

Homework 1 Key

- Let us assume that after a really long day, when it was time to type the final form of his equation explaining the black body radiation, Planck made a few typos. Can you tell in each of the following cases if we can still recover the Rayleigh-Jeans and/or Wien's laws despite the blunders of a sleepy Planck?

(a) $\frac{8\pi h v^3}{c} \frac{1}{\exp(\frac{h v}{k T}) - 2}$

$$\lim_{v \rightarrow 0} = \frac{8\pi h v^3}{c^3} \frac{1}{\exp(\frac{h v}{k T}) - 2} = -\frac{8\pi h v^3}{c^3}$$

does not model rayleigh jeans, has negative

$$\lim_{v \rightarrow \infty} = \frac{8\pi h v^3}{c^3} \frac{1}{\exp(\frac{h v}{k T}) - 2} = \frac{8\pi h v^3}{c^3} \frac{1}{\exp(\frac{h v}{k T})}$$

yes, models wien -2 is negligible when $\lim_{v \rightarrow \infty}$.

(b) $\lim_{v \rightarrow 0} = \frac{8\pi h v^4}{c^3} \frac{1}{\exp(\frac{h v^2}{k T}) - 1} = \frac{8\pi h v^4}{c^3} \frac{1}{\frac{h v^2}{k T}} = \frac{8\pi v^2 k T}{c^3}$

Do taylor expansion and this models the rayleigh jean equation

$$\lim_{v \rightarrow \infty} = \frac{8\pi h v^4}{c^3} \frac{1}{\exp(\frac{h v^2}{k T}) - 1} = \frac{8\pi h v^4}{c^3} \frac{1}{\exp(\frac{h v^2}{k T})}$$

does not model wien; -1 is negligible when $\lim_{v \rightarrow \infty}$

(c) $\lim_{v \rightarrow 0} = \frac{16\pi h v^3}{c^3} \frac{1}{\exp(\frac{2 h v}{k T}) - 1} = \frac{16\pi h v^3}{c^3} \frac{1}{\frac{2 h v}{k T}} = \frac{8\pi v^2 k T}{c^3}$

Do taylor expansion and does model rayleigh jean

$$\lim_{v \rightarrow \infty} = \frac{16\pi h v^3}{c^3} \frac{1}{\exp(\frac{2 h v}{k T}) - 1} = \frac{16\pi h v^3}{c^3} \frac{1}{\exp(\frac{2 h v}{k T})}$$

-1 is negligible when $\lim_{v \rightarrow \infty}$ does not model wien

- Two students were asked to study the photoelectric effect using the same metal and light source. They ran their experiments separately and got the following results:

After checking these results their advisor said that they are both correct. Can you explain this? Hint: What are "x" and "y" in the previous figures?

Left measures frequency and right measures wavelength. right graph is

the shape of $1/x$

3. Calculate the De Broglie wavelength of:

(a) 1g bullet with velocity 350ms^{-1}

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \cdot 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \cdot \text{s}}{(1 \cdot 10^{-3} \text{kg})(350 \text{m/s})}$$

$$\lambda = 1.893 \cdot 10^{-33} \text{ m}$$

(b) H_2 molecule with kinetic energy of $(3/2) \cdot kT$ at $T=20 \text{ K}$

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{6.626 \cdot 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \cdot \text{s}}{p}$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$p = \sqrt{2mK} = \sqrt{2m(\frac{3}{2}kT)} \quad \text{k is Boltzmann constant}$$

$$p = \sqrt{2 \left(\frac{2.016 \cdot 10^{-3} \text{kg}}{1 \text{mol}} \right) \left(\frac{1 \text{mol}}{6.022 \cdot 10^{23} \text{molecule}} \right) \frac{3}{2} \cdot 1.38 \cdot 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \cdot 20 \text{K}}$$

$$p = 1.665 \cdot 10^{-24} \text{kg} \cdot \text{m/s}$$

substitute to step 2

$$\lambda = 3.980 \cdot 10^{-10} \text{ m}$$

(c) A chemistry professor (70 kg) running at 150 m/s

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \cdot 10^{-34} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \cdot \text{s}}{(70 \text{kg})(150 \text{m/s})}$$

$$\lambda = 6.310 \cdot 10^{-38} \text{ m}$$

4. Which of these expressions are consistent with Heisenberg's uncertainty principle? Explain your answers.

Correct is C. different setting different directionality

b - cannot measure the both momentum and position exactly a- can never be less than one-half of the reduced Planck constant.

$$5. v = \frac{m_e q_e^4}{8 \epsilon_0^2 h^3} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

Suppose that instead of an electron we form an H-like atom with a proton

and an “electronium”, an hypothetical particle that has the same charge as the electron, but twice its mass

- (a) Will all the emission frequencies in the vacuum change due to this change?

yes v will be 2x because 2x mass

- (b) Would it be possible to measure the emission lines of this hypothetical atom in some conditions that guarantee that all the lines have the same frequencies they had in the “normal” H atom in the vacuum?
not in laboratory