

Vertex: Every individual data element is called a vertex or a node. In the above image 1,2,3,4,5 & 6 are the vertices.

Edge: It is a connecting link between two nodes or vertices. Each edge has two ends and is represented as (startingVertex, endingVertex).

Undirected Edge: It is a bidirectional edge.

Directed Edge: It is a unidirectional edge.

Weighted Edge: An edge with value (cost) on it.

Degree: The total number of edges connected to a vertex in a graph.

Indegree: The total number of incoming edges connected to a vertex.

Outdegree: The total number of outgoing edges connected to a vertex.

Self-loop: An edge is called a self-loop if its two endpoints coincide with each other.

Adjacency: Vertices are said to be adjacent to one another if there is an edge connecting them.

**

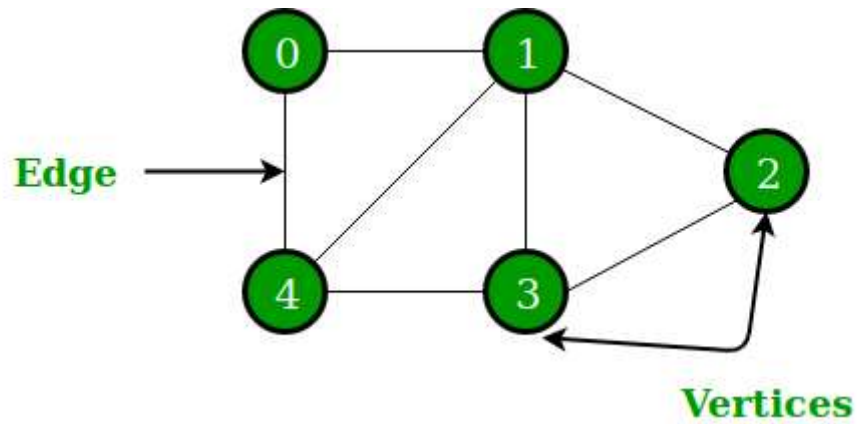
```

In [ ]: struct TreeNode {
        int Data;
        TreeNode *left;
        TreeNode *right;
      };
  
```

Graph Representation:

1. Adjacency list
2. Matrix representation

Representation of Unweighted Undirected Graph



```

'''
v: vertices
e: edges

0: [1,4]
1: [0,4,3,2]
4: [0,1,3]
3: [4,1,2]
2: [1,3]

SC: (v+2e)

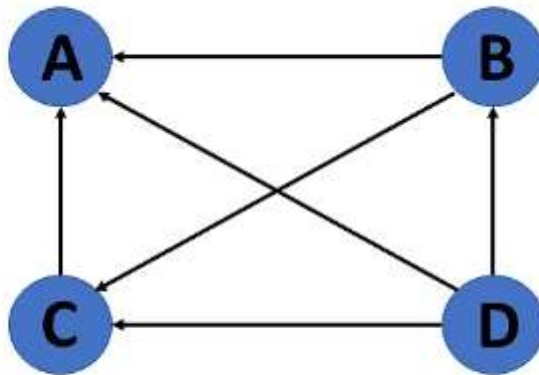
    0  1  2  3  4
0  0  1  0  0  1
1  1  0  1  1  1
2  0  1  0  1  0
3  0  1  1  0  1
4  1  1  0  1  0

SC: (v^2)
'''

```

In []:

Representation of Unweighted Directed Graph



A: []
 B: [A,C]
 C: [A]
 D: [A,B,C]

SC: $(v+2e)$

	A	B	C	D
A	0	0	0	0
B	1	0	1	0
C	1	0	0	0
D	1	1	1	0

SC: (v^2)

