Recovering Astronomical Observations Through Interferometry Using an RBF Model*

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Summary

The interferometry problem addresses the estimation of an unknown quantity exploiting the interference among measurements from different sources. In Astronomy these measurement are in the Fourier domain. The objective of this work is to propose a probabilistic approach for reconstructing interferometry images. In this way, we propose a visibility model given by a sum of basis functions and then assume a complex Gaussian measurements noise.
The first stage of this work is to test the method with synthetic images, then, we will use the method for real images. Finally, we will analyze the results and define the future work.

Interferometry Basics

The supporting concept behind interferometry is the Van Cittert-Zernike theorem [1] [2] , this theorem states: *If V is the visibility function from a wavefront, I is his intensity and A is the reception area of measurement instrument, then:*

$$V(u,v) = \int\int A(x,y) I(x,y) \exp\{-2\pi j(ux+vy)\} dxdy$$

This that means, the visibility function of a wave and its intensity is a pair of Fourier transform/anti-transform. [3]

However, in real-world applications, we only use a subset of measurements, because we have a finite set of antennas as figure 1 shows. Then, the challenge is to reconstruct the image with a subset of data.

(a) (b)

Figure 1: (a) Aerial view of Chajnantor plateau, located at an altitude of 5000 meters in Chilean Andes, where the array of ALMA antennas is located. (b) This 12-meter-diameter antenna was manufactured by Mitsubishi Electric Corporation.
[source] http://www.almaobservatory.org/

Proposed Model

We assume a model for the image I and then calculate the induced model for the measuremets.
Let an image $I : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{Z}_{[0,255]}$. The CLEAN method assumes that the image is a sum of Radial Base Function (RBF) [4], this means:

$$I(x,y) = \sum_{i=1}^{N_B} \beta_i \psi_i(x,y)$$
$$\forall i = 1, ..., N_B \ \beta_i \in \mathbb{R}; \psi_i : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$$

Where $\psi(x,y)$ is a basis function and have the form:

$$\psi_i(x,y) = \frac{1}{\sqrt{2\pi} l} \exp\left\{ -\frac{1}{2} \left(\frac{\left\| \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} C_x^i \\ C_y^i \end{bmatrix} \right\|_2}{l} \right)^2 \right\}$$

This model induce a model for the measurements. This is:

$$V(u,v) = \sum_{i=1}^{N_B} \alpha_i \phi_i(u,v)$$
$$\forall i = 1, ..., N_B \ \alpha_i \in \mathbb{R}; \phi_i : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{C}$$

where:

$$\alpha_i = \beta_i l$$

$$\phi_i(u,v) = \exp\left\{ \frac{-2\pi^2 \left(u^2+v^2\right)}{1/l^2} + j2\pi(C_x^i u + C_y^i v) \right\}$$

Finally, we assume a complex-valued Gaussian observation noise in the measurements [5]. Therefore, the observation model:

$$V_{obs}(u,v) = \sum_{i=1}^{N_B} \alpha_i \phi_i(u,v) + \eta(0, C, P)$$

Learning the model through optimization

The proposed model will be fitted through maximum likelihood. In this way, let $U = \{u_k, v_k\}_{k=1}^{N_s}$ be the set of measurements locations and $V = \{V_k\}_{k=1}^{N_s}$ their respective visibilities, then the Negative Log-Likelihood function is given by:

$$\text{NLL} = \frac{1}{N_s} \sum_{k=1}^{N_s} log(\pi|\Sigma|) + \frac{1}{2} e^T \Sigma^{-1} e$$

where: $e = \begin{bmatrix} V_k - V(u_k, v_k) \\ \bar{V}_k - \bar{V}(u_k, v_k) \end{bmatrix}$

Finally, the optimal parameters are given by:

$$\theta^* = \underset{M,K,l,\alpha_i,C_x^i,C_y^i}{\text{argmin}} \ \text{NLL}(M,K,l,\alpha_i,C_x^i,C_y^i)$$

