Homework 6 Key

1. Which of the following functions are well-behaved over the given intervals?

   (a) $g(x) = \exp(ax), x \in [0, 1]$ yes, it is continuous when you take the limit, normalized when you take the integral, and differentiable when you take the derivative.

   (b) $g(x) = \frac{1}{x}, x \in [1, 2]$ yes, it is continuous when you take the limit, normalized when you take the integral, and differentiable when you take the derivative.

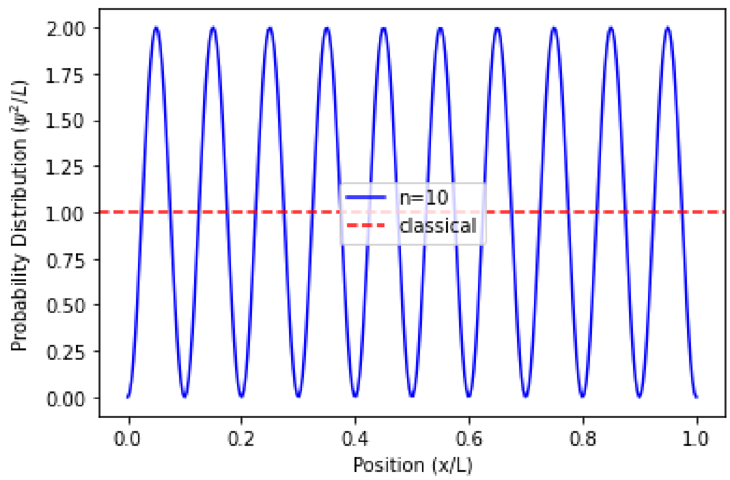
   (c) $g(x) = |x|, x \in [-1, 1]$ No, it is not discontinuous at x=0, sharp turn.

2. $\Lambda \approx 16\frac{E}{U_0}(1 - \frac{E}{U_0})\exp(-2L\sqrt{\frac{2}{m\hbar^2}(U_0 - E)}$

   (a) Reducing E and U_0 by 10%:
   $\Lambda' \approx 16\frac{.9E}{.9U_0}(1 - \frac{.9E}{.9U_0})\exp\left(-2L\sqrt{\frac{2}{m\hbar^2}(.9U_0 - .9E)}\right)$
   $\Lambda' \approx \exp\left(-2L\sqrt{\frac{2}{m\hbar^2}(U_0 - E)}\sqrt{0.9}\right)$

   (b) Reducing L by 10%:
   $\Lambda \approx 16\frac{E}{U_0}(1 - \frac{E}{U_0})\exp(-2 * 0.9L\sqrt{\frac{2}{m\hbar^2}(U_0 - E)}$
   $\sqrt{0.9} < 0.9$ so B

3. Make a publication-quality plot of the normalized probability distribution for a particle in a 1D box of length L in its n = 10 state versus dimensionless position (x = x/L). Superimpose the classical probability distribution on the same graph. Make sure both axes have numerical values and proper tick marks.



4. Come back next week!

5. 3D harmonic oscillator has potential energy given by:

$$V(x,y,z) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + \frac{1}{2}k_z z^2$$

   (a) Find eigenvalues of 3D harmonic oscillator.

   $\hat{H}\psi = E\psi$

   $\hat{H} = \frac{\hbar^2}{2m}\frac{d}{dx} + \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + \frac{1}{2}k_z z^2$

   $\hat{H}_x = \frac{\hbar^2}{2m}\frac{d}{dx} + \frac{1}{2}k_x x^2$

   $\hat{H}_y = \frac{\hbar^2}{2m}\frac{d}{dx} + \frac{1}{2}k_y y^2$

   $\hat{H}_z = \frac{\hbar^2}{2m}\frac{d}{dx} + \frac{1}{2}k_z z^2$

   $E = (n + \frac{1}{2})h\nu$ for a 1D harmonic oscillator

   $E = (n_x + \frac{1}{2})h\nu + (n_y + \frac{1}{2})h\nu + (n_z + \frac{1}{2})h\nu$

   $E = (n_x + n_y + n_z + \frac{3}{2})h\nu$ for 3D harmonic oscillator

   (b) For an anisotropic 3D harmonic oscillator $k_x = k_y = \frac{1}{3}k_z$. Calculate the degeneracy for lowest 10 levels.

