {
    int q = cnt[arr[pos]];
    cnt[arr[pos]]--;
    caras[q]--;

    if (ok.count(q)) {
        ok[q]--;
        if (ok[q] == 0) {
            ok.erase(q);
        }
    }
    if (!ok.empty()) {
        current_answer = (*ok.end()--).first;
    }
    else current_answer = 1;
}

int main() {
    cin.sync_with_stdio(false);
    cin.tie(NULL);

    int n, q;
    cin >> n >> q;
    BLOCK_SIZE = static_cast<int>(sqrt(n));

    for(int i = 0; i < n; i++)
        cin >> arr[i];

    for(int i = 0; i < q; i++) {
        cin >> queries[i].first.first >> queries[i].first.second;
        queries[i].first.first--;
        queries[i].first.second--;

        queries[i].second = i;
    }

    // Sort queries using Mo's special comparator we defined.
    sort(queries, queries + q, mo_cmp);
}
```

## Policy-based set

```
// Lib - Ordered Set - Policy-based Set
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef tree<
    int,
    null_type,
    less<int>,
    rb_tree_tag,
    tree_order_statistics_node_update> pbd_set;
// map: tA -> tB com comparador less<tA>
// pode usar como um map normalmente
// s.find_by_order(k) :: retorna iterador para o k-esimo elemento (0-index) (ou s.end())
// s.order_of_key(x) :: retorna a qtd de elementos estritamente menores que x
int main() {
```

```
// Set up current segment [mo_left, mo_right].
int mo_left = 0, mo_right = -1;

for(int i = 0; i < q; i++) {
    // [left, right] is what query we must answer now.
    int left = queries[i].first.first;
    int right = queries[i].first.second;

    // Usual part of applying Mo's algorithm: moving mo_left
    // and mo_right.
    while(mo_right < right) {
        mo_right++;
        add(mo_right);
    }
    while(mo_right > right) {
        remove(mo_right);
        mo_right--;
    }

    while(mo_left < left) {
        remove(mo_left);
        mo_left++;
    }
    while(mo_left > left) {
        mo_left--;
        add(mo_left);
    }

    // Store the answer into required position.
    answers[queries[i].second] = current_answer;
}

// We output answers *after* we process all queries.
for(int i = 0; i < q; i++)
    cout << answers[i] << "\n";
return 0;
}
```

```
pbd_set s;
int t;
scanf("%d", &t);
while(t--) {
    char typ;
    int n;
    scanf("_%c%d", &typ, &n);
    if (typ == 'I')
        s.insert(n);
    else if (typ == 'D')
        s.erase(n);
    else if (typ == 'K') {
        int ans;
        n--;
        if (s.find_by_order(n) != s.end()) {
            ans = *s.find_by_order(n);
            printf("%d\n", ans);
        }
    }
}
```

```

    }
    else
        printf("invalid\n");
}
else{
    int ans = s.order_of_key(n);

```

## Random Numbers

```

#include <algorithm>
#include <chrono>
#include <iostream>
#include <random>
#include <vector>
using namespace std;

const int N = 3000000;

double average_distance(const vector<int> &permutation) {
    double distance_sum = 0;

    for (int i = 0; i < N; i++)
        distance_sum += abs(permutation[i] - i);

    return distance_sum / N;
}

int main() {

```

## Sieve

```

#include <bits/stdc++.h>
using namespace std;

const int M = 10000010;
bool notprime[M];
int fp[M]; // smallest first prime factor
vector<int> primes;

void sieve(int n) {
    assert(n < M);
    notprime[0] = notprime[1] = true;

    for (int i = 2; i <= n; i++) {
        if (!notprime[i]) {
            fp[i] = i;
            for (int j = i+i; j <= n; j += i) {
                notprime[j] = true;
                fp[j] = i;
            }
            primes.push_back(i);
        }
    }
}

```

```

        printf("%d\n",ans);
    }
}
return 0;
}

```

```

mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
vector<int> permutation(N);

for (int i = 0; i < N; i++)
    permutation[i] = i;

shuffle(permutation.begin(), permutation.end(), rng);
cout << average_distance(permutation) << '\n';

for (int i = 0; i < N; i++)
    permutation[i] = i;

for (int i = 1; i < N; i++)
    swap(permutation[i], permutation[uniform_int_distribution<int>(0, i)(rng)]);

cout << average_distance(permutation) << '\n';
}

```

```

bool isPrime(vector<int> &primes, int64_t n) {
    if (n < M) return !notprime[n];

    for (const auto &p : primes) {
        if (n % p == 0)
            return false;
    }

    return true;
}

int main() {
    int n;

    cin >> n;

    sieve(n);

    return 0;
}

```

## Strongly Connected Components

```
// Strongly Connected Components - Kosaraju and Sharir Algorithm
#include <bits/stdc++.h>
using namespace std;