Simulations

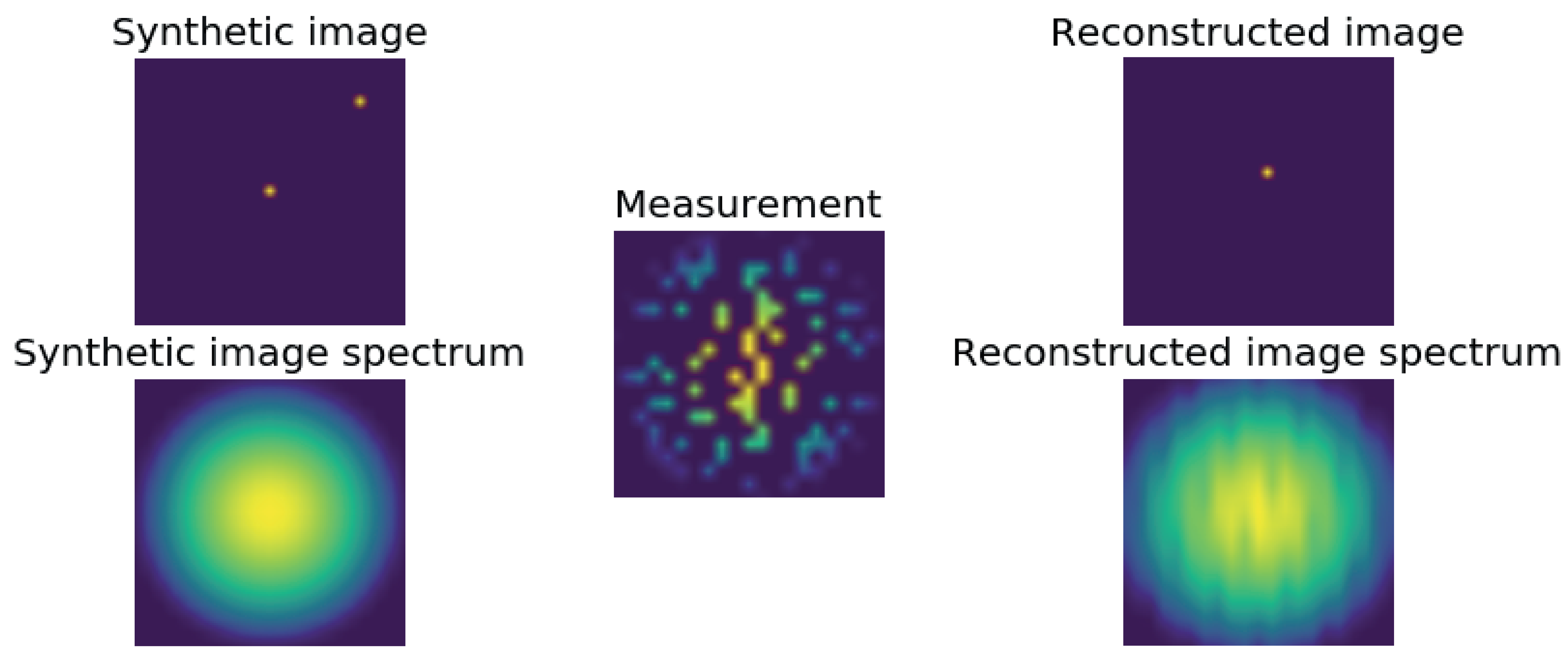
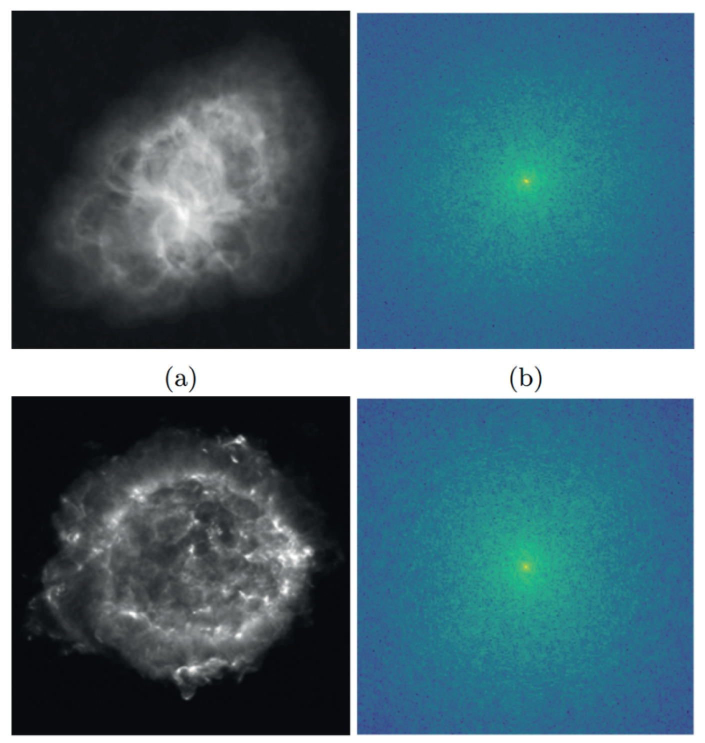


Figure 2: Simulated data with two RBF's. Synthetic image, Synthetic image spectrum, Measurement, Reconstructed image spectrum, reconstructed image. Error rate: 7.6%

The figure 2 shows the results for a synthetic image. This reconstruction was making with approximate 30% of the signal spectrum, simulating 30 antennas in random positions. Theses figures are, Synthetic Image, Synthetic Image Spectrum, Measurement, Reconstructed spectrum, Reconstructed image.

In the simulations the error rate is around to 5%-10%. In the figure 1, the reconstructed image have a close Gaussian width, but the space domain's center it doesn't in the correct location. However, one of theses center is relative close to the real center and the other center is 2 pixels away.

Future Work



(a) (b) (c) (d)

Figure 3: Real data interferometry. This images has been reconstructed using CLEAN method. First Column: Real Image; Second Column: Real Image Spectrum

The results in synthetic images are promising. The principal problem is that the optimization algorithm finds multiple local minima. This would improve if we will compute the gradient of the objective function.

Another future work is to test the algorithm with real images.

At the same time, we will improve the model by considering different widths for the basis functions. Because the real data seems to be a sum of different basis functions with variable widths (see figure 3). For this kind of data, we need setting a large amount of basis functions.

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