In []:

```

In [4]: # Adjancey List Repr
# dict<int, List>
# map<int, vector<int>>
class UndirectedGraph:

    def __init__(self):
        self.__data = {} # empty dict/map

    def add(self, v1, v2):

        if v1 not in self.__data:
            self.__data[v1] = []
        self.__data[v1].append(v2)

        if v2 not in self.__data:
            self.__data[v2] = []
        self.__data[v2].append(v1)

    def print(self):
        print(self.__data)

g = UndirectedGraph()
g.add(0,1)
g.add(0,4)
g.add(1,4)
g.add(1,3)
g.add(1,2)
g.add(4,3)
g.add(3,2)
g.print()

# 0: [1,4]
# 1: [0,4,3,2]
# 4: [0,1,3]
# 3: [4,1,2]
# 2: [1,3]

```

```
{0: [1, 4], 1: [0, 4, 3, 2], 4: [0, 1, 3], 3: [1, 4, 2], 2: [1, 3]}
```

```

In [8]: # Adjancecy List Repr
# dict<int, List>
# map<int, vector<int>>
class DirectedGraph:

    def __init__(self):
        self.__data = {} # empty dict/map

    def add(self, v1, v2):
        """
        Create an edge from v1->v2
        """

        if v1 not in self.__data:
            self.__data[v1] = []
        self.__data[v1].append(v2)

        if v2 not in self.__data:
            self.__data[v2] = []

    def print(self):
        print(self.__data)

g = DirectedGraph()
g.add('B', 'A')
g.add('B', 'C')
g.add('C', 'A')
g.add('D', 'A')
g.add('D', 'B')
g.add('D', 'C')
# A: []
# B: [A,C]
# C: [A]
# D: [A,B,C]
g.print()

{'B': ['A', 'C'], 'A': [], 'C': ['A'], 'D': ['A', 'B', 'C']}

```

In []:

Matrix Repr

```
In [27]: # dict<int, List>
# map<int, vector<int>>
class UndirectedGraph:

    def __init__(self, num_vertices):
        self.__data = []
        for _ in range(num_vertices):
            self.__data.append( [0 for _ in range(num_vertices) ] )

    def add(self, v1, v2):
        self.__data[v1][v2] = 1
        self.__data[v2][v1] = 1

    def print(self):
        for row in self.__data:
            print(row)

g = UndirectedGraph(5)
g.add(0,1)
g.add(0,4)
g.add(1,4)
g.add(1,3)
g.add(1,2)
g.add(4,3)
g.add(3,2)
g.print()

# 0: [1,4]
# 1: [0,4,3,2]
# 4: [0,1,3]
# 3: [4,1,2]
# 2: [1,3]
```

```
[0, 1, 0, 0, 1]
[1, 0, 1, 1, 1]
[0, 1, 0, 1, 0]
[0, 1, 1, 0, 1]
[1, 1, 0, 1, 0]
```

In []:

In []:

```
In [ ]: ### Traversal
- DFS: stack, recursion
- BFS: queue, iteration
```

In []:

BFS: Using queue


```

In [26]: import queue
# Adjancecy List Repr
# dict<int, List>
# map<int, vector<int>>
class UndirectedGraph:

    def __init__(self):
        self.__data = {} # empty dict/map

    def add(self, v1, v2):

        if v1 not in self.__data:
            self.__data[v1] = []
        self.__data[v1].append(v2)

        if v2 not in self.__data:
            self.__data[v2] = []
        self.__data[v2].append(v1)

    def traverse(self):
        # // q
        # // set
        # // add first element to q and set
        # // while q is not empty:
        # // check if neighbour not in set:
        # // push to set and q
        q = queue.Queue()
        visited = set() # hash set O(1)
        if len(self.__data) == 0:
            print("{}")
            return

        curr = next(iter(self.__data.keys())) # get one element
        q.put(curr)
        visited.add(curr)
        while not q.empty(): # O(v)
            curr = q.get()
            print(curr)
            neighbours = self.__data[curr]

            for v in neighbours: # v
                if v not in visited:
                    visited.add(v)
                    q.put(v)

# TC: O(v^2) worst case (all vertices are connected)
# TC: O(e)
# SC: O(v)

g = UndirectedGraph()
g.add(0,1)
g.add(0,4)
g.add(1,4)
g.add(1,3)
g.add(1,2)
g.add(4,3)
g.add(3,2)