const int N = 100100;
vector<int> g[N], gr[N];
bool used[N];
vector<int> order, component;

void dfs1(int v) {
    used[v] = true;
    for (const auto &u : g[v]) {
        if (!used[u]) {
            dfs1(u);
        }
    }
    order.push_back(v);
}

void dfs2(int v) {
    used[v] = true;
    component.push_back(v);
    for (const auto &u : gr[v]) {
        if (!used[u]) {
            dfs2(u);
        }
    }
}

void kosaraju(int n) {
    memset(used, false, sizeof(used));
    for (int v = 0; v < n; v++) {
        if (!used[v]) {
            dfs1(v);
        }
    }
}
```

## Suffix Automata

```
// Suffix Automaton
// LCS problem
#include <bits/stdc++.h>
using namespace std;

struct SuffixAutomaton {
    vector<map<char, int>> edges; // edges[i] : edges of state i
    vector<int> link; // suffix link of state i
    vector<int> lenght; // lenght of longest string in class of state i
    int last, sz;
    vector<bool> term;

    SuffixAutomaton() {
        // add the initial node
        edges.push_back(map<char, int>());
        link.push_back(-1); // suffix link of first state is dummy
        lenght.push_back(0);
        term.push_back(false);
        last = 0; // initiate with index of first state
        sz = 0; // lenght of longest suffix added to automaton
    }
}
```

```

    }

    memset(used, false, sizeof(used));
    component.clear();

    for (int i = 0; i < n; i++) {
        int v = order[n - 1 - i];
        if (!used[v]) {
            dfs2(v);
            cout << "Component:";
            for (const auto &u : component) {
                cout << " " << u;
            }
            cout << endl;
            component.clear();
        }
    }
}

int main() {
    int n, m;

    cin >> n >> m;

    for (int i = 0, x, y; i < m; i++) {
        cin >> x >> y; // x -> y
        g[x].push_back(y);
        gr[y].push_back(x); // reverse graph
    }

    kosaraju(n);
    return 0;
}
```

```
SuffixAutomaton(string s) : SuffixAutomaton() {
    for (const auto &ch : s) {
        extend(ch);
    }
    find_terminals();
}

void extend(char ch) {
    // create new state for new equivalence (end-points) class
    edges.push_back(map<char, int>());
    sz++;
    lenght.push_back(sz);
    link.push_back(0);
    term.push_back(false);

    int r = edges.size() - 1;
    int p = last;

    while (p >= 0 && edges[p].find(ch) == edges[p].end()) {
        edges[p][ch] = r;
        p = link[p];
    }
}
```



```

    if (p != -1) {
        int q = edges[p][ch];
        if (lenght[p] + 1 == lenght[q]) {
            link[r] = q;
        } else {
            edges.push_back(edges[q]);
            lenght.push_back(lenght[p] + 1);
            link.push_back(link[q]);
            term.push_back(false);
            int qq = edges.size() - 1;
            link[q] = qq;
            link[r] = qq;

            while (p >= 0 && edges[p][ch] == q) {
                edges[p][ch] = qq;
                p = link[p];
            }
        }
    }

    last = r;
}

void find_terminals() {
    term = vector<bool>(edges.size(), false);
    int p = last;
    while (p >= 0) {
        term[p] = true;
        p = link[p];
    }
}

int64_t lcs(SuffixAutomaton &sa, string t) {
    int st = 0, len = 0, best = 0, best_i = 0;

    for (int i = 0, sz = t.size(); i < sz; i++) {
        char ch = t[i];

        while (st != 0 && sa.edges[st].count(ch) == 0) {
            st = sa.link[st];
            len = sa.lenght[st];
        }

        if (sa.edges[st].count(ch) > 0) {
            st = sa.edges[st][ch];
            len++;
        }

        if (best < len) {
            best = len;
            best_i = i;
        }
    }

    cout << t.substr(best_i-best+1, best) << endl;

    return best;
}

void dfs(SuffixAutomaton &sa, vector<int64_t> &occur, vector<int64_t> &words, int st) {
    if (occur[st] > 0)

```

```

        return;

        int64_t occ = 0, wrd = 0;

        if (sa.term[st]) {
            occ++;
            wrd++;
        }

        for (const auto &p : sa.edges[st]) {
            dfs(sa, occur, words, p.second);
            occ += occur[p.second];
            wrd += words[p.second] + occur[p.second];
        }

        occur[st] = occ;
        words[st] = wrd;
    }

    string find_kth_substr(SuffixAutomaton &sa, int k) {
        vector<int64_t> occur(sa.edges.size()), words(sa.edges.size());
        dfs(sa, occur, words, 0);

