WorkoutSpot Algorithms

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This article intends to serve as a reference for the formulae used in WorkoutSpot. The code (in WorkoutSpot) should have links to the original formulae (as does this document), however referenced formulae may have inconsistent variables, or components of a formula may be scattered.

1   Notation and Values

This document uses the following symbols and notation.

altitude (height): h

latitude: ϕ

longitude: λ

semi-major axis: a (6378137 meters is used for Earth) [1, Fixed radius]

semi-minor axis: b (6356752 meters is used for Earth) [1, Fixed radius]

2 Distances between Earth Coordinates

`-[WSCoordinateAnalysis stepSpace]`

This method uses a modified Haversine formula to find the distance d between two coordinates on Earth. The radius r of Earth is computed at each of the input latitudes ϕ using a geocentric radius formula.

Radius formula [1, Geocentric radius]:

$$r = \sqrt{\frac{\left(a^2 \cos\left(\phi\right)\right)^2 + \left(b^2 \sin\left(\phi\right)\right)^2}{\left(a \cos\left(\phi\right)\right)^2 + \left(b \sin\left(\phi\right)\right)^2}} \tag{1}$$

To decrease the number of vector operations (i.e., operations that must be performed on non-constant values), distribute the exponents. This allows $\sin\phi$ and $\cos\phi$ to be squared once, then used in both the numerator, and denominator.

$$r = \sqrt{\frac{a^4 \cos^2\left(\phi\right) + b^4 \sin^2\left(\phi\right)}{a^2 \cos^2\left(\phi\right) + b^2 \sin^2\left(\phi\right)}} \tag{2}$$

Haversine formula [2, Formulation]:

$$d = 2r \arcsin\left(\sqrt{\sin^2\left(\frac{\phi_2 - \phi_1}{2}\right) + \cos\left(\phi_1\right)\cos\left(\phi_2\right)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}\right) \tag{3}$$

I modified the formula to use the radii of each location, instead of a single radius; the modification is straight forward: $2r \rightarrow (r_1 + r_2)$

Modified haversine formula:

$$d = (r_1 + r_2) \arcsin\left(\sqrt{\sin^2\left(\frac{\phi_2 - \phi_1}{2}\right) + \cos\left(\phi_1\right)\cos\left(\phi_2\right)\sin^2\left(\frac{\lambda_2 - \lambda_1}{2}\right)}\right) \tag{4}$$

3 Earth Coordinates to XYZ

`-[WSCoordinateAnalysis globeMapForAltitudes:]`

This method uses a geodetic to ECEF formula [3, From geodetic to ECEF coordinates] to convert latitude ϕ, longitude λ, altitude h to XYZ coordinates, where $(0, 0, 0)$ is the center of Earth.

$$e^2 = 1 - \frac{b^2}{a^2} \tag{5}$$

$$N(\phi) = \frac{a}{\sqrt{1 - e^2 \sin^2(\phi)}} \tag{6}$$

$$X = (N(\phi) + h)\cos(\phi)\cos(\lambda) \tag{7}$$

$$Y = (N(\phi) + h)\cos(\phi)\sin(\lambda) \tag{8}$$

$$Z = \left(\frac{b^2}{a^2}N(\phi) + h\right)\sin(\phi) \tag{9}$$

4   Geodetic Parameterization

Linear Scalar Parameterization:

$$x = x_1 + (x_2 - x_1) t \tag{10}$$

For the below equations assume there is an initial point $P_1 (\phi_1, \lambda_1)$, and final point $P_2 (\phi_2, \lambda_2)$. This will involve the point $P_0 (\phi_0, \lambda_0)$, where the great circle of P_1 and P_2 intersects the equator. Lastly, the point $P (\phi, \lambda)$ will be computed.

Initial (P_1) Course, α_1 [4, Course]:

$$\tan \alpha_1 = \frac{\cos \phi_2 \sin (\lambda_2 - \lambda_1)}{\cos \phi_1 \sin \phi_2 - \sin \phi_1 \cos \phi_2 \cos (\lambda_2 - \lambda_1)} \tag{11}$$

Final (P_2) Course, α_2 [4, Course]:

$$\tan \alpha_2 = \frac{\cos \phi_2 \sin (\lambda_2 - \lambda_1)}{- \cos \phi_2 \sin \phi_1 + \sin \phi_2 \cos \phi_1 \cos (\lambda_2 - \lambda_1)} \tag{12}$$

Equator (P_0) Course, α_0 [4, Finding way-points]:

$$\sin \alpha_0 = \sin \alpha_1 \cos \phi_1 \tag{13}$$

Equator (P_0) Course (with increased accuracy when the great circle nears the poles), α_0 [4, Finding way-points]:

$$\tan \alpha_0 = \frac{\sin \alpha_1 \cos \phi_1}{\sqrt{\cos^2 \alpha_1 + \sin^2 \alpha_1 \sin^2 \phi_1}} \tag{14}$$

Angular Distance (between P_0 and P_1), σ_{01} [4, Finding way-points]:

$$\tan \sigma_{01} = \frac{\tan \phi_1}{\cos \alpha_1} \tag{15}$$

Angular Distance (between P_0 and P_2), σ_{02} [4, Finding way-points]:

$$\tan \sigma_{02} = \frac{\tan \phi_2}{\cos \alpha_2} \tag{16}$$

Equator (P_0) Longitude, λ_0 [4, Finding way-points]:

$$\tan (\lambda_0 - \lambda_1) = \frac{\sin \alpha_0 \sin \sigma_{01}}{\cos \sigma_{01}} \tag{17}$$

Select an angular distance, σ; we'll be using the equation from (10) to select a value between the angular distances from above.

$$\sigma = \sigma_1 + (\sigma_2 - \sigma_1) t \tag{18}$$

Computed (P_0) Latitude, ϕ [4, Finding way-points]:

$$\sin \phi = \cos \alpha_0 \sin \sigma \tag{19}$$

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Computed (P_0) Latitude (with increased accuracy around the poles), ϕ [4, Finding way-points]:

$$\tan \phi = \frac{\cos \alpha_0 \sin \sigma}{\sqrt{\cos^2 \sigma + \sin^2 \alpha_0 \sin^2 \sigma}} \tag{20}$$

Computed (P_0) Longitude, λ [4, Finding way-points]:

$$\tan (\lambda - \lambda_0) = \frac{\sin \alpha_0 \sin \sigma}{\cos \sigma} \tag{21}$$

References

[1] https://en.wikipedia.org/wiki/Earth_radius

[2] https://en.wikipedia.org/wiki/Haversine_formula

[3] https://en.wikipedia.org/wiki/Geographic_coordinate_conversion

[4] https://en.wikipedia.org/wiki/Great-circle_navigation