```

```
g.traverse()
```

```
# 0: [1,4]
# 1: [0,4,3,2]
# 4: [0,1,3]
# 3: [4,1,2]
# 2: [1,3]
```

```
0
1
4
3
2
```

In []:

```
class UndirectedGraph{
    Map<Integer,List<Integer>> graph;
    public UndirectedGrap(){
        graph = new HashMap<>();
    }

    public void add(self, v1, v2){
        graph.computeIfAbsent(v1,value->new ArrayList<>()).add(v2);
        graph.computeIfAbsent(v2,value->new ArrayList<>()).add(v1);
    }

    public void traverse(){
        Queue<Integer> que = new LinkedList<>();
        Set<Integer> vis = new HashSet<>();
        que.add(0);
        vis.add(0);
        while(que.size()>0){
            Integer front = que.remove();
            System.out.print(front+" ");
            for(Integer adj: graph.get(front)){
                if(!vis.contains(adj)){
                    que.add(adj);
                    set.add(adj);
                }
            }
        }
    }
}
```

```
public class UndirectedGraph {
    private HashMap<Integer, ArrayList<Integer>> data;

    public UndirectedGraph() {
        data = new HashMap<>();
    }

    public void add(int v1, int v2) {
        if (!data.containsKey(v1)) {
            data.put(v1, new ArrayList<>());
        }
        data.get(v1).add(v2);

        if (!data.containsKey(v2)) {
            data.put(v2, new ArrayList<>());
        }
        data.get(v2).add(v1);
    }

    public void traverse() {
        HashSet<Integer> visited = new HashSet<>();
        Queue<Integer> queue = new LinkedList<>();

        // Add the first element to the queue and visited set
        int startVertex = data.keySet().iterator().next();
        visited.add(startVertex);
        queue.offer(startVertex);

        while (!queue.isEmpty()) {
            int currentVertex = queue.poll();
            System.out.print(currentVertex + " ");

            ArrayList<Integer> neighbors = data.get(currentVertex);
            for (int neighbor : neighbors) {
                if (!visited.contains(neighbor)) {
                    visited.add(neighbor);
                    queue.offer(neighbor);
                }
            }
        }
    }
}
```

In []:

In []:

DFS

```

In [33]: import queue
          # Adjancecy List Repr
          # dict<int, List>
          # map<int, vector<int>>
          class UndirectedGraph:

              def __init__(self):
                  self.__data = {} # empty dict/map

              def add(self, v1, v2):

                  if v1 not in self.__data:
                      self.__data[v1] = []
                  self.__data[v1].append(v2)

                  if v2 not in self.__data:
                      self.__data[v2] = []
                  self.__data[v2].append(v1)

              def traverse(self):
                  if len(self.__data) == 0:
                      print("{}")
                      return

                  curr = next(iter(self.__data.keys())) # get one element
                  visited = set()
                  self._traverse(curr, visited)

              def _traverse(self, curr, visited):
                  if curr in visited:
                      return

                  print(curr) # process current node
                  visited.add(curr)

                  # recur for neighbours that are not already visited
                  for v in self.__data[curr]:
                      if v not in visited:
                          self._traverse(v, visited)

              def traverse_disconnected(self):
                  if len(self.__data) == 0:
                      print("{}")
                      return

                  visited = set()
                  for node in self.__data.keys(): # traverse each vertex
                      self._traverse(node, visited)

          # for a tree order of print matters
          # def traverse(root):
          #     print(root)
          #     traverse(root.left)
          #     traverse(root.right)

          g = UndirectedGraph()
          g.add(0,1)

```

```
g.add(0,4)
g.add(1,4)
g.add(1,3)
g.add(1,2)
g.add(4,3)
g.add(3,2)
g.traverse()
```

```
0
1
4
3
2
```

In []:

Disconnected Graph