        // find kth substring

        int st = 0;
        string t = "";

        int64_t prev_k = k;
        while (k > 0) {
            int64_t acc = 0, tmp;

            for (const auto &p : sa.edges[st]) {
                tmp = acc;
                acc += words[p.second];
                if (acc >= k) {
                    st = p.second;
                    k -= tmp + occur[p.second];
                    t += p.first;
                    break;
                }
            }

            if (k == prev_k) {
                t = "No_such_line.";
                break;
            }

            prev_k = k;
        }

        return t;
    }

    int main() {
        ios::sync_with_stdio(false);
        string s, t;

        cin >> s >> t;

        SuffixAutomaton am(s);

        cout << lcs(am, t) << "\n";
    }
}

```

```
// find kth substring in lexicographical order
cout << find_kth_substr(am, 1) << endl;
```

```
    return 0;
}
```

## Suffix Automata's Applications

Problem: Find whether a given string  $w$  is a substring of  $s$ .

Solution: Simply run the automaton.

```
SuffixAutomaton a(s);
bool fail = false;
int n = 0;
for(int i=0;i<w.size();i++) {
    if(a.edges[n].find(w[i]) == a.edges[n].end()) {
        fail = true;
        break;
    }
    n = a.edges[n][w[i]];
}
if(!fail) cout << w << " is a substring of " << s << "\n";
```

Problem: Find whether a given string  $w$  is a suffix of  $s$ .

Solution: Construct the list of terminal states, run the automaton as above and check in the end if the  $n$  is among the terminal states. Let's now look at the dp problems.

Problem: Count the number of distinct substrings in  $s$ .

Solution: The number of distinct substrings is the number of different paths in the automaton. These can be calculated recursively by calculating for each node the number of different paths starting from that node. The number of different paths starting from a node is the sum of the corresponding numbers of its direct successors, plus 1 corresponding to the path that does not leave the node.

Problem: Count the number of times a given word  $w$  occurs in  $s$ .

Solution: Similar to the previous problem. Let  $p$  be the node in the automaton that we end up while running it for  $w$ . This time the number of times a given word occurs is the number of paths starting from  $p$  and ending in a terminal node, so one can calculate recursively the number of paths from each node ending in a terminal node.

Problem: Find where a given word  $w$  occurs for the first time in  $s$ .

Solution: This is equivalent to calculating the longest path in the automaton after reaching the node  $p$  (defined as in the previous solution).

Finally let's consider the following problem where the suffix links come handy.

Problem: Find all the positions where a given word  $w$  occurs in  $s$ .

Solution: Prepend the string with some symbol '\$' that does not occur in the string and construct the suffix automaton. Let's then add to each node of the suffix automaton its children in the suffix tree:

```
children=vector<vector<int>>(link.size());
for(int i=0;i<link.size();i++) {
    if(link[i] >= 0) children[link[i]].push_back(i);
}
```

Now find the node  $p$  corresponding to the node  $w$  as has been done in the previous problems. We can then dfs through the subtree of the suffix tree rooted at  $p$  by using the children vector. Once we reach a leaf, we know that we have found a prefix of  $s$  that ends in  $w$ , and the length of the leaf can be used to calculate the position of  $w$ . All of the dfs branches correspond to different prefixes, so no unnecessary work is done and the complexity is  $O(|s| + |w| + \text{sizeof output})$ .

## Topological Sort

```
// Topological Sort
#include <bits/stdc++.h>
using namespace std;

const int N = 100100;
vector<int> g[N];
bool vis[N];
vector<int> ans;

void dfs(int v) {
    vis[v] = true;
    for (const auto &u : g[v]) {
        if (!vis[u]) {
            dfs(u);
        }
    }
    ans.push_back(v);
}

void topological_sort(int n) {
    memset(vis, false, sizeof(vis));
    for (int v = 0; v < n; v++) {
        if (!vis[v]) {
            dfs(v);
        }
    }
}
```

## Union Find

```
#include <bits/stdc++.h>
using namespace std;

// versao com namespace
namespace UF {
    const int N = 100 * 1000 + 10;
    int parent[N];
    int sz[N];
    void init(int size) {
        assert(size < N);
        for (int i = 0; i < size; i++) {
            parent[i] = i;
            sz[i] = 1;
        }
    }
    int find(int x) {
        if (parent[x] == x) return x;
        else return parent[x] = find(parent[x]);
    }
    void join(int x, int y) {
        x = find(x);
        y = find(y);

        if (x == y) return;

        if (sz[x] < sz[y]) {
            parent[x] = y;
            sz[y] += sz[x];
        } else {

```