   $V(x,y,z) = \frac{1}{2}k_x x^2 + \frac{1}{2}k_y y^2 + \frac{1}{2}k_z z^2$

$E = (n_x + n_y + n_z + \frac{3}{2})h\nu$

$\omega = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $\hbar = \frac{h}{2\pi}$

$E = (n_x + n_y + \frac{1}{\sqrt{3}}n_z + \frac{3}{2})\hbar\omega; \forall n \in \mathbb{N}$ so test out numbers

lowest state $(n_x, n_y, n_z) = 0$: (0,0,0) degeneracy = 1

2nd lowest $(n_x, n_y, n_z) = 1$: (1,0,0);(0,1,0) degeneracy = 2

3rd lowest $(n_x, n_y, n_z) = \sqrt{3}$: (0,0,1) degeneracy = 1

4th lowest $(n_x, n_y, n_z) = 2$: (2,0,0);(0,2,0);(1,1,0) degeneracy = 3

5th lowest $(n_x, n_y, n_z) = 1 + \sqrt{3}$: (1,0,1);(0,1,1) degeneracy = 2

6th lowest $(n_x, n_y, n_z) = 3$: (3,0,0);(0,3,0);(1,2,0);(2,1,0) degeneracy = 4

7th lowest $(n_x, n_y, n_z) = 2\sqrt{3}$: (0,0,2) degeneracy = 1

8th lowest $(n_x, n_y, n_z) = 2 + \sqrt{3}$: (1,1,1);(2,0,1);(0,2,1) degeneracy = 3

9th lowest $(n_x, n_y, n_z) = 4$: (4,0,0);(0,4,0);(3,1,0);(1,3,0);(2,2,0) degeneracy = 5

10th lowest $(n_x, n_y, n_z) = 1 + 2\sqrt{3}$: (1,0,2);(0,1,2)degeneracy = 2

6. A particle of mass m is in a 2D box. Initially (t = 0), the box has sides S1 and S2 with lengths L and 4L, respectively. Then, side S2 starts to shrink according to the expression 4L 0.5Lt. How will the energy gap between the 2nd and 3rd energy levels for this system change as a function of time? For simplicity, you can focus in the cases where t changes in one unit from 0 to 7. (Extra credit if you can work out the fully general case). For a 2D particle in a box of side lengths s1 and s2, the energy is:

   $E = \frac{h^2}{8m}(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2})$

   $E_t = \frac{h^2}{8m}\left(\frac{2}{L} + \frac{1}{(4L-0.5Lt)^2}\right) - \frac{h^2}{8m}\left(\frac{1}{L} + \frac{2}{(4L-0.5Lt)^2}\right)$ for energy levels when $L_x > L_y$ this applies when t=0,1,2,3,4,5

   Degeneracy at t=6

$E_{t=6} = \frac{h^2}{8m}\left(\frac{2}{L} + \frac{2}{(4L-0.5Lt)^2}\right) - \frac{h^2}{8m}\left(\frac{1}{L} + \frac{2}{(4L-0.5Lt)^2}\right)$ for energy level when $L_x = L_y$

$E_{t=6} = \frac{h^2}{8m}\left(\frac{2}{L} + \frac{1}{(4L-0.5Lt)^2}\right) - \frac{h^2}{8m}\left(\frac{2}{L} + \frac{1}{(4L-0.5Lt)^2}\right)$ for energy level when $L_x = L_y$

At t=7 $L_x < L_y$

$E_{t=7} = \frac{h^2}{8m}\left(\frac{1}{L} + \frac{2}{(4L-0.5Lt)^2}\right) - \frac{h^2}{8m}\left(\frac{2}{L} + \frac{1}{(4L-0.5Lt)^2}\right)$

Click here if you are interested in my code.