```

        }
        parent[y] = x;
        sz[x] += sz[y];
    }
}

// versao usando um struct
const int N = 100 * 1000 + 10;
struct union_find {
    int parent[N];
    int sz[N];

    union_find() {
    }

    void init(int size) {
        assert(size < N);
        for (int i = 0; i < size; i++) {
            parent[i] = i;
            sz[i] = 1;
        }
    }

    int find(int x) {
        if (parent[x] == x)
            return x;
        else
            return parent[x] = find(parent[x]);
    }
}

reverse(ans.begin(), ans.end());
// Topological sort is on vector ans
}

int main() {
    int n, m;
    cin >> n >> m;
    for (int i = 0, x, y; i < m; i++) {
        cin >> x >> y; // x -> y
        g[x].push_back(y);
    }

    topological_sort(n);

    cout << "Topological_Order:";
    for (int i = 0; i < n; i++) {
        cout << "_" << ans[i];
    }
    cout << endl;
    return 0;
}
```

```

void join(int x, int y) {
    x = find(x);
    y = find(y);

    if (x == y) return;

    if (sz[x] < sz[y]) {
        parent[x] = y;
        sz[y] += sz[x];
    } else {
        parent[y] = x;
        sz[x] += sz[y];
    }
}
};

```

## XOR Gaussian Elimination

```

// Lib - Gaussian Elimination XOR
// For linear equation systems in modulo 2
#include <bits/stdc++.h>
using namespace std;

const int N = 10, M = 10; // N variables, M equations
const int INF = 1e9;

struct Matrix {
    bitset<N+1> m[M];
    bitset<N+1> &operator[](size_t i) {
        return m[i];
    };
};

struct Vector {
    bitset<N> v;
    int operator[](size_t i) {
        return v[i];
    };
};

// n variables, m equations
int elimination(Matrix &aug, int n, int m, Vector &ans) {
    int where[N];
    memset(where, -1, sizeof(where));

    for (int col = 0, row = 0; col < n && row < m; col++) {
        for (int i = row; i < m; i++) {
            if (aug.m[i][col]) {
                swap(aug.m[i], aug.m[row]);
                break;
            }
        }

        if (!aug.m[row][col]) // independent variable or impossible system
            continue;

        where[col] = row; // assign a row for variable of column col

        // zero elements in column col (except in row)

```

```

int main() {
    ios::sync_with_stdio(false);

    int n;
    cin >> n;

    UF::init(n);

    union_find uf;
    uf.init(n);

    return 0;
}

```

```

    for (int i = 0; i < m; i++) {
        if (i != row && aug.m[i][col]) {
            aug.m[i] ^= aug.m[row];
        }
    }

    row++;
}

// assign answer to variables
for (int j = 0; j < n; j++) {
    if (where[j] != -1) {
        ans.v[j] = aug.m[where[j]][n];
    }
}

// verify if system is impossible
for (int i = 0; i < m; i++) {
    int sum = 0;
    for (int j = 0; j < n; j++) {
        if (where[j] != -1)
            sum ^= (ans.v[j] & aug.m[i][j]);
    }

    if (sum != aug.m[i][n]) {
        cout << aug.m[i] << endl;
        return 0; // impossible system
    }
}

for (int j = 0; j < n; j++) {
    if (where[j] == -1) // infinite solutions
        return INF;
}

return 1;
}

```

```

void printSol(Matrix aug, int n, int m) {
    // print system
    for (int i = 0; i < m; i++) {

```

```

        for (int j = 0; j <= n; j++) {
            cout << aug.m[i][j];
        }
        cout << endl;
    }

    Vector ans;
    cout << "number_of_solutions:" << elimination(aug, n, m, ans) << endl;
    cout << "solution_(x0,...,x" << n-1 << "):";
    for (int j = 0; j < n; j++) {
        cout << ans[j] << " ";
    }
    cout << endl;
}

int main() {
    Matrix aug;
    // inverse order of values of matrix (bitset details)
    aug.m[0] = bitset<N+1>("11101");
    aug.m[1] = bitset<N+1>("01110");
    aug.m[2] = bitset<N+1>("01011");
    aug.m[3] = bitset<N+1>("10111");

```

```

    cout << "System_1:" << endl;
    printSol(aug, 4, 4);

    aug.m[0] = bitset<N+1>("1111");
    aug.m[1] = bitset<N+1>("0001");
    aug.m[2] = bitset<N+1>("0111");
    aug.m[3] = bitset<N+1>("0110");

    cout << "System_2:" << endl;
    printSol(aug, 3, 4);

    aug.m[0] = bitset<N+1>("0000001");
    aug.m[1] = bitset<N+1>("1001011");
    aug.m[2] = bitset<N+1>("1010101");
    aug.m[3] = bitset<N+1>("0010110");
    aug.m[4] = bitset<N+1>("0001110");

    cout << "System_3:" << endl;
    printSol(aug, 6, 5);

    return 0;
}

```